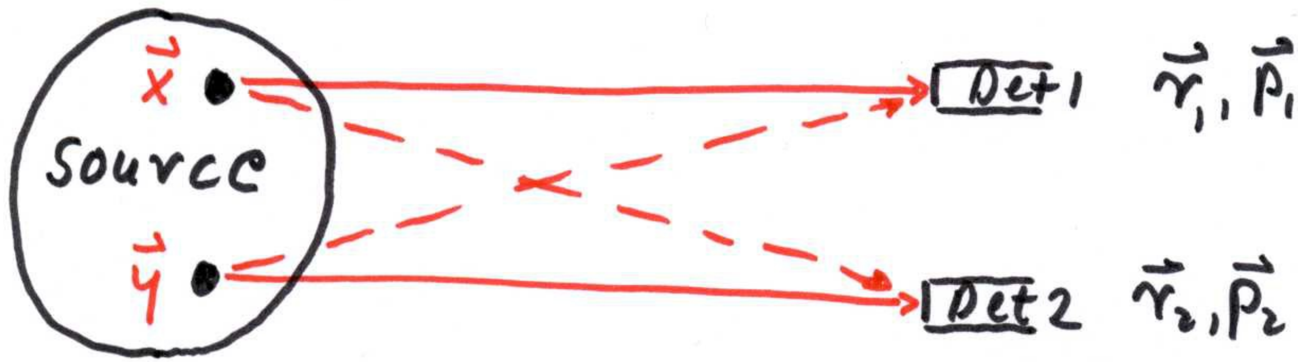
Quark-Gluon Plasma Physics

10. Correlations and Fluctuations

Correlations of identical bosons due to quantum interference

stochastic emission from extended source
consider 2 identical bosons (photons, pions, …)
2 detectors in locations r_1, r_2 observe identical bosons of momenta p_1 and p_2



cannot distinguish solid and dashed paths because of identical particles

→ for plane waves, the probability amplitude for detection of the pair is

$$A_{12} = \frac{1}{\sqrt{2}}[e^{ip_1(r_1-x)}e^{ip_2(r_2-y)} + e^{ip_1(r_1-y)}e^{ip_2(r_2-x)}]$$

with 4-vectors p,r,x,y (to be general for nonstatic source)

square of amplitude: intensity → "intensity interferometry"

probability $A_{12}^2 \propto (1 + \cos(\Delta k \cdot \Delta x))$

is enhanced for momenta close to each other due to Bose-Einstein statistics

Hanbury-Brown Twiss effect

technique of intensity interferometry developed by Hanbury-Brown and Twiss in astrophysics as a means to determine size of distant objects

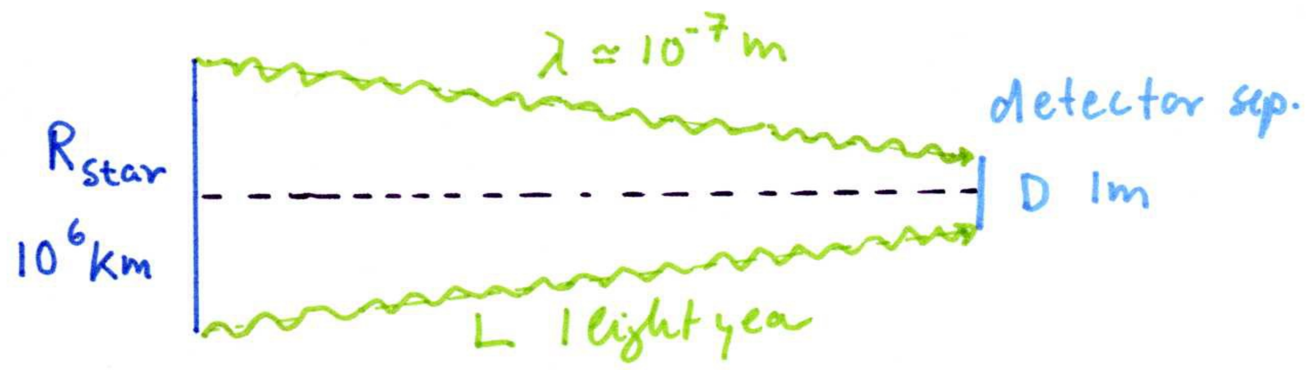
R. Hanbury-Brown and R.Q. Twiss, Phil. Mag. 45 (1954) 663

radiowaves to determine size of galaxies (Cygnus, Cassiopeia)

- this could be done with classical Michelson (amplitude) interferometry

and Nature 178 (1956) 1046

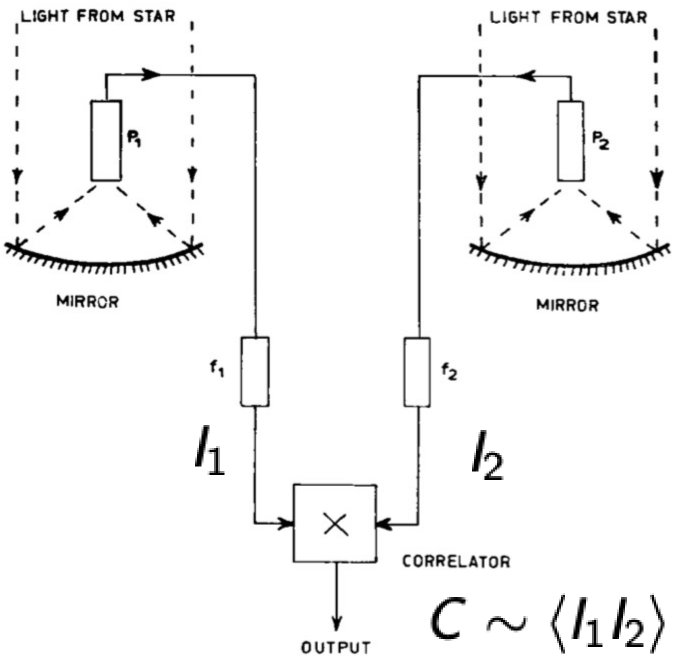
visible light to determine the size of stars (Sirius)



correlated signal varies on characteristic length scale: $D = \lambda \frac{R_{star}}{L}$
varying separation of detectors one learns the apparent angle between the sources

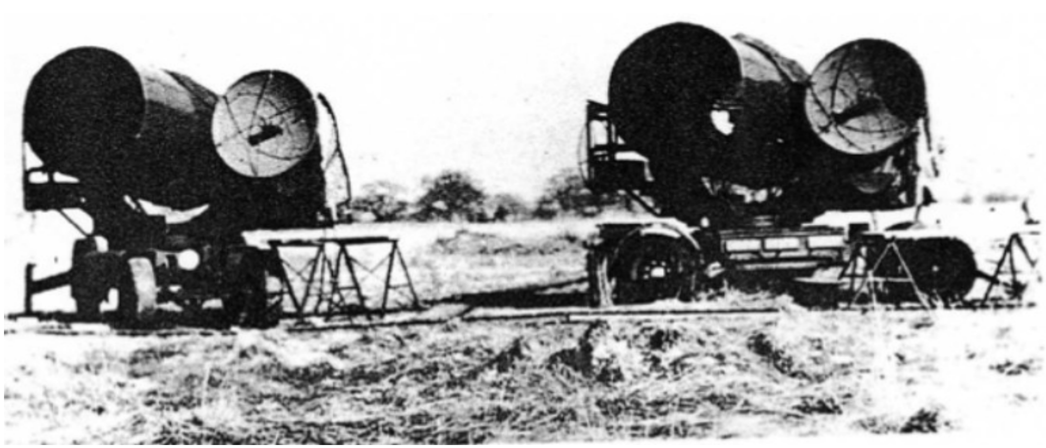
→ with detector separation of the order of meters, star size of order 10^6 km 1 light year away can be determined by intensity interferometry with visible light

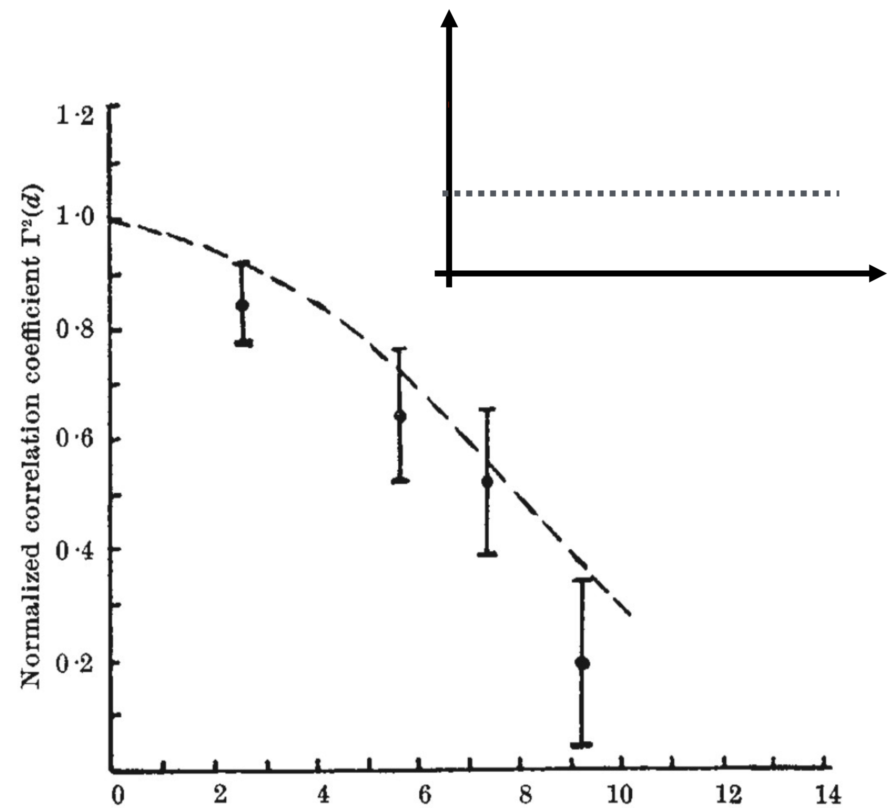
Angular diameter of Sirius from HBT Correlations



A TEST OF A NEW TYPE OF STELLAR INTERFEROMETER ON SIRIUS

By R. HANBURY BROWN
Jodrell Bank Experimental Station, University of Manchester
AND
Dr. R. Q. TWISS
Services Electronics Research Laboratory, Baldock

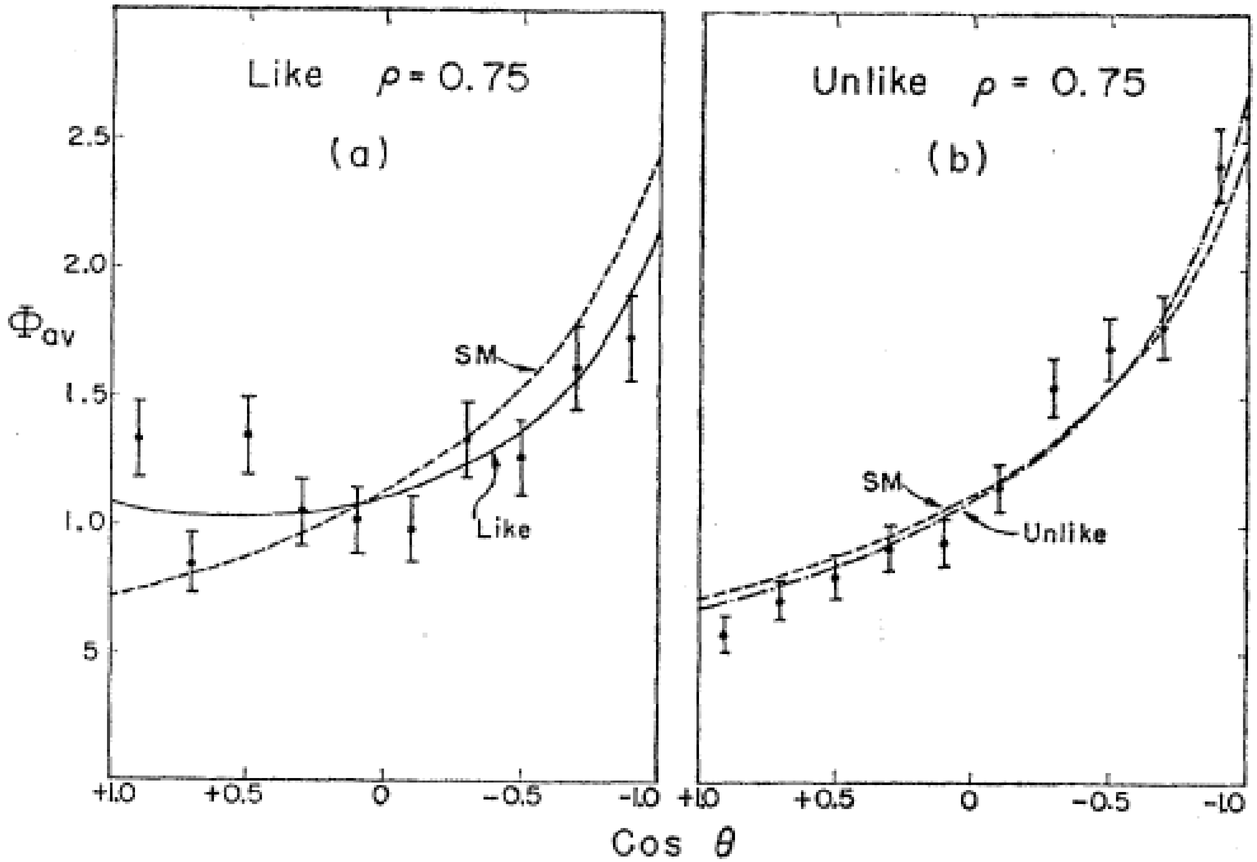




HBT interference in particle physics

when phase space volume smaller than $\Delta p_x \Delta x \approx \hbar$ is considered, chaotic system of identical noninteracting particles exhibits quantum fluctuations following Bose-Einstein (or Fermi-Dirac) statistics

first observation in particle physics: pions with small relative momenta
G. Goldhaber, S. Goldhaber, W.Y. Lee, A. Pais, Phys. Rev. 120 (1960) 300
pion correlations after a $p_{bar}p$ annihilation



angle between 2 pions in the p pbar cm system

look at angular opening between like- and unlike-sign pions in events with 4 charged pions at sqrt(s) = 2.1 GeV

radius r of volume of strong interaction in Compton wavelengths of pion $\frac{\hbar c}{mc^2} = 1.4\,\mathrm{fm}$

found to be 0.5 - 0.75
i.e. 0.7-1.0 fm

HBT interferometry in nuclear and particle physics

after discovery of GGLP systematically used as tool to determine source size in particular in collisions of high energy nuclei

useful references:

G.I. Kopylov, M.I. Podgoretsky, Sov. J. Nucl. Phys. 15 (1972) 219 and ibid 18 (1974) 336 and PLB50 (1974) 472

theory of interference of identical pions, lifetime of source

E. Shuryak, Phys. Lett. B44 (1973) 387

introduces time, i.e. duration of emission

excellent reviews by: G. Baym, Acta Phys. Polon. B29 (1998) 1839

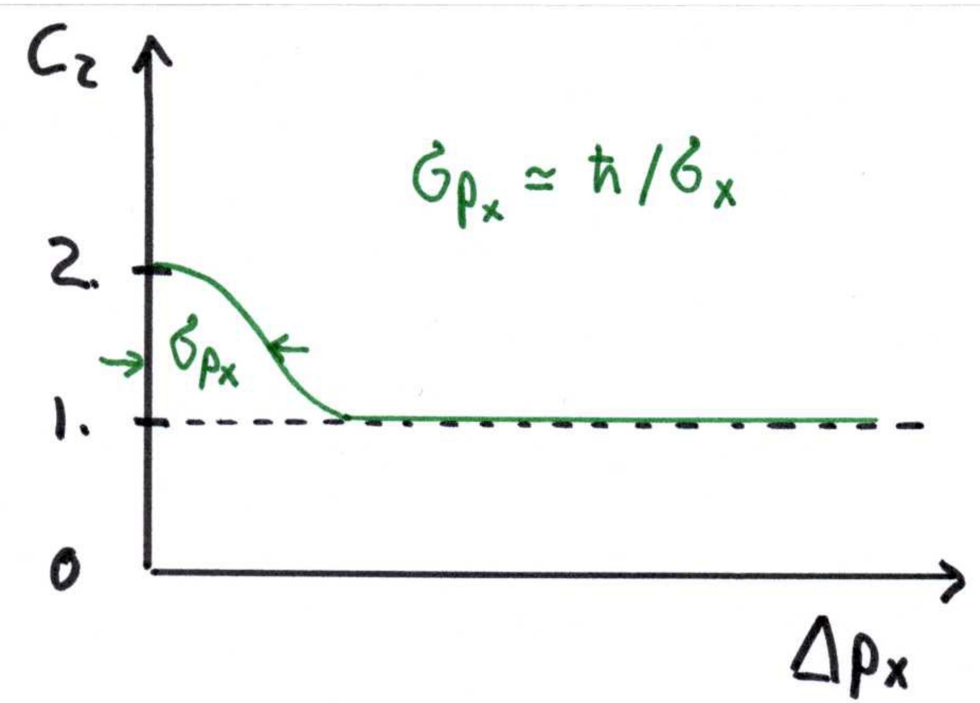
U. W. Heinz, B. V. Jacak, Ann. Rev. Nucl .Part. Sci. 49 (1999) 529-579, nucl-th/9902020 [nucl-th]

2-particle correlation function

typical particle physics experiment: vary momenta at fixed location of detector

$$C_2(\vec{p}_1 - \vec{p}_2) = \frac{d^6N/d\vec{p}_1 d\vec{p}_2}{d^3N/d\vec{p}_1 \cdot d^3N/d\vec{p}_2} = 1 + f(\vec{p}_1 - \vec{p}_2)$$

get uncorrelated denominator typically from event mixing



in heavy ion collisions typical dimensions 1-10 fm,
→ leads to interference at momentum differences of 20-200 MeV/c

for a stochastic source we get a 2-particle correlation function

$$C_2(q) = P_{12} = \int d^4x d^4y \mid A_{12} \mid^2 \rho(x)\rho(y)$$

with a 4-momentum difference $\mathrm{q} \equiv \mathrm{p_1} - \mathrm{p_2}$
and momentum independent source functions $\rho(\mathrm{x}), \rho(\mathrm{y})$
transformation to relative coordinates and integration leads to

$$C_2(q) = 1 + \mid \tilde{\rho}(q) \mid^2$$

$$\tilde{\rho}(\mathrm{q}) = \int \mathrm{d^4x}\rho(\mathrm{x})\mathrm{e^{-iqx}}$$

Fourier transform of space-time density distribution

example: Gaussian space-time density distribution $\rho(\mathrm{x})$

$$\rho(x) = c\ \exp(-\frac{|\vec{x}|^2}{2r_0^2} - \frac{t^2}{2\tau_0^2})$$

$$\longrightarrow \qquad \tilde{\rho}(q) = c'\ \exp(-\frac{|\vec{q}|^2 r_0^2}{2} - \frac{q_0^2\tau_0^2}{2})$$

and $C_2(q) = 1 + \exp(-r_0^2|\vec{q}|^2 - \tau_0^2 q_0^2)$ with norm $\tilde{\rho}(0) = 1$

note that the 3d rms radius is $r_{rms} = \sqrt{3}r_0$

Coupling of spatial information and time

time and space information are coupled, i.e. q_0 and q(vector) are not independent in the pair rest frame $q^* = (0, \vec{q^*}) = (0, 2\vec{k}^*)$ with $\vec{k}^* = \vec{p_1^*} = -\vec{p_2^*}$

(in general $\vec{k}^* = \frac{m_2\vec{p_1^*} - m_1\vec{p_2^*}}{m_1 + m_2}$)

if $\tilde{\mathrm{v}}$ is the velocity of the pair in the overall cm frame, $\gamma^2 = \frac{1}{1 - v^2/c^2}$

and $\vec{k}_T^* = \vec{q}_T/2\,,\ k_L^* = q_L/2\gamma$ (here T and L refer to the direction of the pair)

and $\vec{v} \cdot \vec{q} = v\, q_L$ but $\frac{\vec{v}}{c} = \frac{\vec{p}_1 + \vec{p}_2}{E_1 + E_2}$

→ $\vec{v} \cdot \vec{q} = E_1 - E_2 = q_0$

→ $C_2(q) = 1 + exp(-4r_0^2 k_T^{*2} - 4\rho^2\gamma^2 k_L^{*2})$

where $\rho^2 = r_0^2 + (v\tau_0)^2$

in direction of the pair, spatial and temporal information cannot be distinguished for identical particle

Coulomb correction

in addition to quantum correlations 2 identical pions also experience correlations due to electromagnetic interaction, i.e. Coulomb repulsion

for 2 pions from a point source this problem was already solved for beta-decay (Gamov function)

$$\psi_{\mathrm{C}}(0) = \left(\frac{2\pi\eta}{\mathrm{e}^{2\pi\eta} - 1}\right)^{1/2} \quad \text{with} \quad \eta = \frac{\mathrm{zz'e^2}}{\mathrm{v_{rel}}}$$

was used also in early days of HBT, but approximation is too crude

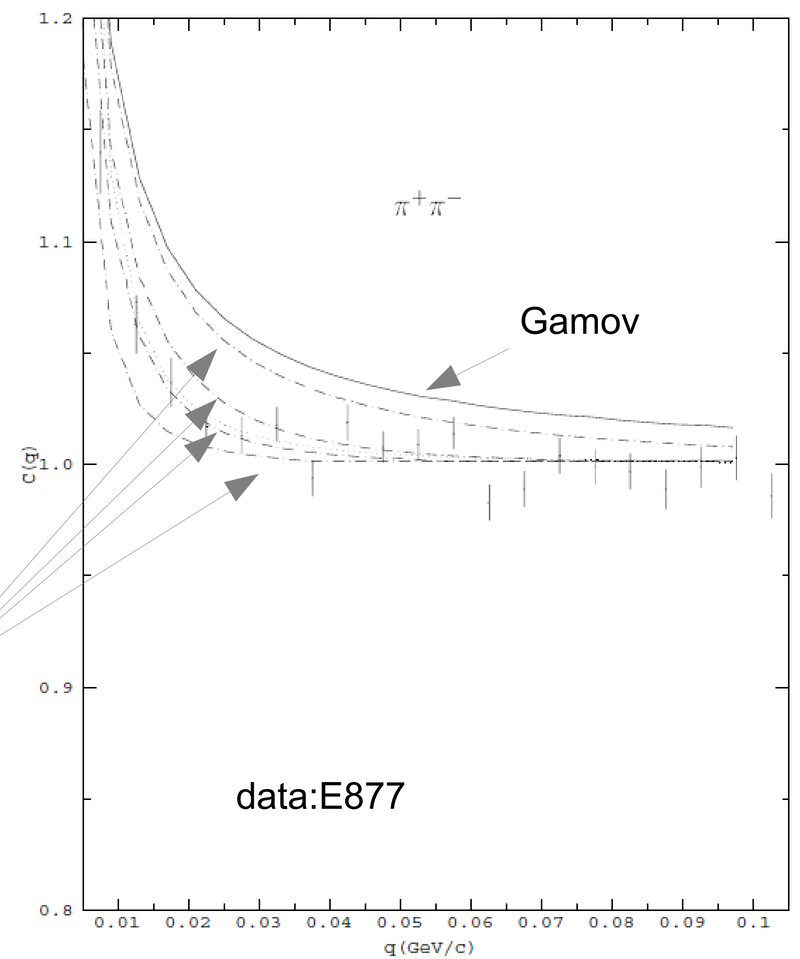
(P. Braun-Munzinger, G. Baym, Nucl. Phys. A610 (1996) 286c)

look at $\pi^+\pi^-$ correlation to see Coulomb effect

need to integrate over source of proper size

examples R = 1,5,9,18 fm

a recursive problem, since source size from HBT is determined after Coulomb correction – typically very few iteration steps



3-dimensional correlations functions in pp and heavy ion collisions

go from momentum difference in cartesian coordinates to so-called Bertsch-Pratt variables (G. Bertsch, Phys. Rev. C37 (1988) 1896)
define z-axis as beam axis

$$q_{long} = \vec{q} \cdot \hat{e}_z = q_z$$

and in transverse direction distinguish direction parallel to pair momentum

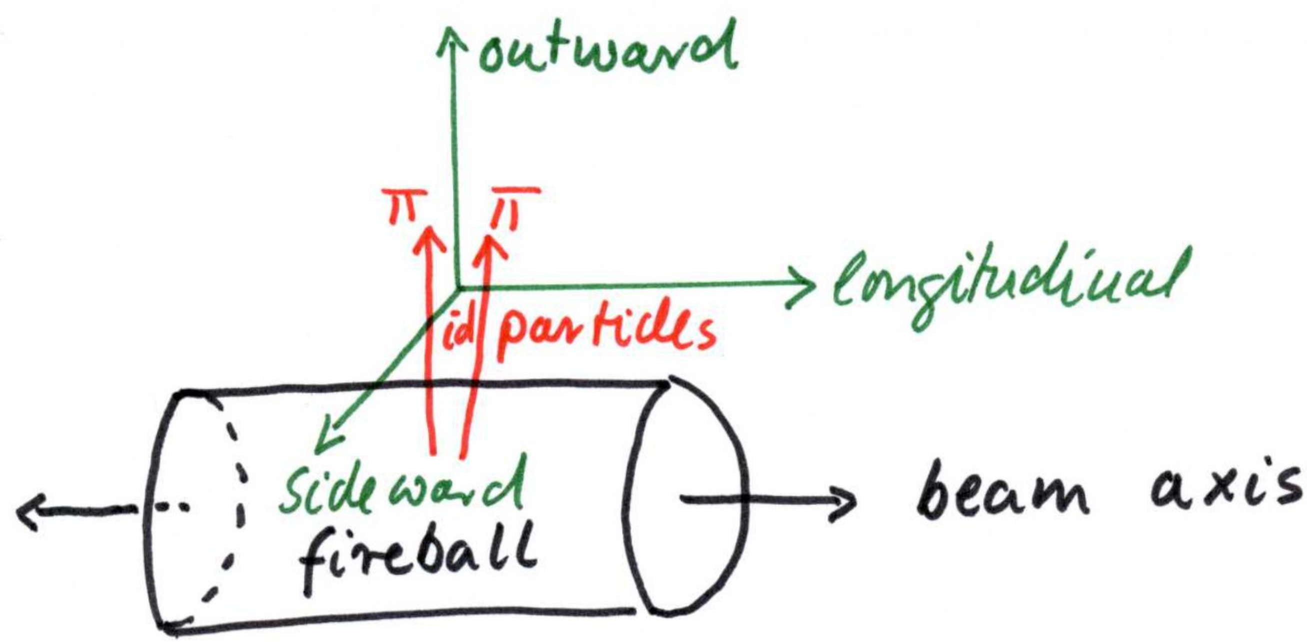
$$\hat{e}_{out} = \frac{\vec{p}_{t_1} + \vec{p}_{t_2}}{|\vec{p}_{t_1} + \vec{p}_{t_2}|}$$

$$q_{out} = \vec{q} \cdot \hat{e}_{out}$$

and perpendicular to it

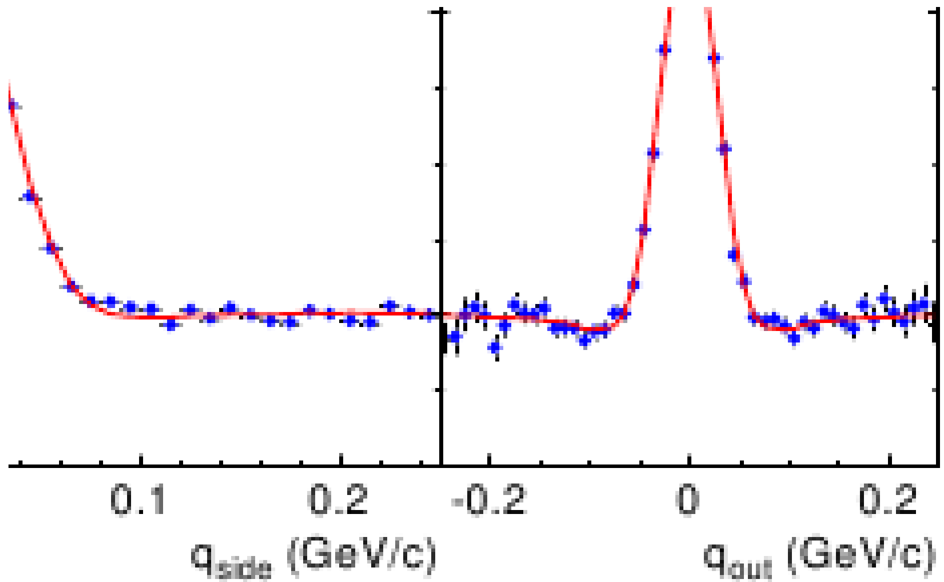
$$\hat{e}_{side} = \hat{e}_{out} \times \hat{e}_z$$

$$q_{side} = \vec{q} \cdot \hat{e}_{side}$$



Example: raw measured ππ correlation function

CERES/NA45 at CERN SPS - central 158 A GeV/c PbAu collisions



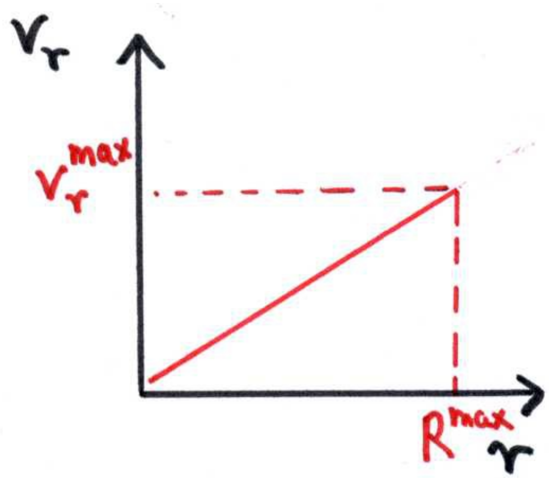
characteristic width order 30 MeV/c

fit function of form $C_2 = 1 + \lambda exp(-R^2_{long}q^2_{long} - R^2_{side}q^2_{side} - R^2_{out}q^2_{out})$
to the exp. correlation function
with correlation strength Θ= 1 for chaotic source and
0 for fully coherent source

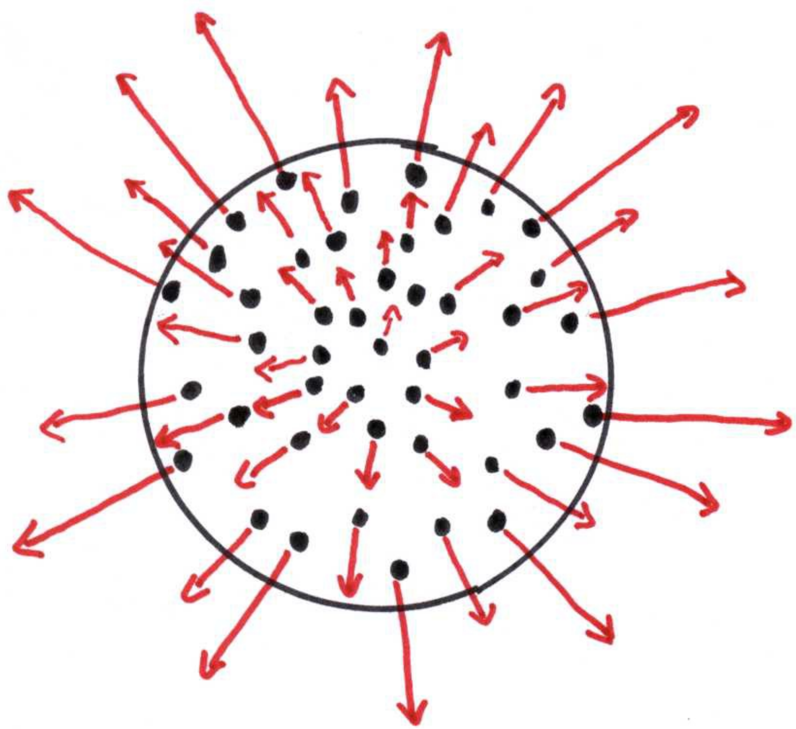
Dynamical source with space-momentum correlations

collective expansion introduces space-momentum correlations leading to an apparent reduction in source radius

simple example:
source with T=0 emitting particles radially with a Hubble-like velocity profile



the apparent source from
2-particle correlations looks pointlike!
only particles emitted from the same space point have the same momentum, i.e. a small momentum difference



Dynamical source with space-momentum correlations

effect of hydrodynamic expansion on radius parameters extracted from 2-pion correlations

S. Pratt, Phys. Rev. Lett. 53 (1984) 1219
A.N. Makhlin, Y.M. Sinyukov, Z. Physik C39 (1988) 69
Y.M. Sinyukov, Nucl. Phys. A498 (1989) 151c

<u>picture:</u> thermal source that expands hydrodynamically with velocity v(r) and corresponding ∩(r) and B(r)

look at 2-pion correlation function for different slices in pair transverse momentum

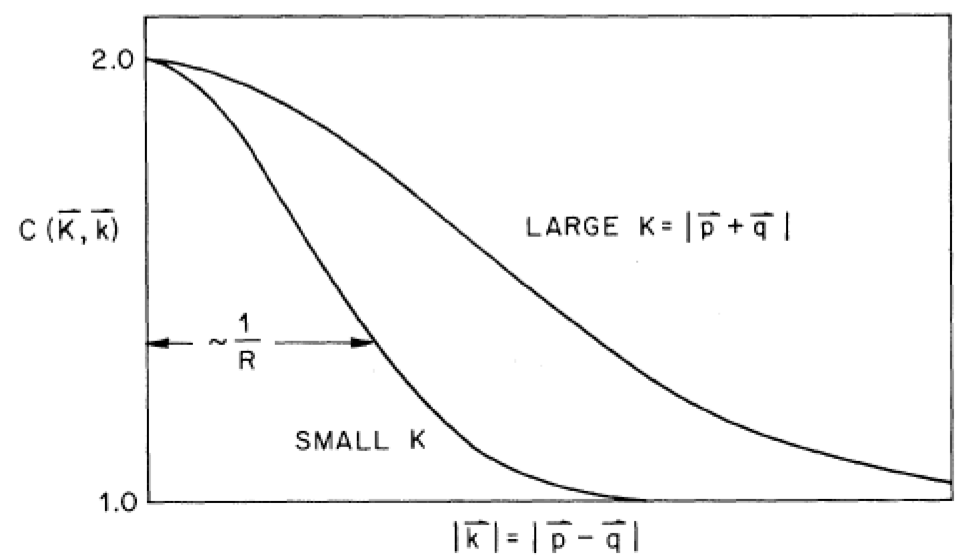
$$\vec{k} = \frac{1}{2}(\vec{k}_1 + \vec{k}_2) \quad \text{or also} \quad \mathrm{k_t} = \frac{1}{2}(\mathrm{k_{t,1}} + \mathrm{k_{t,2}})$$

find that R(k) decreases with k, as ratio of energy of collective expansion over thermal energy increases

$$R(k) = R[(y \tanh y)^{-1} - \sinh^{-2} y]$$

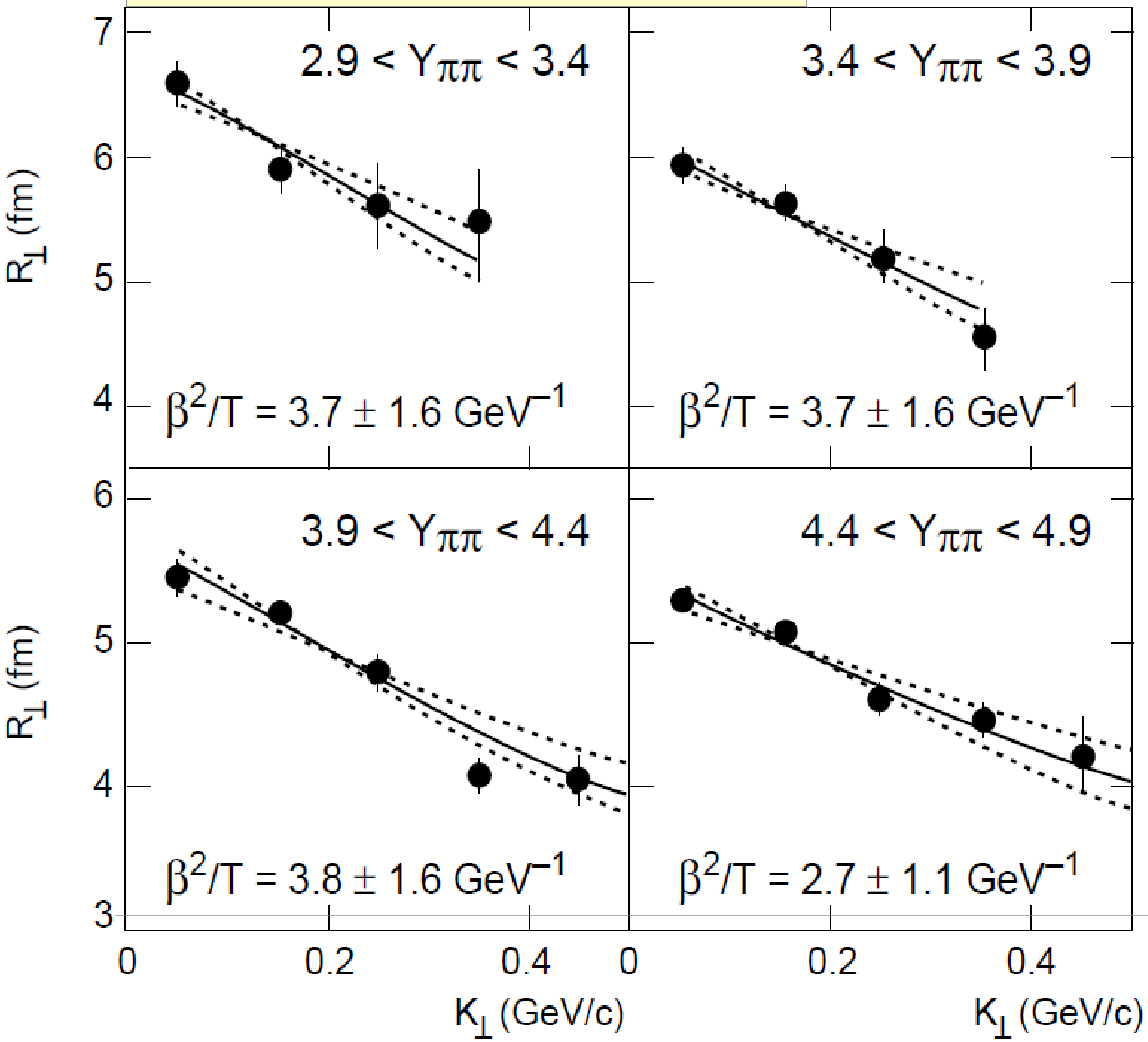
$$\text{with} \qquad y = k\gamma\beta/T$$

faster pions are more likely to be emitted near the point on spherical shell expanding with velocity ∩in direction of k (S.Pratt)



First successful hydrodynamic description of HBT data

data: NA49 at SPS, 200 GeV/c Pb+Pb
thesis H. Appelshäuser
Eur. Phys. J. C2 (1998) 661



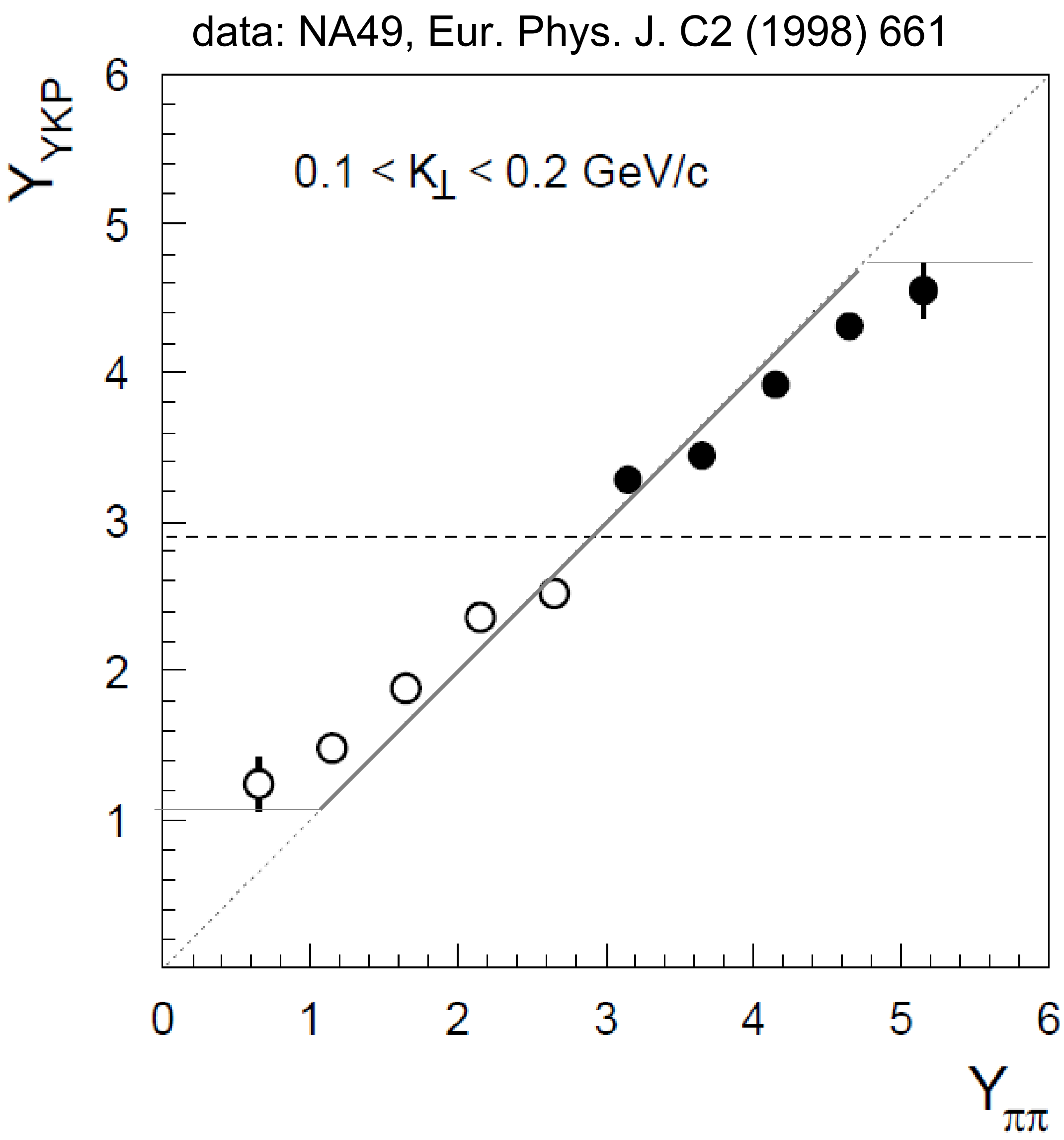
$$R_\perp = \sigma_x (1 + \frac{m_t \beta^2}{T\, cosh(y_{YPK} - y_{\pi\pi})})^{-1/2}$$

fit function U.Heinz et al.
expanding source with
expansion velocity ∩,
temperature T and
pair transverse mass

$$m_t^2 = m_\pi^2 + k_t^2$$

→ consistent description w.
b = 0.5, T = 120 MeV
σ_x = 8.2 fm

Velocity of source emitting pions relative to pair velocity



consistent with boost invariant expansion over ± 1.5 units of y (source rapidity equal pair rapidity) central fireball with Bjorken expansion 3 units long

R_{long} - Longitudinal Expansion of Fireball

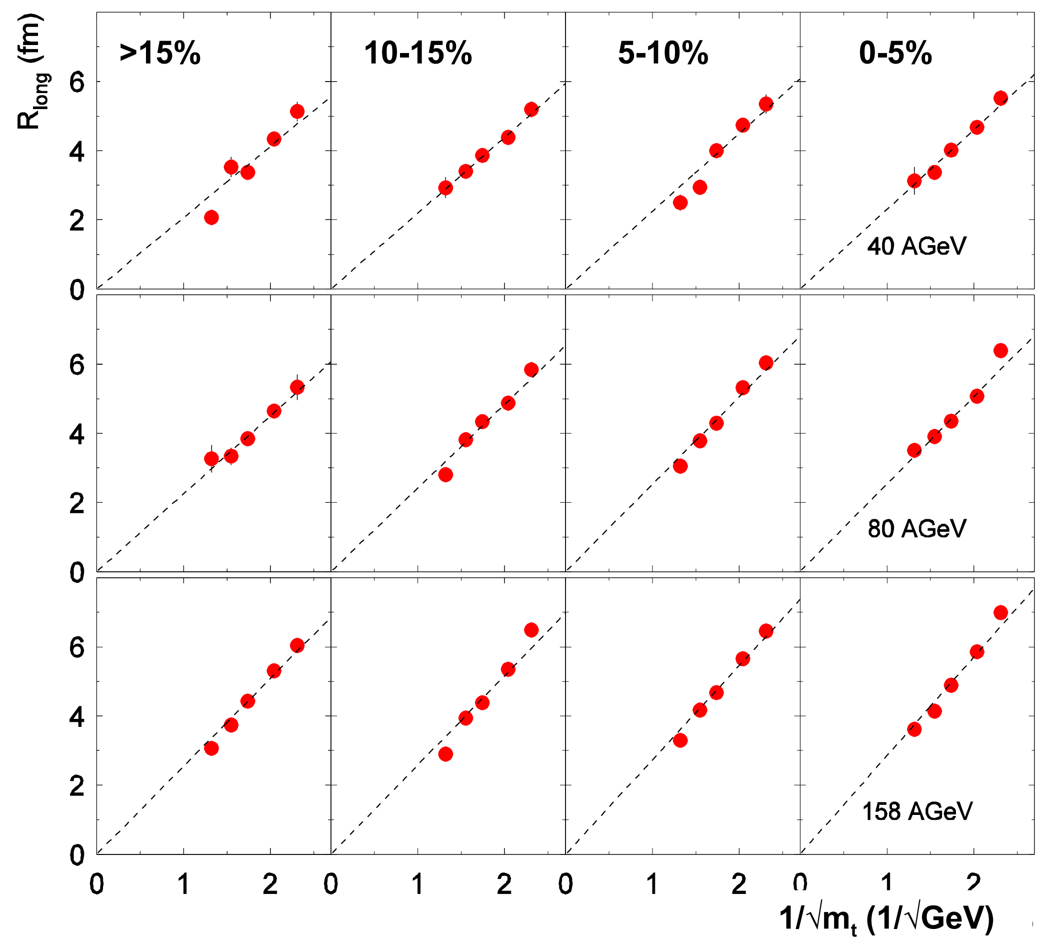
due to finite T, pions are correlated over an interval in rapidity and due to expansion this grows linearly with duration of expansion
R_{long} is longitudinal correlation length

$$R_{long} \approx \tau \underbrace{\sqrt{T_f/m_t}}_{\text{thermal velocity}} \qquad \text{Y.Sinyukov}$$

duration of expansion (lifetime) τ
of the system can be estimated from the transverse momentum dependence of R_{long}

$$\tau = 6 - 8 \text{ fm/c}$$
$$\text{for } T_f = 120 - 160 \text{ MeV}$$

CERES Pb-Au *Nucl. Phys. A714 (2003) 124*



Hubble plot of nuclear fireball

R_{side} – transverse expansion and geometry

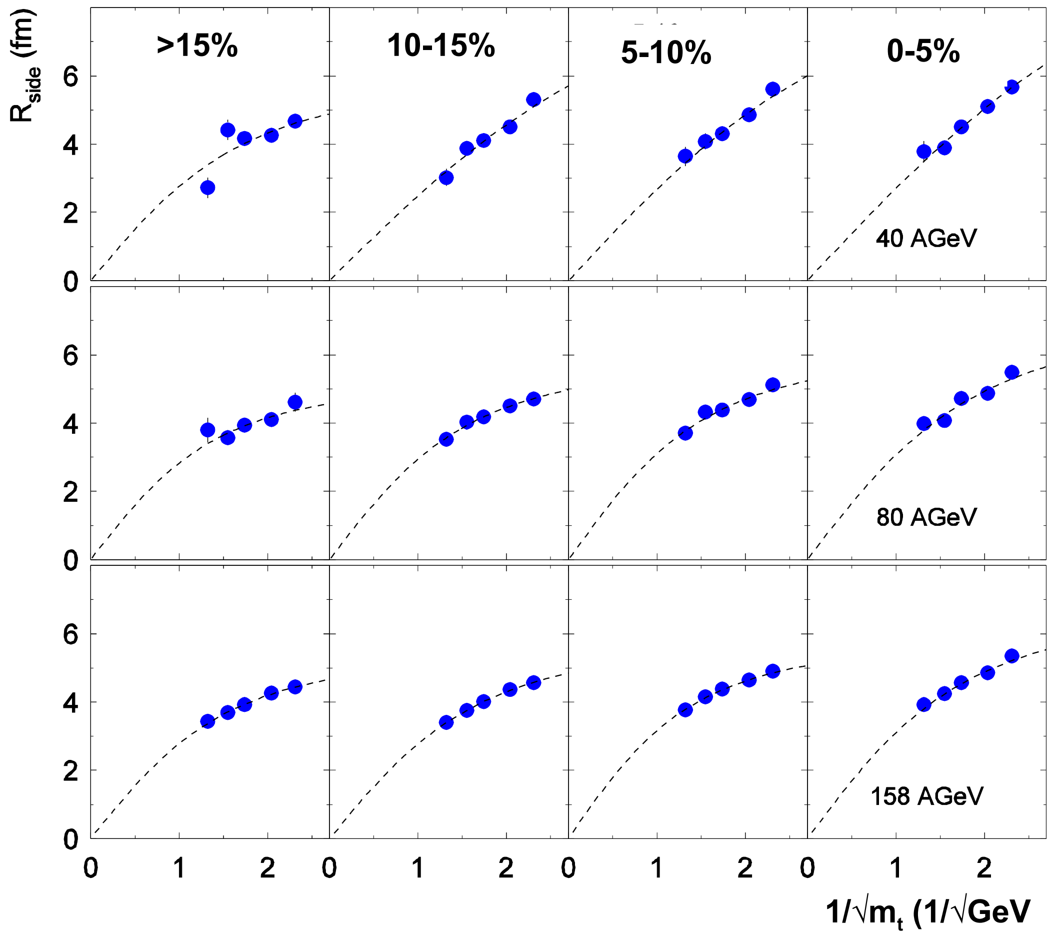
$$R_{side} = R_{geo} / \sqrt{1 + \eta_f^2 m_t / T_f}$$

η_f^2 :
strength of transverse expansion
(U. Heinz, B. Tomasik, U.Wiedemann)

$$\langle \beta_t \rangle = 0.5 - 0.6$$
$$\text{for } T_f = 160\text{-}120 \text{ MeV}$$

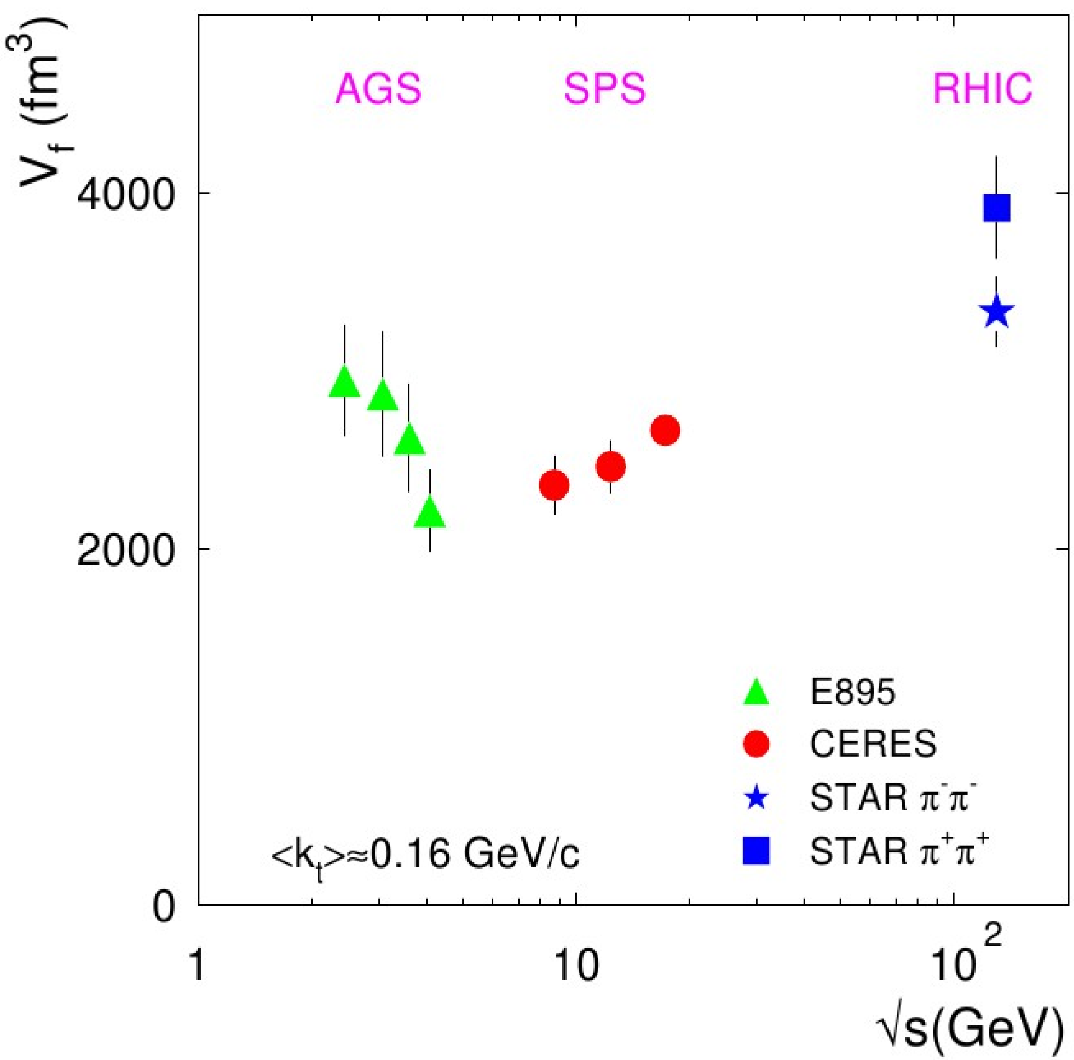
$$R_{geo} = 5.5 - 6 \text{ fm}$$

CERES Pb-Au Nucl. Phys. A714 (2003) 124



Freeze-out volume vs. beam energy

estimate freeze-out volume V_f: $$V_f = (2\pi)^{3/2} R_{side}^2 R_{long}$$



note: this is volume of 0.88 units of rapidity

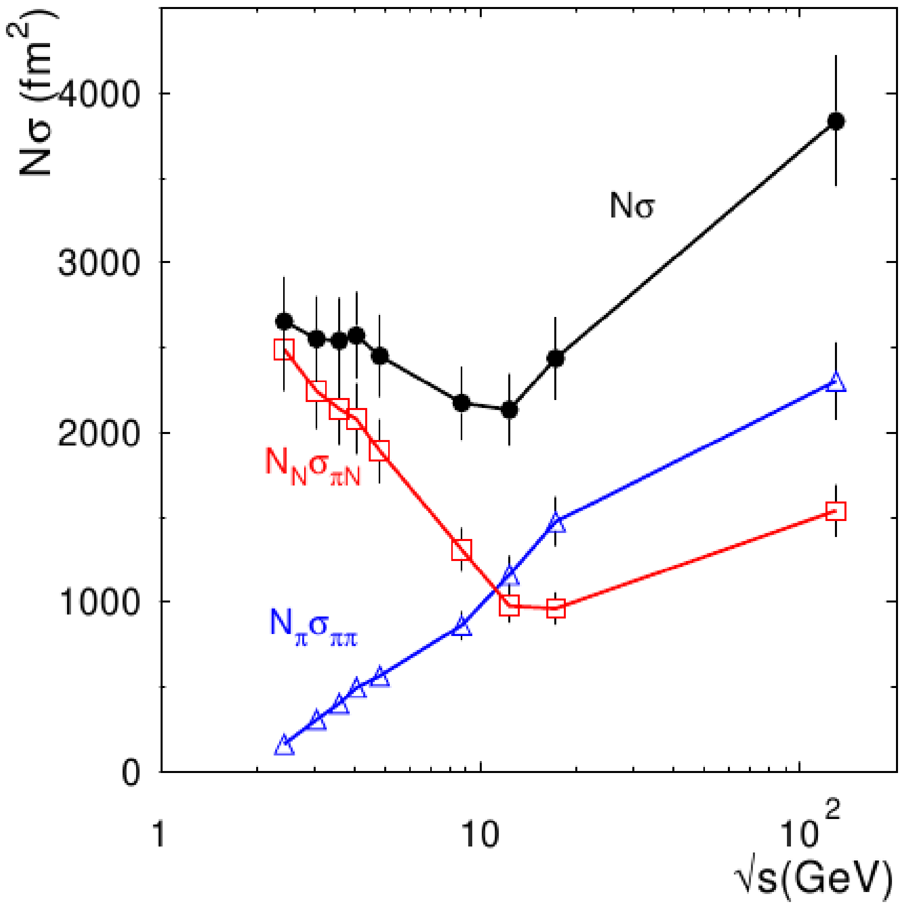
- surprise: non-monotonic behaviour
- minimum between AGS and SPS
- explained by falling nucleon density and increasing pion density together with $\sigma_{\pi N} >> \sigma_{NN}$

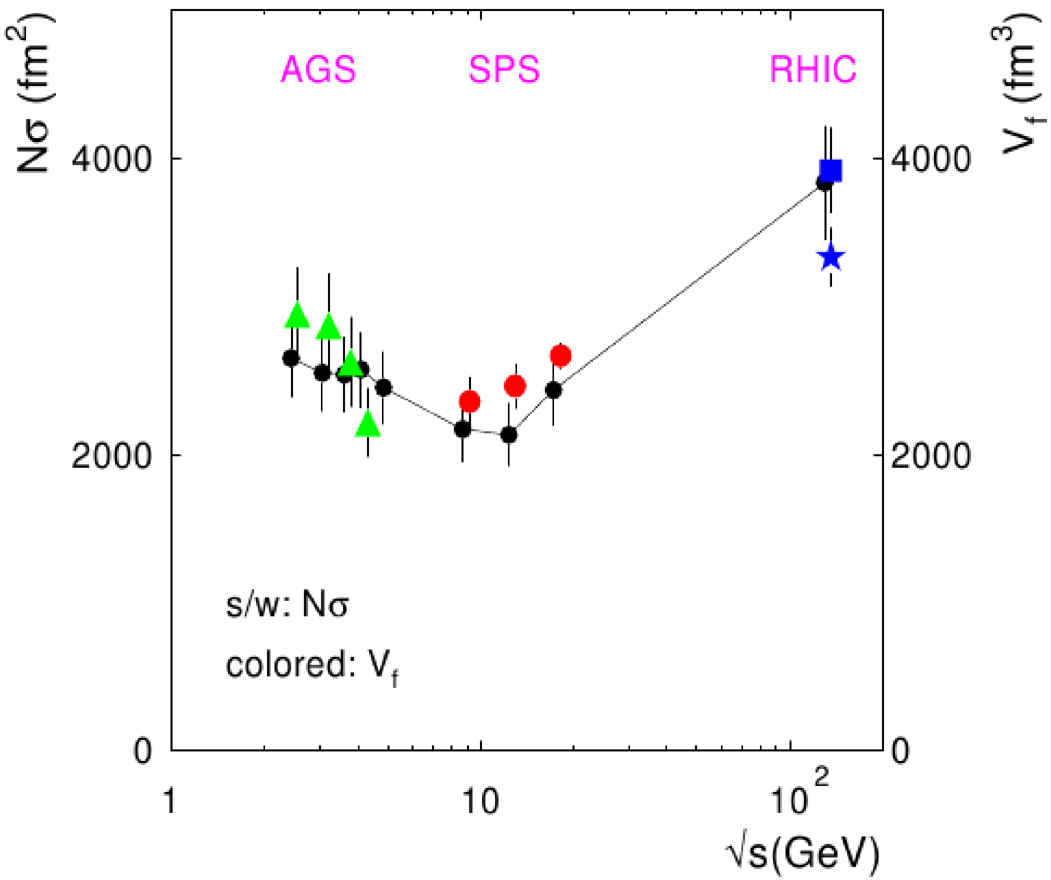
what governs pion freeze-out?

pion mean free path: $\lambda_f = 1/(\rho_f \cdot \sigma) = V_f/(N \cdot \sigma)$

$$N \cdot \sigma \approx N_N \cdot \sigma_{\pi N} + N_\pi \cdot \sigma_{\pi\pi}$$

CERES Phys. Rev. Lett. 90 (2003) 022301

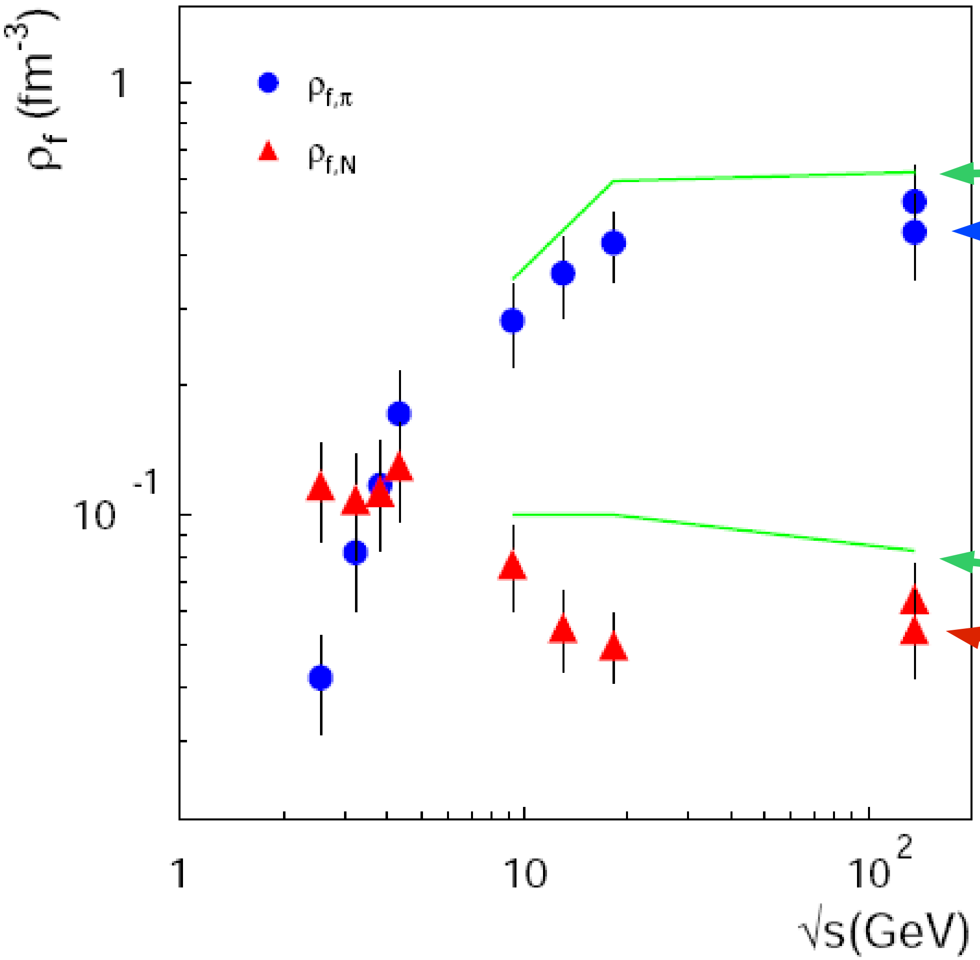




universal freeze-out at mean free path $\lambda_f \approx 1$ fm - small vs system size

Densities at chemical and thermal freeze-out

HBT gives density at thermal freeze-out



pion density at chemical freeze-out

pion density at thermal freeze-out

nucleon density at chemical freeze-out

nucleon density at thermal freeze-out

$$1/2 - 1/3\rho_0$$

volume appears only to grow 30 % between chemical and thermal freeze-out
get from the 2 densities, the velocity and final radii: isentropic expansion for 0.9-2.3 fm
T_f = 158 – 132 MeV, rate of cooling near T_c: (13 ± 1) % per fm/c

R_{out} – duration of pion emission

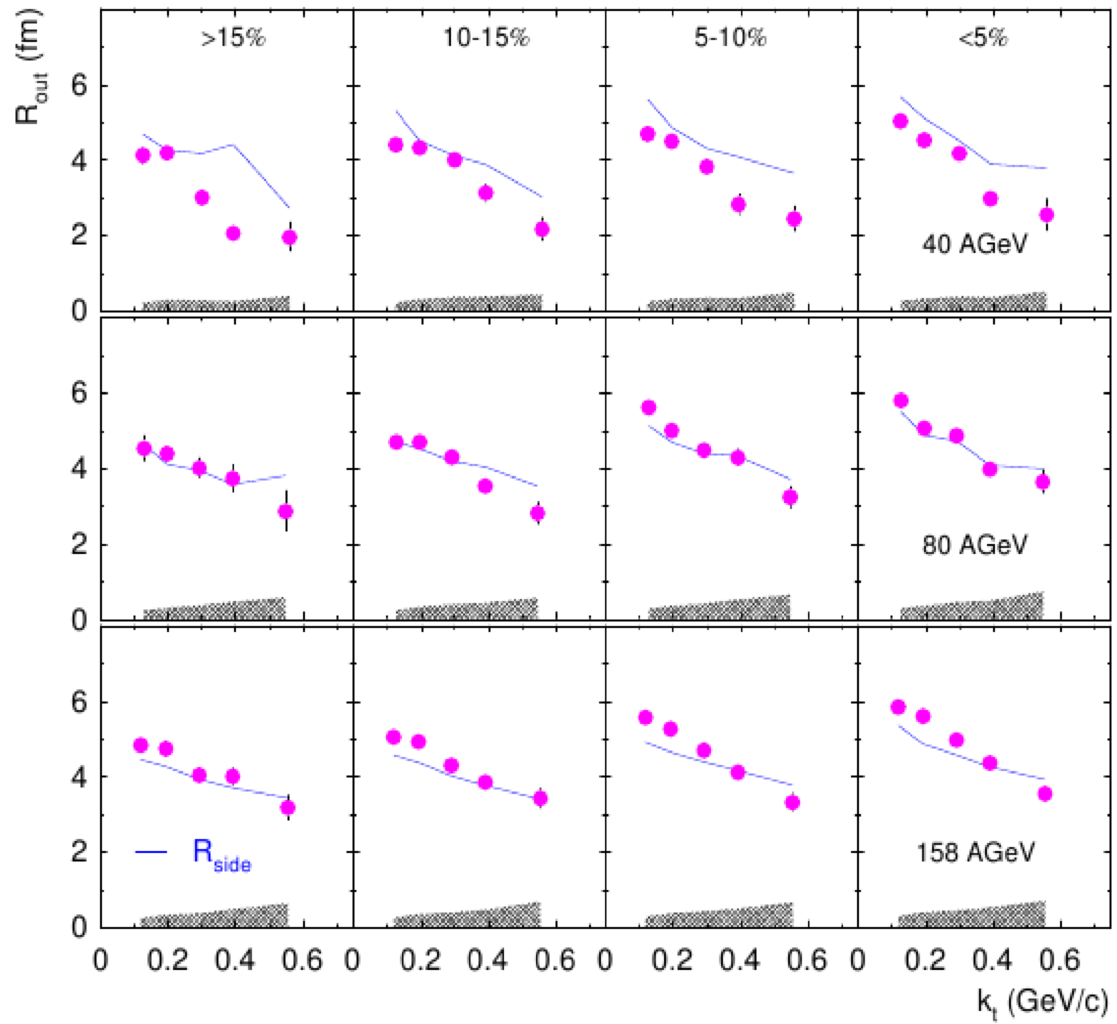
generally: $R_{out} \approx R_{side}$

at 158 AGeV:
short but finite emission duration

$$\Delta\tau^2 = \frac{1}{\beta_t^2}(R_{out}^2 - R_{side}^2)$$

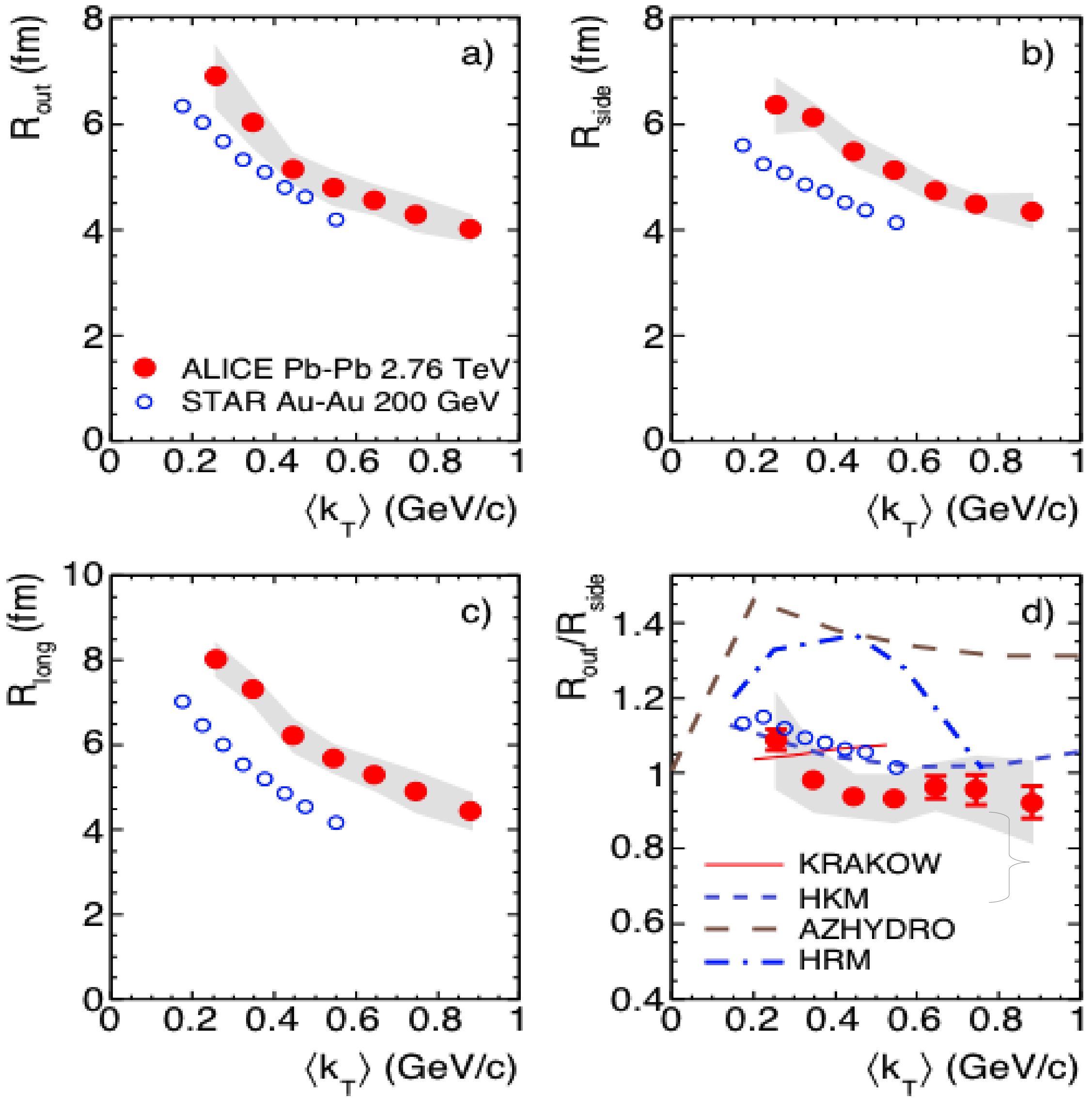
Δτ ≈ 2 fm/c i.e. short, consistent with small density change

CERES H.Appelshäuser, Nucl.Phys. **A714** (2003) 124



ALICE PbPb collisions at the LHC at $\sqrt{s_{nn}}$ = 2.76 TeV

ALICE, Phys. Lett. B696 (2011) 328



HBT radii larger at LHC than at RHIC
larger radial flow

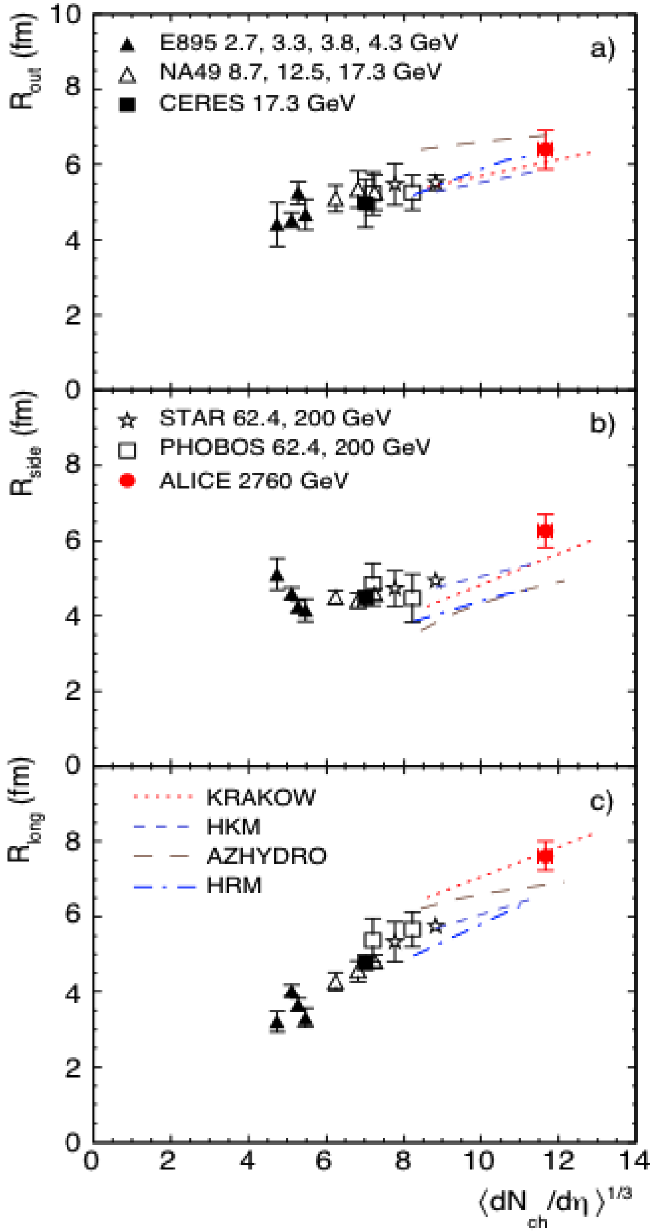
k_t dependence shows shape typical for hydrodynamically expanding source
rather similar to RHIC data
but radii are larger
R_{out}/R_{side} dropping below 1
reproduced reasonably well by Krakow and Kiev hydro models

hydro models

HBT radii much bigger than 1-d charge radii of colliding nuclei

Updated energy dependence of HBT radii

Phys. Lett. B696(2011)328



$<k_t> = 0.3$ GeV/c

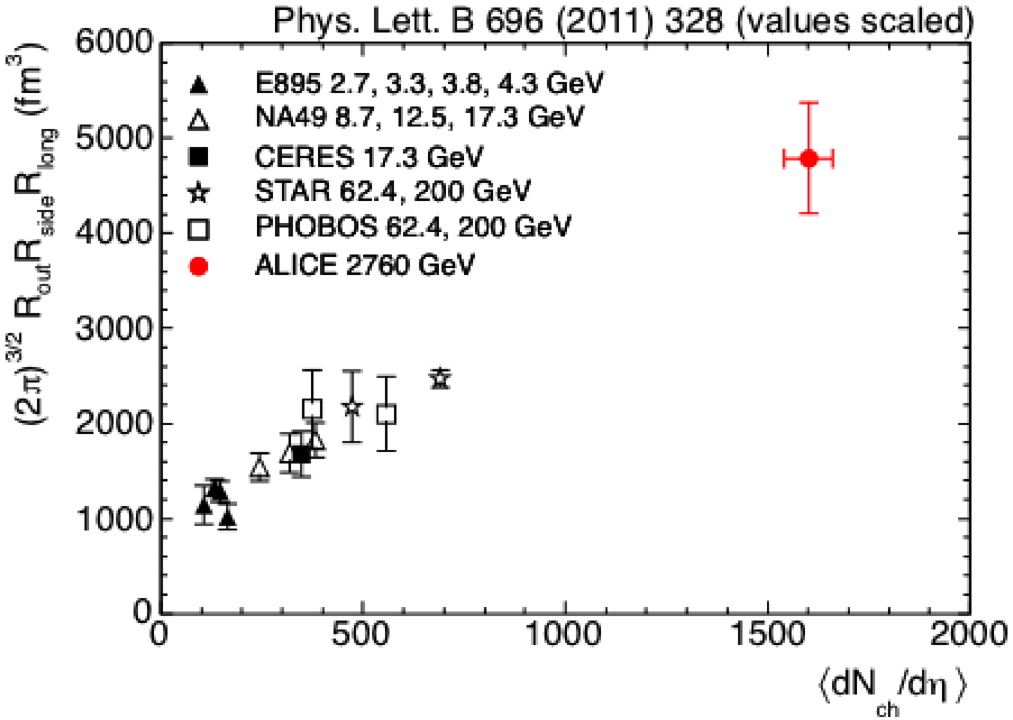
- significant extension of reach of world data by ALICE at higher energies radii grow as cube root of multiplicity indicating freeze-out at constant density
- hydro models reproduce growth reasonably well

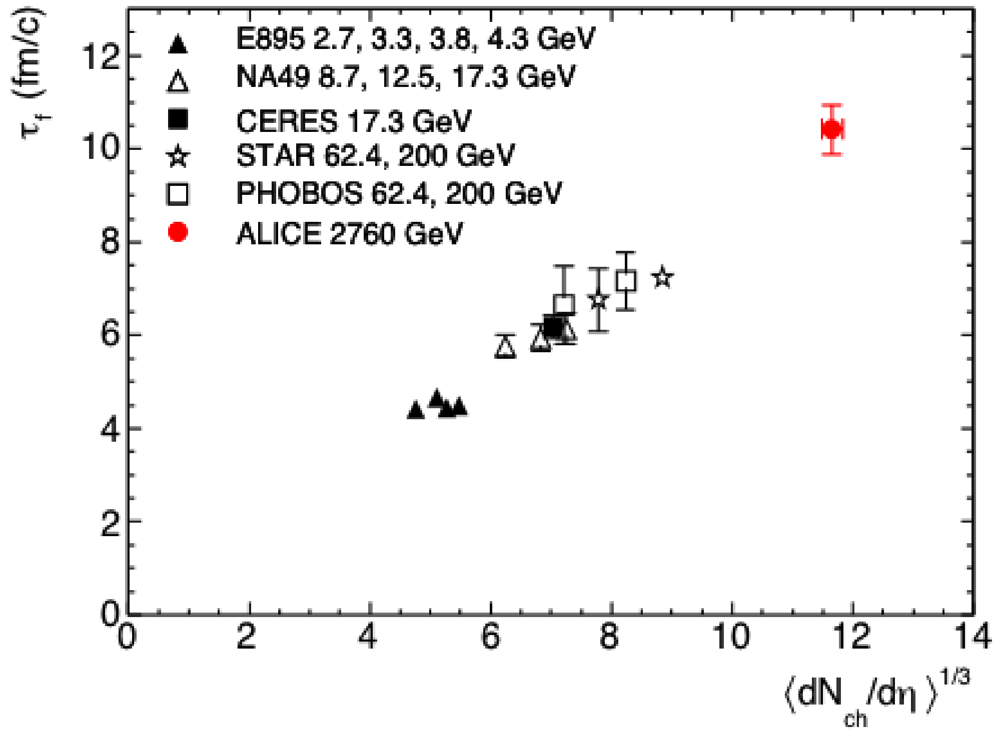
freeze-out volume and duration of expansion

$$\text{coherence volume V} = (2\pi)^{3/2} R_{side}^2 R_{long}$$

from R_{long}: expansion at LHC 10 fm/c

$$R_{long} = \tau_f \sqrt{T/m_t}$$



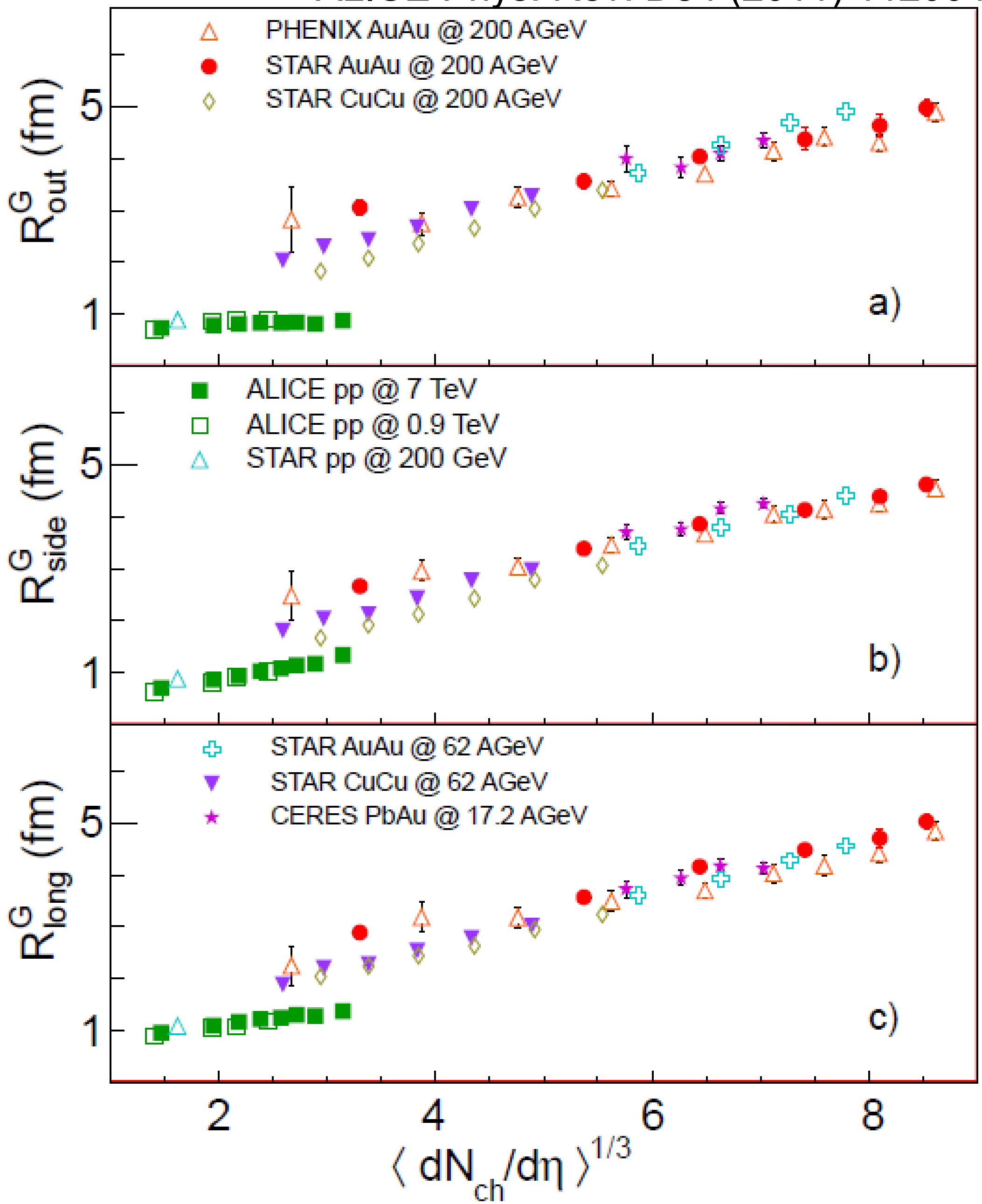


huge growth at LHC

ALICE, Phys. Lett. B696(2011)328

Volume in pp and heavy ion collisions different

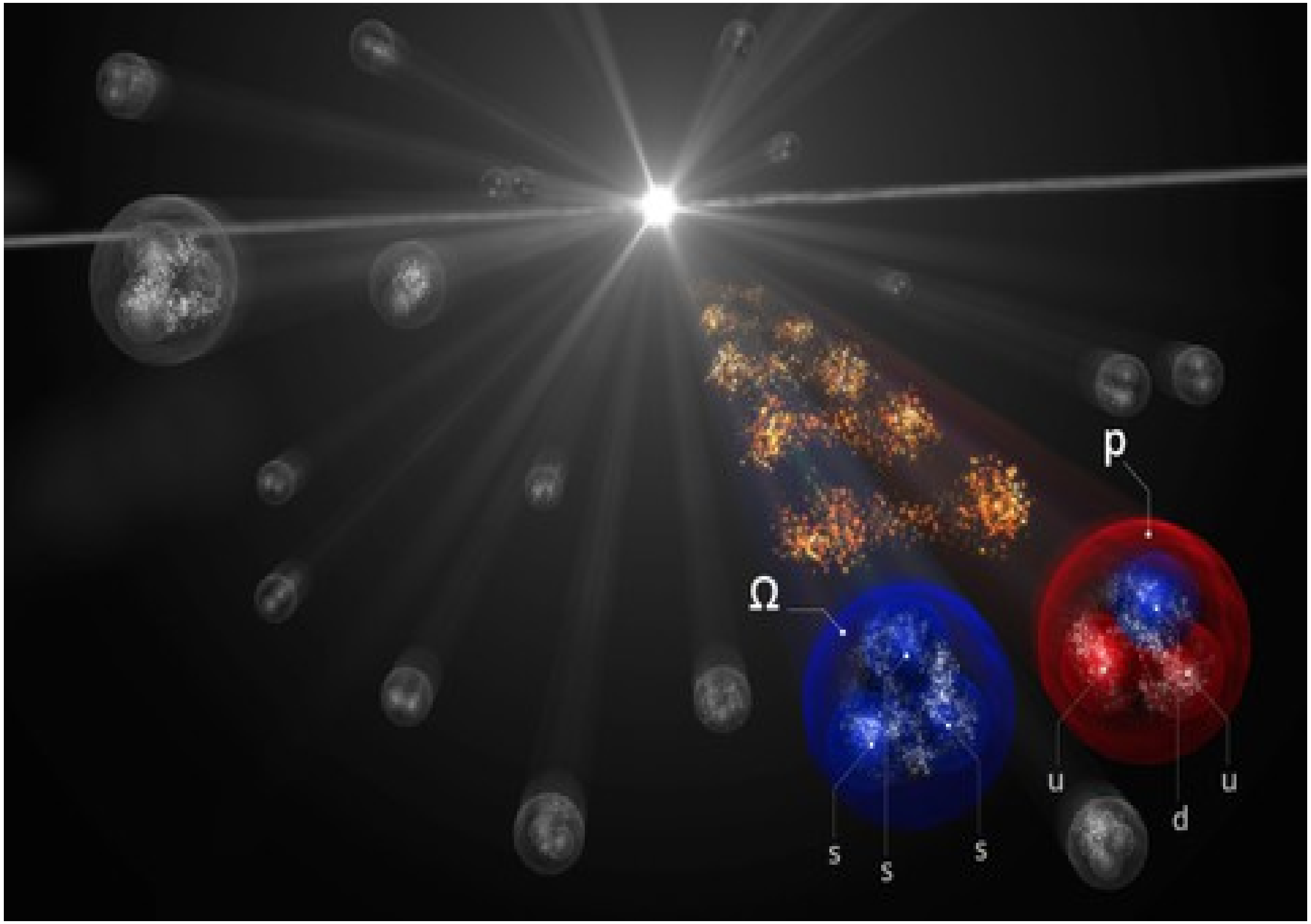
ALICE Phys. Rev. D84 (2011) 112004



PbPb coll.scale with cube root of multiplicity i.e. freeze-out at constant density for different centralities and energies
pp falls on different curves, i.e. at a given multiplicity pp physics different

Unveiling the strong interaction among hadrons at the LHC

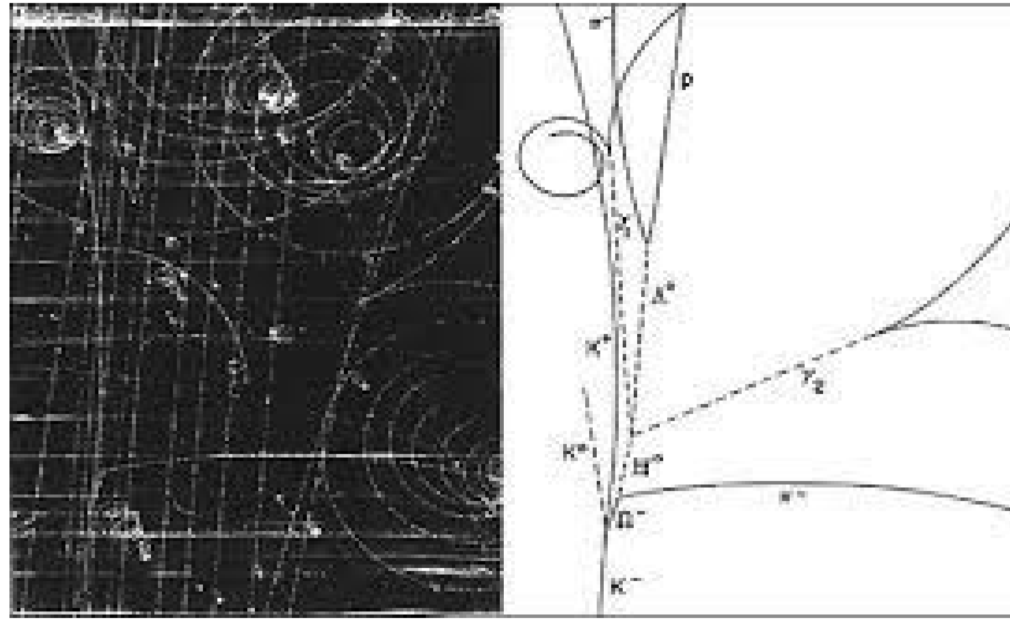
ALICE coll., Nature 588 (2020) 232-238, arXiv:2005.11495 [nucl-ex]



the Ω baryon and its interaction with nucleons

	Lifetime
$\Omega^- \rightarrow \Xi^0 + \pi^-$ sss uss $\bar{u}$d	$0.8x10^{-10}\,s$
$\Xi^0 \rightarrow \Lambda^0 + 2\gamma$ uss uds	$2.9x10^{-10}\,s$
$\Lambda^0 \rightarrow p + \pi^-$ uds uud $\bar{u}$d	$2.6x10^{-10}\,s$

source: Wikipedia



BNL 80 inch hydrogen bubble chamber, 1 event

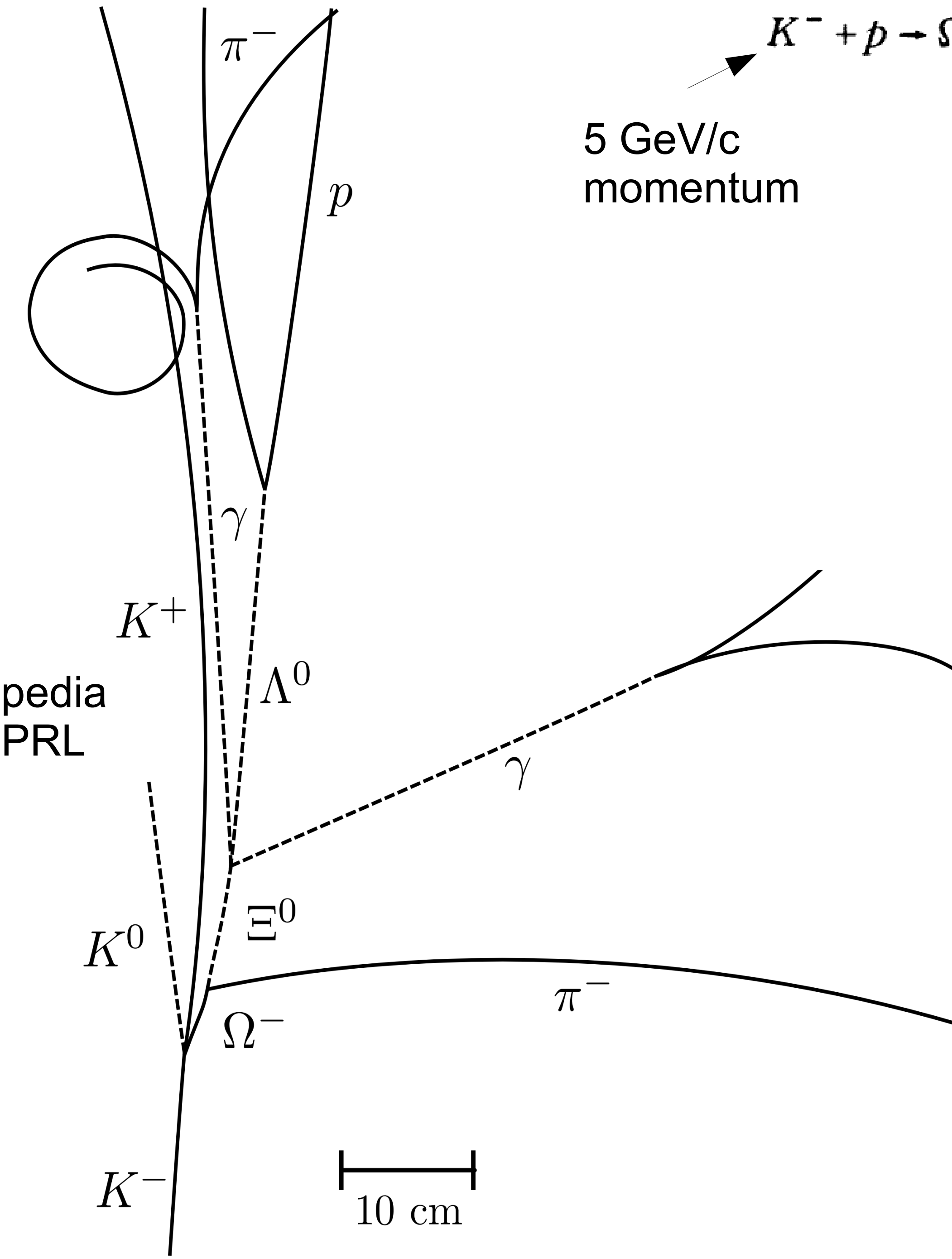
VOLUME 12, NUMBER 8 PHYSICAL REVIEW LETTERS 24 FEBRUARY 1964

OBSERVATION OF A HYPERON WITH STRANGENESS MINUS THREE*

V. E. Barnes, P. L. Connolly, D. J. Crennell, B. B. Culwick, W. C. Delaney, W. B. Fowler, P. E. Hagerty,† E. L. Hart, N. Horwitz,† P. V. C. Hough, J. E. Jensen, J. K. Kopp, K. W. Lai, J. Leitner,† J. L. Lloyd, G. W. London,‡ T. W. Morris, Y. Oren, R. B. Palmer, A. G. Prodell, D. Radojičić, D. C. Rahm, C. R. Richardson, N. P. Samios, J. R. Sanford, R. P. Shutt, J. R. Smith, D. L. Stonehill, R. C. Strand, A. M. Thorndike, M. S. Webster, W. J. Willis, and S. S. Yamamoto
Brookhaven National Laboratory, Upton, New York
(Received 11 February 1964)

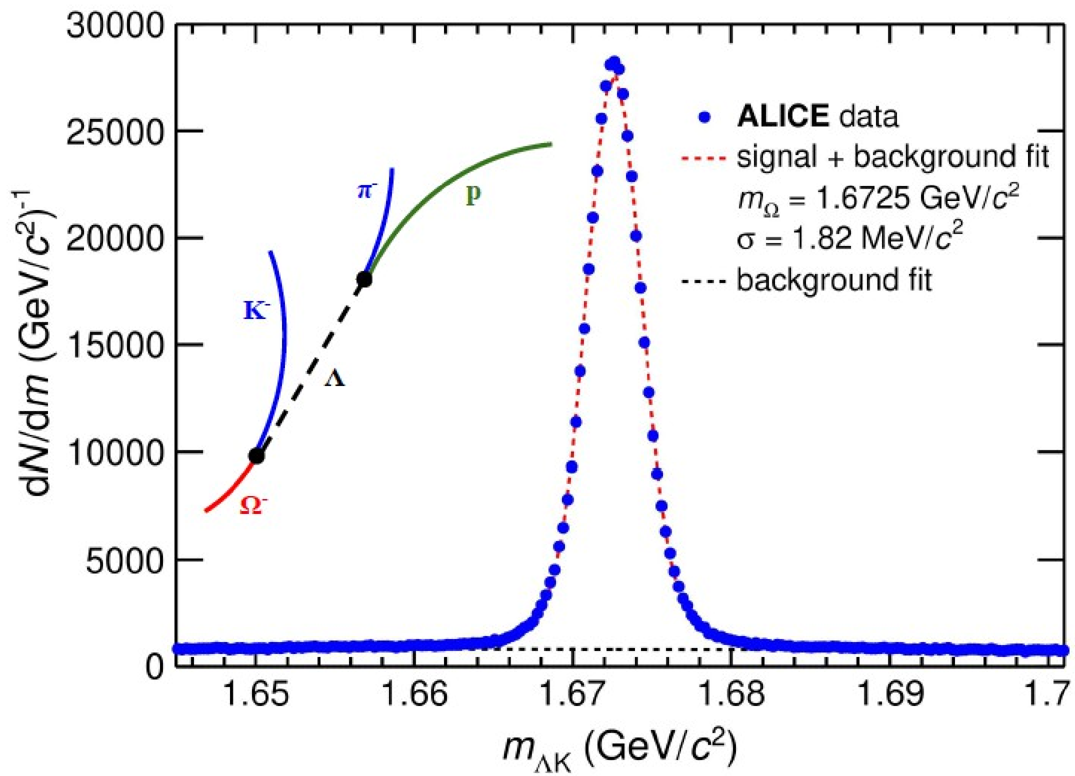
$$K^- + p \rightarrow \Omega^- + K^+ + K^0$$
$$\Omega^- \rightarrow \Xi^0 + \pi^-$$
$$\Xi^0 \rightarrow \Lambda^0 + \pi^0$$
$$\pi^0 \rightarrow \gamma_1 + \gamma_2$$
$$\gamma_2 \rightarrow e^+ + e^-$$
$$\gamma_1 \rightarrow e^+ + e^-$$
$$\Lambda^0 \rightarrow \pi^- + p.$$

5 GeV/c momentum



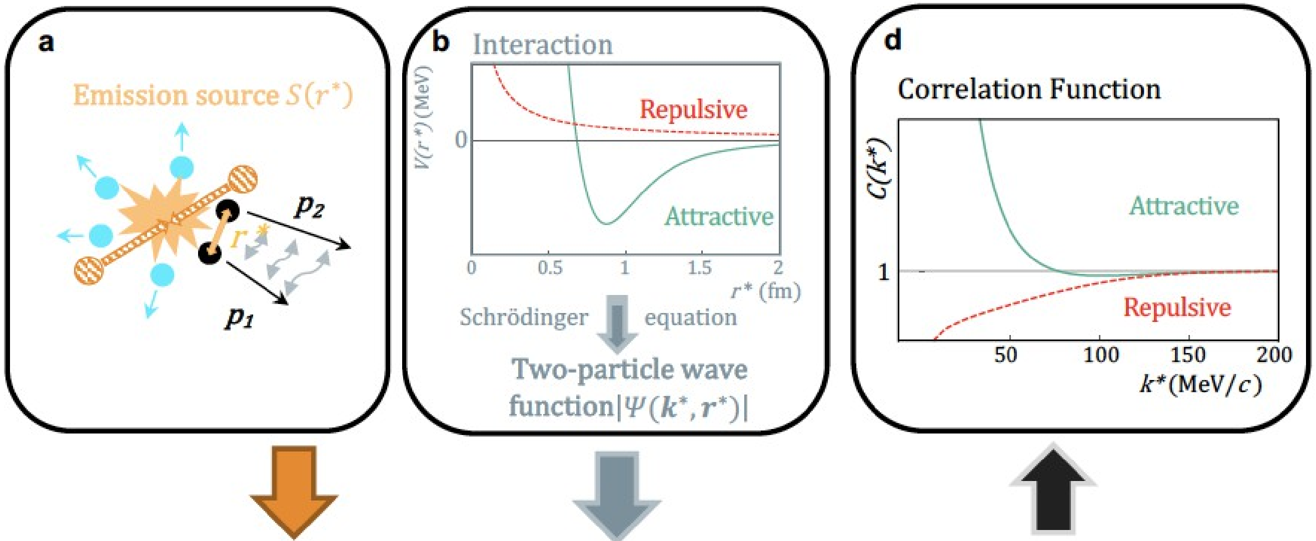
source: Wikipedia and original PRL

identifying the Ω baryon through invariant mass reconstruction in the ALICE experiment



note the very low background under the Ω peak and the very narrow peak width

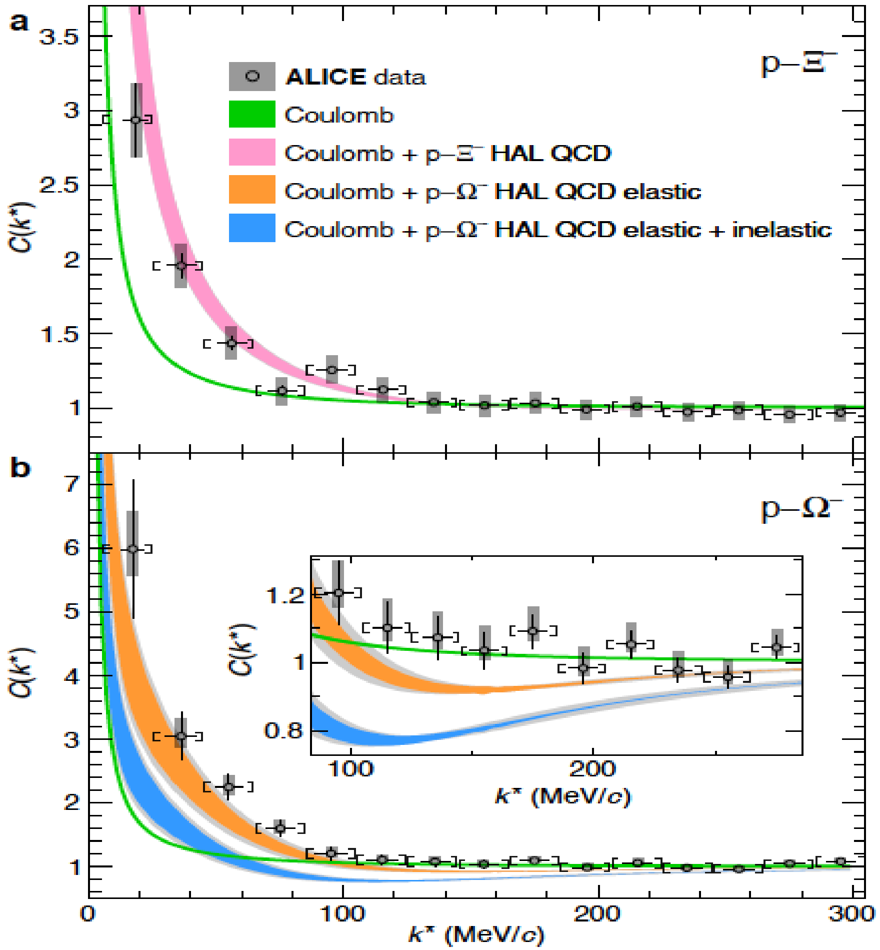
determining the baryon-baryon interaction from 2-particle correlation measurements



$$C(k^*) = \int S(r^*) \; |\Psi(k^*, r^*)|^2 \, d^3r^* = \xi(k^*) \cdot \frac{N_{\mathrm{same}}(k^*)}{N_{\mathrm{mixed}}(k^*)}$$

Schematic representation of the correlation method. a, A collision of two protons generates a particle source $S(r^*)$ from which a hadron–hadron pair with momenta $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$ emerges at a relative distance r^* and can undergo final-state interaction before being detected. Consequently, the relative momentum k^* is either reduced or increased via an attractive or a repulsive interaction, respectively. **b**, Example of attractive (green) and repulsive (dotted red) interaction potentials, V(r^*), between two hadrons, as a function of their relative distance. Given a certain potential, a non-relativistic Schrödinger equation is used to obtain the corresponding two-particle wave function, $\psi(\boldsymbol{k}^*, \boldsymbol{r}^*)$. **c**, The equation of the calculated (second term) and measured (third term) correlation function $C(k^*)$, where $N_{\mathrm{same}}(k^*)$ and $N_{\mathrm{mixed}}(k^*)$ represent the k^* distributions of hadron–hadron pairs produced in the same and in different collisions, respectively, and $\xi(k^*)$ denotes the corrections for experimental effects. **d**, Sketch of the resulting shape of $C(k^*)$. The value of the correlation function is proportional to the interaction strength. It is above unity for an attractive (green) potential, and between zero and unity for a repulsive (dotted red) potential.

Experimental p–Ξ^- and p–Ω^- correlation functions. **a**, **b**, Measured p–Ξ^- (**a**) and p–Ω^- (**b**) correlation functions in high multiplicity pp collisions at $\sqrt{s} = 13$ TeV. The experimental data are shown as black symbols. The black vertical bars and the grey boxes represent the statistical and systematic uncertainties. The square brackets show the bin width and the horizontal black lines represent the statistical uncertainty in the determination of the mean k^* for each bin. The measurements are compared with theoretical predictions, shown as coloured bands, that assume either Coulomb or Coulomb + strong HAL QCD interactions. For the p–Ω^- system the orange band represents the prediction considering only the elastic contributions and the blue band represents the prediction considering both elastic and inelastic contributions.



attraction in both channels, no evidence for a bound state

1st measurement of interaction between a proton and multi-strange baryons