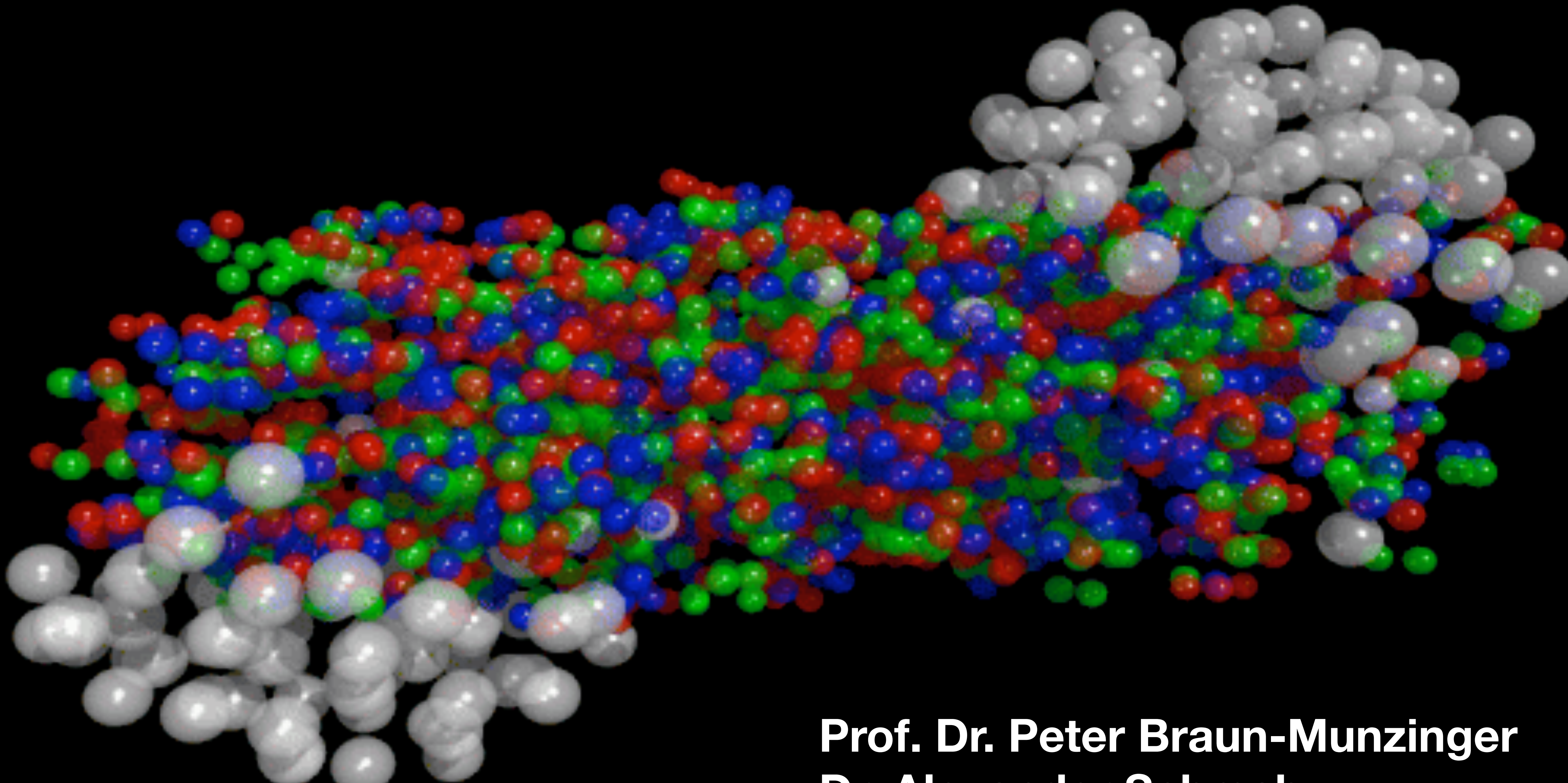
Quark-Gluon Plasma Physics

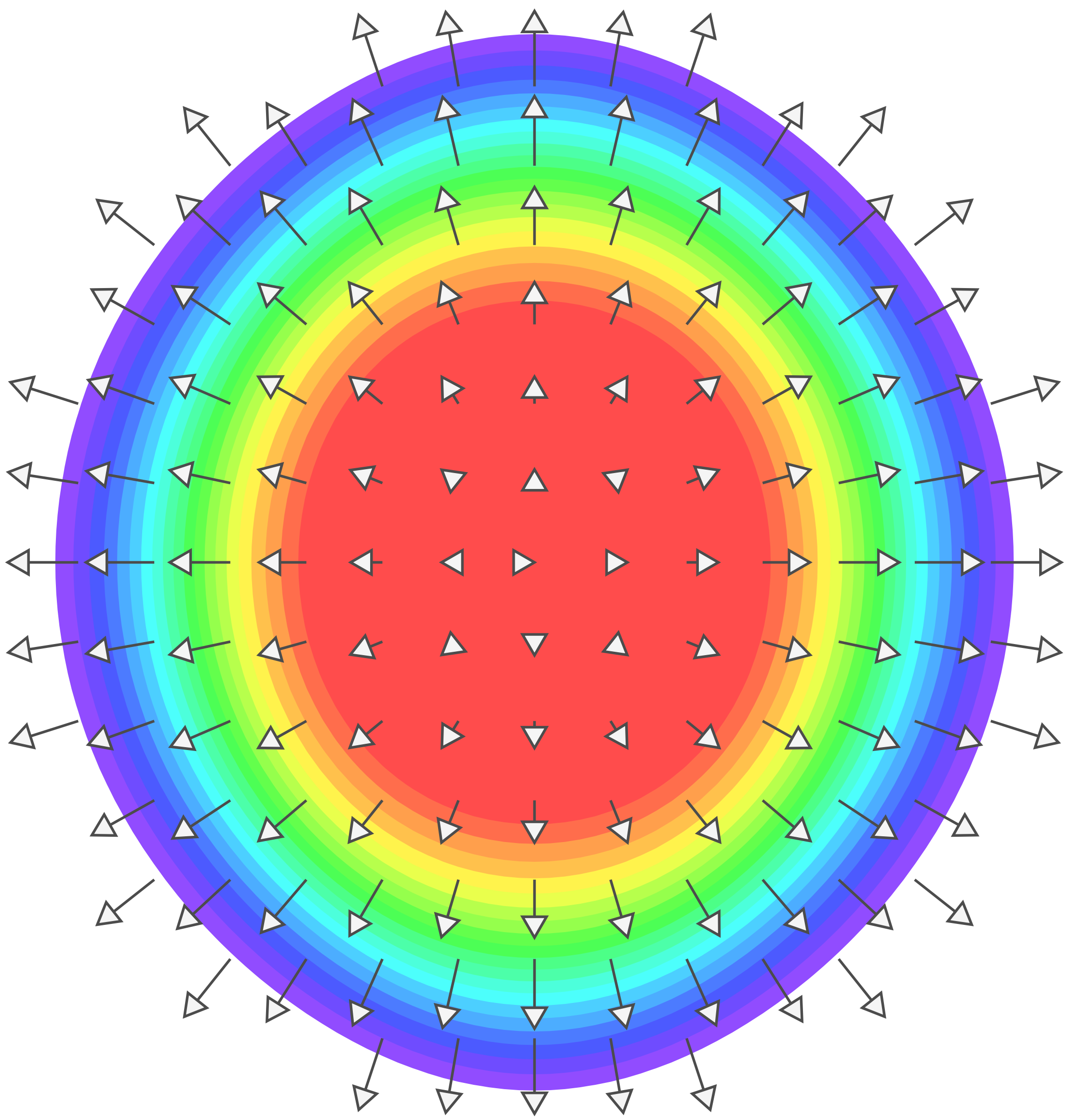
7. Space-time evolution of the QGP



Prof. Dr. Peter Braun-Munzinger
Dr. Alexander Schmah
Heidelberg University

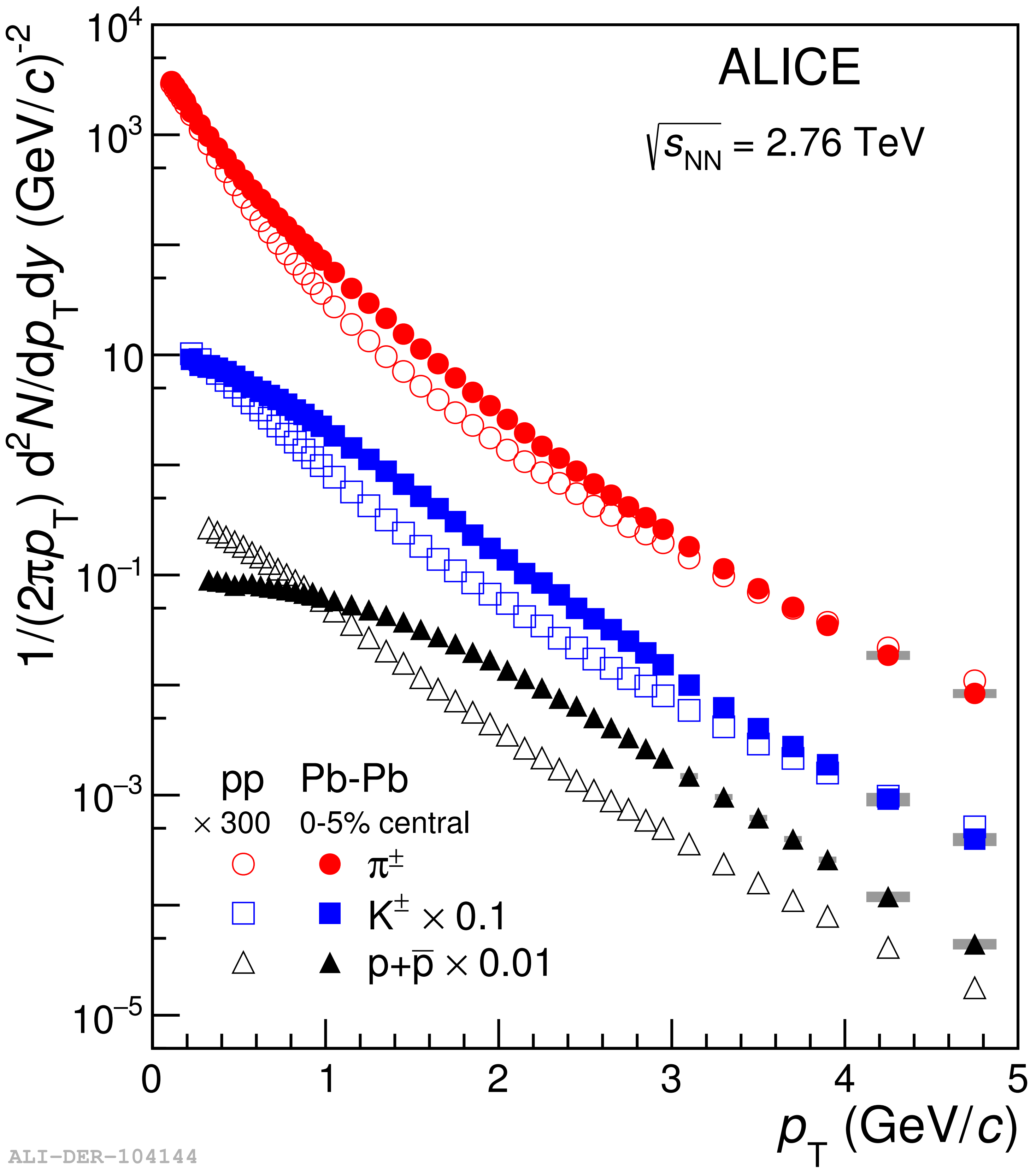
SS 2021

Evidence for collective behavior in heavy-ion collisions

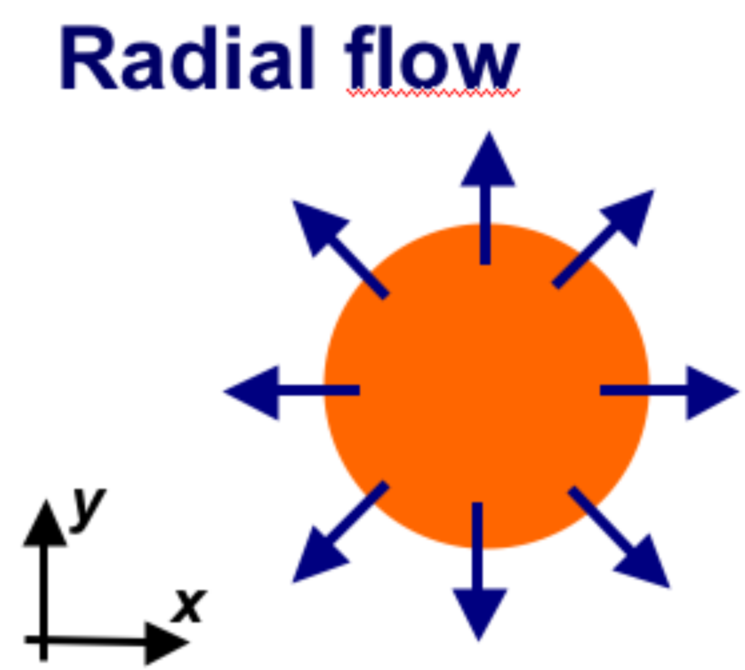


- Shape of low-p_T transverse momentum spectra for particles with different masses
- Azimuthal anisotropy of produced particles
- Source sizes from Hanbury Brown-Twiss correlations
- …

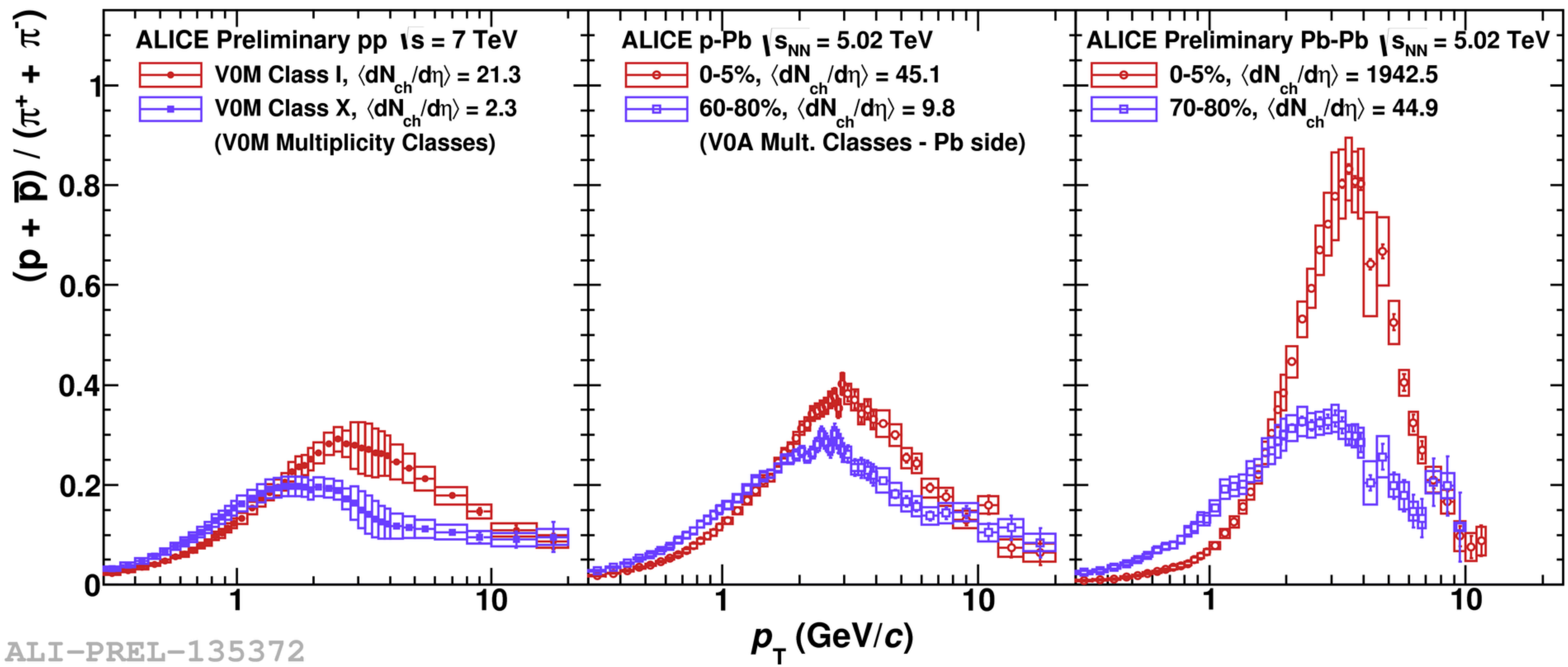
Evidence for radial flow



- Shape is different in pp and A-A
- Stronger effect for heavier particles



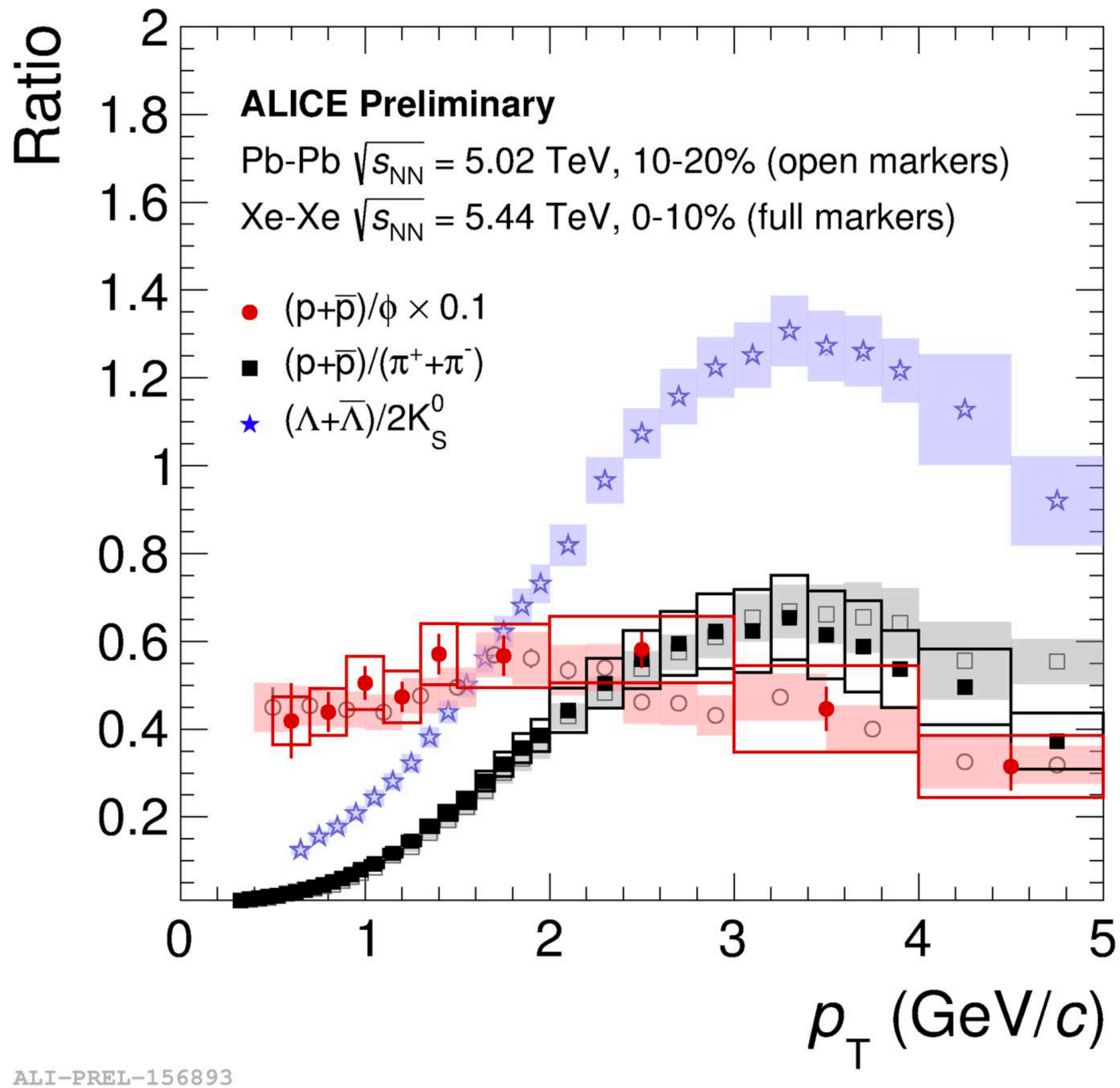
Ratio of particle spectra



$$p = \beta \gamma m$$

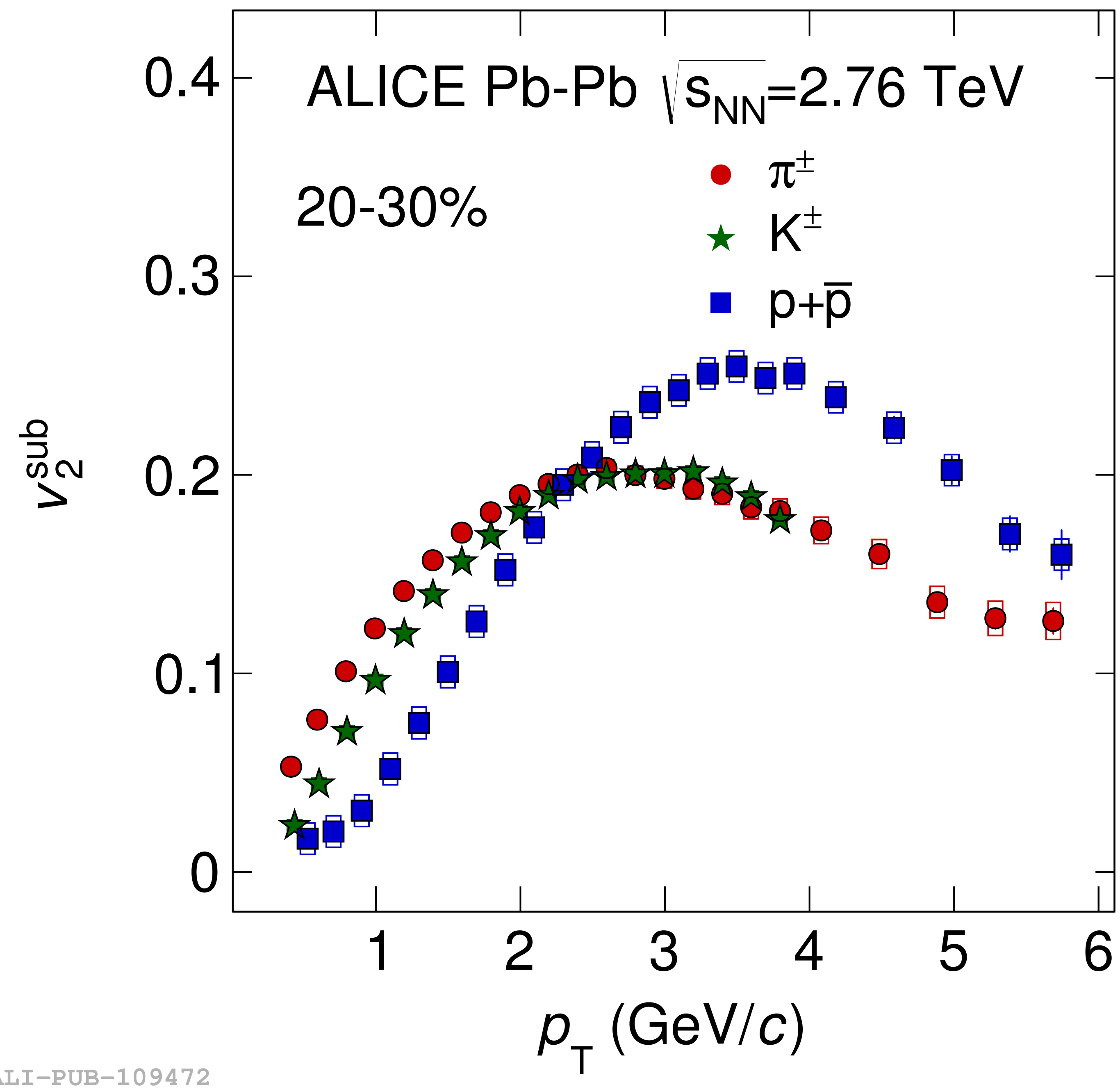
Centrality dependence of radial flow: effect increases with more central events

Baryon to Meson Ratios

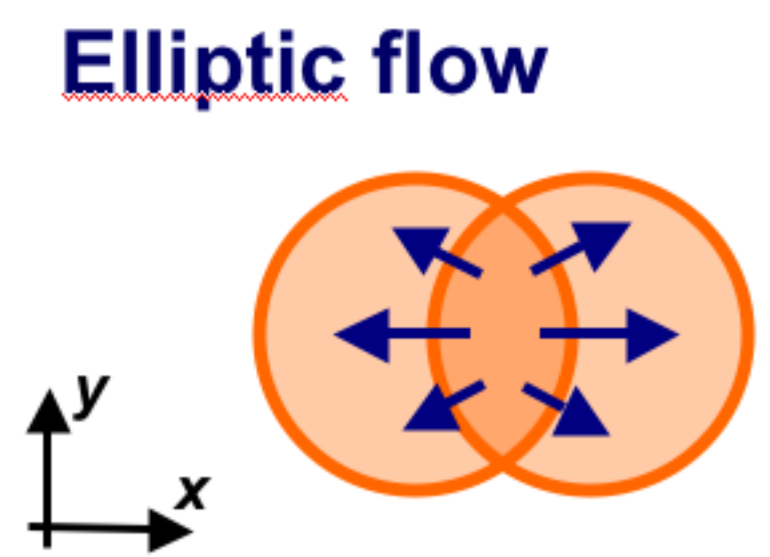


Ratio is almost flat for p/ϕ (similar mass)

Evidence for elliptic flow



Good explanation:
Azimuthal variation of the flow velocity



Basics of relativistic hydrodynamics

See e.g. Ollitrault, arXiv:0708.2433

Standard thermodynamics: P, T, μ constant over the entire volume

Hydrodynamics assumes *local* thermodynamic equilibrium: $P(x^\mu)$, $T(x^\mu)$, $\mu(x^\mu)$

Local thermodynamic equilibrium only possible if mean free path between two collisions much shorter than all characteristic scales of the system:

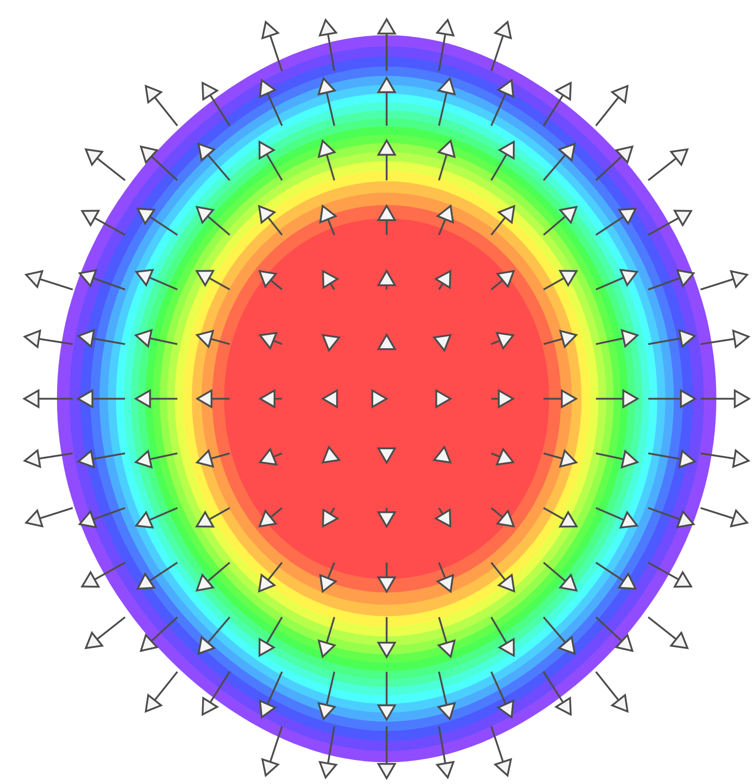
$$\lambda_{\mathrm{mfp}} \ll L$$

This is the limit of non-viscous hydrodynamics.

4-velocity of a fluid element:

$$u = \gamma(1, \vec{\beta}), \quad u^\mu u_\mu = 1$$

$$\gamma = \frac{1}{\sqrt{1 - \vec{\beta}^2}}$$



Number conservation

Mass conservation in nonrelativistic hydrodynamics:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0 \qquad \text{[continuity equation]}$$

conserved quantity, e.g. baryon number

Lorentz contraction in the relativistic case: $\rho \quad \rightarrow \quad n\gamma = nu^0$

The continuity equation then reads: $$\frac{\partial (nu^0)}{\partial t} + \vec{\nabla}(n\vec{u}) = 0$$

nu^0 : baryon density

$n\vec{u}$: baryon flux

The conservation of *n* can be written more elegantly as

$$\partial_\mu (nu^\mu) = 0$$

For a general 4-vector a we have:

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} = (\frac{\partial}{\partial t}, \vec{\nabla}), \quad \partial^\mu \equiv \frac{\partial}{\partial x_\mu} = (\frac{\partial}{\partial t}, -\vec{\nabla}), \qquad \partial_\mu a^\mu = (\frac{\partial}{\partial t}, \vec{\nabla}) \cdot (a^0, \vec{a}) = \frac{\partial a^0}{\partial t} + \vec{\nabla}\vec{a}$$

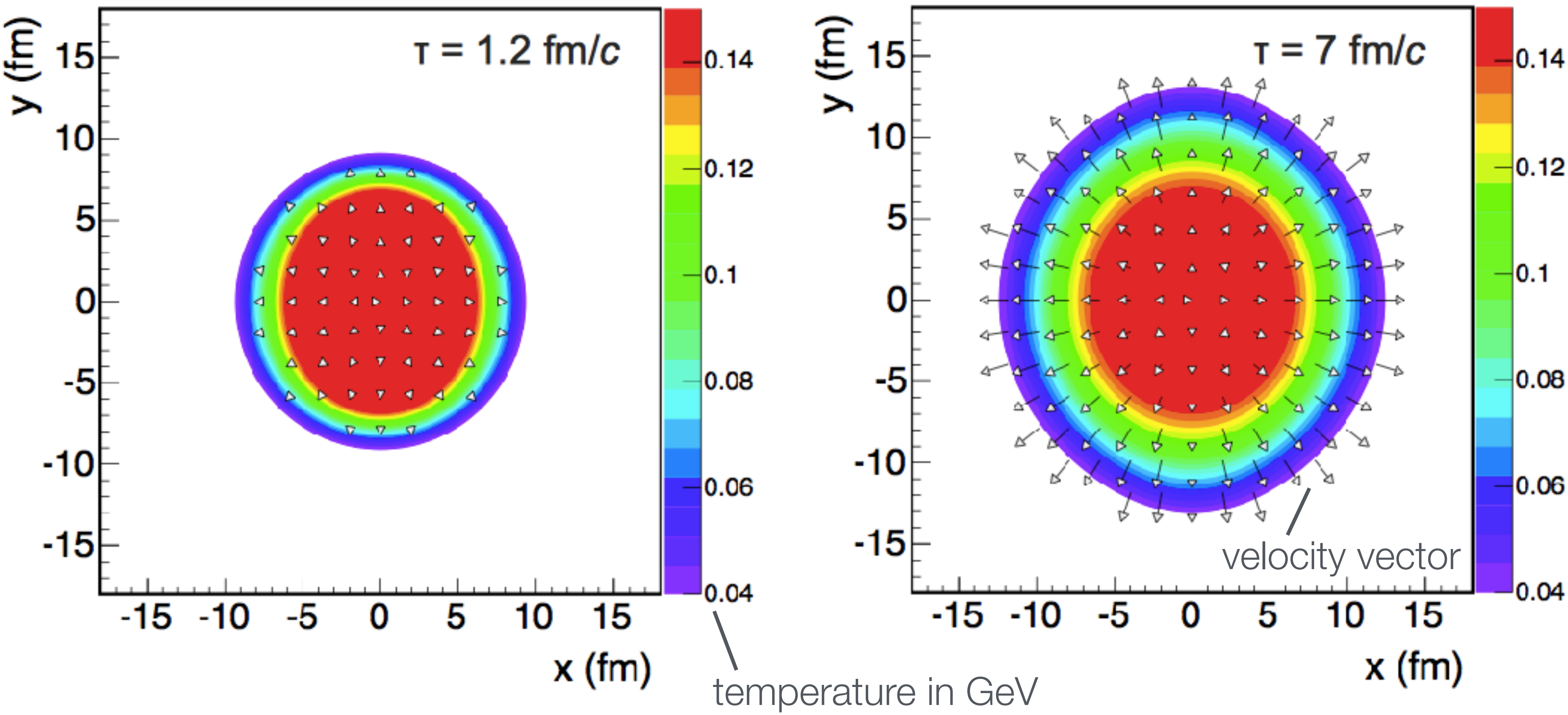
covariant derivative (for ∂_μ)

contravariant derivative (for ∂^μ)

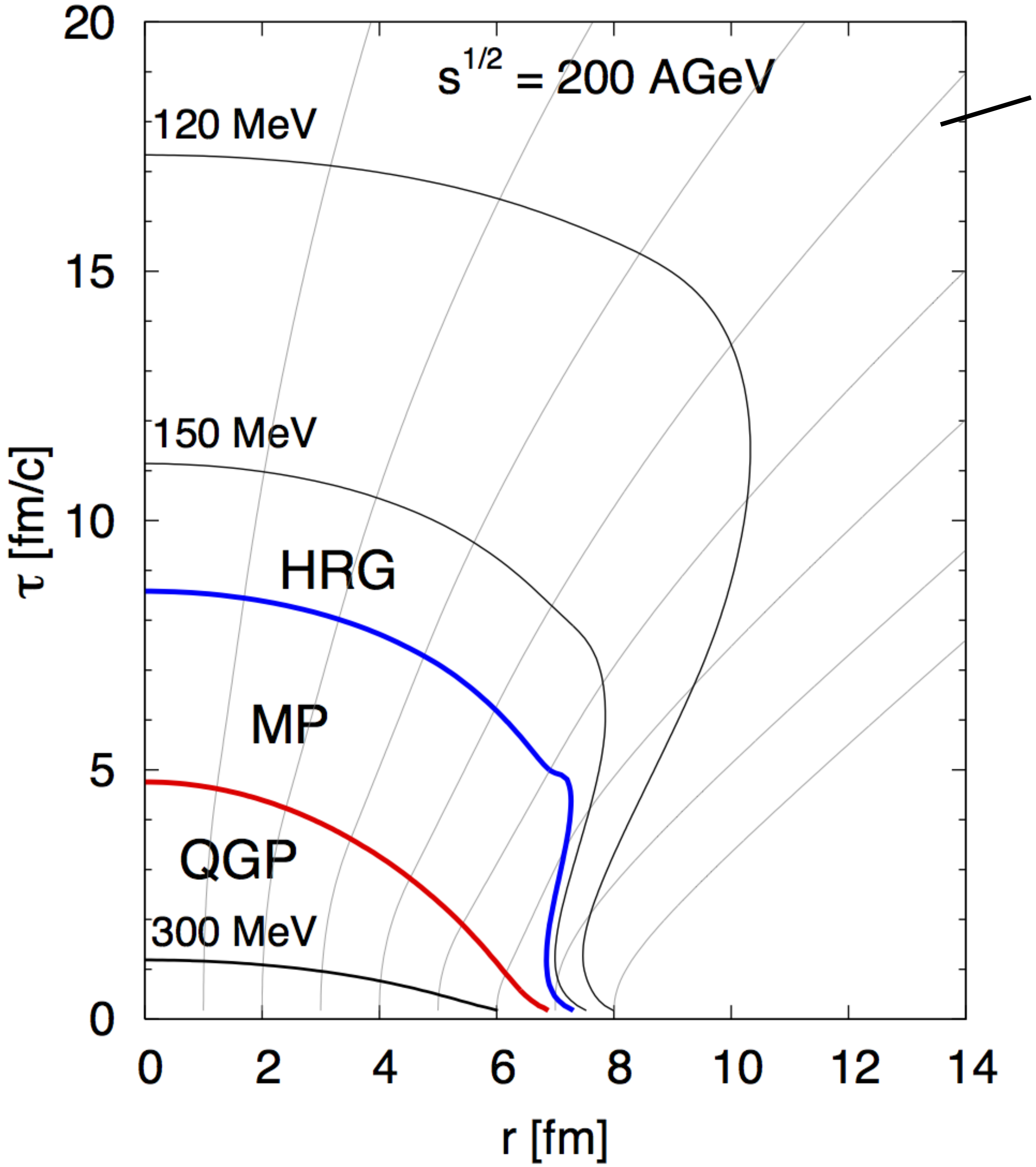
Transverse expansion

Transverse expansion of the fireball in a hydro model (temperature profile)

2+1 d hydro: Bjorken flow in longitudinal direction



Temperature Contours and Flow lines



flow lines indicate motion of fluid cells

Viscosity

Pitch drop experiment, started in Queensland, Australia in 1927

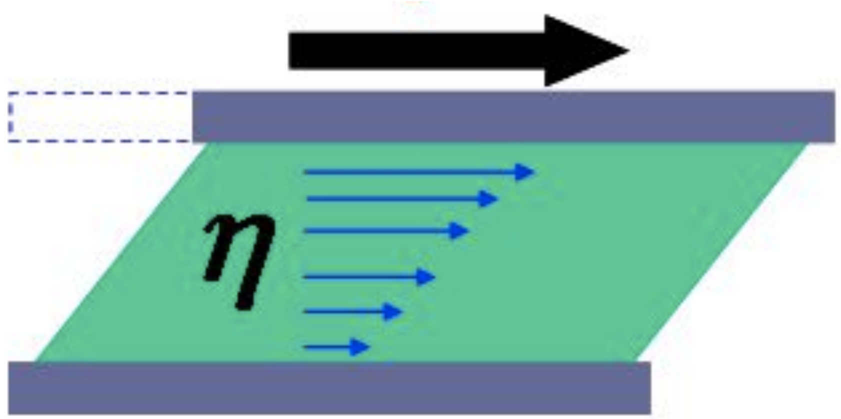
Date	Event	Duration		
		Years	Months	
1927	Hot pitch poured			
October 1930	Stem cut			
December 1938	1st drop fell	8.1	98	
February 1947	2nd drop fell	8.2	99	
April 1954	3rd drop fell	7.2	86	
May 1962	4th drop fell	8.1	97	
August 1970	5th drop fell	8.3	99	
April 1979	6th drop fell	8.7	104	
July 1988	7th drop fell	9.2	111	
November 2000	8th drop fell[A]	12.3	148	
April 2014	9th drop[B]	13.4	156	

Meaningful comparison of different fluids: η/s

https://en.wikipedia.org/wiki/Pitch_drop_experiment

Shear and bulk viscosity

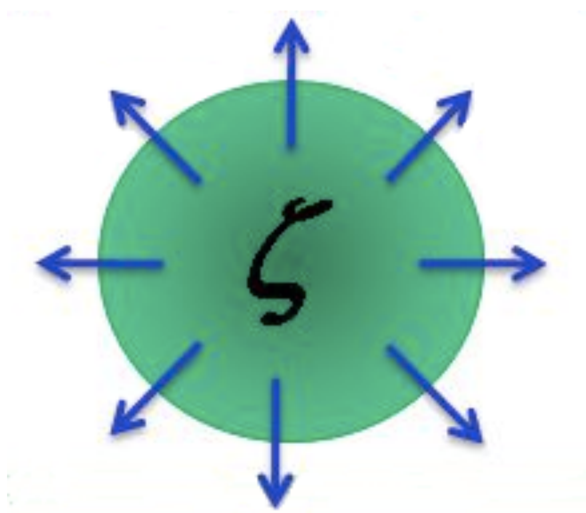
Shear viscosity



Acts against buildup of flow anisotropies (v_2, v_3, v_4, v_5, ,..)

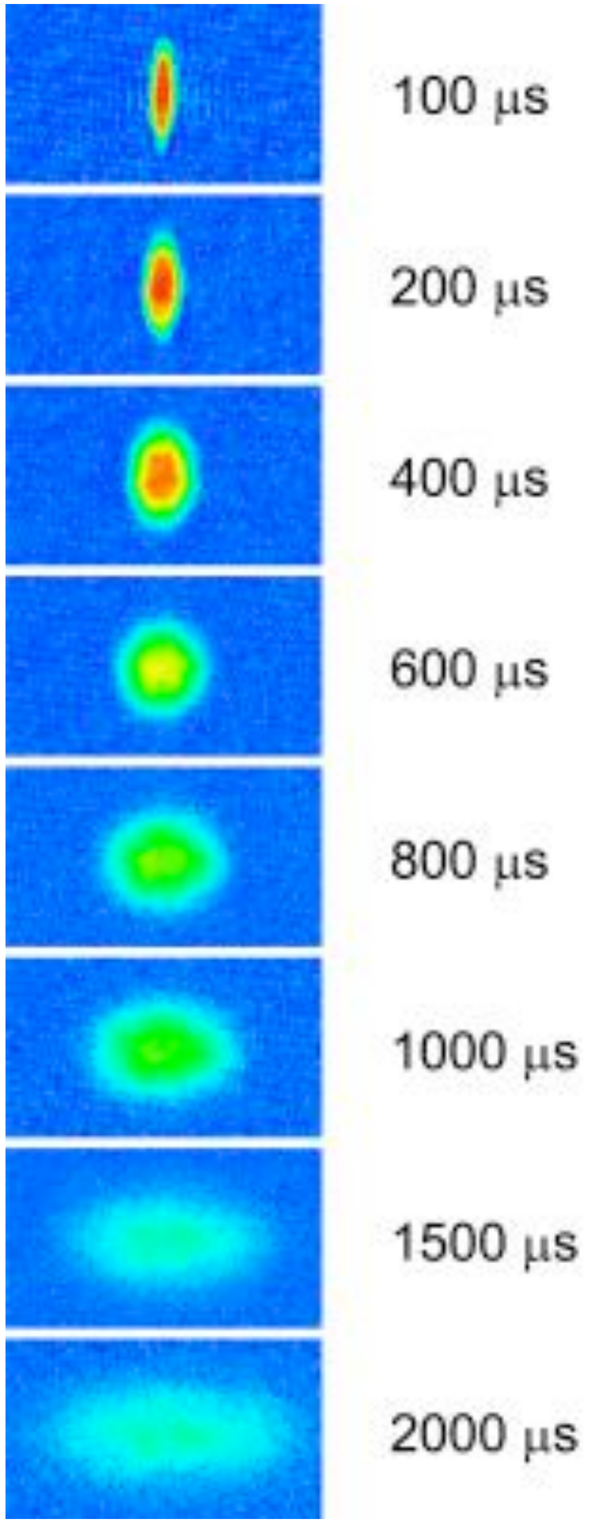
η/s: shear viscosity per entropy density ratio

Bulk viscosity



Acts against buildup of radial flow

Universal aspects of the underlying physics



- Strongly-interacting degenerate gas of fermionic ^{6}Li atoms at 0.1 μK
- Cigar-shaped cloud initially trapped by a laser field
- Anisotropic expansion upon abruptly turning off the trap: Elliptic flow!
- η/s can be extracted: [PhD thesis Chenglin Cao]

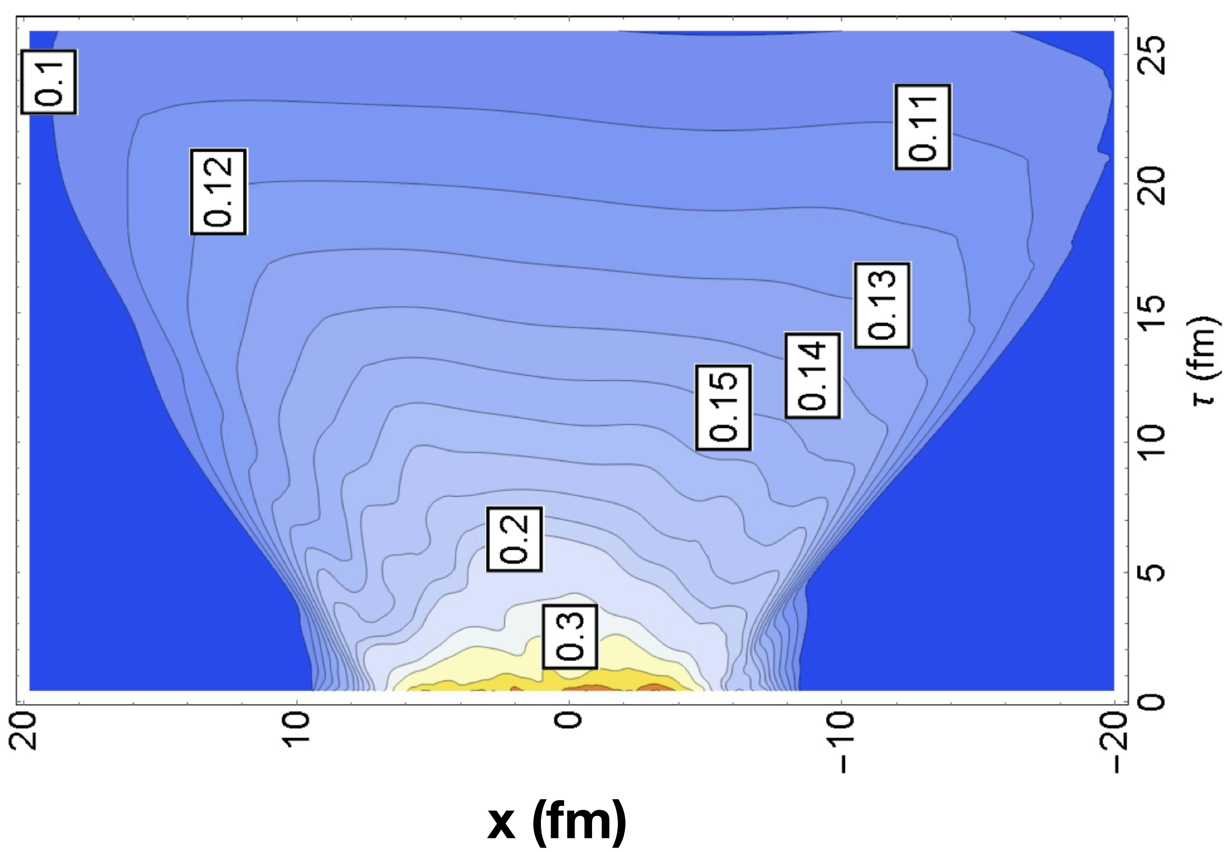
$$(\eta/s)_{^6\mathrm{Li\ gas}} \approx 0.4 = 5 \times \frac{1}{4\pi}$$

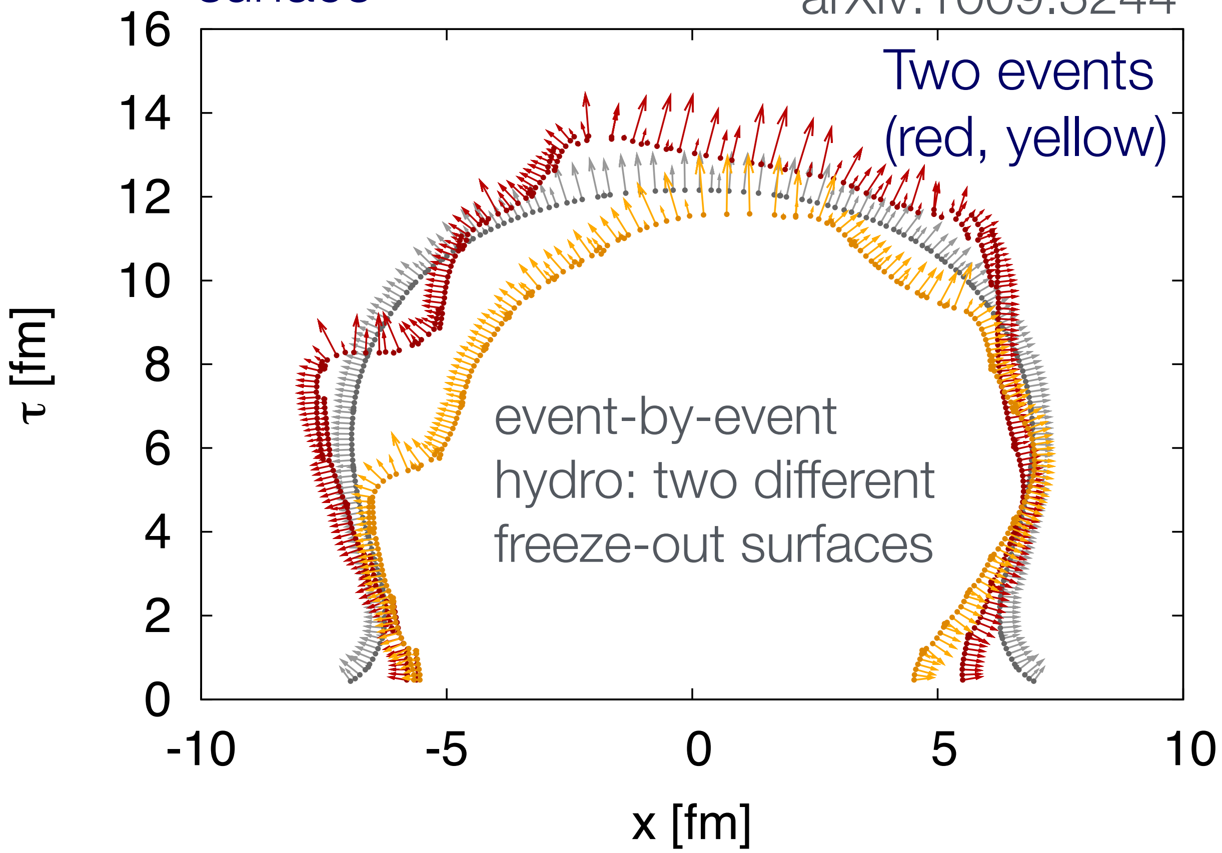
The ultimate goal is to unveil the universal physical laws governing seemingly different physical systems (with temperature scales differing by 19 order of magnitude)

John Thomas, https://www.physics.ncsu.edu/jet/research/stronginter/index.html

Hydrodynamic modeling of heavy-ion collisions: State of the art

- Equation of state from lattice QCD
- (2+1)D or (3+1)D viscous hydrodynamics
- Fluctuating initial conditions (event-by-event hydro)
- Hydrodynamic evolution followed by hadronic cascade





The blast-wave model: A Simple model to describe the effect of radial flow on particle spectra

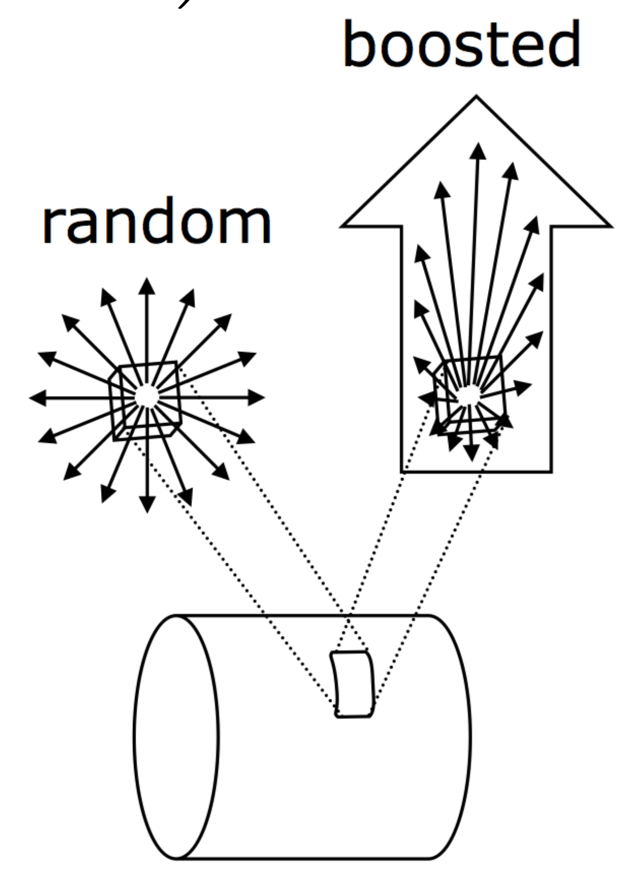
Transverse velocity profile: $$\beta_T(r) = \beta_s \left(\frac{r}{R}\right)^n$$

Superposition of thermal sources with different radial velocities:

$$\frac{1}{m_T}\frac{dn}{dm_T} \propto \int_0^R r\,dr\, m_T I_0\left(\frac{p_T \sinh\rho}{T}\right) K_1\left(\frac{m_T\cosh\rho}{T}\right)$$

$\rho := \mathrm{arctanh}(\beta_T)$ "transverse rapidity"

$I_0,\ K_1$: modified Bessel functions



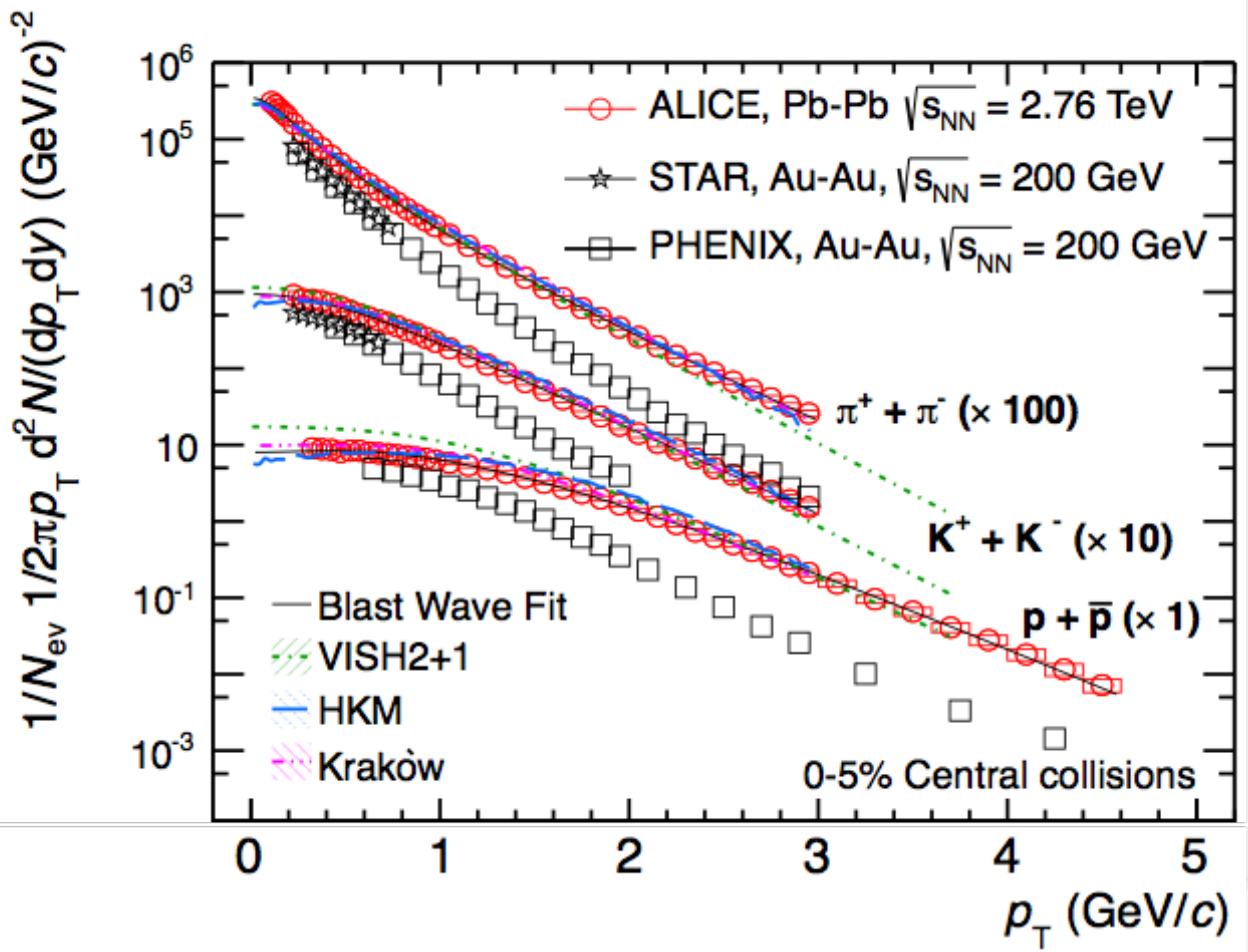
Schnedermann, Sollfrank, Heinz, Phys.Rev.C48:2462-2475,1993

K. Reygers, A. Schmah, A. Berdnikova, X. Sun

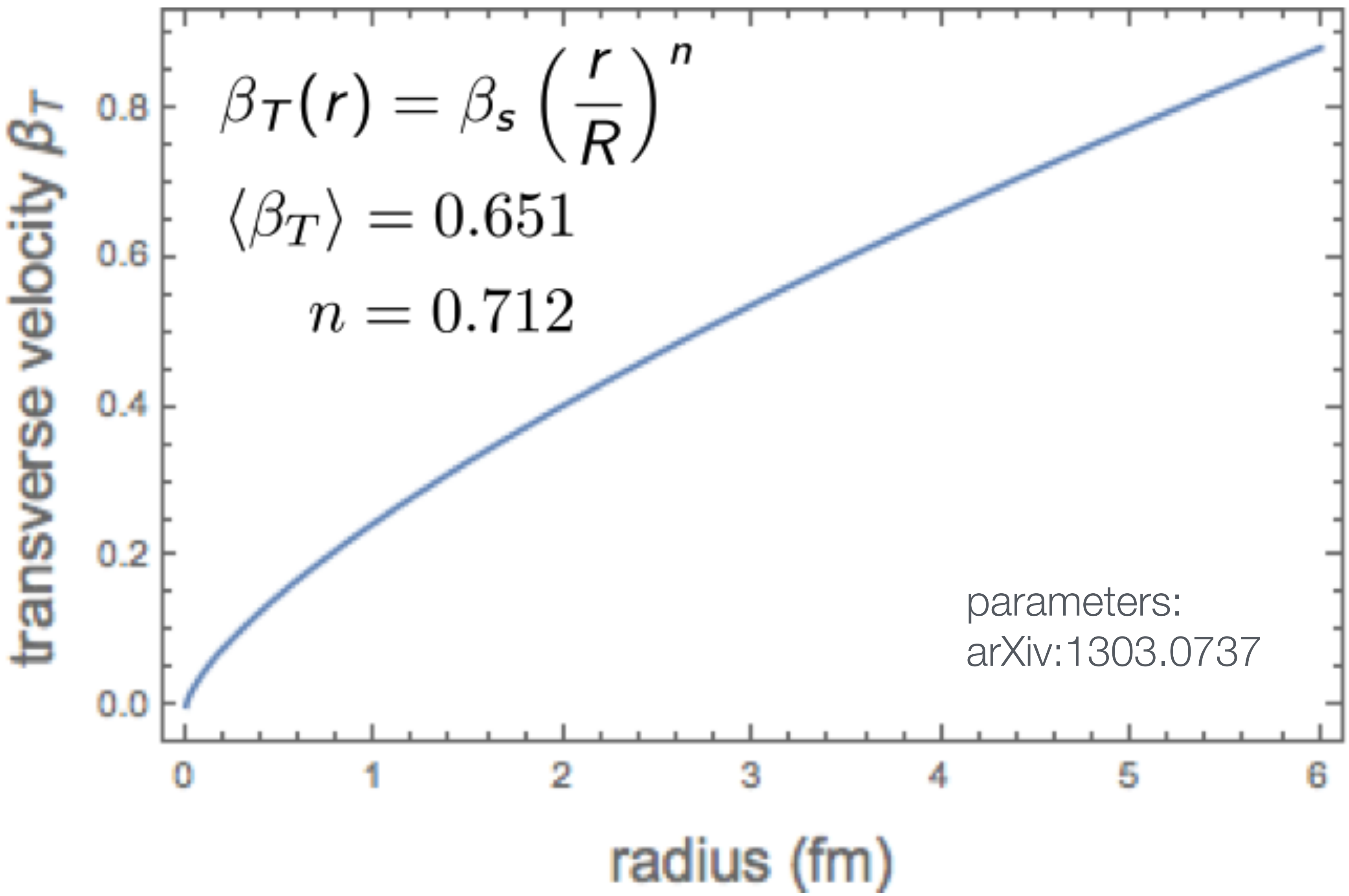
10.1103/PhysRevC.101.064905

Freeze-out at a 3d hyper-surface, typically instantaneous, e.g.: $$t_f(r,z) = \sqrt{\tau_f^2 + z^2}$$

Comparison of π, K, p spectra with hydro and blast-wave models



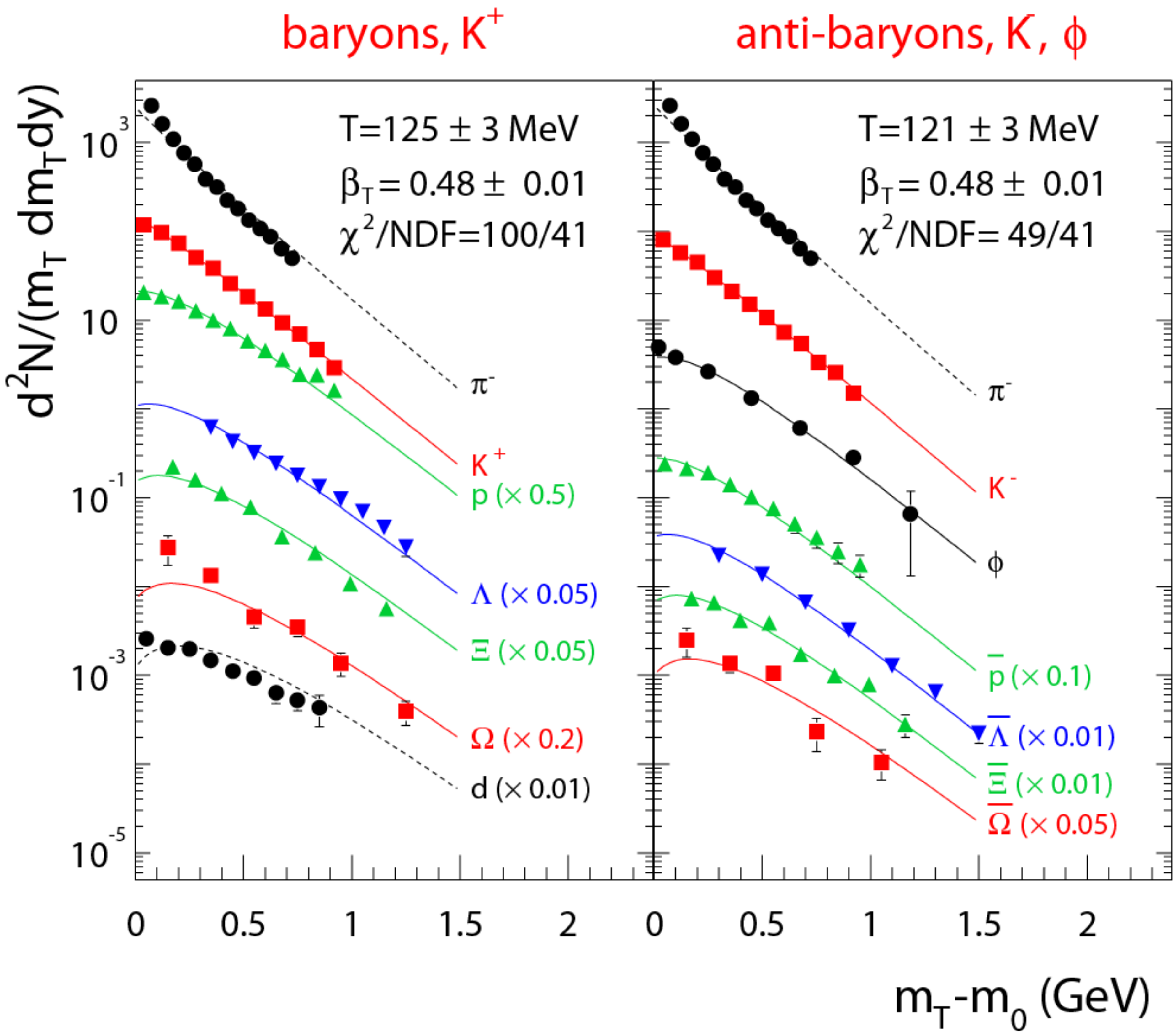
Example: Radial Flow Velocity Profile from Blast-wave Fit to 2.76 TeV Pb-Pb Spectra (0-5%)

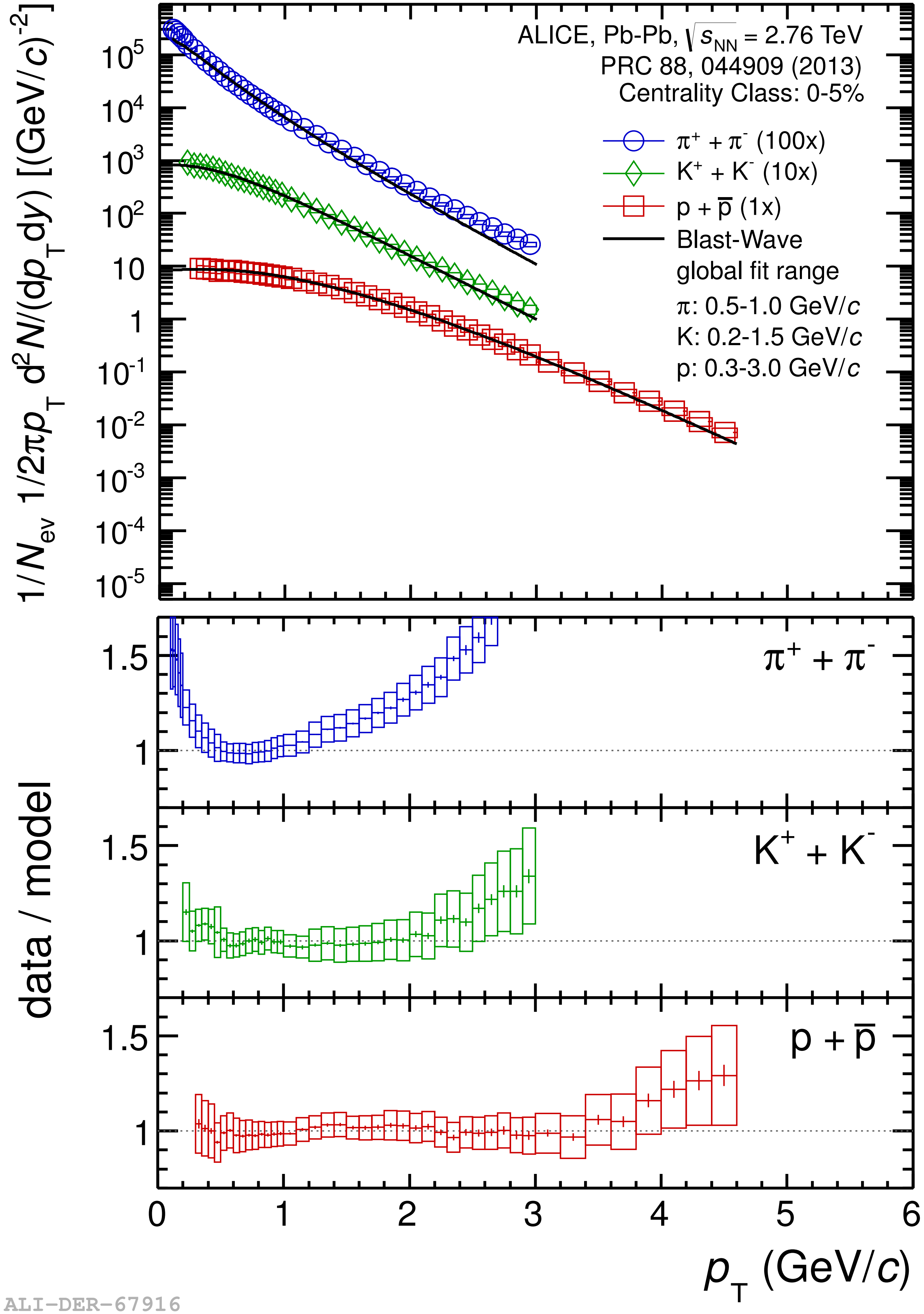


$$\langle \beta_T \rangle = \frac{\int_0^R \int_0^{2\pi} r dr d\varphi \beta_T(r)}{\int_0^R \int_0^{2\pi} r dr d\varphi} = \frac{2}{n+2} \beta_s$$

$$\langle \beta_T \rangle = 0.651, n = 0.712$$
$$\rightarrow \quad \beta_s = 0.8$$

Blast-wave fit for CERN SPS data (NA49)





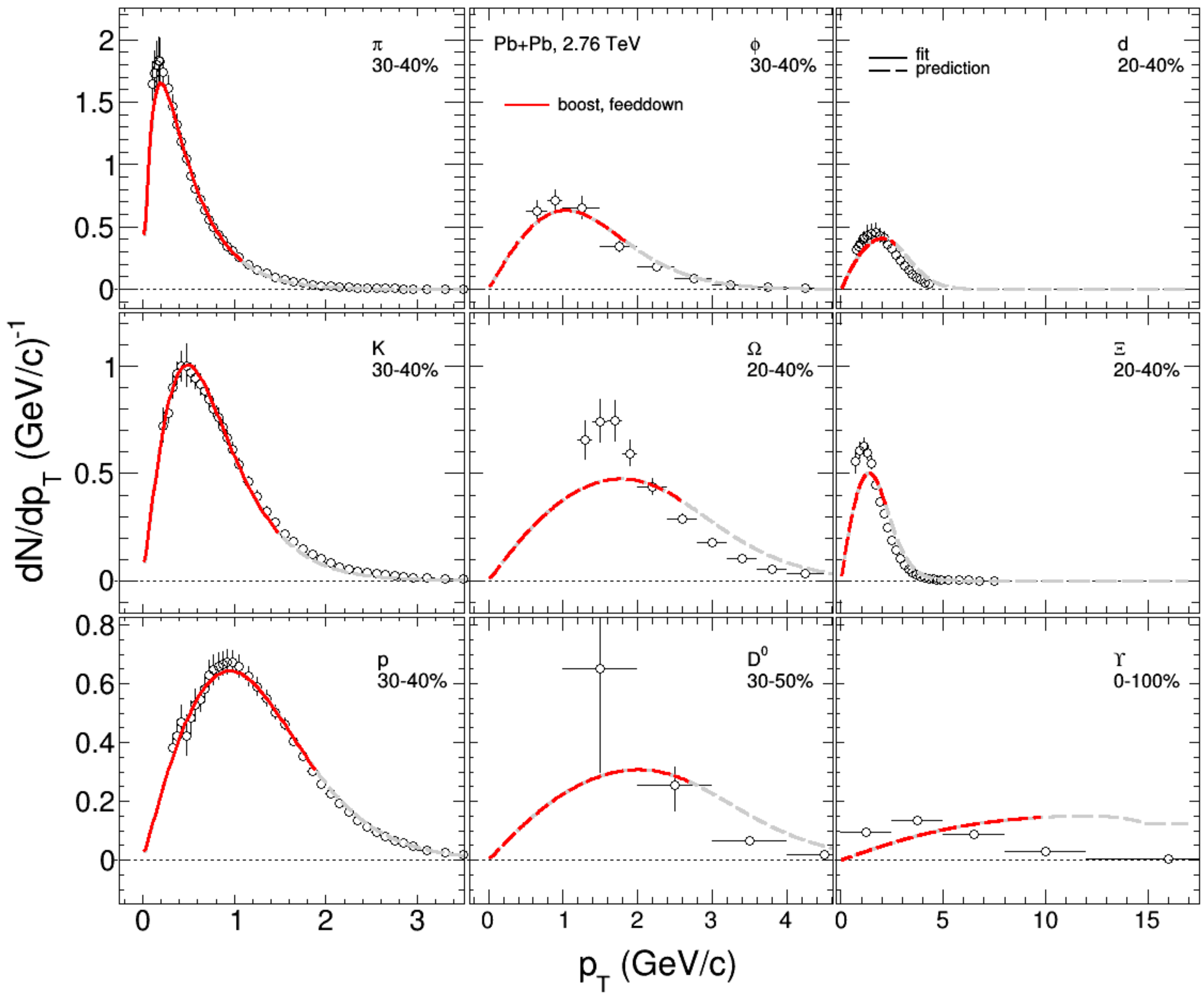
Blast-wave fit LHC

Works well for K and p

For pions, the deviation at low transverse momentum is currently the subject of intense investigations with no final conclusion yet.

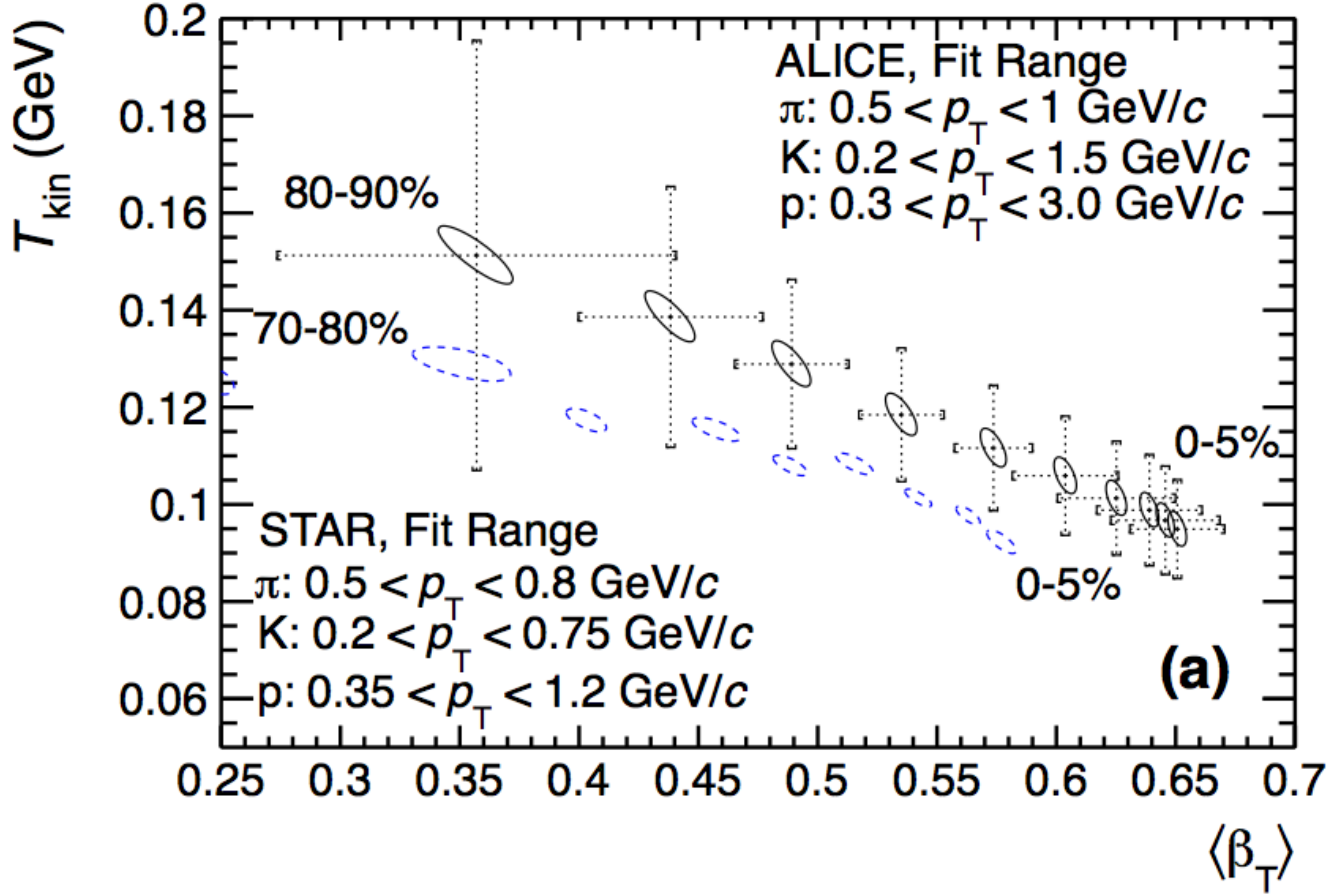
At very high p_T the deviations are mainly due to jets (high p_T parton-parton scattering).

Most recent results on blast-wave



A. Schmah, K. Reygers, X. Sun, A. Berdnikova, N. Gruenwald
Publication in process

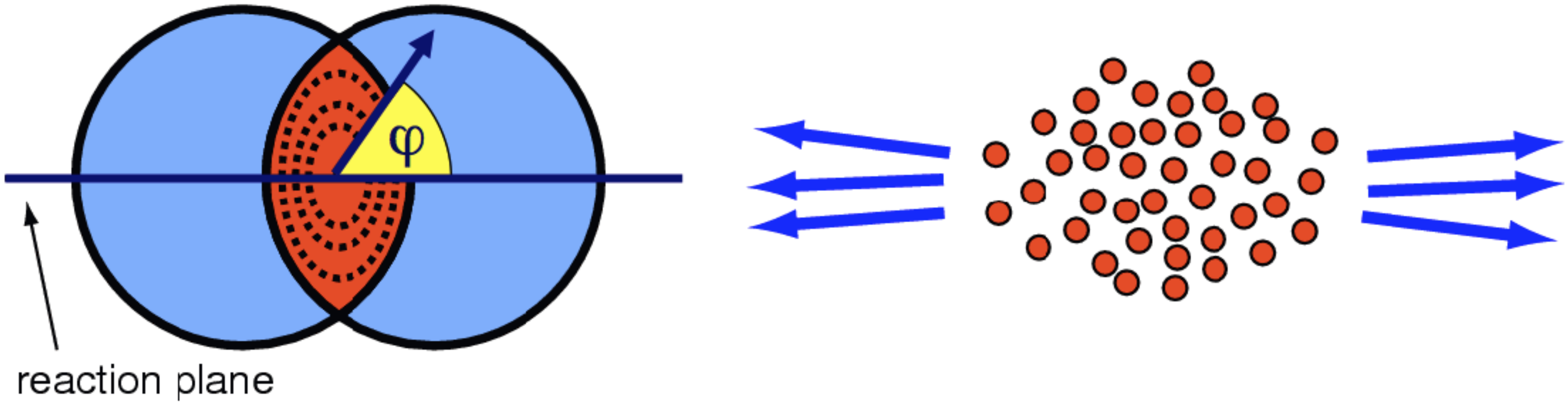
T und ⟨β⟩ for different centralities at RHIC and the LHC



10% larger flow velocities in central collisions at the LHC than at RHIC

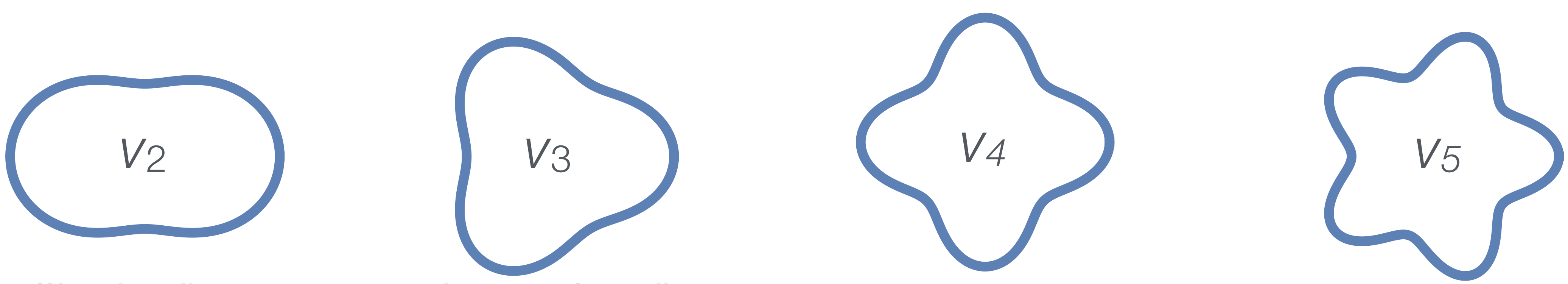
Elliptic flow and higher flow harmonics

Azimuthal distribution of produced particles



$$\frac{dN}{d\varphi} \propto 1 + 2\sum_{n=1}^{\infty} v_n \cos[n(\varphi - \Psi_n)]$$

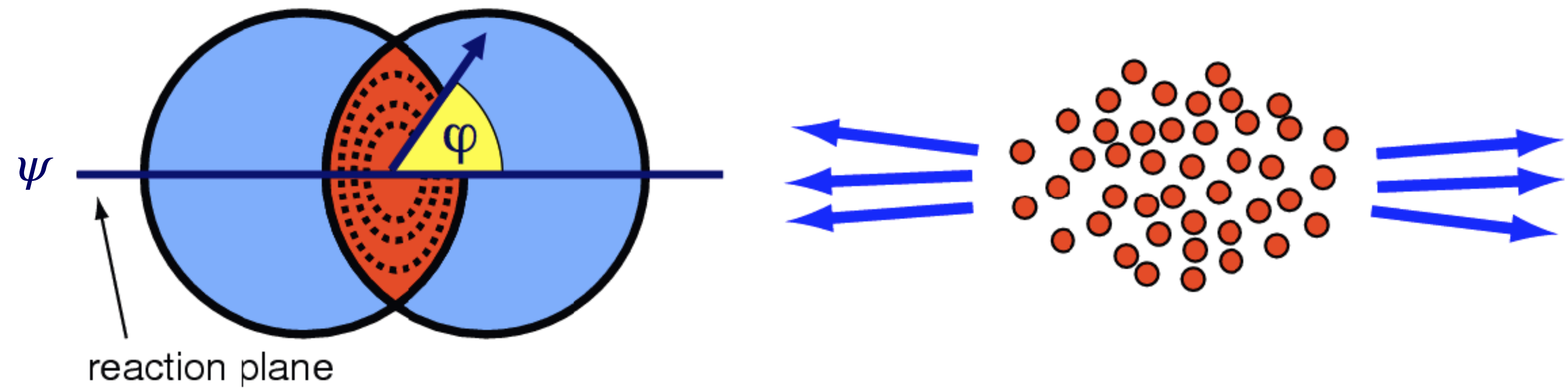
Fourier coefficients: $v_n(p_T, y) = \langle \cos[n(\varphi - \Psi_n)] \rangle$



elliptic flow | triangular flow

$f(\varphi) = 1 + 2v_n \cos(n\varphi)$

Event plane reconstruction



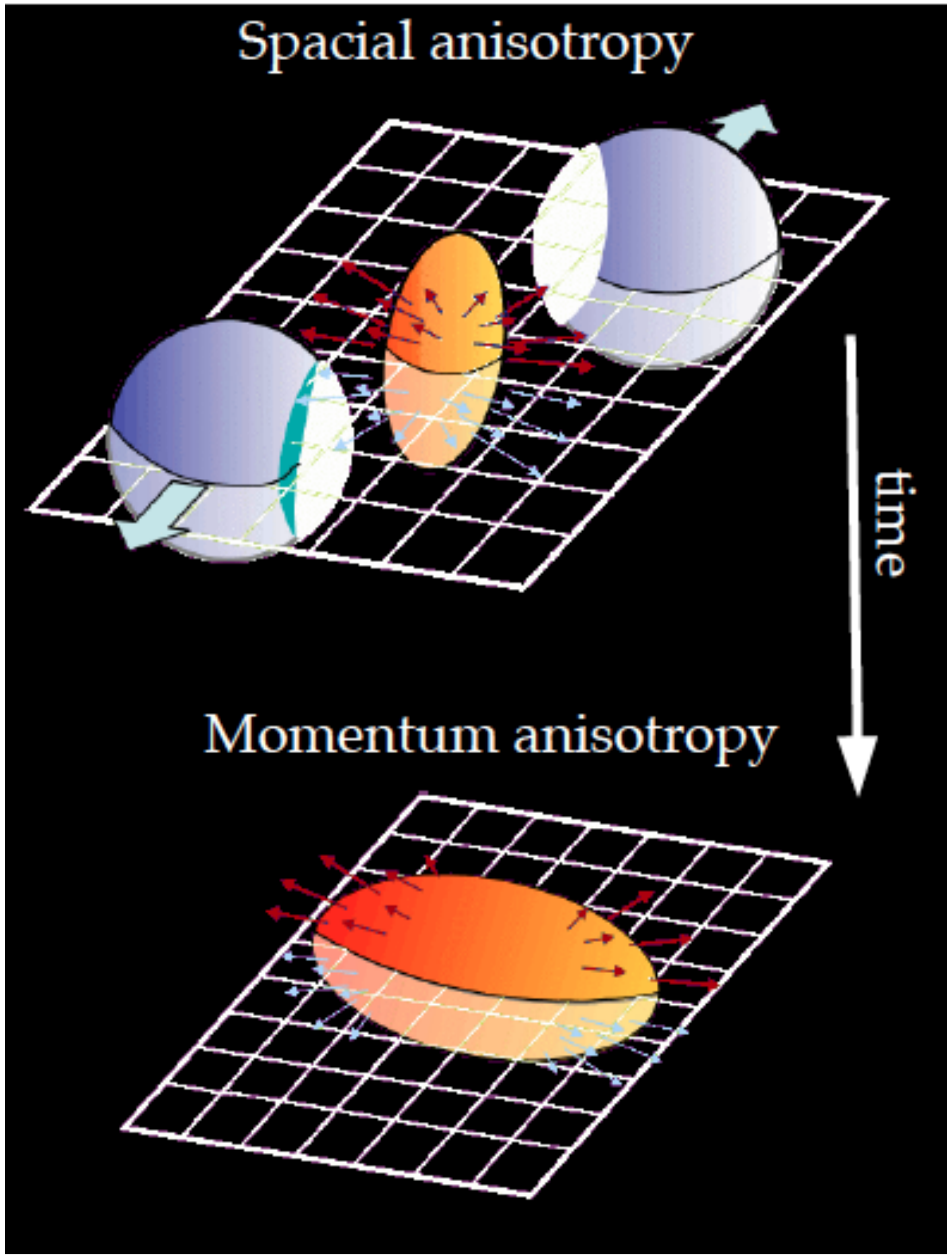
- Particles fly dominantly into the event (reaction) plane
- Use the produced particles to calculate event plane angle:

$$\Psi_2 = \frac{1}{2}\left(tan^{-1} \frac{\sum_i w_i sin(2\phi_i)}{\sum_i w_i cos(2\phi_i)} \right)$$

- w_i: weights, usually the pT of the particle

Elliptic flow - Space-Time Evolution

Non-central heavy-ion collision



Hydrodynamic evolution

$$v_2 = \frac{\langle p_x^2 \rangle - \langle p_y^2 \rangle}{\langle p_x^2 \rangle + \langle p_y^2 \rangle}$$

$$v_2 = < cos(2\phi - 2\Psi) >$$

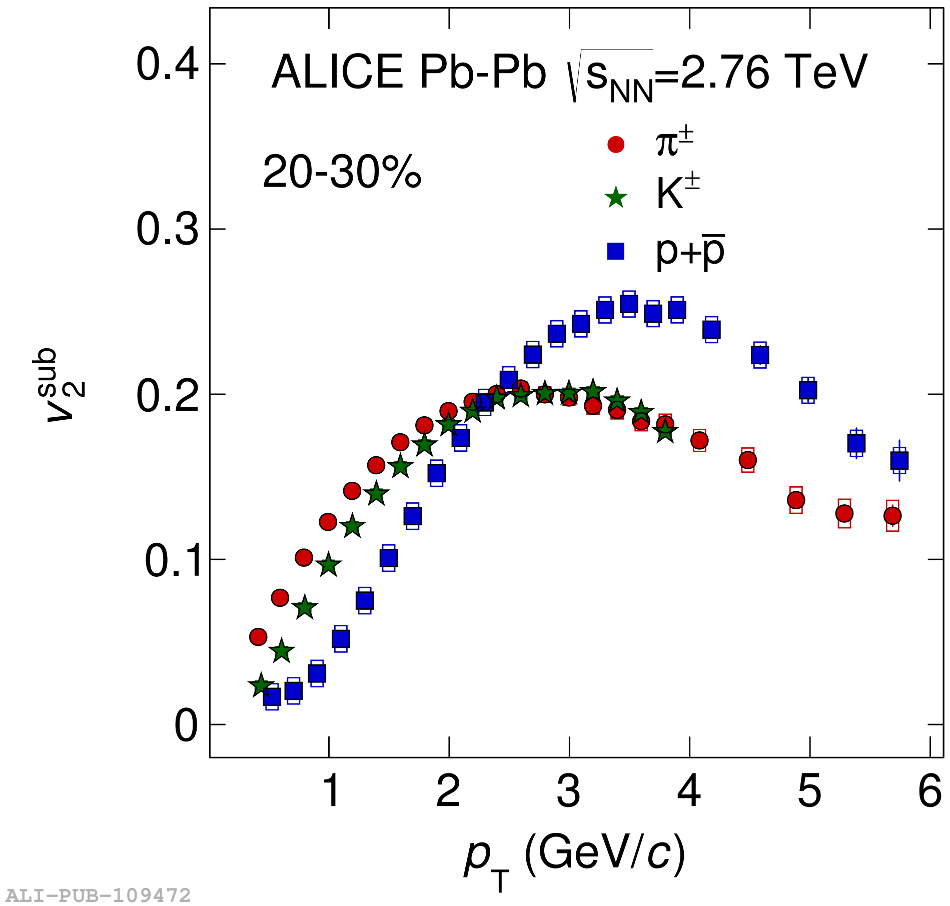
v_2 = elliptic flow

ϕ: particle azimuthal angle

ψ: event plane

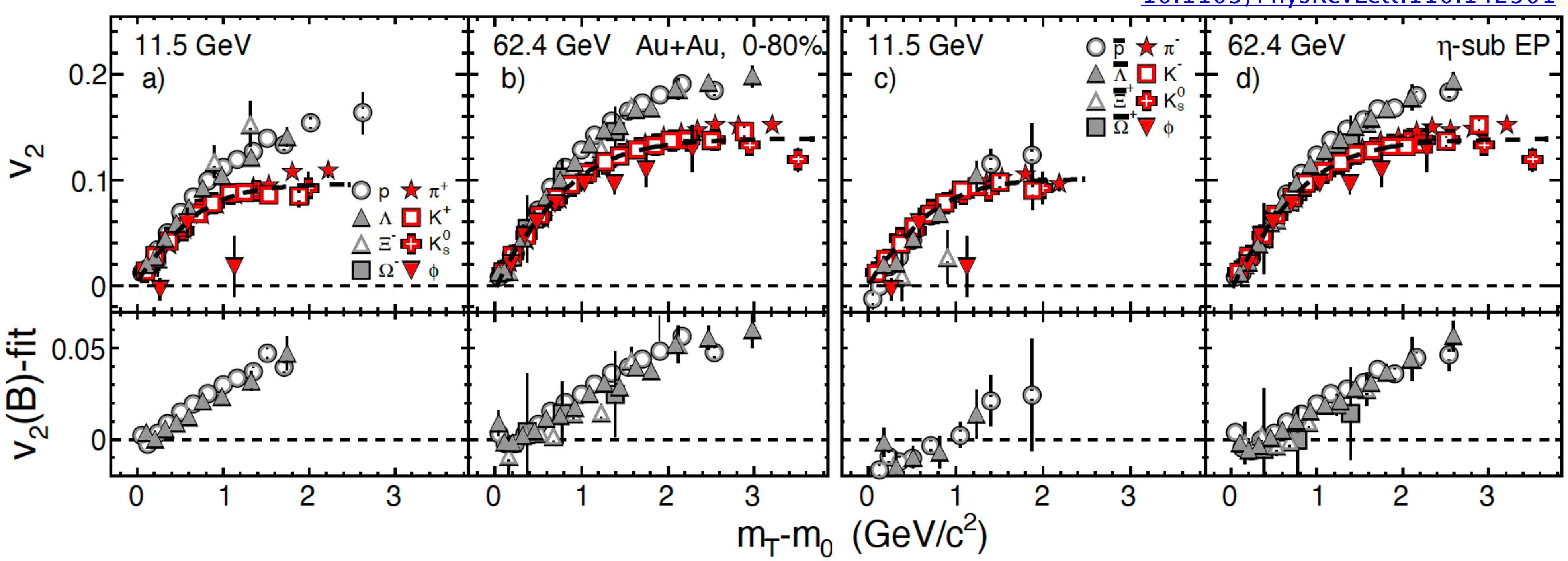
- Anisotropic expansion can tell us a lot about the medium!
  → Kind of constituents
  → Interaction between the constituents
  → Viscosity

Elliptic flow - Data



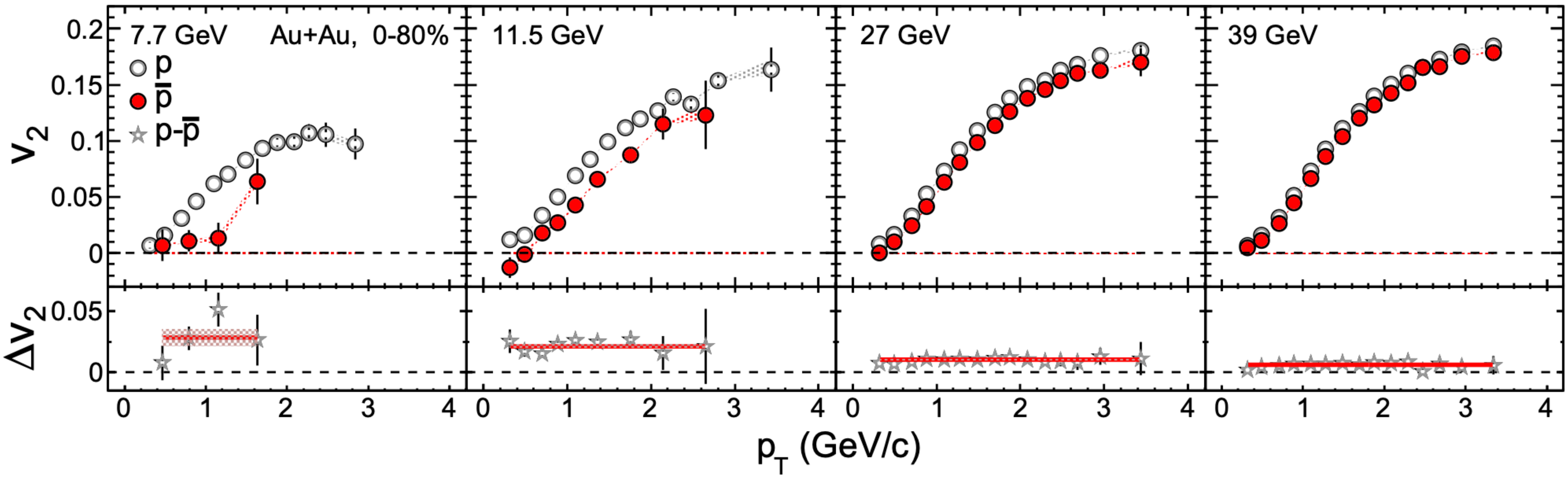
- v_2 increases with p_T at low transverse momenta: direct correlation due to boosting
- At large p_T other effects like jets are getting important
- Difference at low p_T: mass dependence
- Difference at larger p_T: quark-number dependence
- Other way of plotting: using m_T instead of p_T, almost cancelation of radial flow effects

10.1103/PhysRevLett.110.142301



Difference in elliptic flow between particles and anti-particles

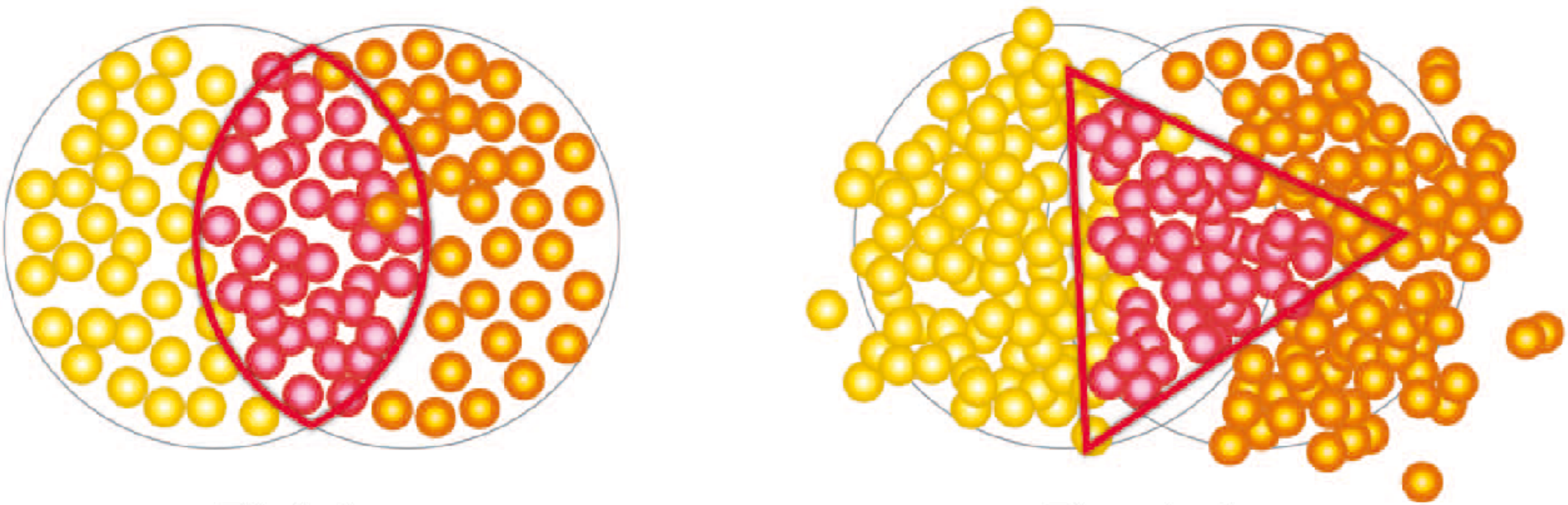
10.1103/PhysRevLett.110.142301



- Observation at lower collision energies: particle and anti-particle elliptic flow isn't the same anymore!
- Difference is most likely a result of baryon stopping. Global thermalization doesn't hold anymore on some scale.

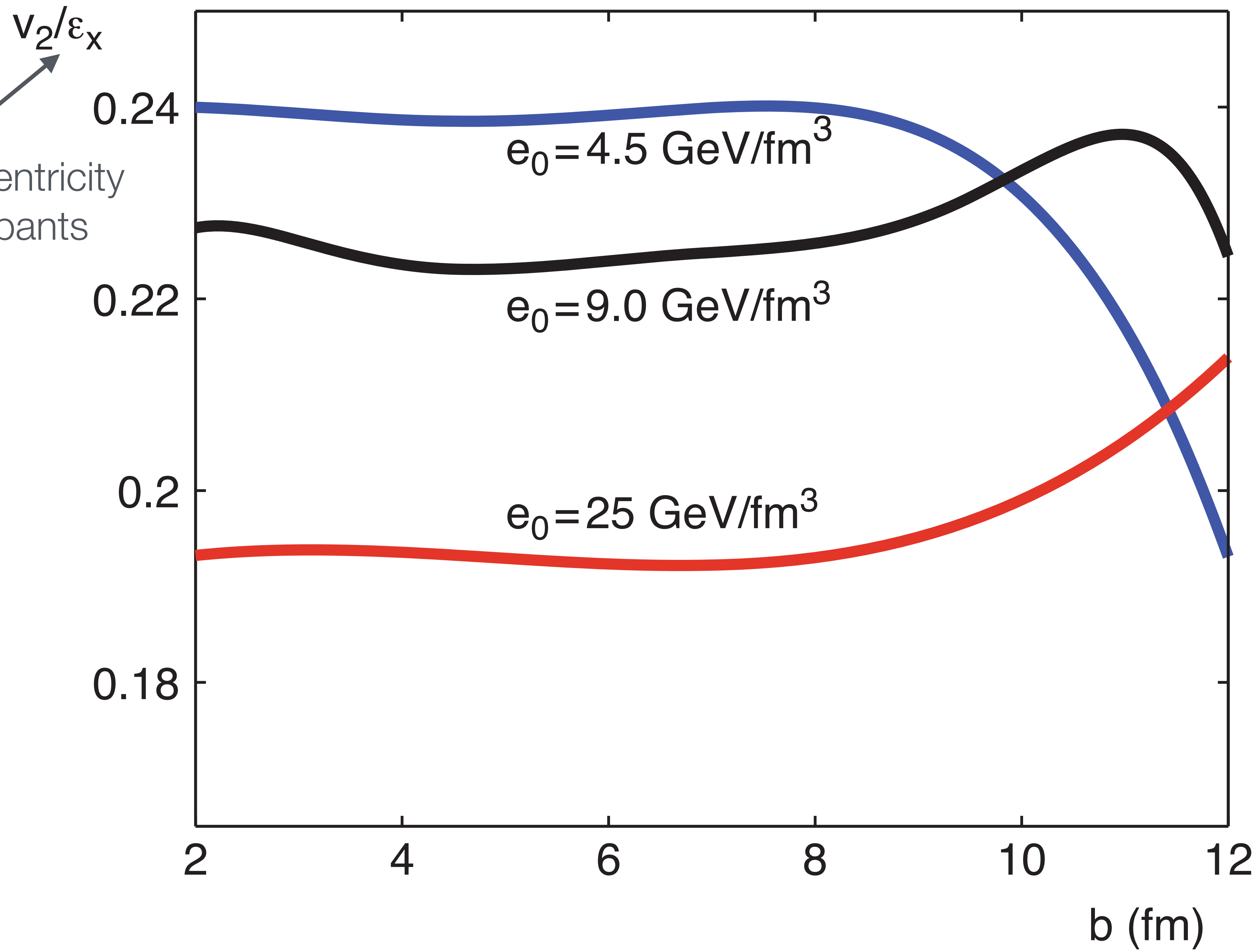
Origin of odd flow components (v_3, v_5, …)

- v_2 is related to the geometry of the overlap zone
- Higher moments result from fluctuations of the initial energy distribution



Müller, Jacak, http://dx.doi.org/10.1126/science.1215901

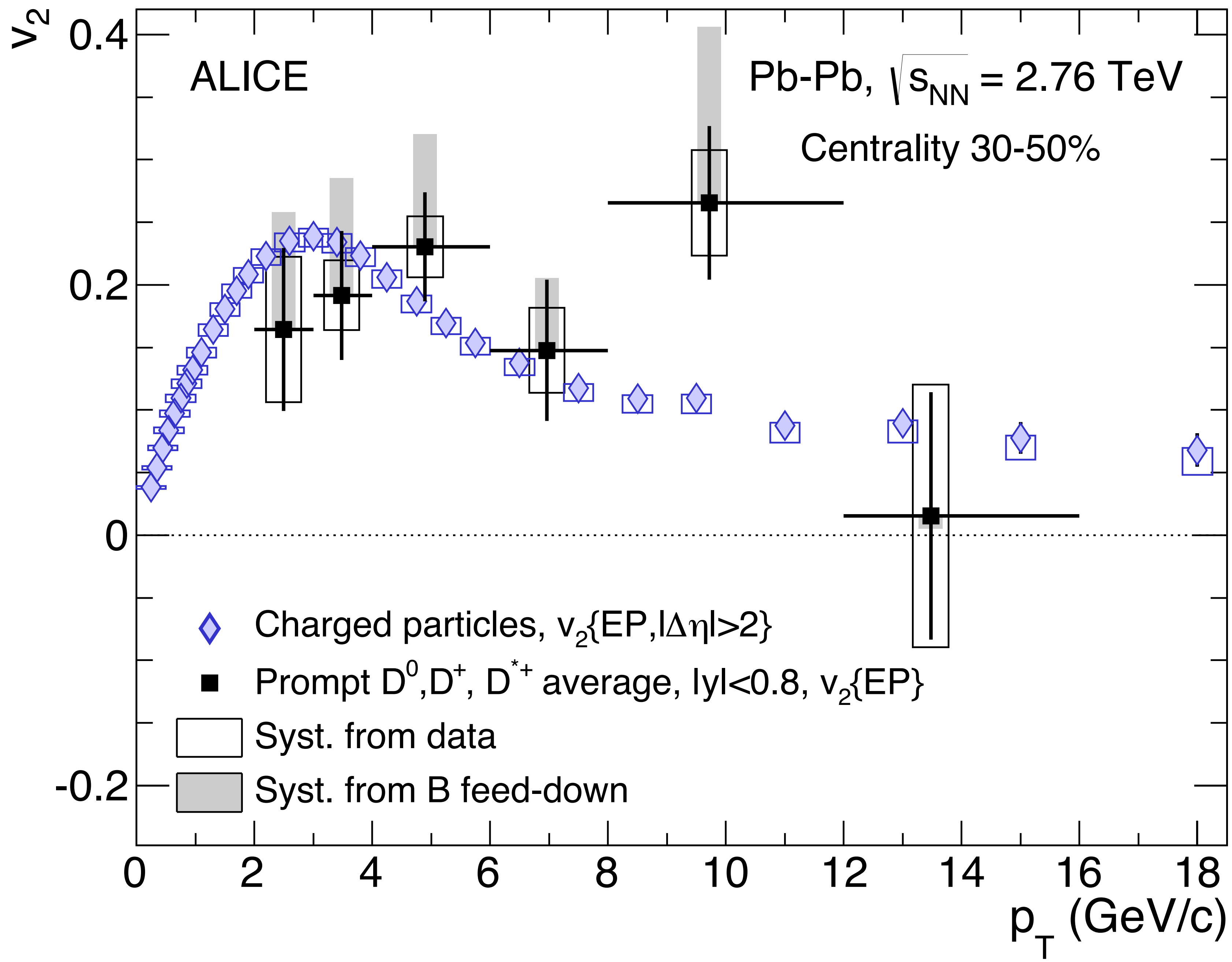
Hydrodynamic models: v_2/ε approx. constant



ε_x: initial eccentricity of the participants

Ideal hydrodynamics gives $v_2 \approx 0.2 - 0\text{-}25\ \varepsilon$

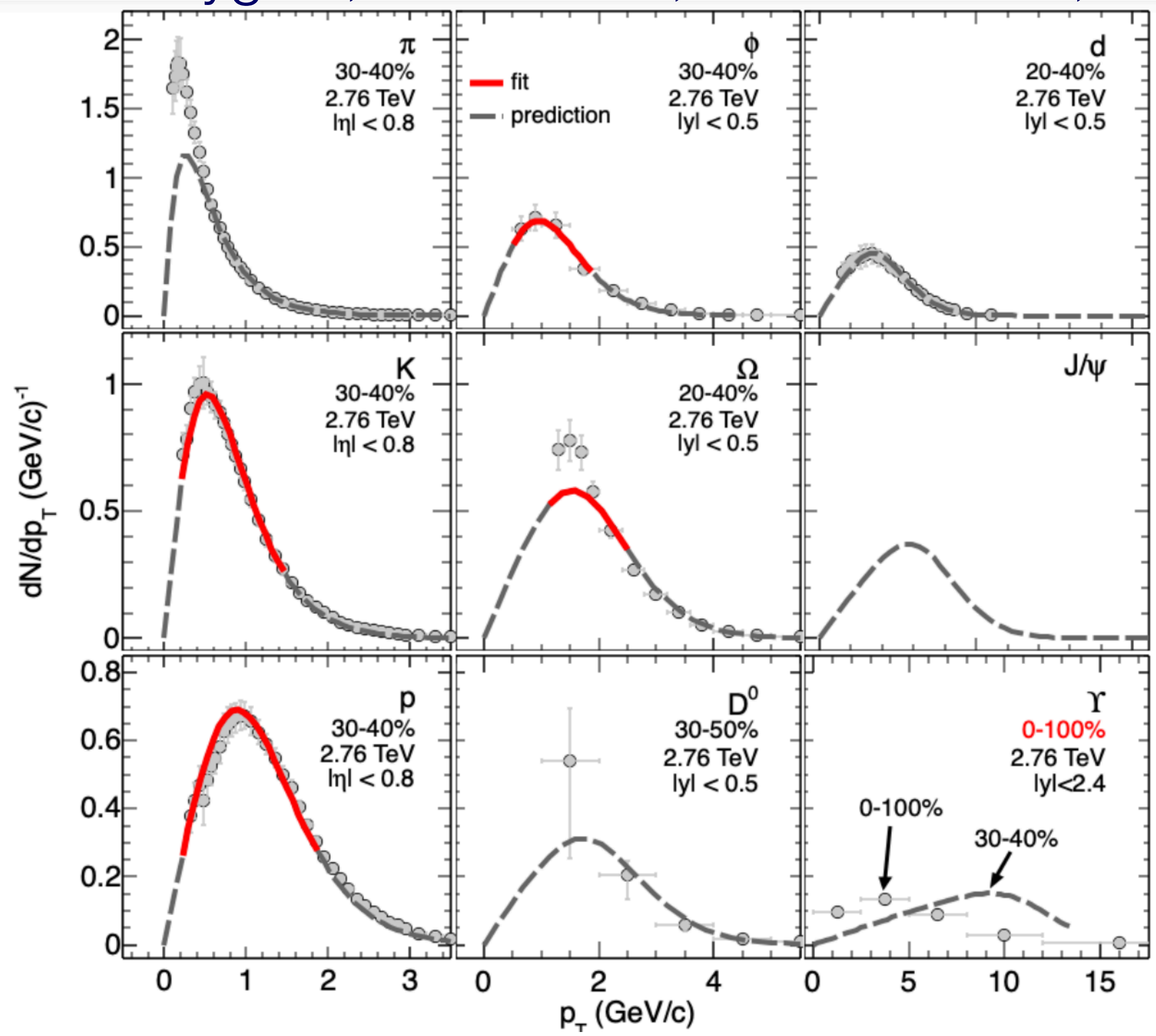
D meson v_2 in Pb-Pb: Heavy quarks seem to flow, too!



Given their large mass, it is not obvious that charm quarks take part in the collective expansion of the medium

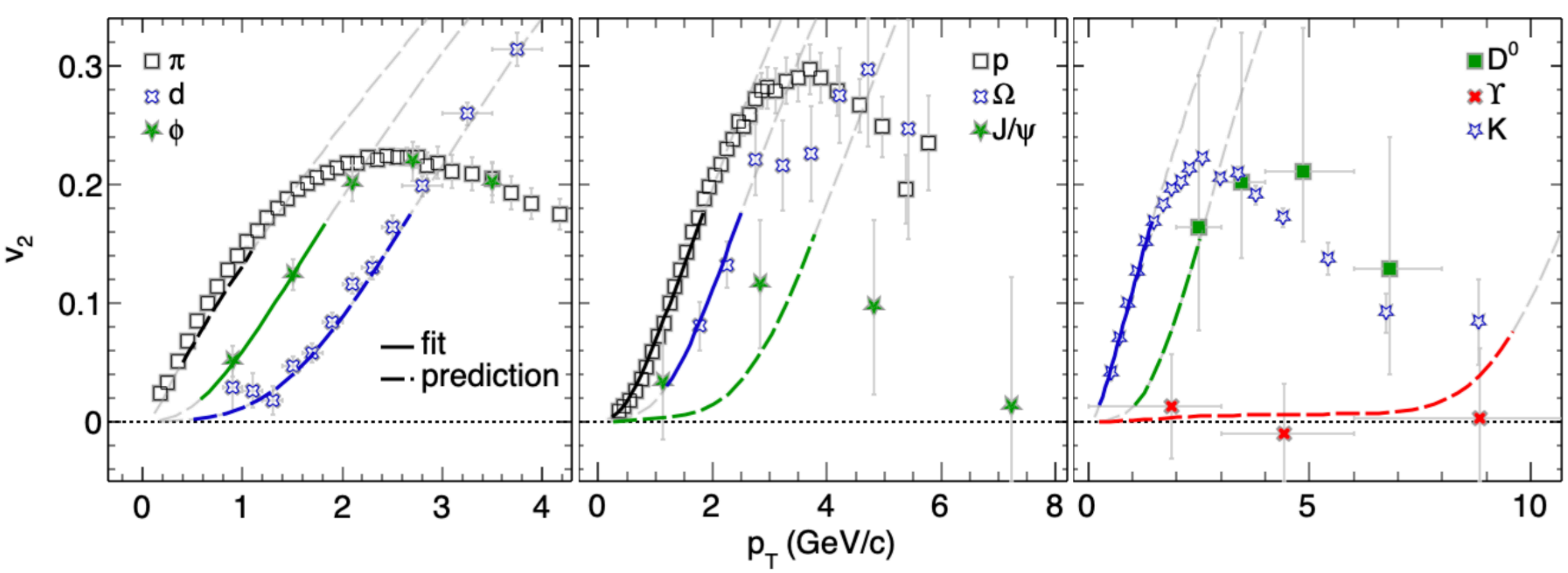
Flow of heavy quarks

K. Reygers, A. Schmah, A. Berdnikova, X. Sun



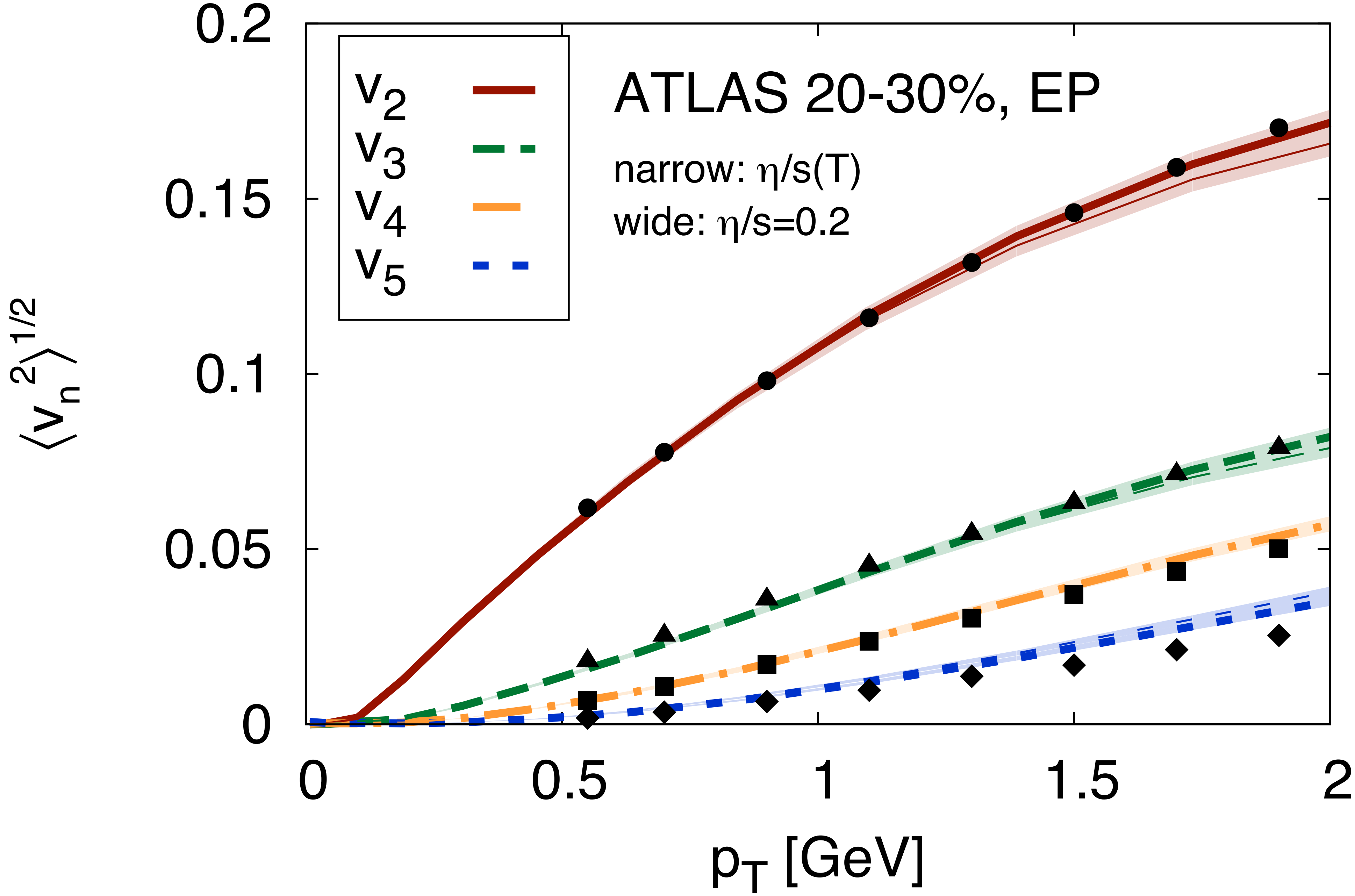
- State of the art blast-wave
- Simultaneous fit to spectra and elliptic flow data
- Good description of Υ

10.1103/PhysRevC.101.064905



η/s from comparison to data

arXiv:1209.6330



Current status (Pb-Pb at $\sqrt{s_{NN}}$ = 2.76 TeV): arXiv:1301.2826

$$(\eta/s)_{\mathrm{QGP}} \approx 0.2 = 2.5 \times \frac{1}{4\pi} \qquad (20\%\ \text{stat. err., } 50\%\ \text{syst. err.})$$