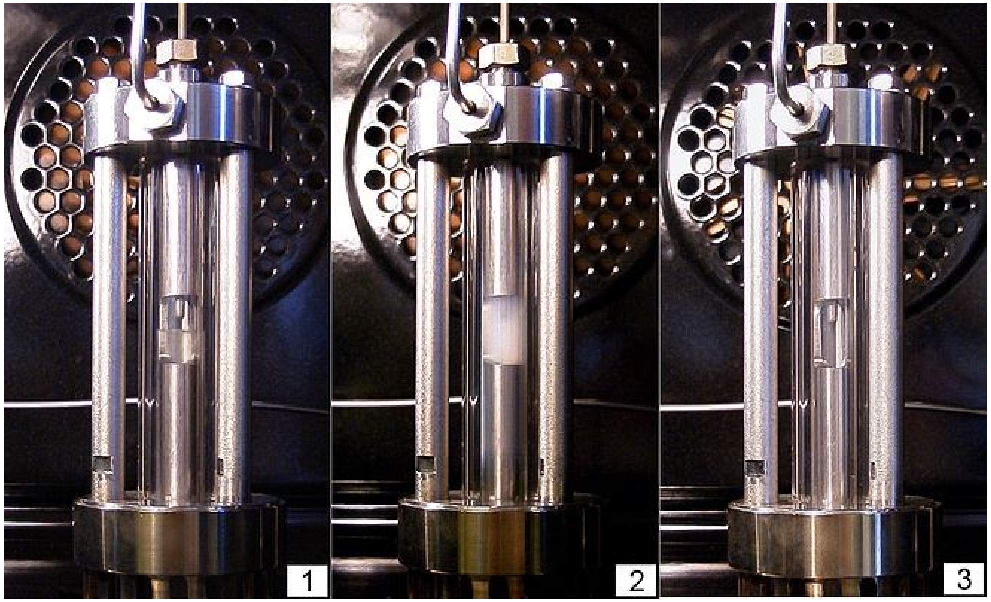
Quark-Gluon Plasma Physics

12. Search for critical phenomena

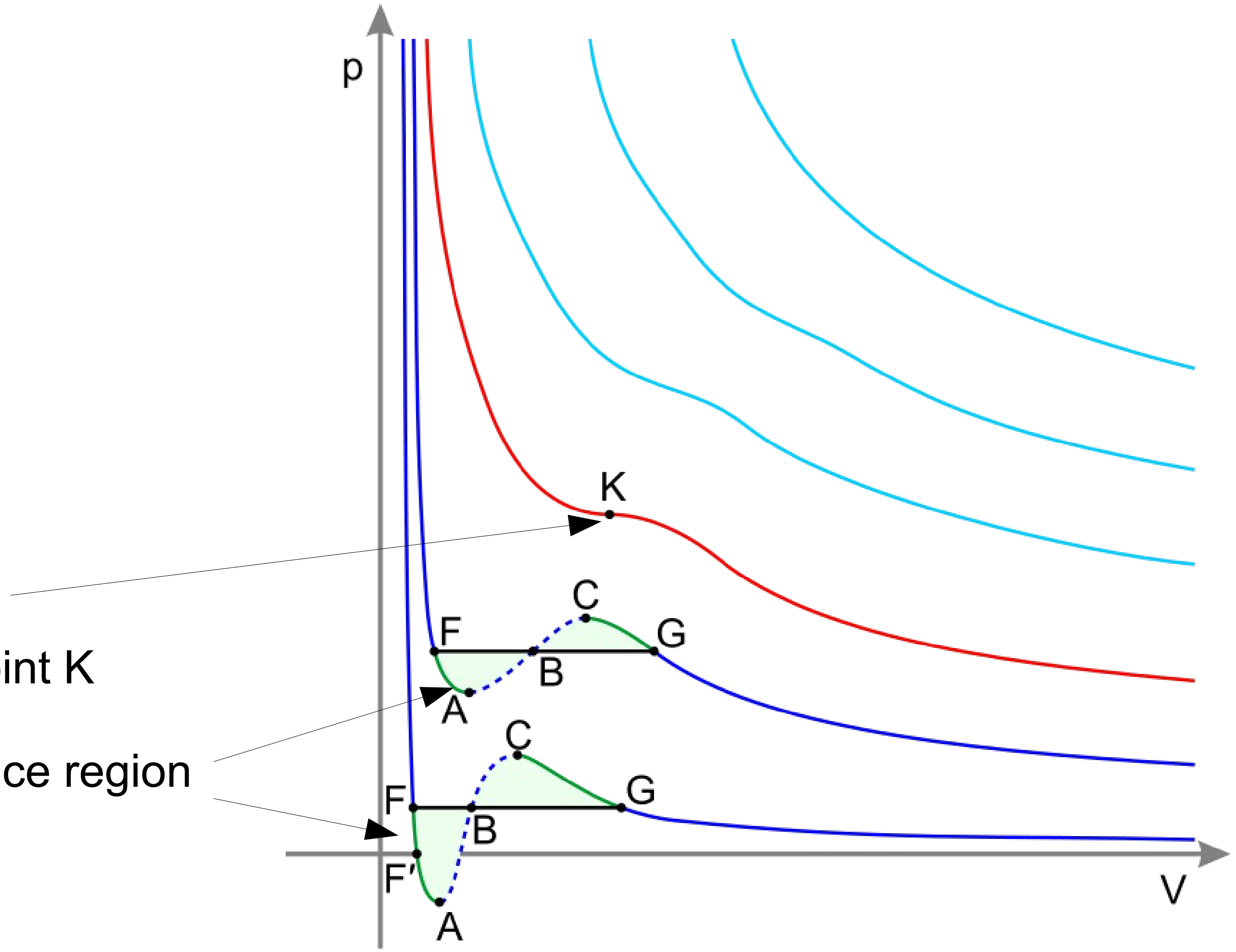
critical endpoints, the end of phase transitions
example of a liquid-gas phase transition



source: wikipedia

[1] Subcritical ethane, liquid and gas phase coexist.
[2] Critical point (32.17 °C, 48.72 bar), opalescence.
[3] Supercritical ethane, fluid

phase diagram and Maxwell construction

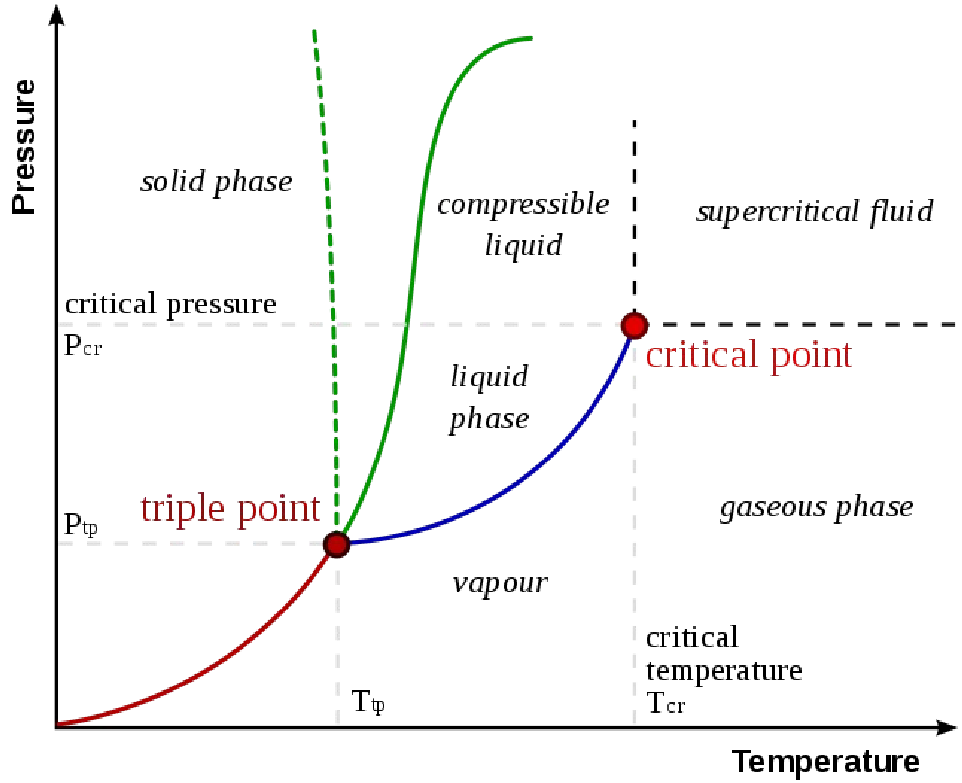


critical point K

coexistence region

source: Wikipedia

pressure-temperature diagram, triple point and critical point

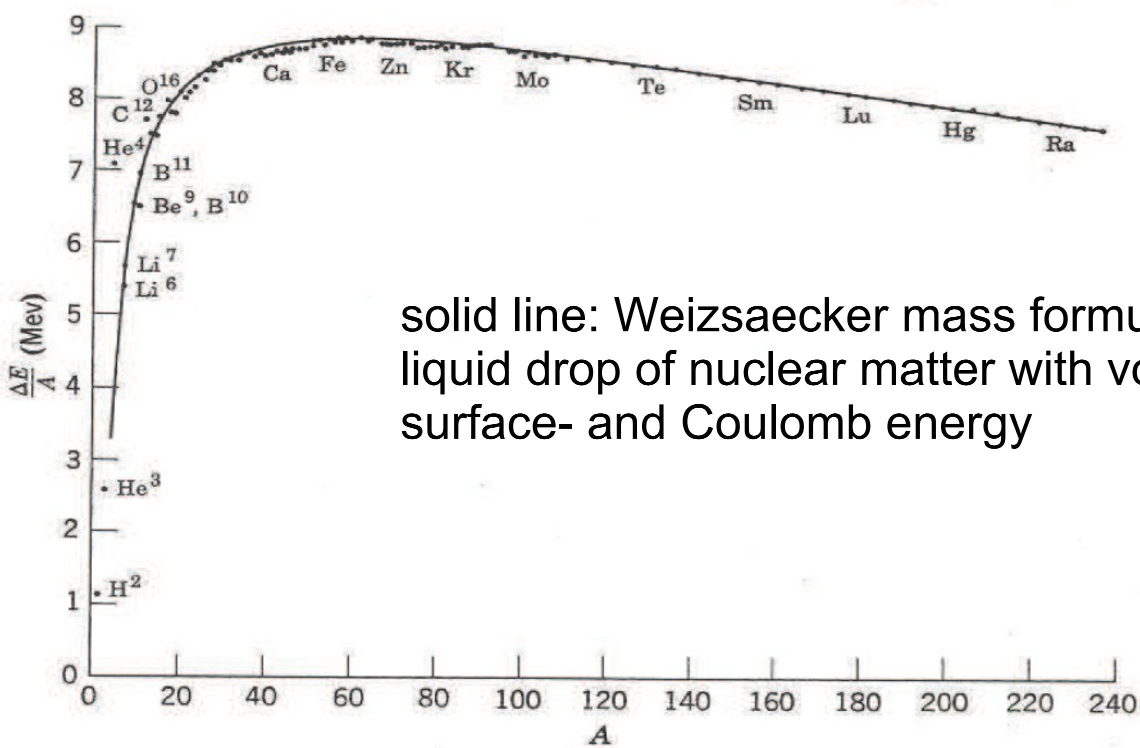


as one approaches the critical point from below, the distinction between liquid and vapor phase becomes ever smaller, leading to large (critical) fluctuations and long-range correlations (critical opalescence)

is there a critical point in the QCD phase diagram?

not obvious, QCD matter is very special

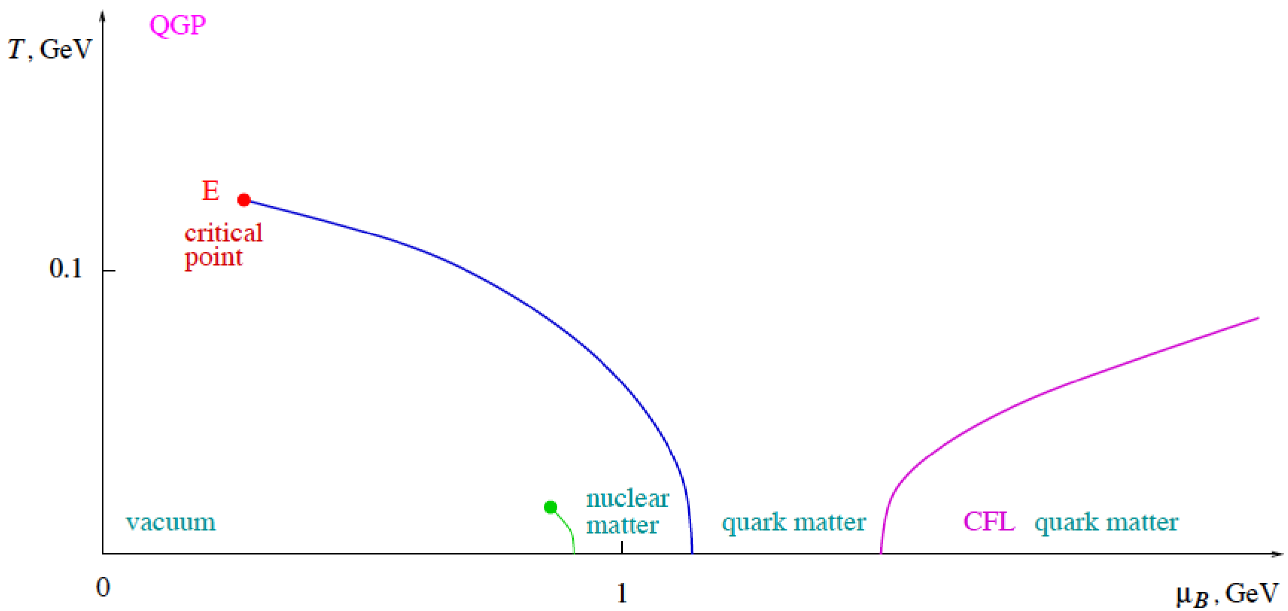
as is well known, QCD matter near T = 0 and nuclear matter density $n_0 = 0.16/fm^3$ can be described as a liquid (Niels Bohr's liquid drop model) with binding energy/nucleon of about 16 MeV (8 MeV in a finite nucleus).



solid line: Weizsaecker mass formula based on a liquid drop of nuclear matter with volume-, surface- and Coulomb energy

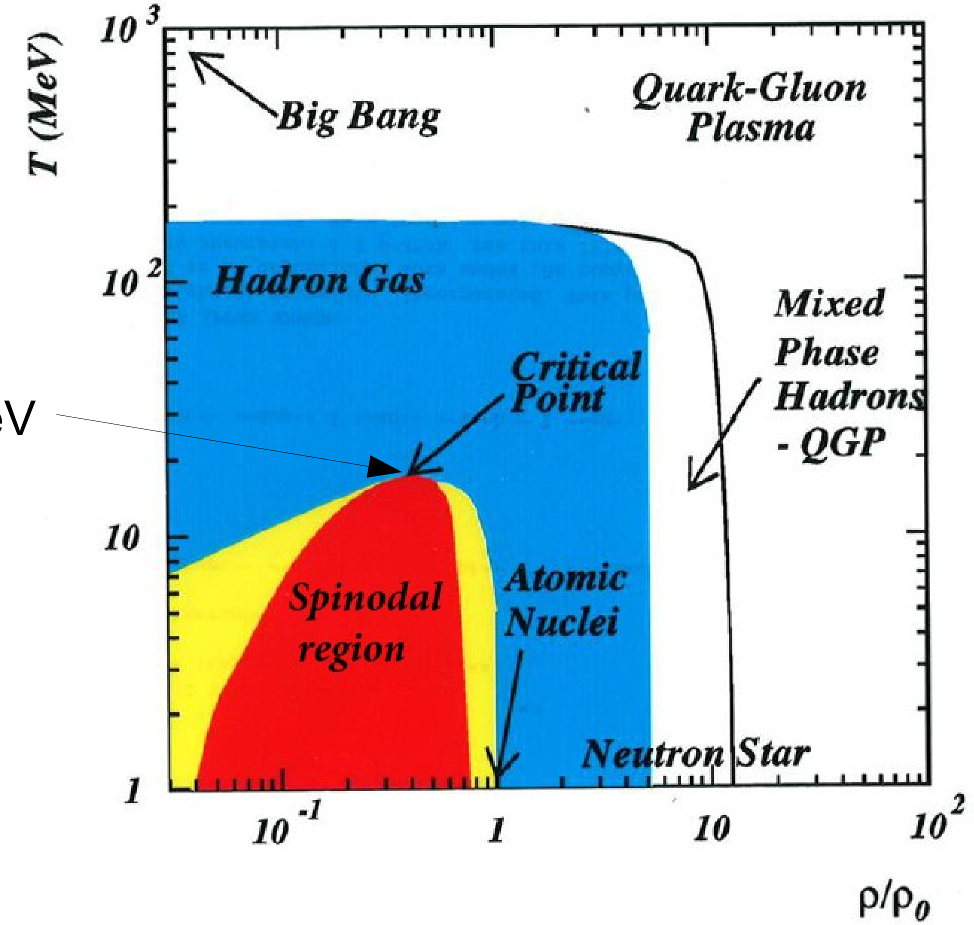
a theorists view

Misha Stephanov, Prog.Theor.Phys.Suppl. 153 (2004) 139-156, Int.J.Mod.Phys.A 20 (2005) 4387-4392



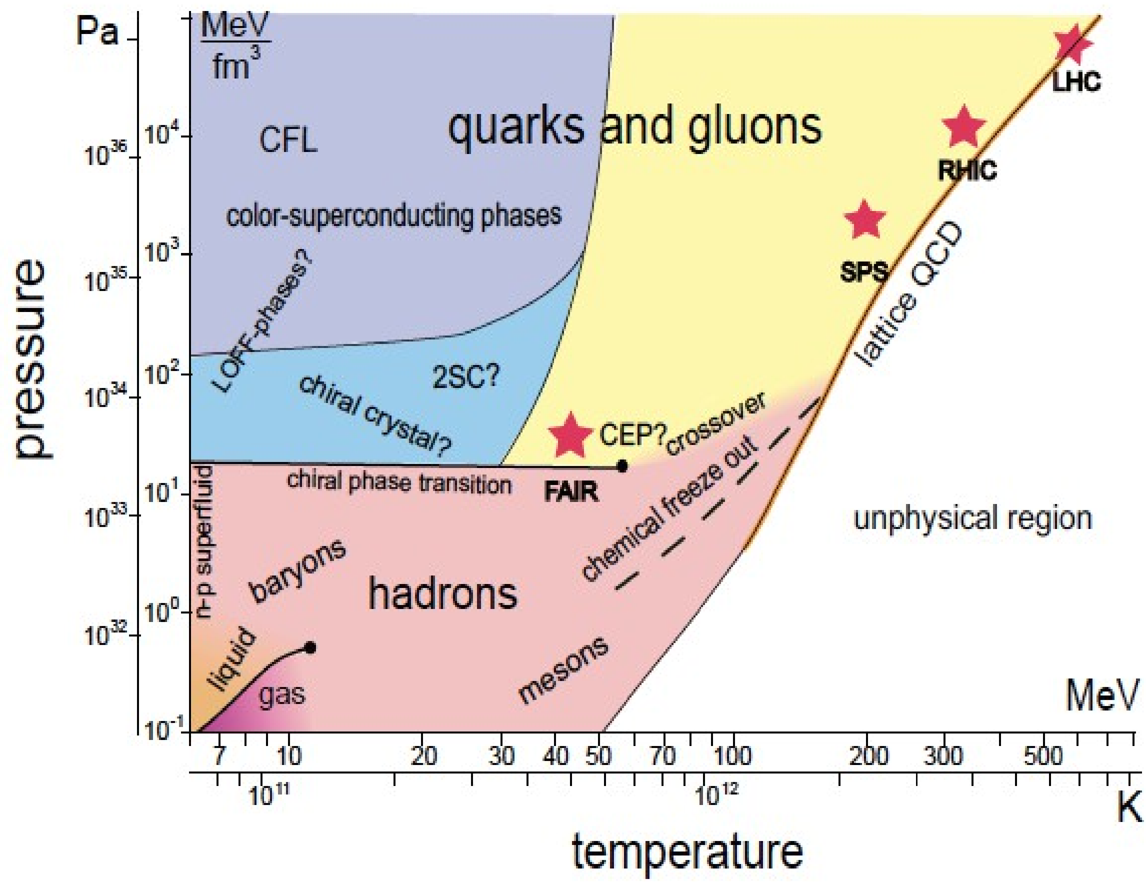
a low energy nuclear physicist's view

critical temperature T_c = 16 MeV

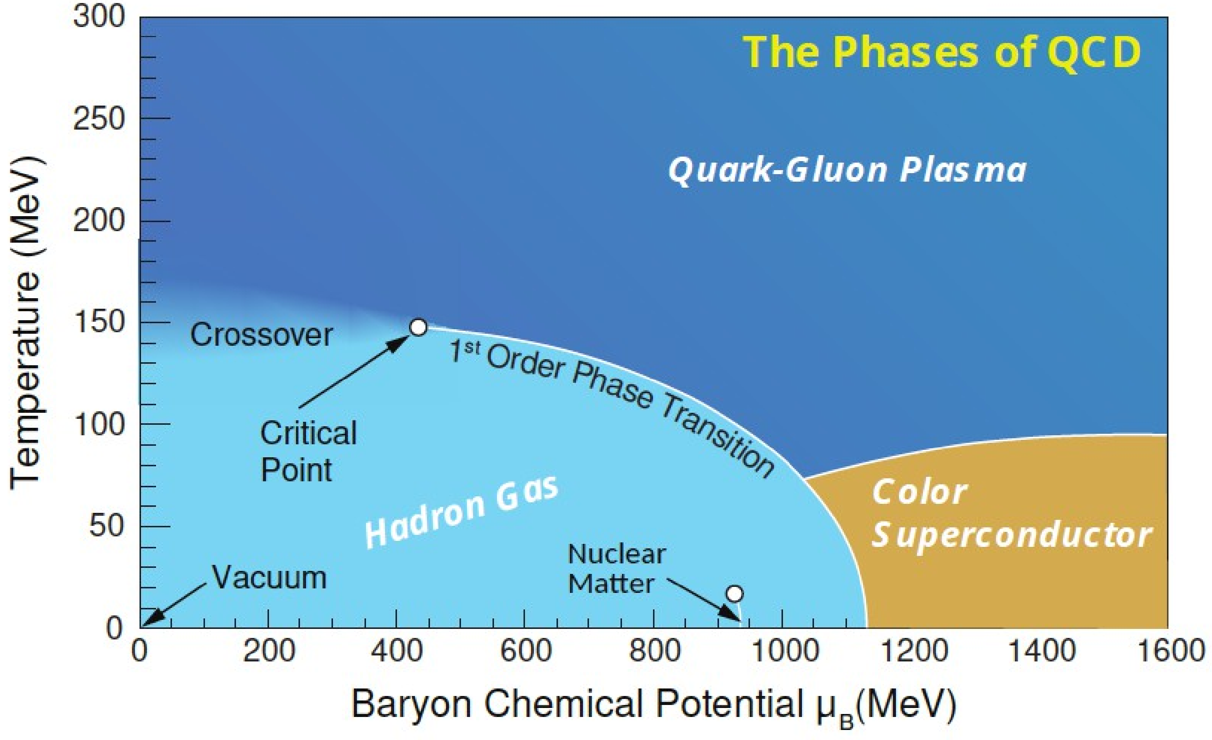


B. Borderie, J.D. Frankland, Prog.Part.Nucl.Phys. 105 (2019) 82-138

QCD phase diagram in the P-T plane



Wambach, Heckmann, Buballa, Acta Phys.Polon.Supp. 5 (2012) 909-916 arXiv:1906.00936v2 [nucl-th]

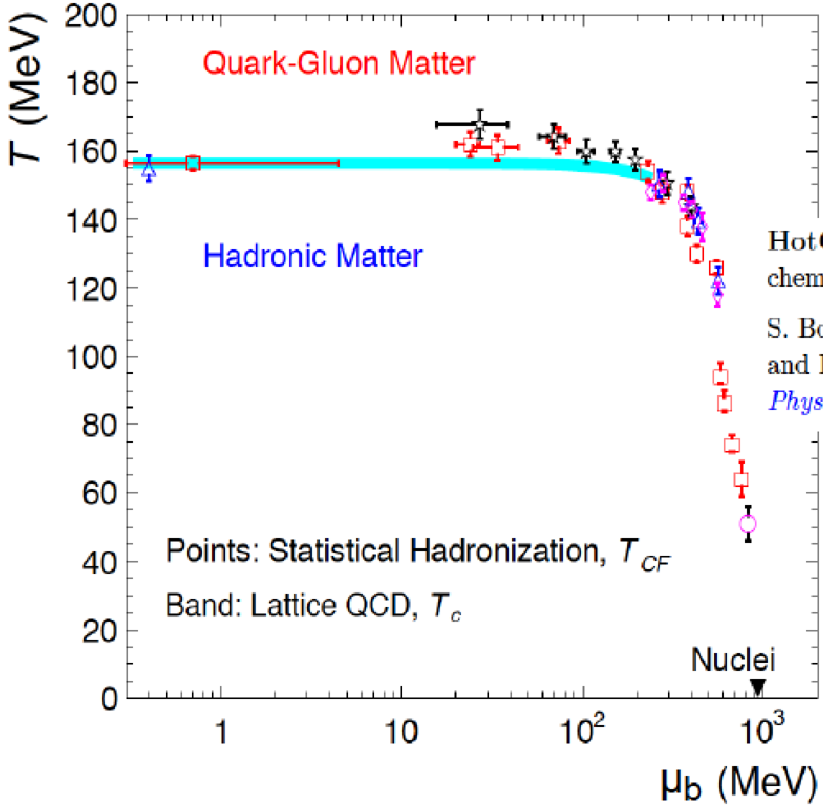


expectations for measurements near the QCD critical point

as one lowers the collision energy towards the (hypothetical) critical point, i.e. as the baryon chemical potential is increased, expect non-monotonous behavior and increased fluctuations.

the QGP phase diagram, LatticeQCD, and hadron production data

note: data from all coll. at SIS, AGS, SPS, RHIC and LHC
each entry is result of years of experiments, variation of μ_b via variation of cm energy



quantitative agreement of chemical freeze-out parameters with most recent LQCD predictions for baryo-chemical potential < 300 MeV

HotQCD Collaboration, A. Bazavov *et al.*, "Chiral crossover in QCD at zero and non-zero chemical potentials," *Phys. Lett. B* **795** (2019) 15–21, arXiv:1812.08235 [hep-lat].

S. Borsanyi, Z. Fodor, J. N. Guenther, R. Kara, S. D. Katz, P. Parotto, A. Pasztor, C. Ratti, and K. K. Szabo, "QCD Crossover at Finite Chemical Potential from Lattice Simulations," *Phys. Rev. Lett.* **125** no. 5, (2020) 052001, arXiv:2002.02821 [hep-lat].

cross over transition at μ_B = 0 MeV, no experimental confirmation

should the transition be 1st order for large μ_B (large net baryon density)?

then there must be a critical endpoint in the phase diagram

experimental determination of phase boundary at
T_c = 156.6 ± 1.7 (stat.) ± 3 (syst.) MeV and μ_b = 0 MeV
Nature 561 (2018) 321

in LQCD results, no critical point is observed for μ_b < 3 T_c

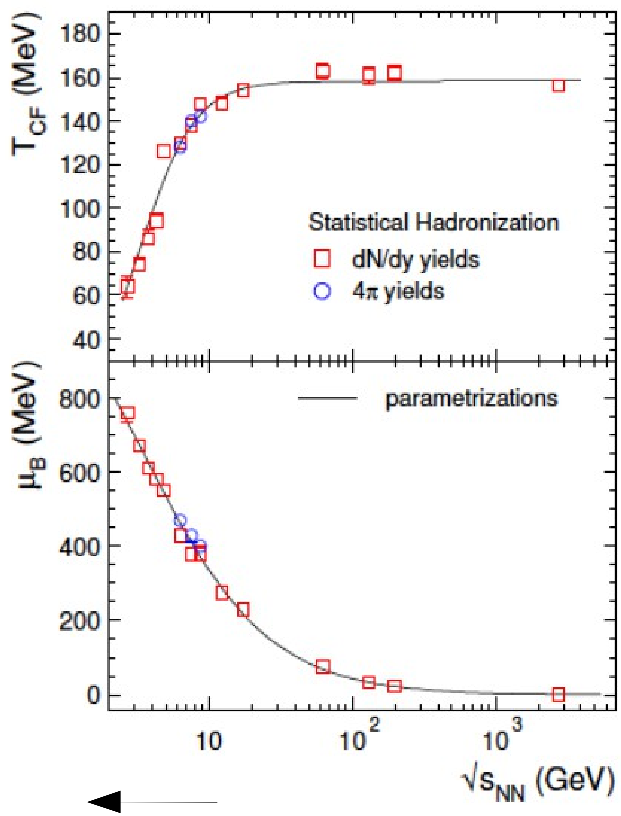
Beam energy dependence of hadron yields from AGS to LHC

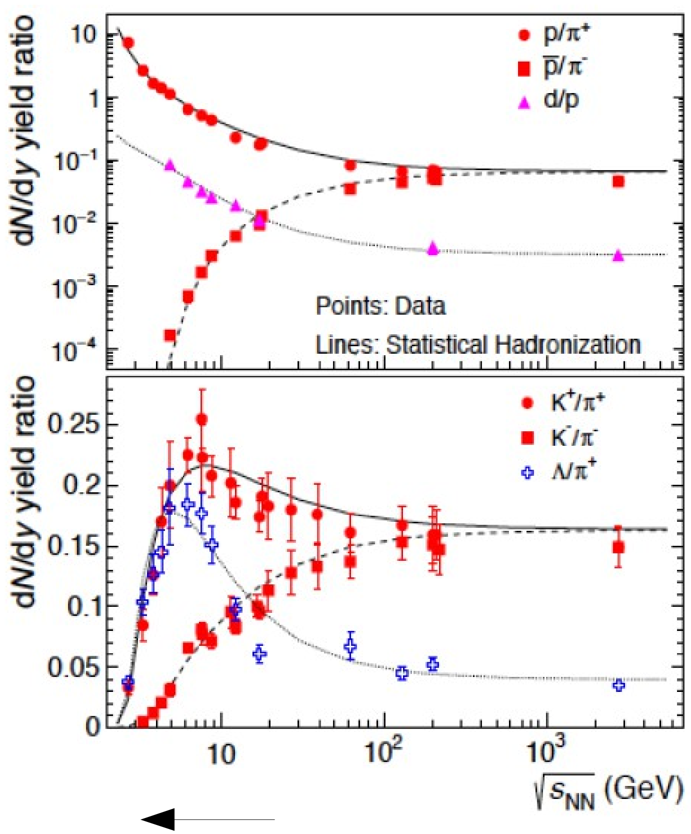
fits work equally well at lower beam energies
following the obtained T and μ_b evolution, features of proton/pion, kaon/pion, deuteron/proton and Lambda/pion ratios reproduced in detail

salient features:

a limiting temperature is reached

nuclear matter becomes increasingly transparent





arrows indicate possible critical region, but with the present data accuracy, no deviations from 'average' thermodynamic behavior is observed

critical point, long range correlations, fluctuations

- near a critical point in the phase diagram of a substance, there are long range correlations, resulting in increased fluctuations, see, e.g., https://en.wikipedia.org/wiki/Critical_point_(thermodynamics))
- for independent observables such as uncorrelated protons and anti-protons, the fluctuations in the net proton number (protons – anti-protons) are Gaussian distributed
- search for critical endpoint implies looking for deviations from Gaussian distributions in the net-proton number
- the shape of any distributions is determined by its moments (and vice-versa)
- the Gauss distribution is the only (known) distribution that has only a 1^{st} moment (mean) and a 2^{nd} moment (variance), all other moments vanish
- look for non-vanishing higher moments in the net-proton distributions measured event-by-event in nuclear collisions at high energy

a note on cumulants

cumulants are used to describe the moments of a distribution

for a mathematical definition, see, e.g.
https://en.wikipedia.org/wiki/Cumulant#Definition

usually one uses moment generating functions and cumulant generating functions to define cumulants
https://en.wikipedia.org/wiki/Cumulant#Cumulants_and_moments

gaussian distributions, moments and cumulants

cumulants are quantities that are alternate to the moments

knowing all cumulants implies knowing all moments and vice-versa

for central moments:

$$\kappa_2 = \mu_2$$

$$\kappa_3 = \mu_3$$

$$\kappa_4 = \mu_4 - 3{\mu_2}^2$$

$$\kappa_5 = \mu_5 - 10\mu_3\mu_2$$

$$\kappa_6 = \mu_6 - 15\mu_4\mu_2 - 10{\mu_3}^2 + 30{\mu_2}^3$$

$$\mu_1 = 0$$

$$\mu_2 = \kappa_2$$

$$\mu_3 = \kappa_3$$

$$\mu_4 = \kappa_4 + 3\kappa_2^2$$

$$\mu_5 = \kappa_5 + 10\kappa_3\kappa_2$$

$$\mu_6 = \kappa_6 + 15\kappa_4\kappa_2 + 10\kappa_3^2 + 15\kappa_2^3$$

for a gaussian distribution, the 1^{st} 2 cumulants are the mean and the variance

all other cumulants vanish

it is the only known distribution with these properties

Poisson distribution, Skellam distribution

let μ be the mean number of events in a given time or space interval and let the time or position of each event occur independently of time or position of the previous event, then the probability distribution to get k events in a particular measurement is:

$$p(k;\mu) = \frac{\mu^k}{k!} e^{-\mu}.$$

Poisson distribution

the probability distribution for the difference of two independently distributed Poisson distributions with means μ_1 and μ_2 is then, with $I_k(x)$ the modified Bessel function, given by:

Skellam distribution

$$p(k;\mu_1,\mu_2) = e^{-(\mu_1+\mu_2)} \left(\frac{\mu_1}{\mu_2}\right)^{k/2} I_{|k|}(2\sqrt{\mu_1\mu_2})$$

J. G. Skellam,
Journal of the Royal Statistical Society,
Series A, 109 (3) (1946) 296

what to measure to look for changes in fluctuations

- in high energy nuclear collisions, go beyond measuring mean values of observables
- this implies to measure, for the same conditions such as energy, centrality, system size etc, in each collision event a particular observable to construct the full distribution of the observable
- as an example, we take as observable the difference between produced protons and anti-protons
- after analyzing many (hundreds of millions of) collision events one obtains not only the mean net proton number $< p - p_{bar} >$ but the full distribution around the mean
- this is called 'event-by-event' analysis
- net protons are used as observable because they connect to net baryon number $< B - B_{bar} >$
- in strong interactions, the net baryon number is a conserved quantity and can be theoretically computed in the lattice QCD approach

net particle distributions and cumulants

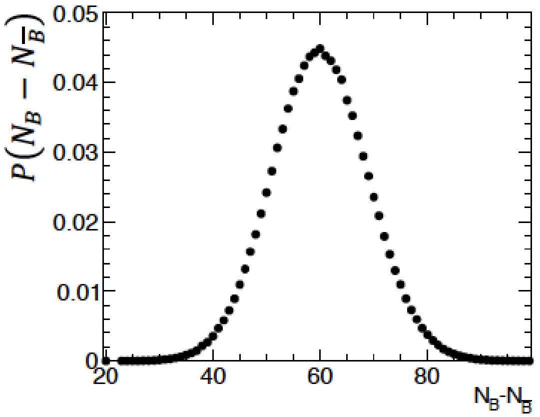
$$X = N_B - N_{\overline{B}}$$

r^{th} central moment:

$$\mu_r \equiv \langle (X - \langle X \rangle)^r \rangle = \sum_X (X - \langle X \rangle)^r P(X)$$

first four cumulants

$$\kappa_1 = \langle X \rangle, \quad \kappa_2 = \mu_2, \quad \kappa_3 = \mu_3, \quad \kappa_4 = \mu_4 - 3\mu_2^2$$



Uncorrelated Poisson limit: $\langle N_B N_{\overline{B}} \rangle = \langle N_B \rangle \langle N_{\overline{B}} \rangle$

Net-Baryons → Skellam

$$\kappa_n = \langle N_B \rangle + (-1)^n \langle N_{\overline{B}} \rangle$$

$$\frac{\kappa_{2n+1}}{\kappa_{2k}} = \tanh\left(\frac{\mu}{T}\right) = \frac{\langle N_B \rangle - \langle N_{\overline{B}} \rangle}{\langle N_B \rangle + \langle N_{\overline{B}} \rangle}$$

$$\kappa_2(X) = \langle X^2 \rangle - \langle X \rangle^2$$

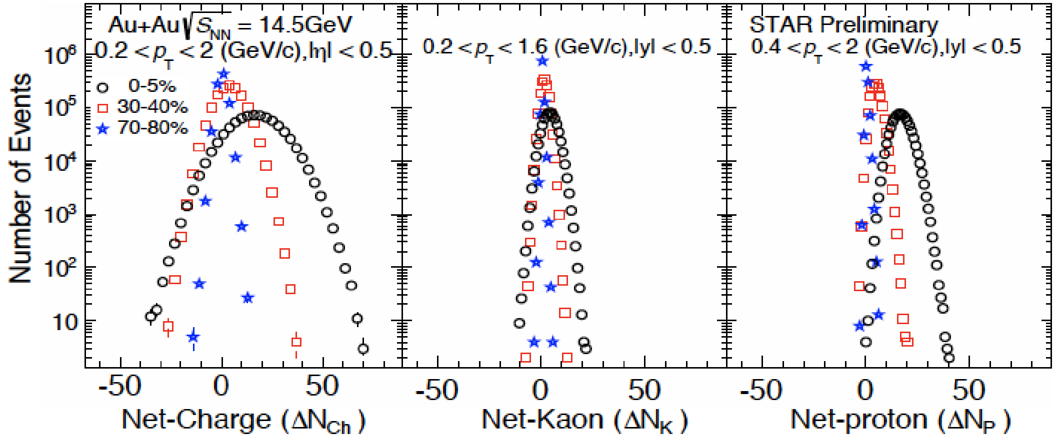
slide courtesy Anar Rustamov, GSI

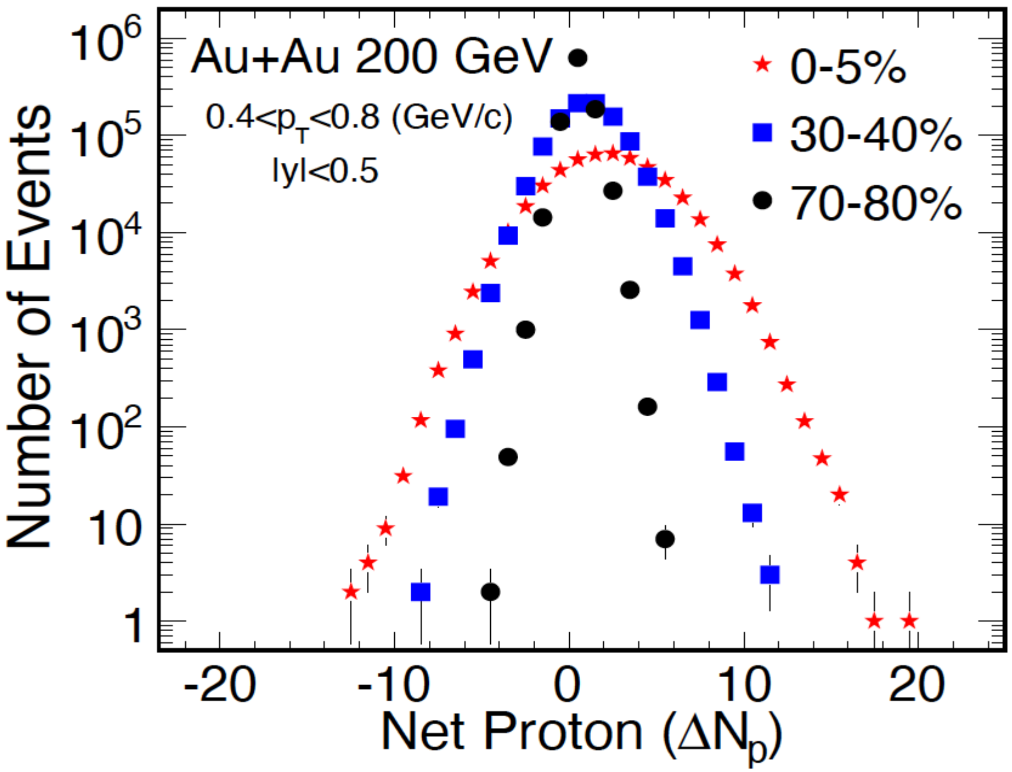
the RHIC beam energy scan (BES) program

BES-II	STAR Events (10^6)	BES-I	STAR Events (10^6)	PHENIX Events (10^6)	$\sqrt{s_{NN}}$ (GeV)	μ_B (MeV)	T_{ch} (MeV)
		2010	238	1681	200	25	166
		2010	45	474	62.4	73	165
		2017	1200		54.4	92	165
		2010	86	154	39	112	164
		2011	32	21	27	156	162
2019	580	2011	15	6	19.6	206	160
2019	320	2014	13		14.5	264	156
2020-21	230	2010	7		11.5	315	152
2020-21	160	2008	0.3		9.2	355	140
2020-21	100	2010	3	2	7.7	420	139

RHIC beam energy scan schedule (as originally approved by BNL management), event statistics, collision energies as well as the corresponding values of the chemical freeze-out temperature and baryonic chemical potential. These values are for the most central Au+Au collisions at RHIC. The 2019 BES-II runs not shown in this Table include a short RHIC electron-cooling test run (3M events at $\sqrt{s_{NN}} = 7.7$GeV) as well as a series of fixed-target mode collisions (50M events at 7.7GeV, 50M events at 3.9GeV, 200M events at 3.2GeV and 300M events at 3.0GeV). In addition to the originally planned energies, a new request for collisions with $\sqrt{s_{NN}}$ at about 17GeV (possibly taking 250 million events in year 2021) has also been put forward.

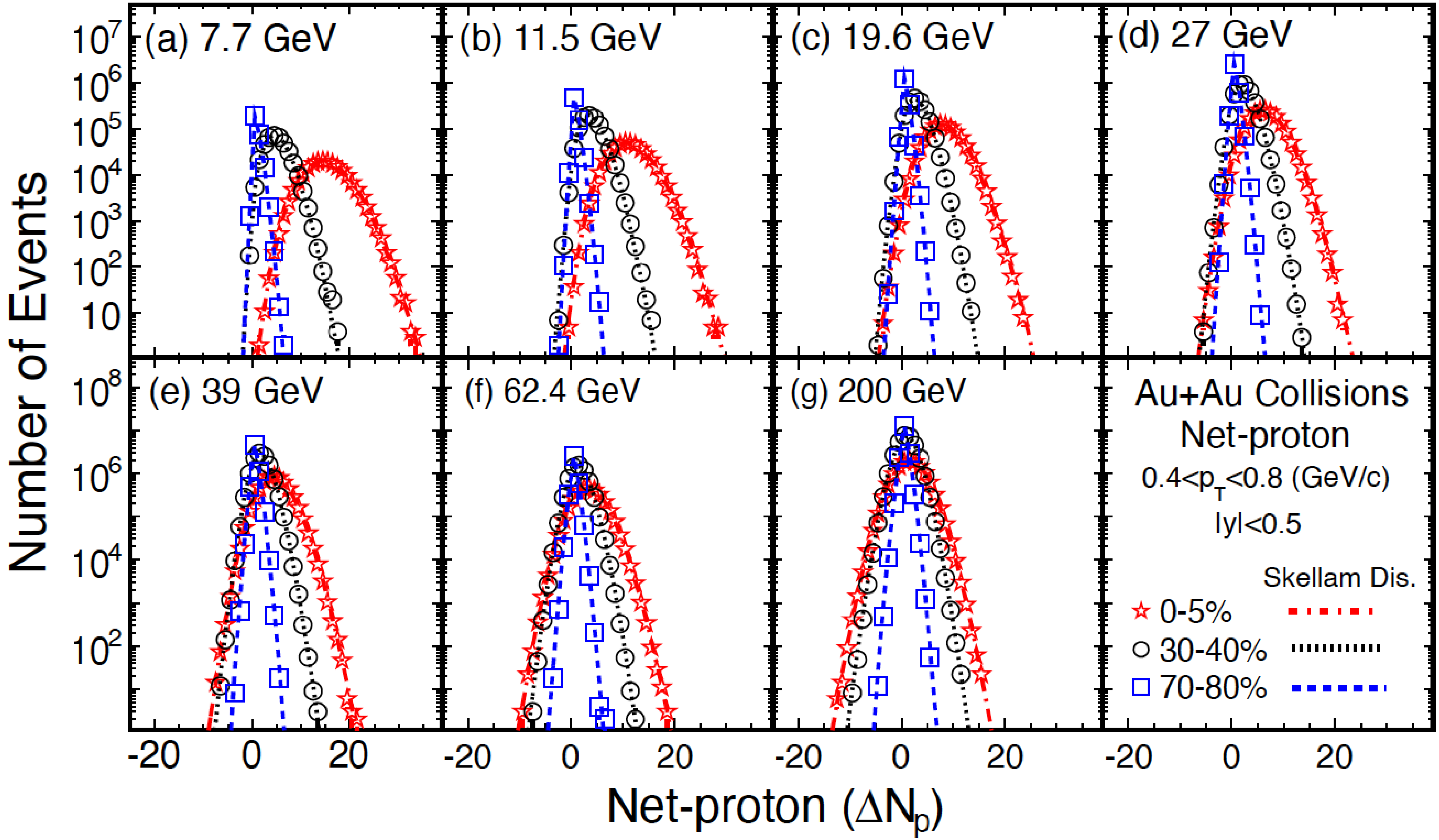
results of net particle measurements from Au-Au collisions at RHIC for different energies and centralities





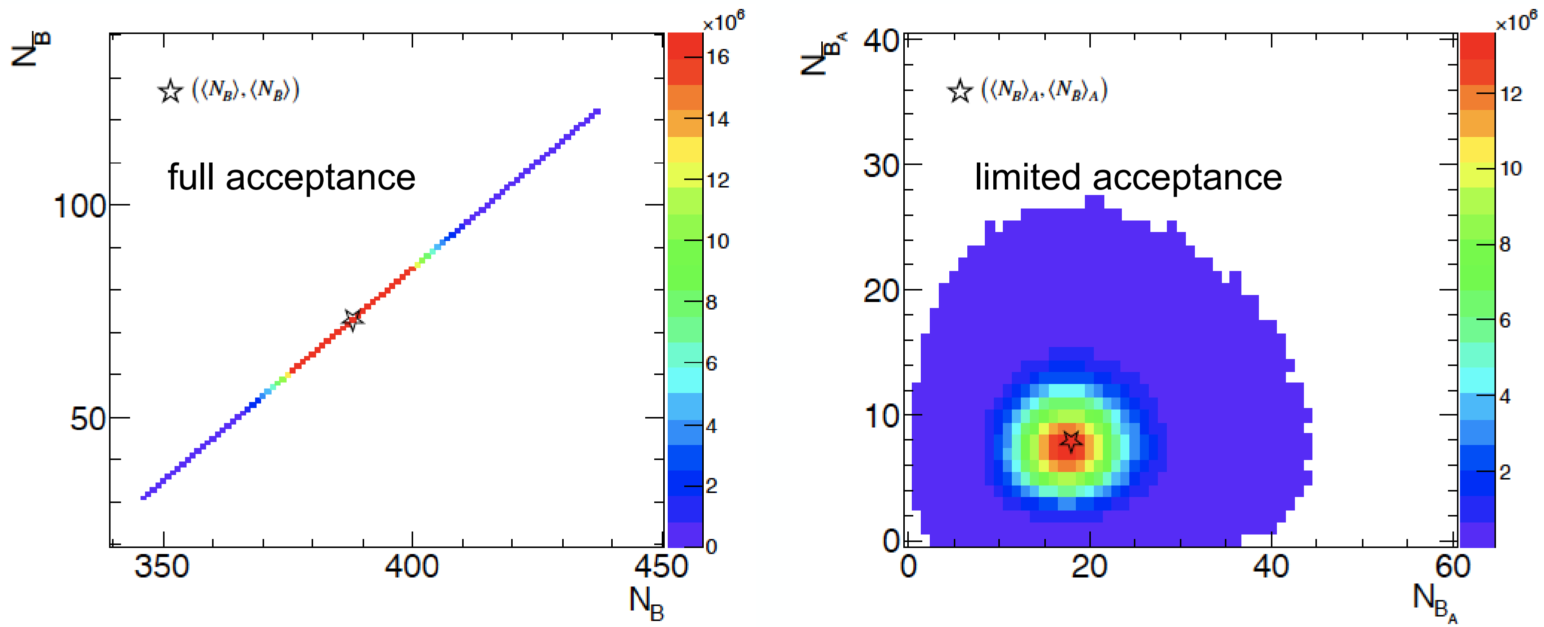
STAR collaboration
arXiv:1004.4959

the energy dependence of the net proton number distribution in Au-Au collisions at RHIC and comparison to the Skellam distribution



STAR coll., Phys.Rev.Lett. 112 (2014) 032302
arXiv:1309.5681 [nucl-ex]

computing net baryon number fluctuations in the SHM under the condition of exact baryon number conservation (canonical ensemble) and without any critical point in the phase diagram

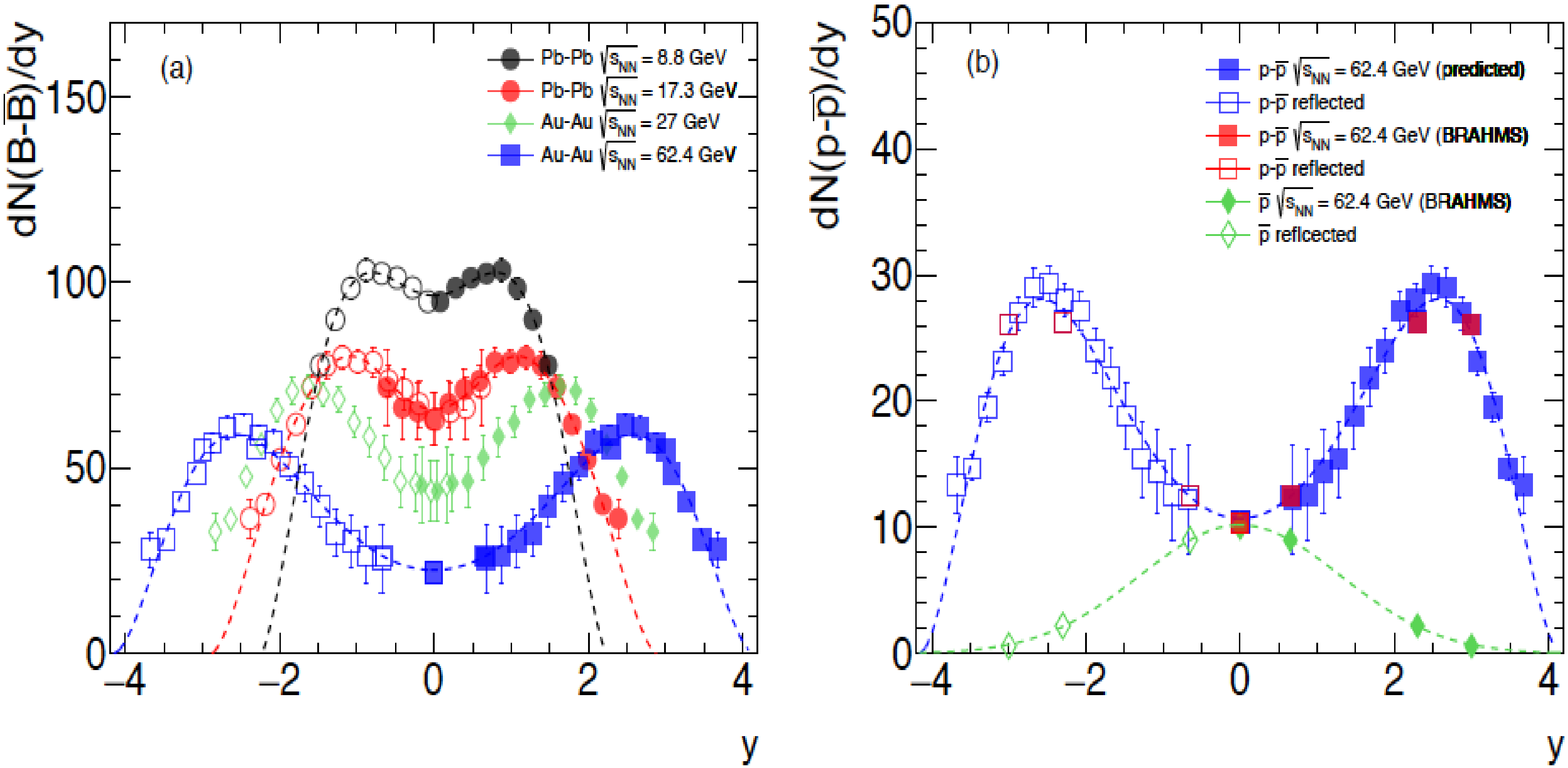


this and the following three slides are taken from:

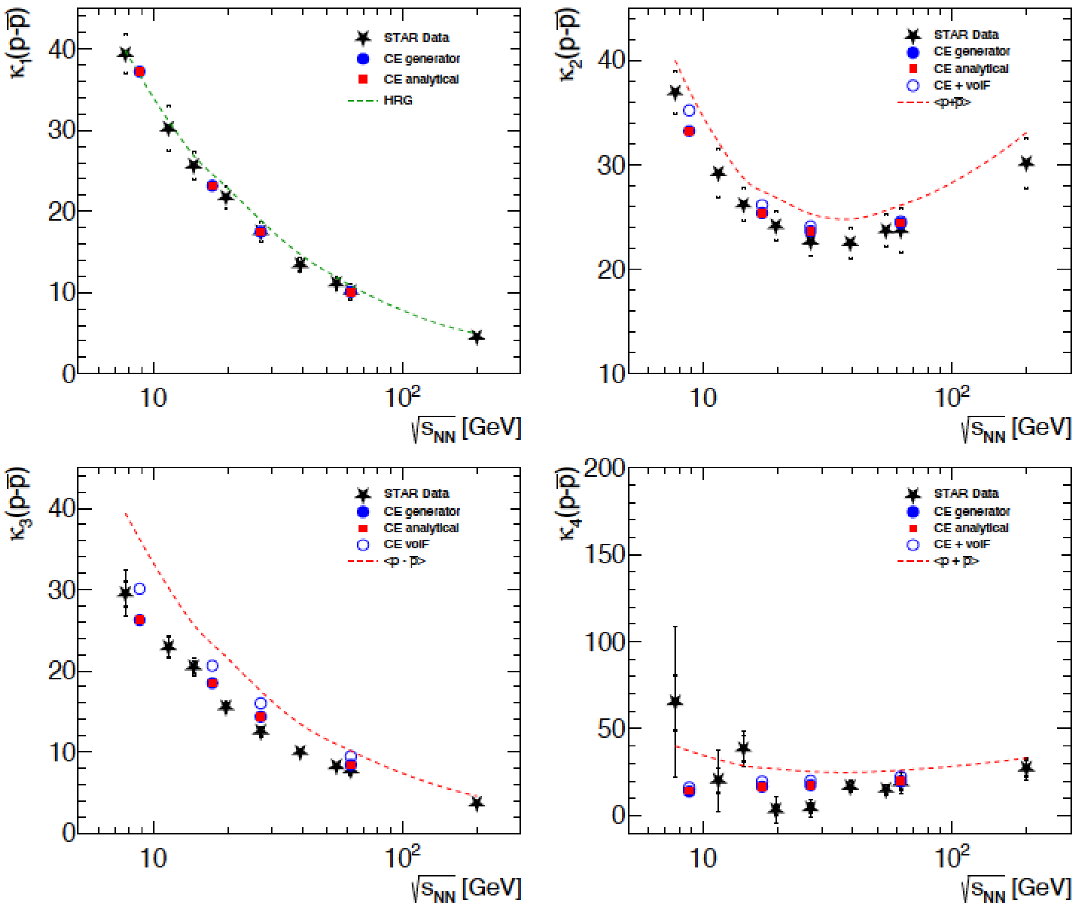
Peter Braun-Munzinger, Bengt Friman, Anar Rustamov, Krzysztof Redlich

Nucl.Phys.A 1008 (2021) 122141

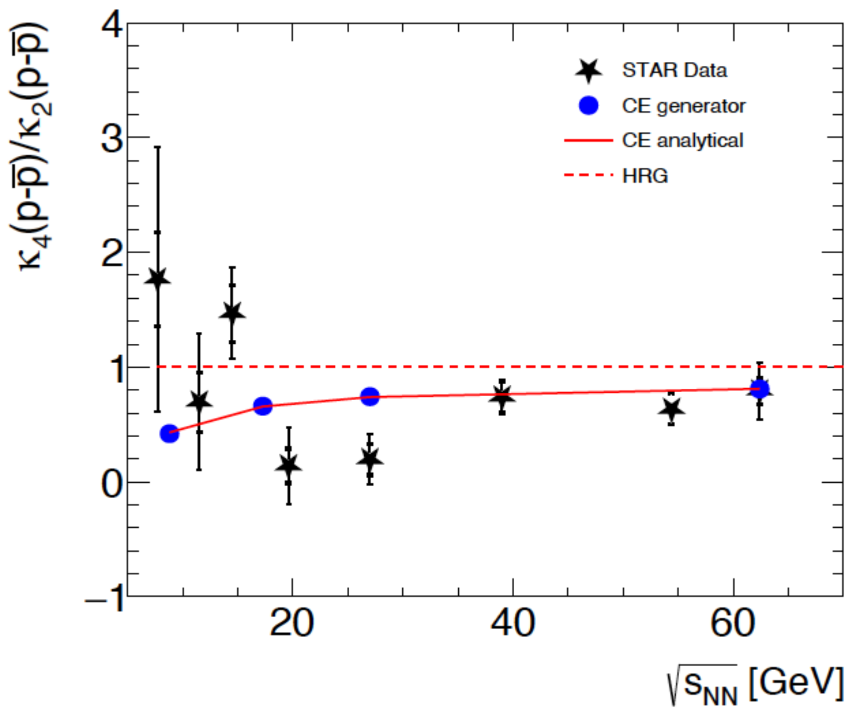
rapidity dependence of net baryon number in relativistic nuclear collisions

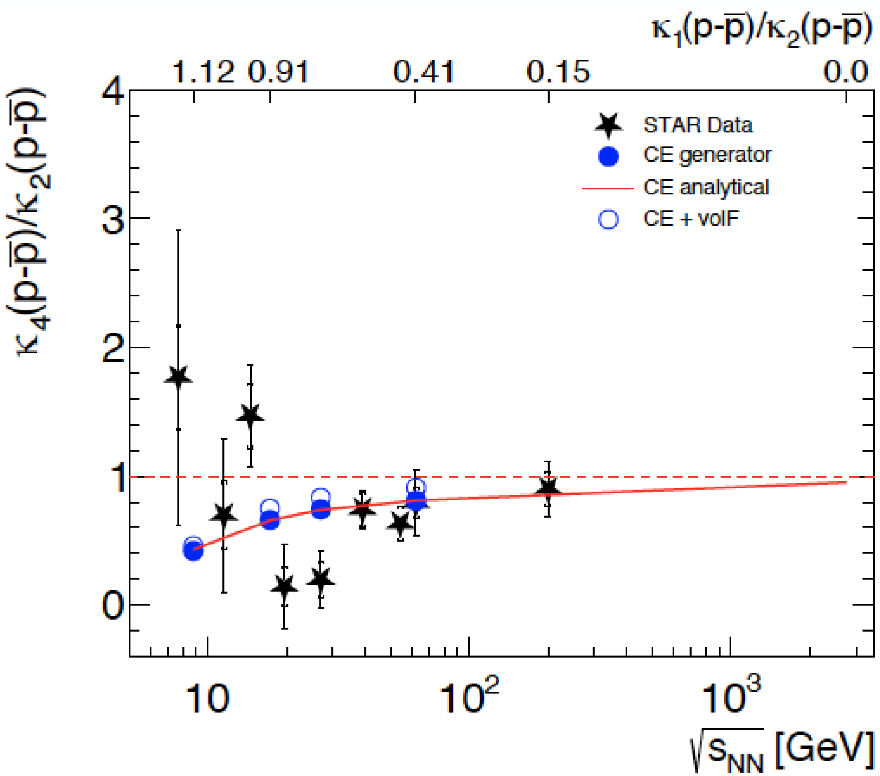


computing the moments and cumulants of the net proton distributions and comparison to the STAR data



detailed comparison in the energy region 7.7 < $\sqrt{s_{NN}}$ < 65 GeV for the ratio of 4[th] to 2[nd] cumulant





some significant fluctuations but no statistically significant deviation from our model predictions which assume no critical point

...such is life. hard work over many years but no sign yet of a critical point.

new measurements have just been completed with much improved statistics. over the next year(s) analysis will tell what nature has in store.

in the mean time, we have learned much about the mechanism of baryon production in such collisions with completely new insights and found unexpected long-range correlations

a new paper is in preparation on this!

important literature

- Misha Stephanov, Krishna Rajagopal, Edward Shuryak
  Phys.Rev.Lett. 81 (1998) 4816-4819, hep-ph/9806219 [hep-ph]
  Phys.Rev.D 60 (1999) 114028, hep-ph/9903292 [hep-ph]

- STAR Collaboration, Mohamed Abdallah et al., 2101.12413 [nucl-ex] and refs. there

- P. Braun-Munzinger, B. Friman, K. Redlich, A. Rustamov, J. Stachel
  Nucl. Phys. A 1008 (2021) 122141, 2007.02463 [nucl-th] and refs. there

- Volodymyr Vovchenko, Volker Koch, Chun Shen
  2107.00163 [hep-ph] and refs. there

- Adam Bzdak, Shinichi Esumi, Volker Koch, Jinfeng Liao, Misha Stephanov, Nu Xu
  Phys. Rept. 853 (2020) 1-87, 1906.00936 [nucl-th], and refs. there