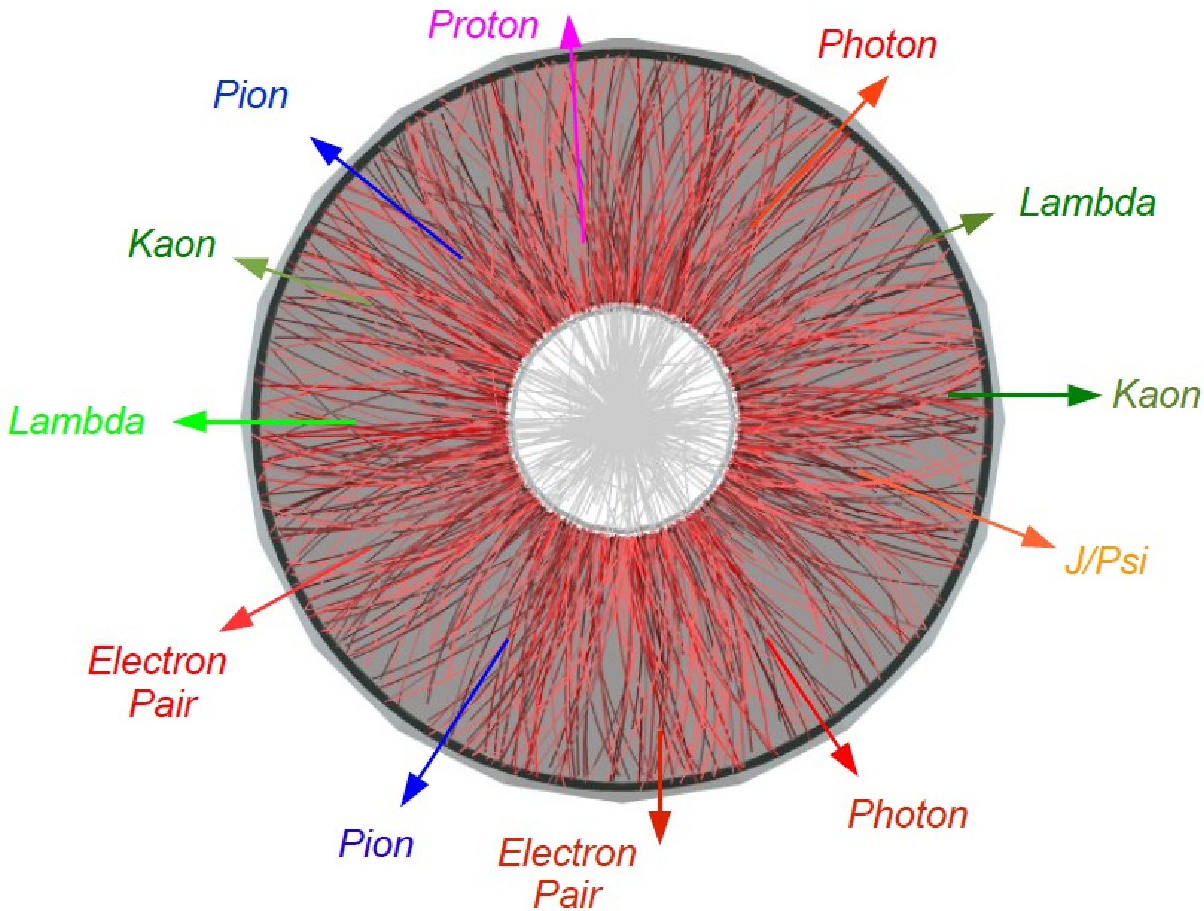
Quark-Gluon Plasma Physics

4. statistical hadronization and strangeness

Hadronization of the nuclear fireball



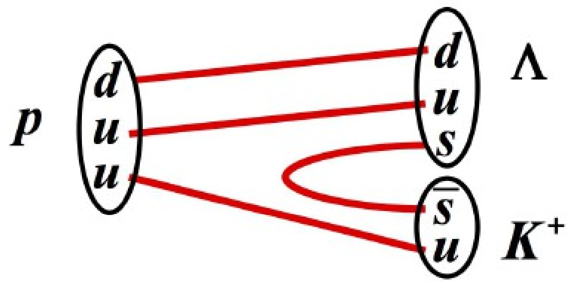
the fireball properties can be determined by measurement of the emitted particles
in this chapter as first species: hadrons with up,down,strange constituent quarks

strangeness in hadronic reactions

$$K^+ = (u\bar{s}),\ K^- = (\bar{u}s),\ K^0 = (d\bar{s}),\ \bar{K}^0 = (\bar{d}s),\ \phi = (s\bar{s}),$$

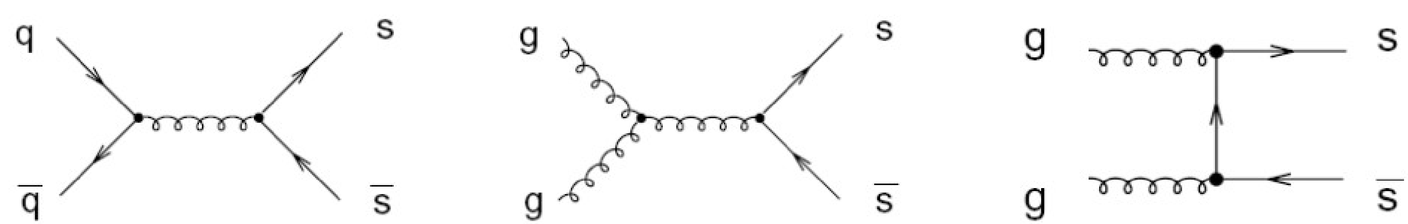
$$\Lambda = (uds),\ \Sigma = (qqs),\ \Xi = (qss),\ \Omega^- = (sss)$$

$$p + p \to p + K^+ + \Lambda, \quad Q = m_\Lambda + m_{K+} - m_p \approx 670\,\text{MeV}$$



$$p + p \to p + p + \Lambda + \bar{\Lambda}, \quad Q = 2m_\Lambda \approx 2230\,\text{MeV}$$

strangeness in the QGP



$$Q_{\mathrm{QGP}} \approx 2m_s \approx 200\,\mathrm{MeV}$$

strangeness in the QGP

Expectation for strangeness production in heavy ion collisions where QGP is produced:

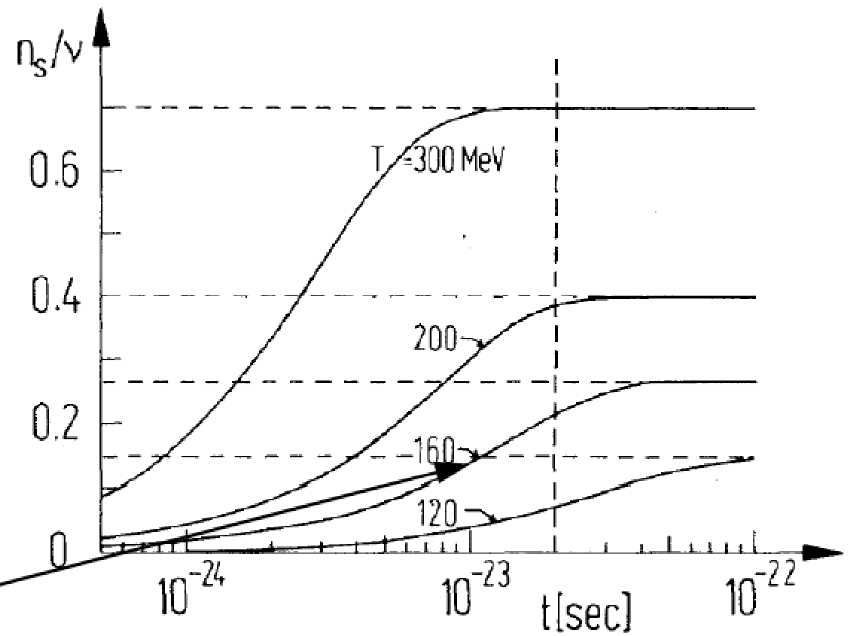
in QGP strangeness gets into equilibrium on a fast time scale

J. Rafelski, B. Müller,
Phys. Rev. Lett. 48 (1982) 1066

there should be more strangeness in heavy ion collisions than in elementary collisions if a QGP is formed

enhanced production of strange hadrons one of the earliest predicted signature of QGP

but relevant for hadron production is critical temperature of QCD phase transition

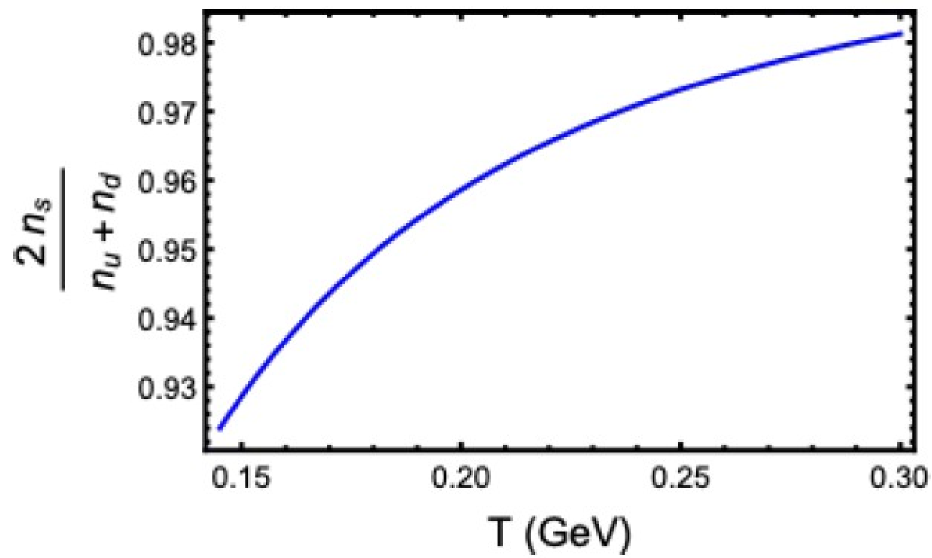


ratio of strange quark to baryon number abundance in a QGP for various temperatures

quark composition of the QGP

$$n_i = g_i \frac{4\pi}{(2\pi)^3} \int_0^\infty \frac{p^2\,\mathrm{d}p}{\exp\left(\frac{\sqrt{p^2+m^2}-\mu}{T}\right) \pm 1} = \frac{g_i}{2\pi^2} m^2 T \sum_{k=1}^{\infty} \frac{(\mp 1)^{k+1}}{k} \lambda^k K_2\left(\frac{km}{T}\right)$$

$$\lambda = e^{\mu/T}$$



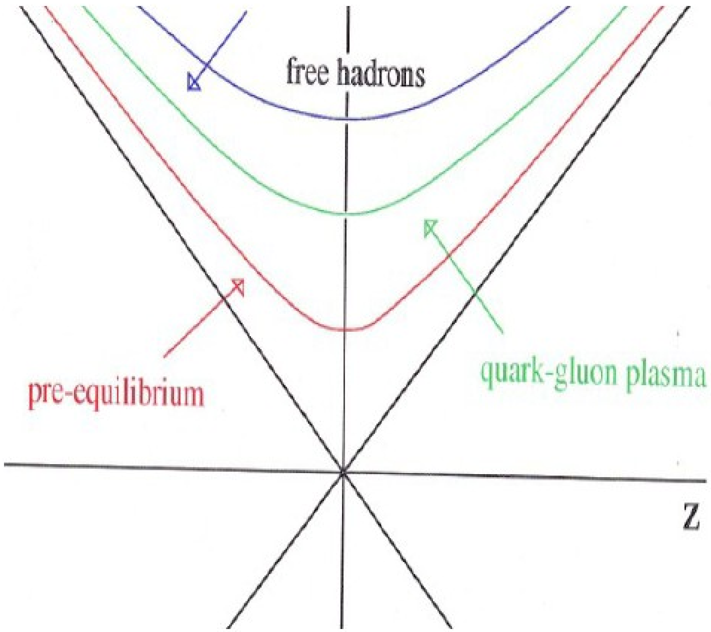
$$\frac{2(n_s + n_{\bar{s}})}{n_u + n_{\bar{u}} + n_d + n_{\bar{d}}} \approx 0.92\text{–}0.98$$

chemical freeze-out

nuclear fireball evolves
(as sketched in lecture 1)
it cools and expands,
when it hits T_c, it hadronizes,
maybe cools and expands further

and finally falls apart
when mean free path large as
compared to interparticle distance
"kinetic freeze-out": momentum
distributions are frozen in
- no more elastic scattering $\}$ T_{kin}

"chemical" or **"hadro-chemical freeze-out"**: abundancies of hadrons
are frozen in
– no more inelastic scattering $\}$ T_{cf}

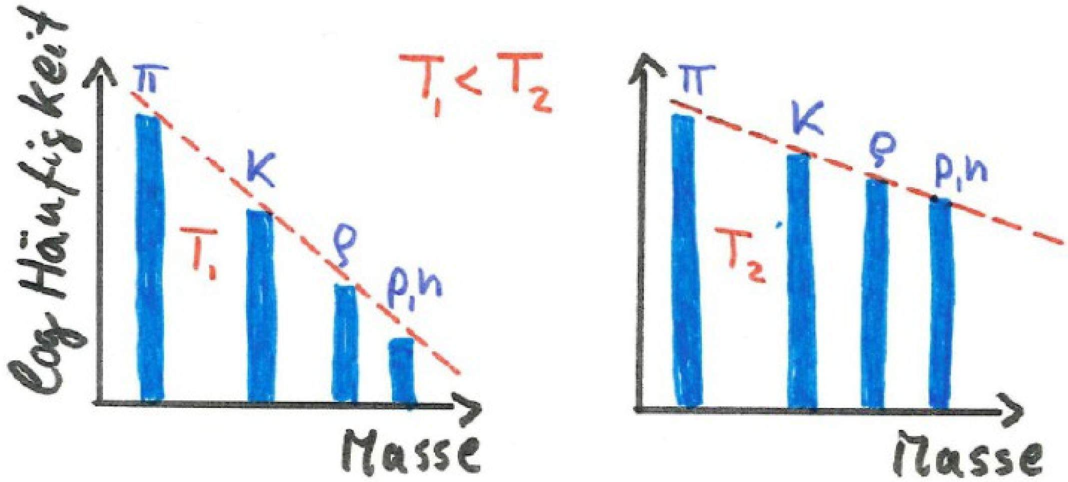


natural ordering: $T_c > T_{cf} > T_{kin}$

T_c is critical temperature of QGP phase transition

thermal energy and population of hadronic states

equivalence of energy and mass



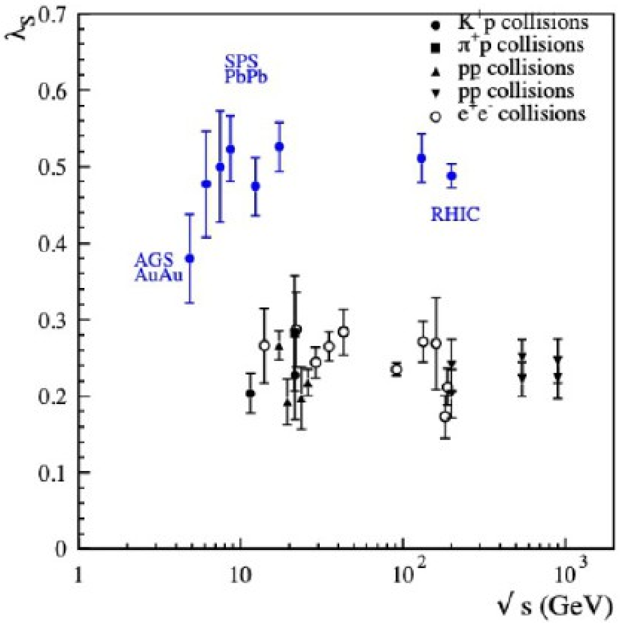
assume phase space is filled thermally (Boltzmann) at hadronization:

abundance of hadron species $\propto m^{3/2} \exp(-m/T)$

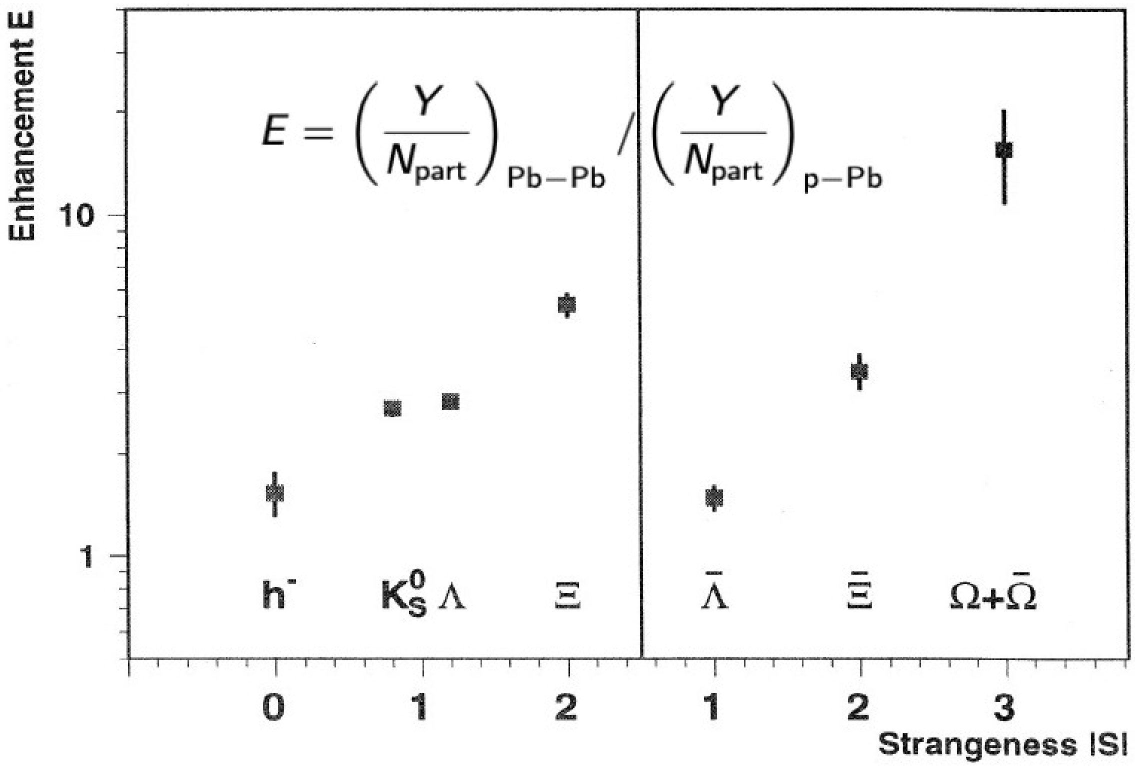
determined by temperature (and density) at time of production of hadrons
= hadronization

valence strange quarks for different colliding systems

$$\lambda_s = \frac{2\langle s\bar{s}\rangle}{\langle u\bar{u}\rangle + \langle d\bar{d}\rangle}$$

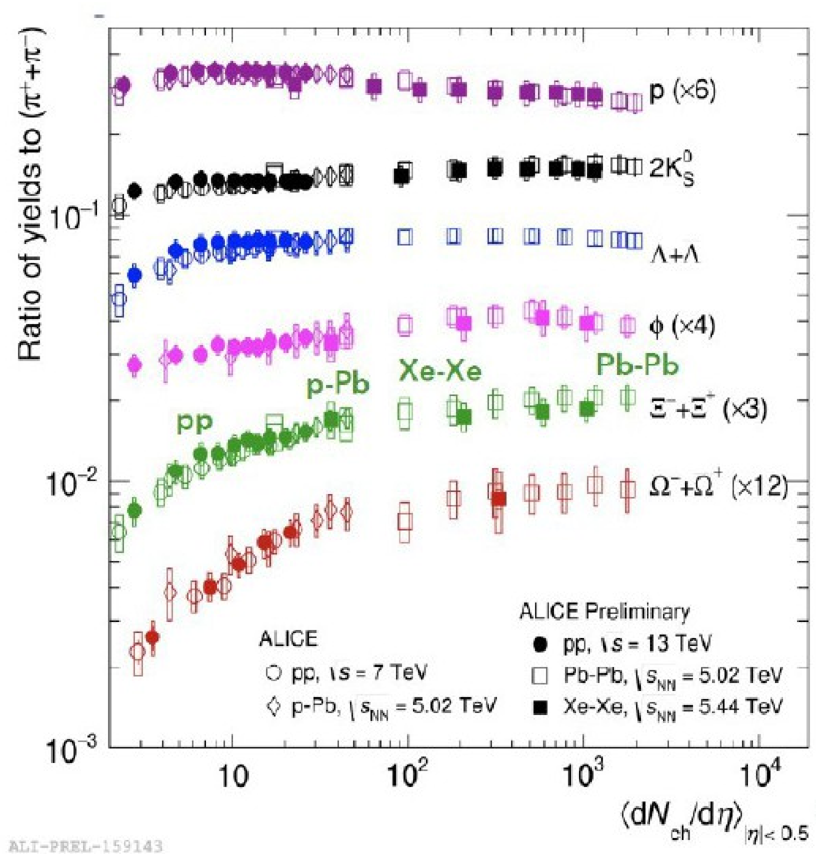


strangeness enhancement in Pb-Pb relative to pPb collisions



data from SPS experiments WA97 and NA57, $\sqrt{s_{NN}}$ = 17.3 GeV

strangeness enhancement in Pb-Pb relative to pp and p-Pb data from LHC experiment ALICE, $\sqrt{s_{NN}}$ = 5.0 – 7.0 TeV

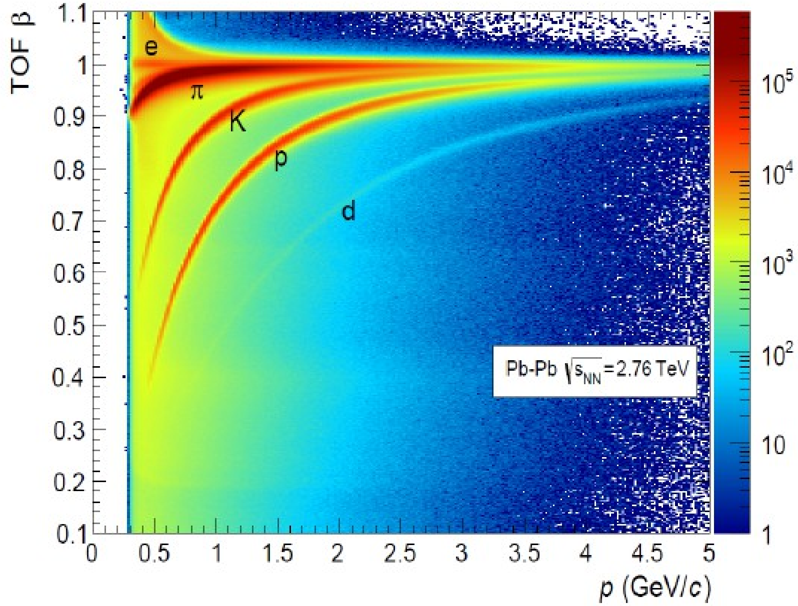


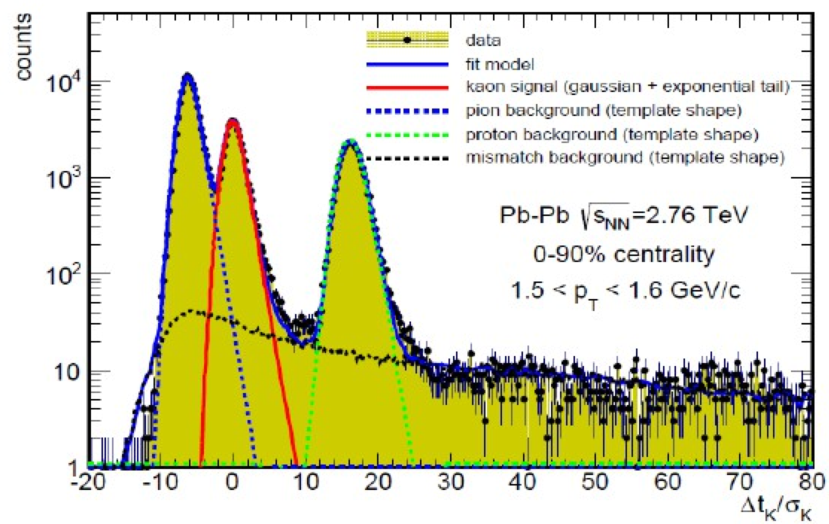
ALICE coll., Nature Phys. 13 (2017) 535-539
arXiv: 1606.07424 [nucl-ex]

note: enhancement is reduced compared to enhancement measured at SPS energy

measurement of production yields of identified hadrons

identification via time-of-flight plus momentum measurement:





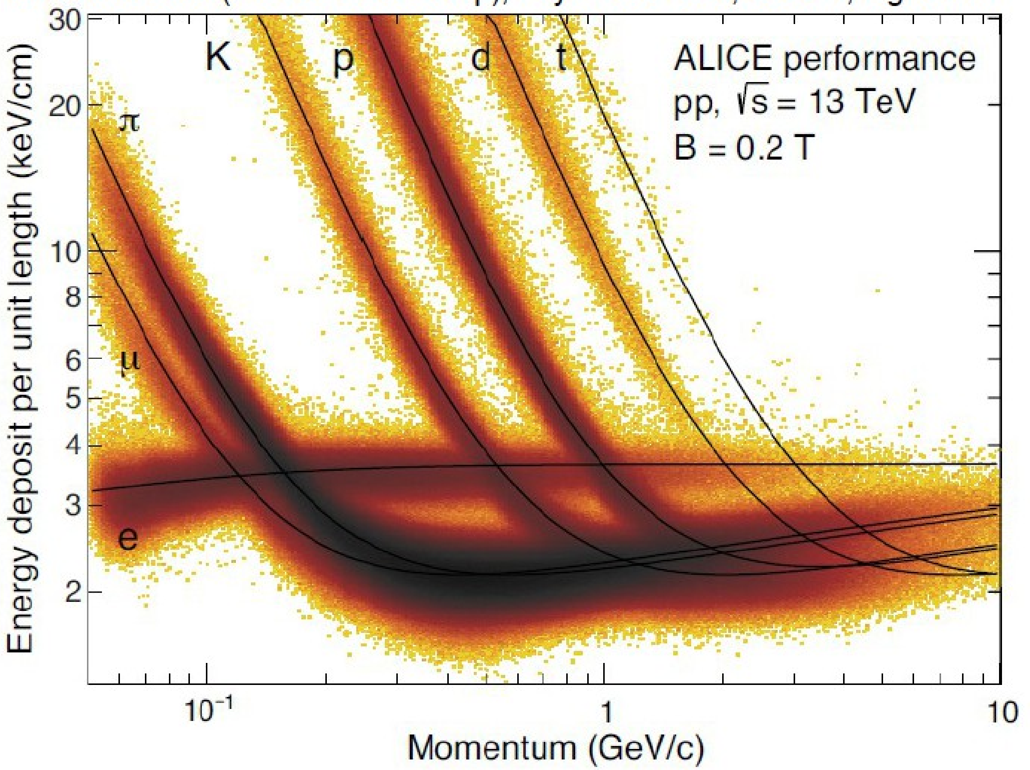
works well as long at ∩/c < 1
but then fades quickly

measurement of production yields of identified hadrons

Identification via specific energy loss:

example: ALICE TPC 150 space points per track

M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001, Fig. 34.15



measurement of production yields of identified hadrons

Identification via invariant mass of decay products

$$
\begin{aligned}
M^2 &= \left[\begin{pmatrix} E_1 \\ \vec{p}_1 \end{pmatrix} + \begin{pmatrix} E_2 \\ \vec{p}_2 \end{pmatrix} \right]^2 = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2 \\
&= m_1^2 + m_2^2 + 2E_1 E_2 - 2\vec{p}_1 \cdot \vec{p}_2 \\
&= m_1^2 + m_2^2 + 2E_q E_2 - 2p_1 p_2 \cos\vartheta
\end{aligned}
$$

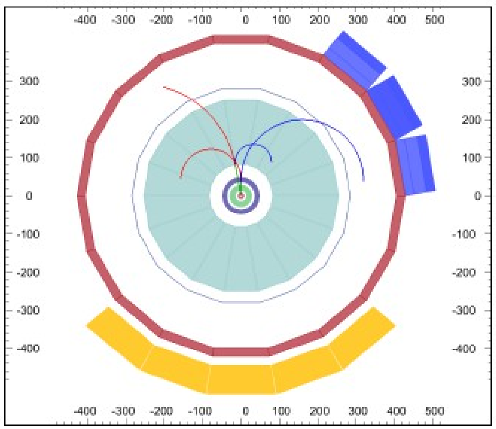
measurement of production yields of identified hadrons

electromagnetic decays:

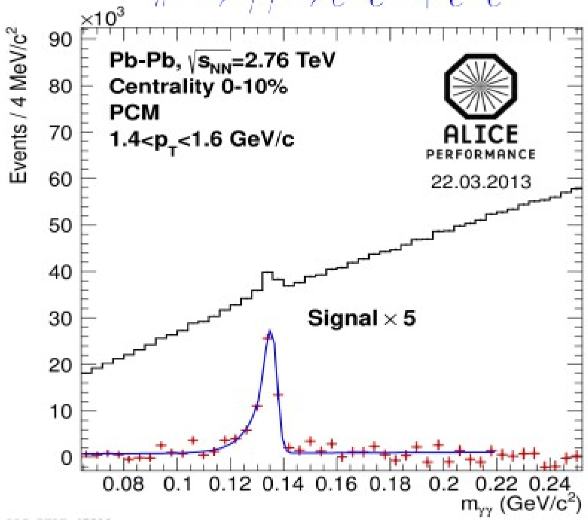
$$\pi^0 \to \gamma\gamma \qquad \mathrm{m}_{\pi^0} = 0.135\mathrm{GeV},\ \mathrm{BR} = 0.988,\ \mathrm{c}\tau = 25.1\,\mathrm{nm}$$
$$\eta \to \gamma\gamma \qquad \mathrm{m}_{\eta} = 0.548\mathrm{GeV},\ \mathrm{BR} = 0.393,\ \mathrm{c}\tau = 0.2\,\mathrm{nm}$$

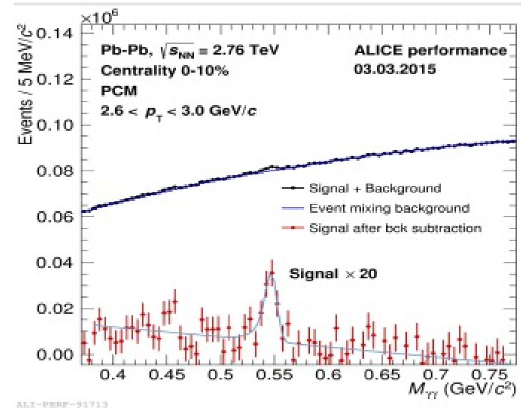
happen practically in the interaction point/target

detect photons in calorimeter
or via e+e- from conversion in detector material



$$\pi^0 \to \gamma\gamma \to e^+e^- + e^+e^-$$





$$\eta \to \gamma\gamma \to e^+e^- + e^+e^-$$

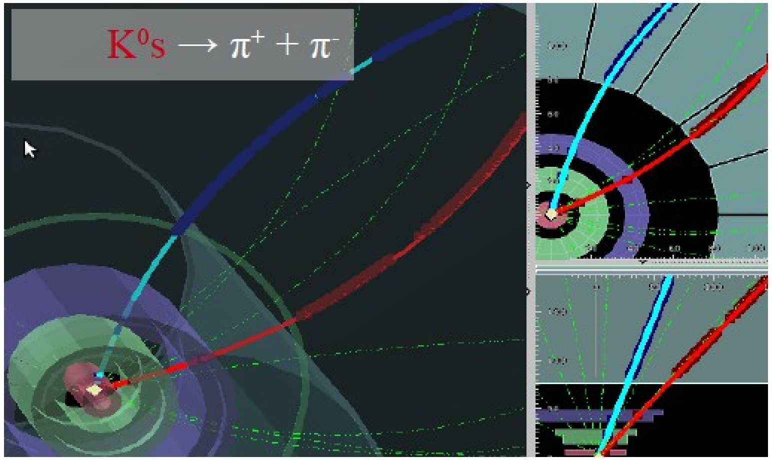
measurement of production yields of identified hadrons

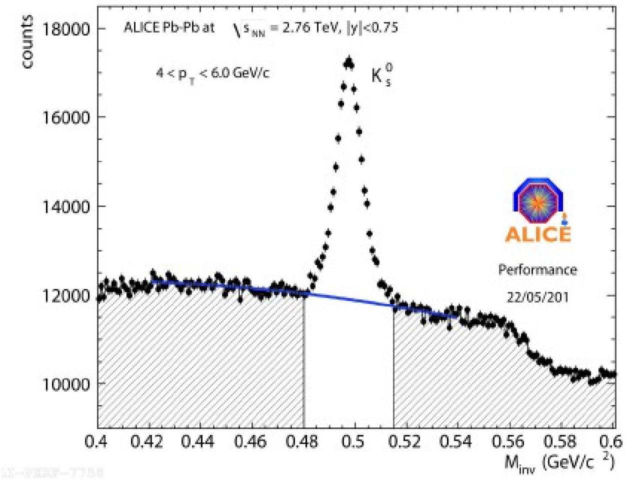
Identification via invariant mass of weak decay products

$$K^0_s \rightarrow \pi^+ + \pi^- \quad (\mathrm{B.R.}\,68\%) \quad c\tau = 2.68\,\mathrm{cm}$$
$$\Lambda \rightarrow p + \pi^- \quad (\mathrm{B.R.}\,64\%) \quad c\tau = 7.89\,\mathrm{cm}$$

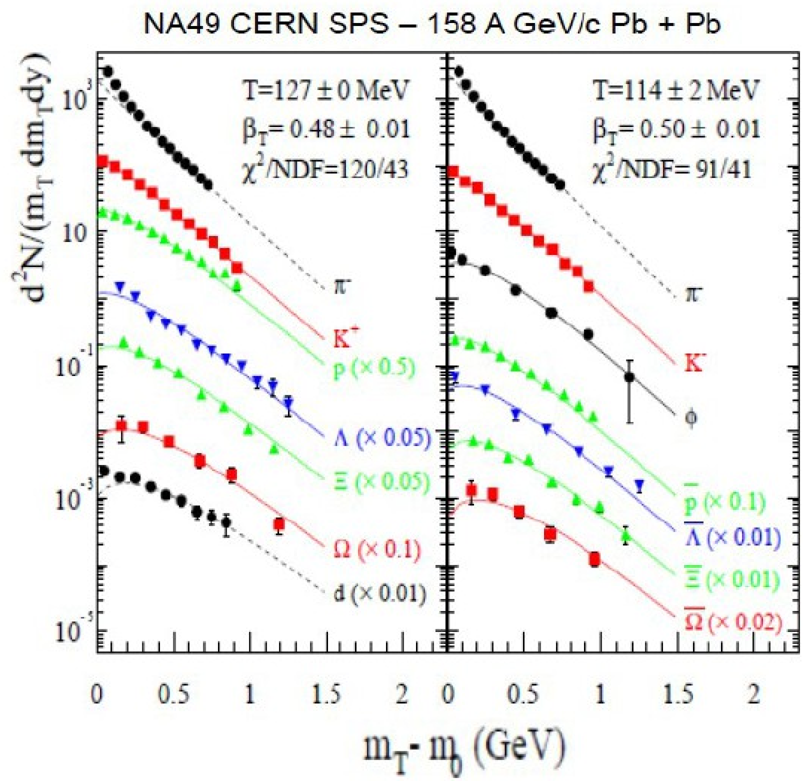
look for secondary decay vertex of a neutral object a few 10 cm away from interaction point

works up to very high momentum!





hadron production in central PbPb collisions at the CERN SPS

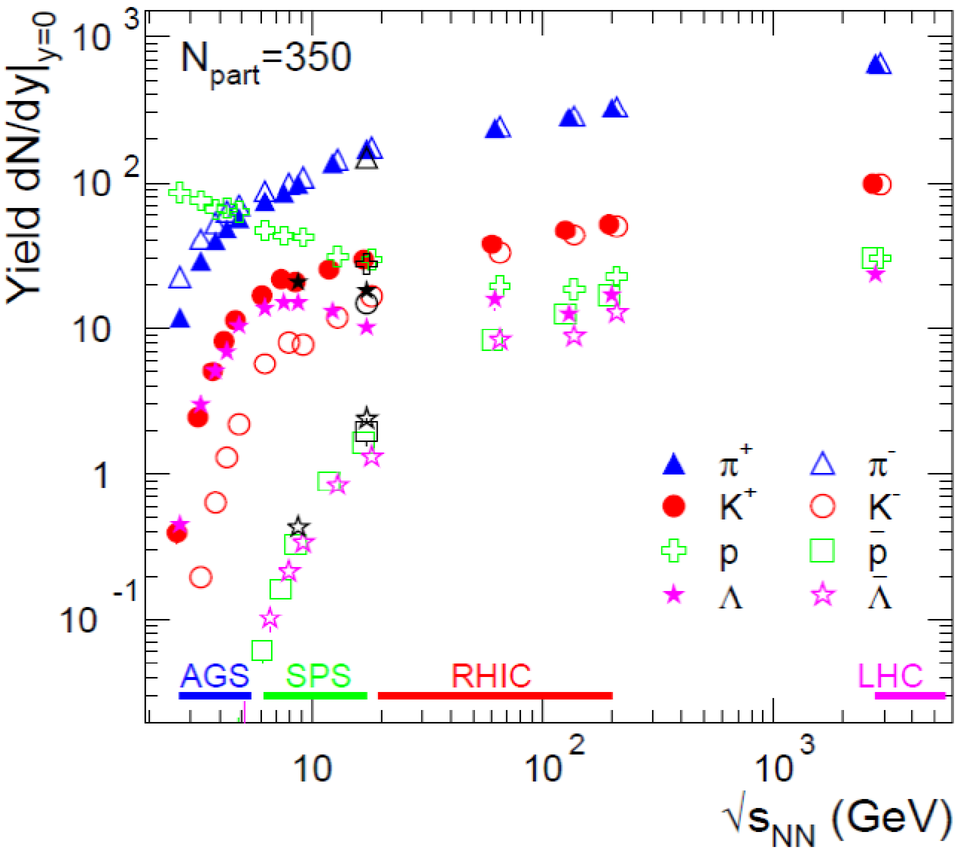


measure p_t spectra and integrate/extrapolate over all values of p_t from 0 to infinity to obtain particle yield

between 5 different experiments a comprehensive data set for 158 A GeV PbPb collisions

particle production in central AA collisions

A. Andronic arXiv:1407.5003



a summary of 25 years of experimental research

systematic trends with beam energy:
mesons rise and level off
baryons drop
antibaryons rise steeply

can we understand all of these?

4.2 statistical hadronization model and hadron yields

Idea: Freeze-out of the QGP creates an equilibrated hadron resonance gas

The HRG then freezes out with a characteristic temperature T_{ch} which determines the yields of different particle species

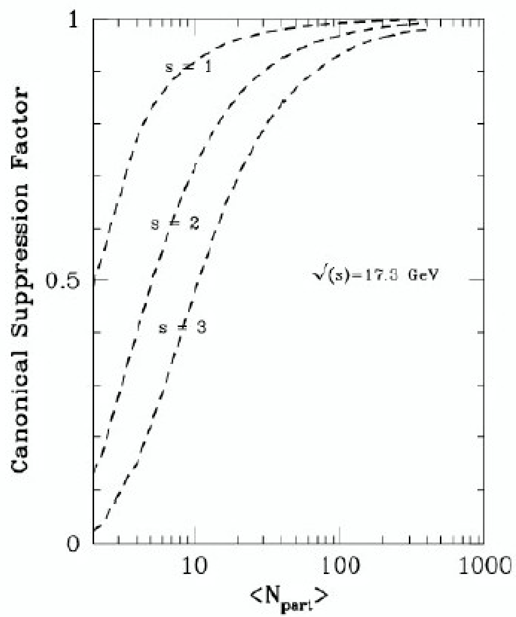
What is the appropriate statistical ensemble for the theoretical treatment?

When multiplicity is low, conservation laws must be implemented locally event-by-event (Hagedorn 1971)

Braun-Munzinger, Redlich, Stachel, nucl-th/030401

When number of produced particles large, conservation of additive quantum numbers can be implemented on average (use of chemical potential)

grand-canonical ensemble – large volume limit of canonical thermodynamics



A. Tounsi, K. Redlich, hep-ph/0111159

Canonical suppression factor F_s:

$$n_K^C = n_K^{GC} \cdot F_S$$

$$F_S = \frac{I_K(2n_K^{GC}V)}{I_0(2n_K^{GC}V)}$$

n_K : Density of particles with strangeness K = |S|, S =-1, -2, -3

I_n : Modified Bessel function of the first kind

Already at moderately central Pb-Pb collisions the grand canonical ansatz is justified

grand canonical ensemble and application to data from high energy heavy ion collisions

partition function: $\ln Z_i = \frac{V g_i}{2\pi^2}\int_0^\infty \pm p^2 dp \ln(1 \pm \exp(-(E_i - \mu_i)/T))$

particle densities: $n_i = N/V = -\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2\,\mathrm{d}p}{\exp((E_i-\mu_i)/T) \pm 1}$

for every conserved quantum number there is a chemical potential:

$$\mu_i = \mu_B B_i + \mu_S S_i + \mu_{I_3} I_i^3$$

but can use conservation laws to constrain V, μ_S, μ_{I_3}

baryon number: $\mathrm{V}\sum_{\mathrm{i}} \mathrm{n_i B_i} = \mathrm{Z} + \mathrm{N}$ $\rightarrow \mathrm{V}$

strangeness: $V\sum_i n_i S_i = 0$ $\rightarrow \mu_S$

charge: $V\sum_i n_i I_i^3 = \frac{Z-N}{2}$ $\rightarrow \mu_{I_3}$

only 2 free parameters left

$\Rightarrow$ fit at each energy provides T and μ_b

comparison to experimental data

compute primary thermal occupation probability for each particle species

spectrum of hadrons involves all confirmed hadronic states as of PDG compilation

implement all strong decays according to PDG
(example: for T=160 MeV, 80% of all pions come from strong decays)

compute for a grid of (T,μ_b) χ^2 value between statistical ensemble calculation and data

minimize χ^2 to obtain for each beam energy and collision system best set of (T,μ_b)

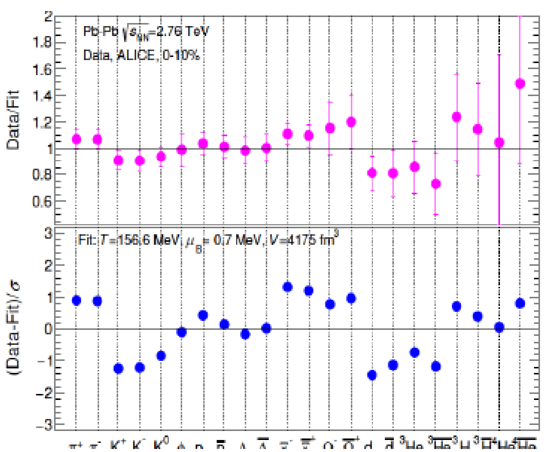
Hadron yields at the LHC compared to statistical model

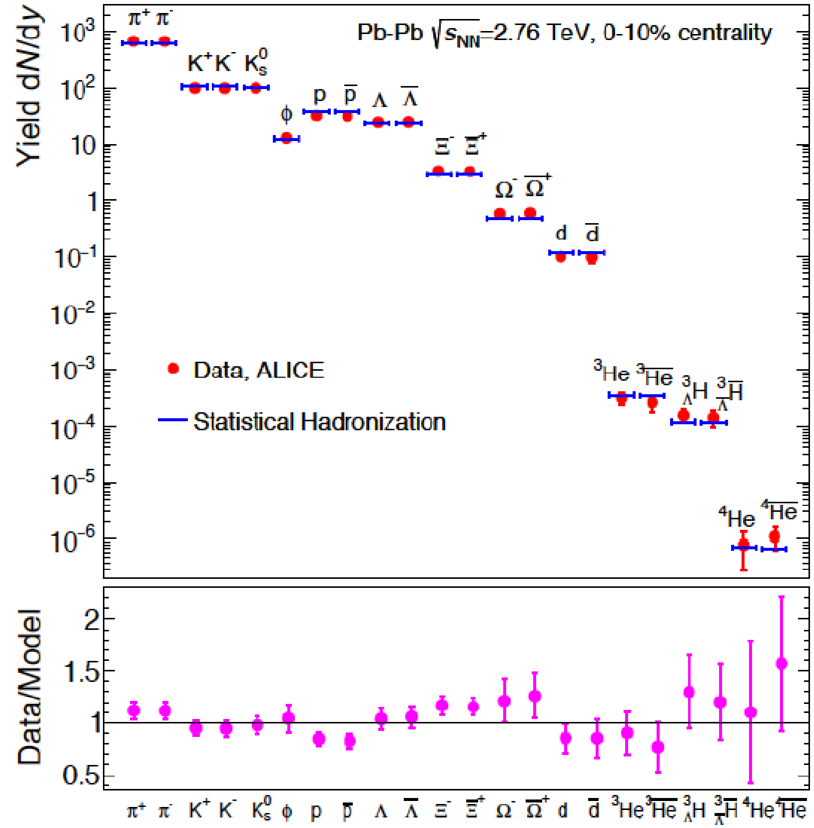
Andronic, Braun-Munzinger, Redlich, Stachel, Nature 561 (2018) 321

data very well reproduced except 2.7 sigma deviation for protons understood in the mean time

S-matrix correction to take account of pion-nucleon phase shifts

Andronic, Braun-Munzinger, Friman, Lo, Redlich, Stachel, arXiv:1808.03102



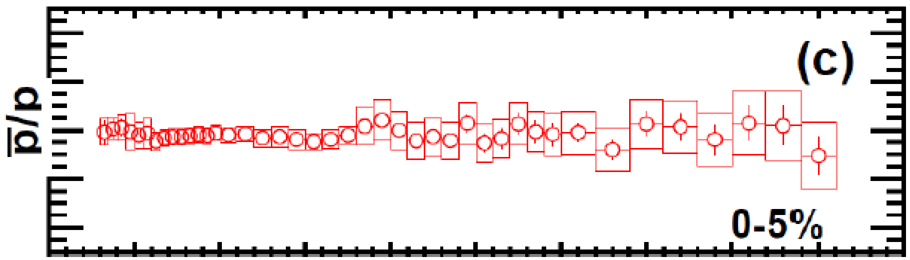


with S-matrix correction

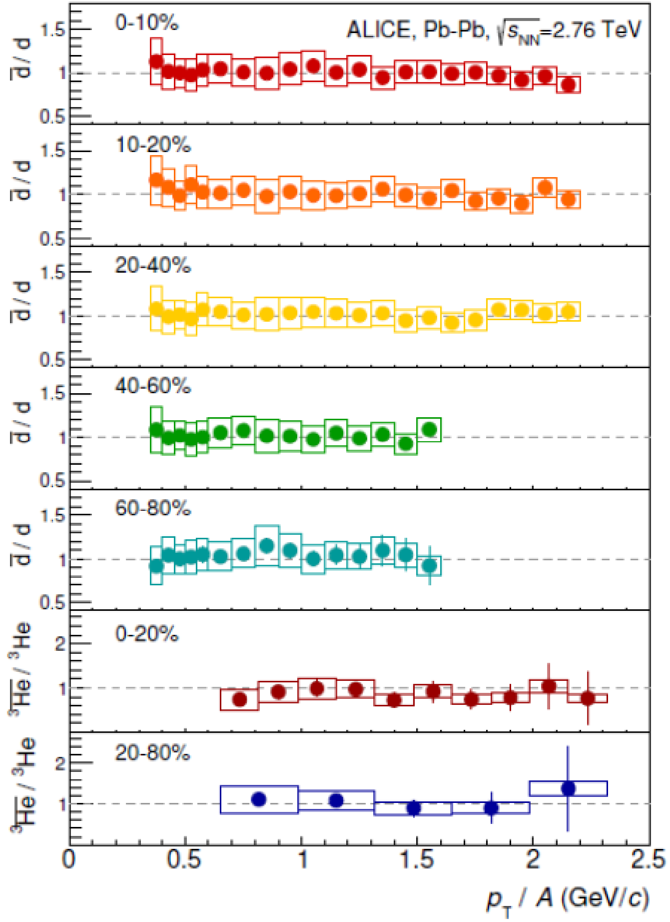
Hadron yields at the LHC – PbPb at 2.76 TeV/nucleon pair

- matter and anti-matter produced in equal proportions at LHC
- consistent with net-baryon free central region, (I_b < 1 MeV)

ALICE, arXiv:1303.0737



ALICE, arXiv:1506.08951



Production of hadrons and (anti-)nuclei at LHC

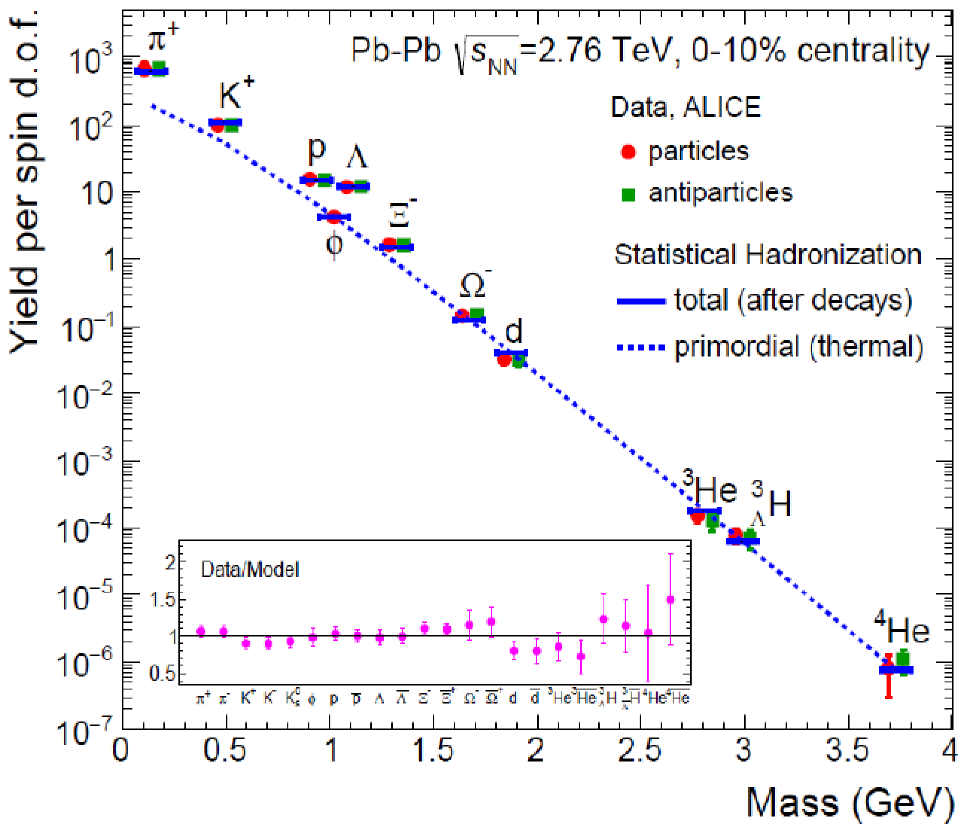
1 free parameter: temperature T

T = 156.5 ± 1.5 MeV

agreement over 9 orders of magnitude with QCD statistical operator prediction (- strong decays need to be added)

- matter and antimatter are formed in equal portions at LHC
- even large very fragile hypernuclei follow the same systematics

A. Andronic, P. Braun-Munzinger, K. Redlich, J. Stachel, Nature 561 (2018) 321



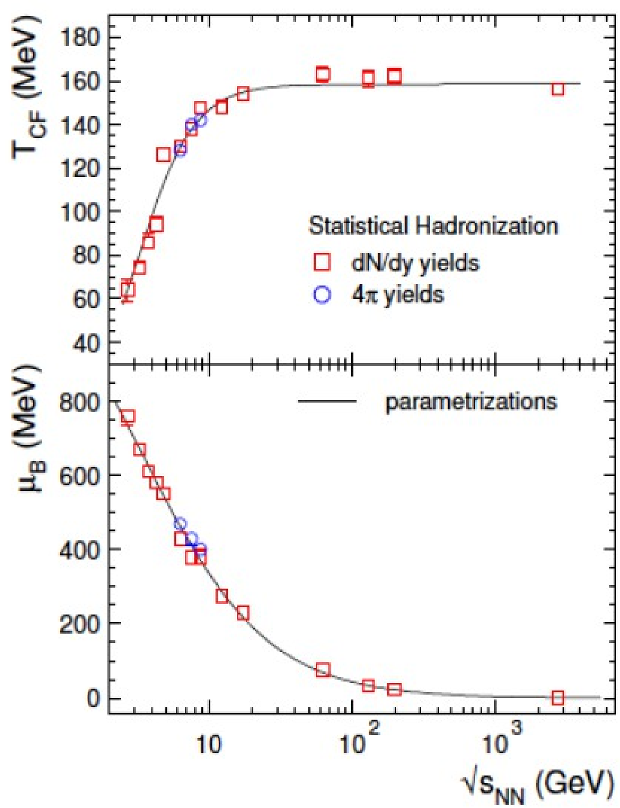
Beam energy dependence of hadron yields from AGS to LHC

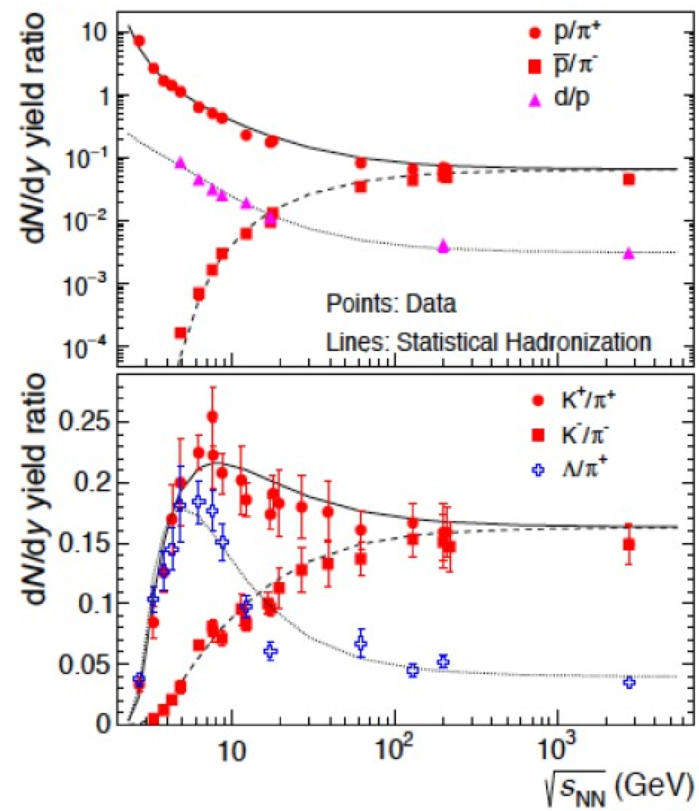
fits work equally well at lower beam energies
following the obtained T and I_b evolution, features of proton/pion, kaon/pion, deuteron/proton and Lambda/pion ratios reproduced in detail

salient features:

a limiting temperature is reached

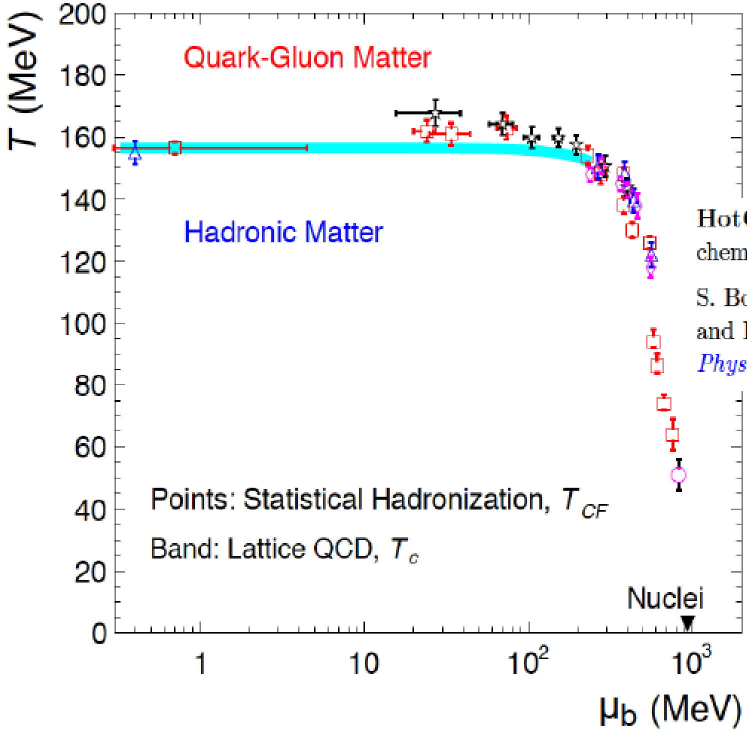
nuclear matter becomes increasingly transparent





the QGP phase diagram, LatticeQCD, and hadron production data

note: data from all coll. at SIS, AGS, SPS, RHIC and LHC
each entry is result of years of experiments, variation of μ_b via variation of cm energy



quantitative agreement of chemical freeze-out parameters with most recent LQCD predictions for baryo-chemical potential < 300 MeV

HotQCD Collaboration, A. Bazavov *et al.*, "Chiral crossover in QCD at zero and non-zero chemical potentials," *Phys. Lett. B* **795** (2019) 15–21, arXiv:1812.08235 [hep-lat].

S. Borsanyi, Z. Fodor, J. N. Guenther, R. Kara, S. D. Katz, P. Parotto, A. Pasztor, C. Ratti, and K. K. Szabo, "QCD Crossover at Finite Chemical Potential from Lattice Simulations," *Phys. Rev. Lett.* **125** no. 5, (2020) 052001, arXiv:2002.02821 [hep-lat].

cross over transition at μ_B = 0 MeV, no experimental confirmation

should the transition be 1st order for large μ_B (large net baryon density)?

then there must be a critical endpoint in the phase diagram

experimental determination of phase boundary at
T_c = 156.6 ± 1.7 (stat.) ± 3 (syst.) MeV and μ_b = 0 MeV
Nature 561 (2018) 321

2-particle collisions not enough – takes about one order of magnitude too long

even when system is initialized in equilibrium at T = 170 MeV, it falls out of equilibrium quickly

<u>simple example:</u>
use a data driven estimate of rate of cooling
near chemical freeze-out (can be explained later) $|\dot{T}/T| = \tau_T^{-1} = (13 \pm 1)\%/\mathrm{fm}$
typical densities at T_{ch} : $\rho_\pi = 0.174/\mathrm{fm}^3(\mathrm{incl.res.}), \rho_K = 0.030/\mathrm{fm}^3 \rho_\Omega = 0.0003/\mathrm{fm}^3$
to maintain equilibrium during 5 MeV temperature drop need a relative rate of change of densities of

$$|\frac{\bar{r}_\Omega}{n_\Omega} - \frac{\bar{r}_K}{n_K}| = \tau_\Omega^{-1} - \tau_K^{-1} = 1.10 - 0.55/\mathrm{fm} = 0.55/\mathrm{fm}$$

so ⁄ density needs to change by 100 % in 1 fm/c
typical reactions with large cross section (10 mb) and rel. velocities of 0.6 give

$$\Omega + \bar{K} \rightarrow \Xi + \pi \qquad \rightarrow \qquad \bar{r}_\Omega/n_\Omega = n_{\bar{K}}\langle v\sigma\rangle = 0.018/\mathrm{fm}$$

$$\pi + \pi \rightarrow K + \bar{K} \quad (\sigma = 3\mathrm{mb}) \qquad \bar{r}_K/n_K = 0.18/\mathrm{fm}$$

much too slow to maintain equlibrium even over drop of T of 5 MeV!
much harder to get into equilibrium!

A possible scenario for rapid equilibration

P. Braun-Munzinger, J. Stachel, C. Wetterich, Phys. Lett. B596 (2004) 61

near phase boundary multiparticle reactions become important
dynamics associated with collective excitations
(key word: critical opalescence at phase transition)
propagation and scattering of these collective excitations expressed in form of multi-hadron scattering

<u>will see:</u> this drives the system into equilibrium very rapidly

Evaluation of multi-strange baryon yield as most challenging test case

consider situation at T_{ch} = 176 MeV first

rate of change of density for n_{in} ingoing and n_{out} outgoing particles

$$r(n_{in}, n_{out}) = \bar{n}(\mathrm{T})^{n_{in}} |\mathcal{M}|^2 \phi$$

with
$$\phi = \prod_{k=1}^{n_{out}} \left(\int \frac{d^3 p_k}{(2\pi)^3 (2E_k)} \right) (2\pi)^4 \delta^4 \left(\sum_k p_k^\mu \right)$$

the phase space factor **Φ** depends on √s

needs to be weighted by the probability *f*(s) that multi-particle scattering occurs at a given value of √s

evaluate numerically in Monte-Carlo using thermal momentum distribution

typical reactions $\Omega + \bar{N} \rightarrow 2\pi + 3K$ $\quad p + \bar{p} \rightarrow 5\pi$

assume cross section equal to the measured one for

$$2\pi + 3K \rightarrow \bar{N} + \Omega$$

at proper energy above threshold, i.e. √s = 3.25 GeV σ = 6.4 mb

compute matrix element and use for rate of

$$r_\Omega = n_\pi^5 (n_K / n_\pi)^3 |\mathcal{M}|^2 \phi$$

Evaluation of multi-strange baryon yield

reaction $2\pi + 3K \rightarrow \bar{N} + \Omega$ leads to

$$r_\Omega = 0.00014 \mathrm{fm}^{-4} \quad \text{or} \quad r_\Omega / n_\Omega = 1/\tau_\Omega = 0.46/\mathrm{fm}$$

can achieve final density starting from only pions and kaons at $\tau = 0$ in an interval of $\tau_\Omega = 2.2$ fm/c

similarly one obtains

for $3\pi + 2K \rightarrow \Xi + \bar{\Lambda}$ that $\tau_\Xi = 0.71\,\mathrm{fm}$

and

for $4\pi + K \rightarrow \Lambda + \bar{N}$ that $\tau_\Lambda = 0.66\,\mathrm{fm}$

why do all particle yields show one common freeze-out T?

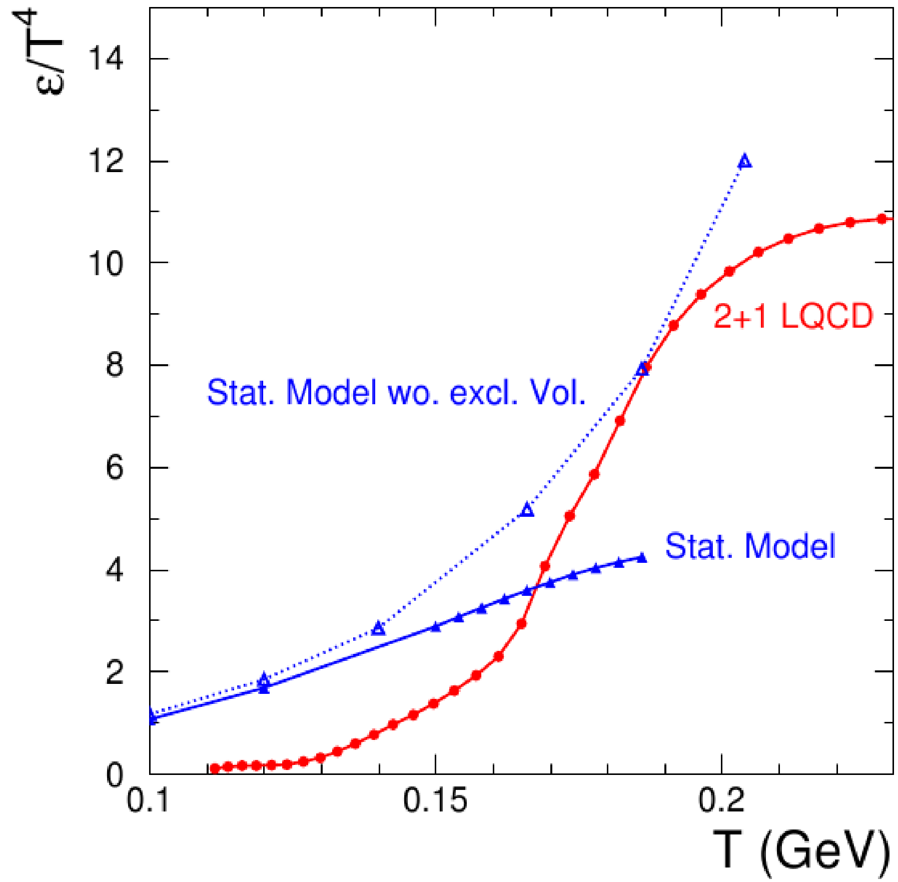
density of particles varies rapidly (factor 2 within 8 MeV) with T near the phase transition due to increase in degrees of freedom.

also: system spends time at T_c

→ volume has to triple (entropy cons.)

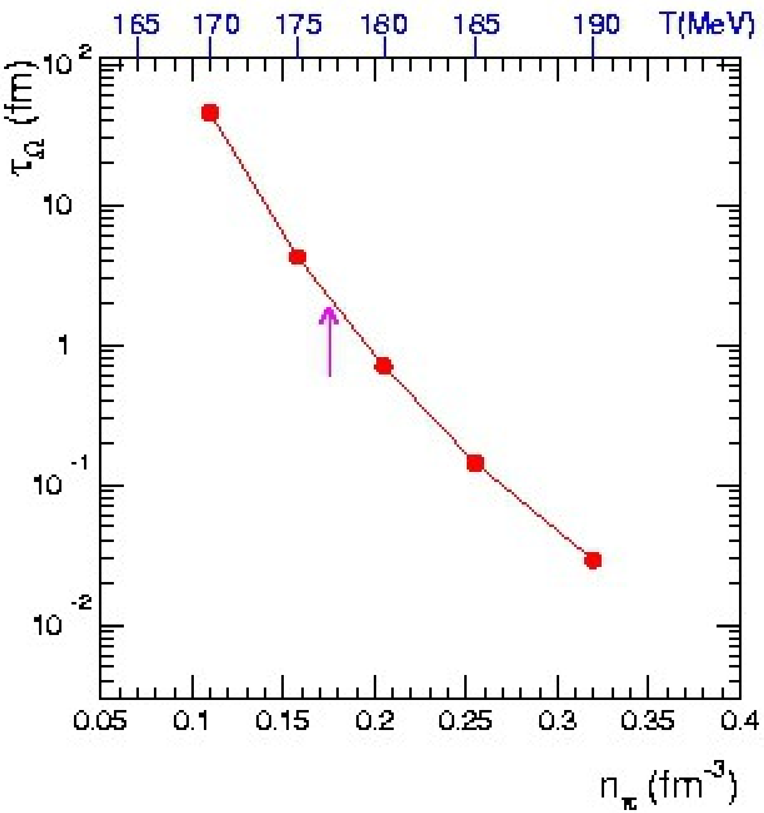
multi-particle collisions are strongly enhanced at high density and lead to chem. equilibrium very near to T_c independent of cross section

all particles freeze out within narrow temperature interval



Lattice QCD by F. Karsch et al. priv. communication

Density dependence of characteristic time for multi-strange baryon production



- near phase transition particle density varies rapidly with T (see previous slide)
- for SPS energies and above reaction such as $2\pi + KKK \rightarrow \Omega\, \bar{N}$ bring multi-strange baryons close to equilibrium rapidly
- in region around T_c equilibration time

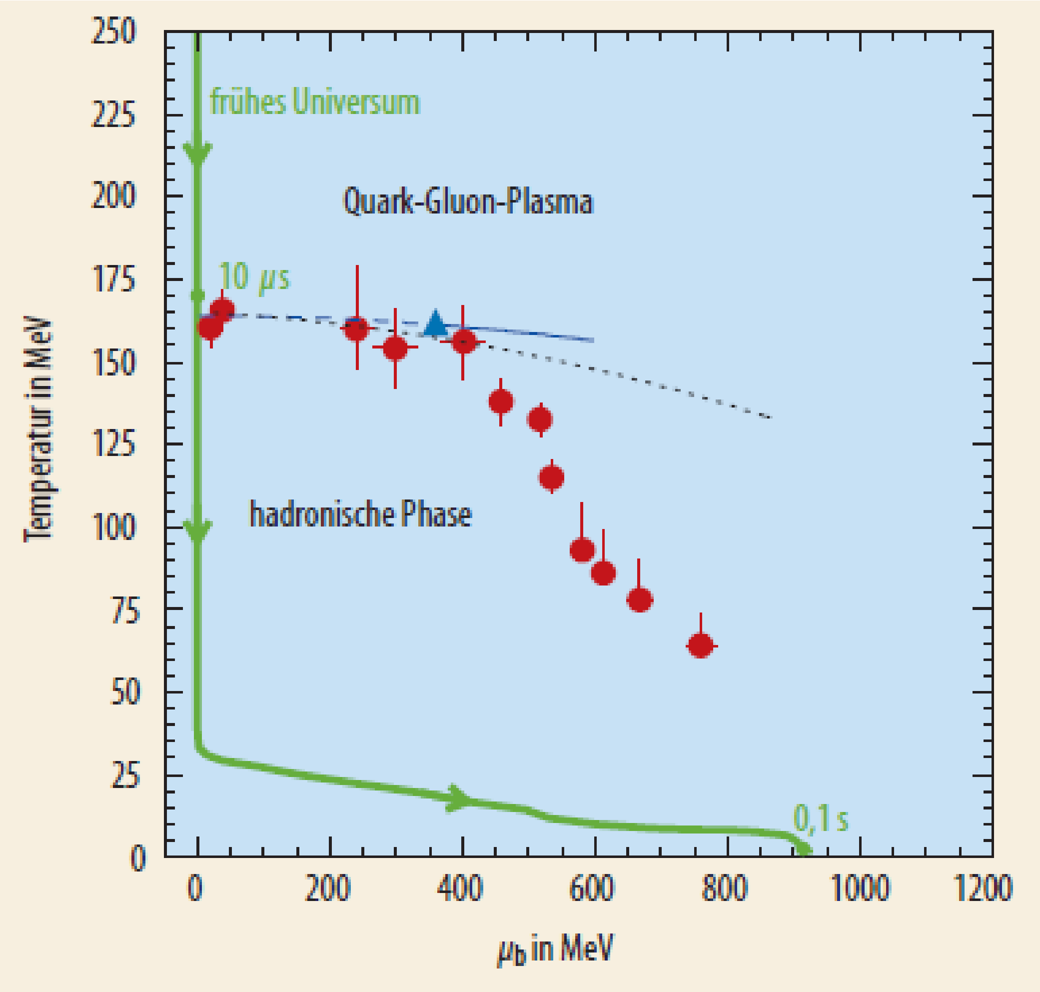
→ τ_Ω is proportional to T^{-60}!

- increase n_π by 1/3: $\tau = 0.2$ fm/c
  (corresponds to increase in T by 8 MeV)
  decrease n_π by 1/3: $\tau = 27$ fm/c

all particles freeze out within a very narrow temperature window due to the extreme temperature sensitivity of multi-particle reactions

in the early universe freeze-out happened after order of 0.1 s

P.Braun-Munzinger, J. Wambach, Rev. Mod. Phys. 81 (2009)1031



isentropic expansion and full chemical equilibrium between hadrons, leptons, photons

plus:
charge neutrality
net lepton number = net baryon number
constant entropy per baryon

75% p
2 10^{-5} d
8 10^{-5} ^{3}He
24.5% ^{4}He
1.5 10^{-10} ^{7}Li

while at LHC factor 300 for every nucleon added

hadro-chemical freeze-out quite different at 0.1 s and $\mu_B \approx 0.9$ GeV