Statistical Methods in Particle Physics / WS 13

Lecture V

Parameter Estimation

Niklaus Berger

Physics Institute, University of Heidelberg

Emmy Noether-Programm

Deutsche Forschungsgemeinschaft DFG

Part V:

Parameter Estimation

5.3. Averaging by the PDG

(See pages 13-16 of the current book)

- Find and select measurements to average:
  Not too old, with uncertainty, not superseded, not questionable

- Perform weighted average, calculate χ^2

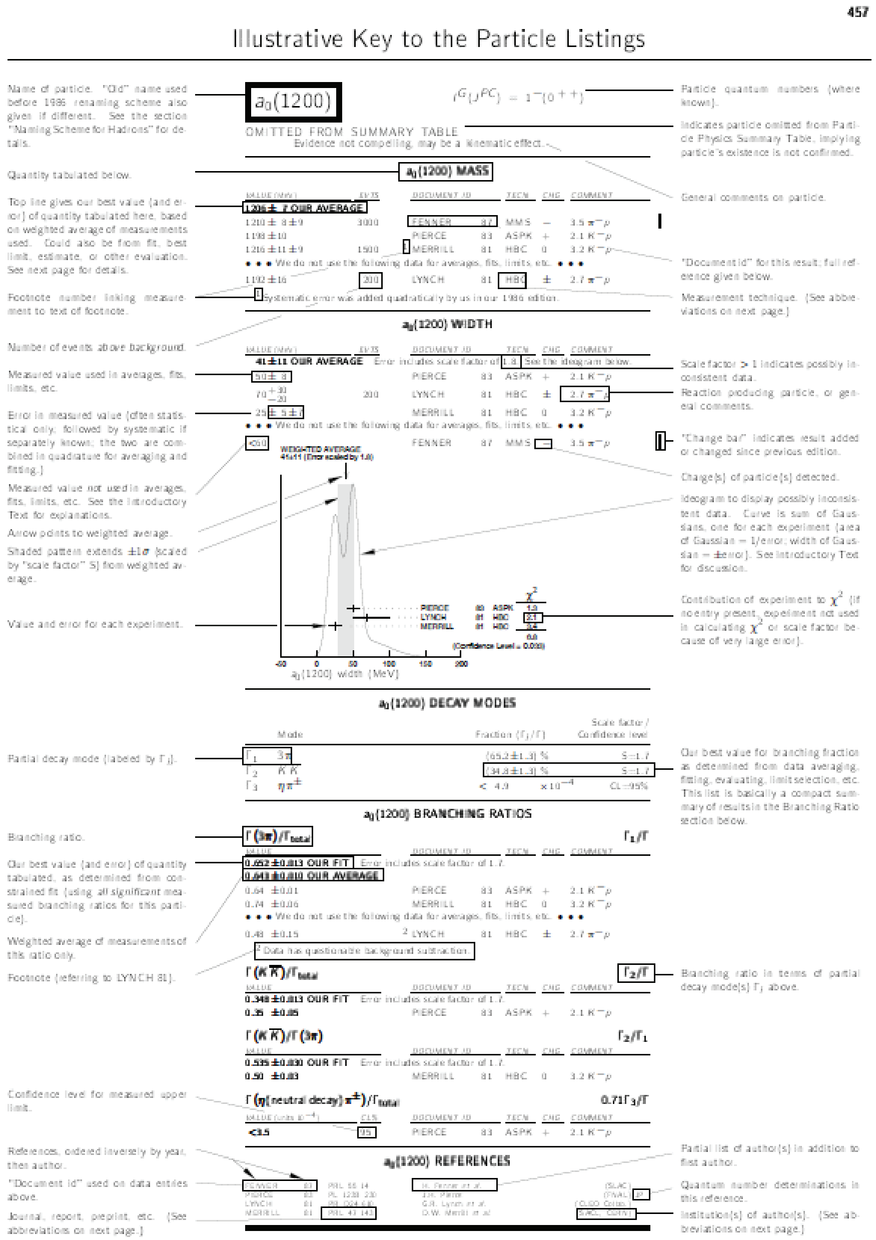
  If $\chi^2/(n-1) < 1$: Use average as is, with standard variance

  If $\chi^2/(n-1) >> 1$: Do not quote an average, measurements are inconsistent

  If $\chi^2/(n-1) > 1$, but not much greater: Scale uncertainty of average by
  $S = (\chi^2/(n-1))^{1/2}$
  If uncertainties vary wildly, calculate S only from those with small uncertainties

  If $\chi^2/(n-1) > 1.25$, show a graph

Reading the PDG Tables (p 457)


457

Illustrative Key to the Particle Listings

Name of particle. "Old" name used before 1986 renaming scheme also given if different. See the section "Naming Scheme for Hadrons" for details.

$a_0(1200)$ — $I^G(J^{PC}) = 1^-(0^{++})$ — Particle quantum numbers (where known).

OMITTED FROM SUMMARY TABLE — Evidence not compelling, may be a kinematic effect. — Indicates particle omitted from Particle Physics Summary Table, implying particle's existence is not confirmed.

Quantity tabulated below. — $a_0(1200)$ MASS

General comments on particle.

Top line gives our best value (and error) of quantity tabulated here, based on weighted average of measurements used. Could also be from fit, best limit, estimate, or other evaluation. See next page for details.

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	CHG	COMMENT
1206 ± 7 OUR AVERAGE						
1210 ± 8 ± 9	3000	FENNER	87	MMS	−	3.5 $\pi^- p$
1198 ± 10		PIERCE	83	ASPK	+	2.1 $K^- p$
1216 ± 11 ± 9	1500	[1] MERRILL	81	HBC	0	3.2 $K^- p$
• • • We do not use the following data for averages, fits, limits, etc. • • •						
1192 ± 16	200	LYNCH	81	HBC	±	2.7 $\pi^- p$

"Document id" for this result; full reference given below.

Measurement technique. (See abbreviations on next page.)

Footnote number linking measurement to text of footnote. — [1] Systematic error was added quadratically by us in our 1986 edition.

Number of events above background.

$a_0(1200)$ WIDTH

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	CHG	COMMENT
41 ± 11 OUR AVERAGE	Error includes scale factor of 1.8. See the ideogram below.					
50 ± 8		PIERCE	83	ASPK	+	2.1 $K^- p$
70^{+30}_{-20}	200	LYNCH	81	HBC	±	2.7 $\pi^- p$
25 ± 5 ± 7		MERRILL	81	HBC	0	3.2 $K^- p$
• • • We do not use the following data for averages, fits, limits, etc. • • •						
<50		FENNER	87	MMS	−	3.5 $\pi^- p$

Measured value used in averages, fits, limits, etc.

Scale factor > 1 indicates possibly inconsistent data.

Reaction producing particle, or general comments.

Error in measured value (often statistical only, followed by systematic if separately known; the two are combined in quadrature for averaging and fitting.)

"Change bar" indicates result added or changed since previous edition.

Charge(s) of particle(s) detected.

Measured value not used in averages, fits, limits, etc. See the Introductory Text for explanations.

Arrow points to weighted average.

Shaded pattern extends ±1σ (scaled by "scale factor" S) from weighted average.

Ideogram to display possibly inconsistent data. Curve is sum of Gaussians, one for each experiment (area of Gaussian = 1/error; width of Gaussian = ±error). See Introductory Text for discussion.

WEIGHTED AVERAGE 41±11 (Error scaled by 1.8)

			χ^2
PIERCE	83	ASPK	1.0
LYNCH	81	HBC	2.1
MERRILL	81	HBC	3.4
			6.6

(Confidence Level = 0.036)

$a_0(1200)$ width (MeV): −50, 0, 50, 100, 150, 200

Contribution of experiment to χ^2 (if no entry present, experiment not used in calculating χ^2 or scale factor because of very large error).

Value and error for each experiment.

$a_0(1200)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Scale factor / Confidence level
Γ_1	3π	(65.2 ± 1.3) %	S=1.7
Γ_2	$K\bar{K}$	(34.8 ± 1.3) %	S=1.7
Γ_3	$\eta\pi^\pm$	< 4.9 × 10^{-4}	CL=95%

Partial decay mode (labeled by Γ_i).

Our best value for branching fraction as determined from data averaging, fitting, evaluating, limit selection, etc. This list is basically a compact summary of results in the Branching Ratio section below.

$a_0(1200)$ BRANCHING RATIOS

Branching ratio.

$\Gamma(3\pi)/\Gamma_{total}$ — Γ_1/Γ

Our best value (and error) of quantity tabulated, as determined from constrained fit (using all significant measured branching ratios for this particle).

Weighted average of measurements of this ratio only.

VALUE		DOCUMENT ID		TECN	CHG	COMMENT
0.652 ± 0.013 OUR FIT	Error includes scale factor of 1.7.					
0.643 ± 0.010 OUR AVERAGE						
0.64 ± 0.01		PIERCE	83	ASPK	+	2.1 $K^- p$
0.74 ± 0.06		MERRILL	81	HBC	0	3.2 $K^- p$
• • • We do not use the following data for averages, fits, limits, etc. • • •						
0.48 ± 0.15		[2] LYNCH	81	HBC	±	2.7 $\pi^- p$

Footnote (referring to LYNCH 81). — [2] Data has questionable background subtraction.

$\Gamma(K\bar{K})/\Gamma_{total}$ — Γ_2/Γ — Branching ratio in terms of partial decay mode(s) Γ_i above.

VALUE		DOCUMENT ID		TECN	CHG	COMMENT
0.348 ± 0.013 OUR FIT	Error includes scale factor of 1.7.					
0.35 ± 0.05		PIERCE	83	ASPK	+	2.1 $K^- p$

$\Gamma(K\bar{K})/\Gamma(3\pi)$ — Γ_2/Γ_1

VALUE		DOCUMENT ID		TECN	CHG	COMMENT
0.535 ± 0.030 OUR FIT	Error includes scale factor of 1.7.					
0.50 ± 0.03		MERRILL	81	HBC	0	3.2 $K^- p$

Confidence level for measured upper limit.

$\Gamma(\eta(\text{neutral decay})\pi^\pm)/\Gamma_{total}$ — $0.71\Gamma_3/\Gamma$

VALUE (units 10^{-4})	CL%	DOCUMENT ID		TECN	CHG	COMMENT
<3.5	95	PIERCE	83	ASPK	+	2.1 $K^- p$

References, ordered inversely by year, then author.

"Document id" used on data entries above.

Journal, report, preprint, etc. (See abbreviations on next page.)

$a_0(1200)$ REFERENCES

FENNER	87	PRL 55 14	H. Fenner et al.	(SLAC)
PIERCE	83	PL 123B 230	J.H. Pierce	(FNAL)
LYNCH	81	PR D24 610	G.R. Lynch et al.	(CIT, CERN)
MERRILL	81	PRL 47 143	D.W. Merrill et al.	(SLAC, CERN)

Partial list of author(s) in addition to first author.

Quantum number determinations in this reference.

Institution(s) of author(s). (See abbreviations on next page.)

Reading the PDG Tables (p 457)

Name of particle. "Old" name used before 1986 renaming scheme also given if different. See the section "Naming Scheme for Hadrons" for details.

$a_0(1200)$

$I^G(J^{PC}) = 1^-(0^{++})$

Particle quantum numbers (where known).

OMITTED FROM SUMMARY TABLE

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25 ± 5 ± 7		MERRILL	81	HBC	0	3.2 $K^- p$
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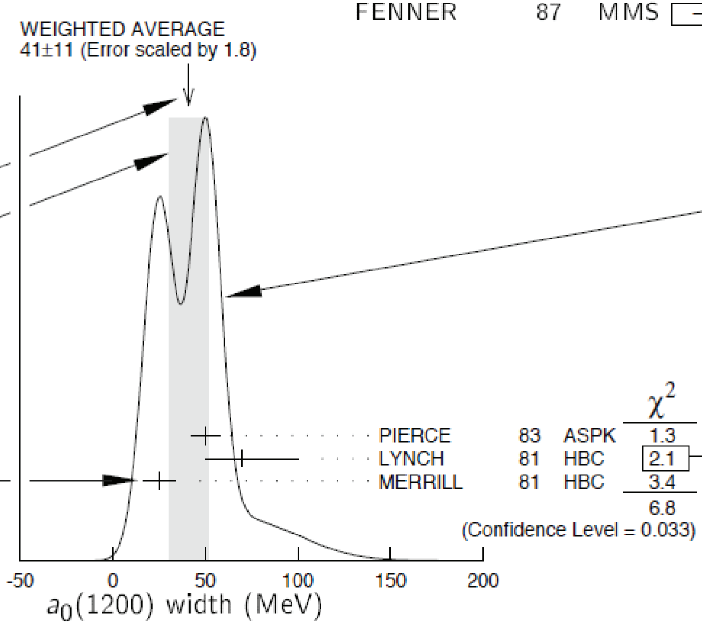
<60		FENNER	87	MMS	−	3.5 $\pi^- p$



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Reading the PDG Tables (p 457)

Partial decay mode (labeled by Γ_i).

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	Mode	Fraction (Γ_i/Γ)	Scale factor/ Confidence level
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Branching ratio.

Our best value (and error) of quantity tabulated, as determined from constrained fit (using *all significant* measured branching ratios for this particle).

Weighted average of measurements of this ratio only.

Footnote (referring to LYNCH 81).

Confidence level for measured upper limit.

Branching ratio in terms of partial decay mode(s) Γ_i above.

$a_0(1200)$ BRANCHING RATIOS

$\Gamma(3\pi)/\Gamma_{total}$ — Γ_1/Γ

VALUE	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
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$\Gamma(\eta(\text{neutral decay})\pi^{\pm})/\Gamma_{total}$ — $0.71\Gamma_3/\Gamma$

VALUE (units 10^{-4})	*CL%*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
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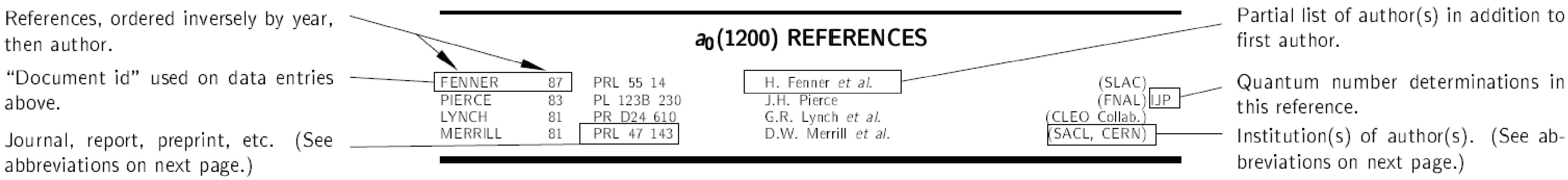
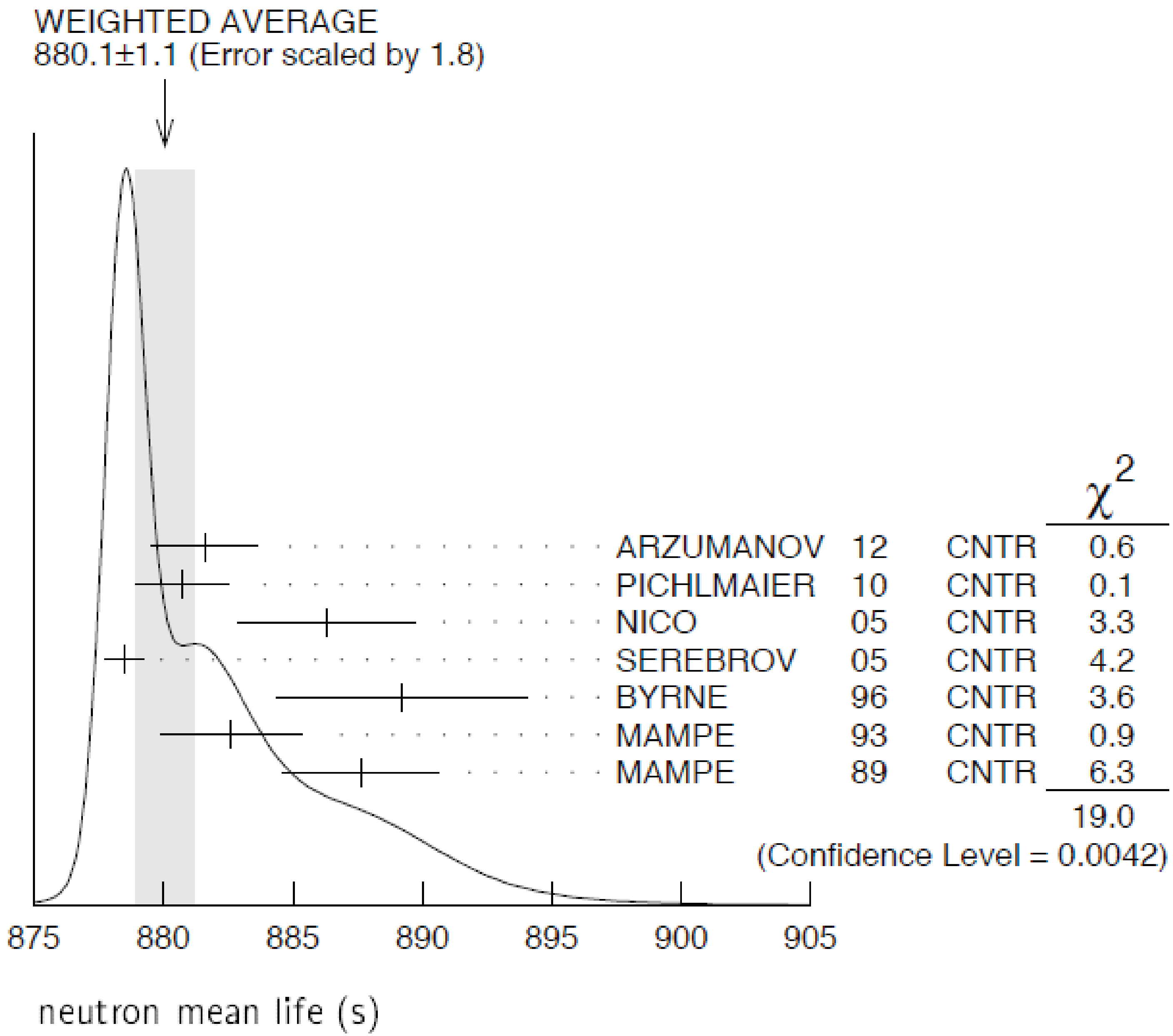


PDG: Neutron lifetime



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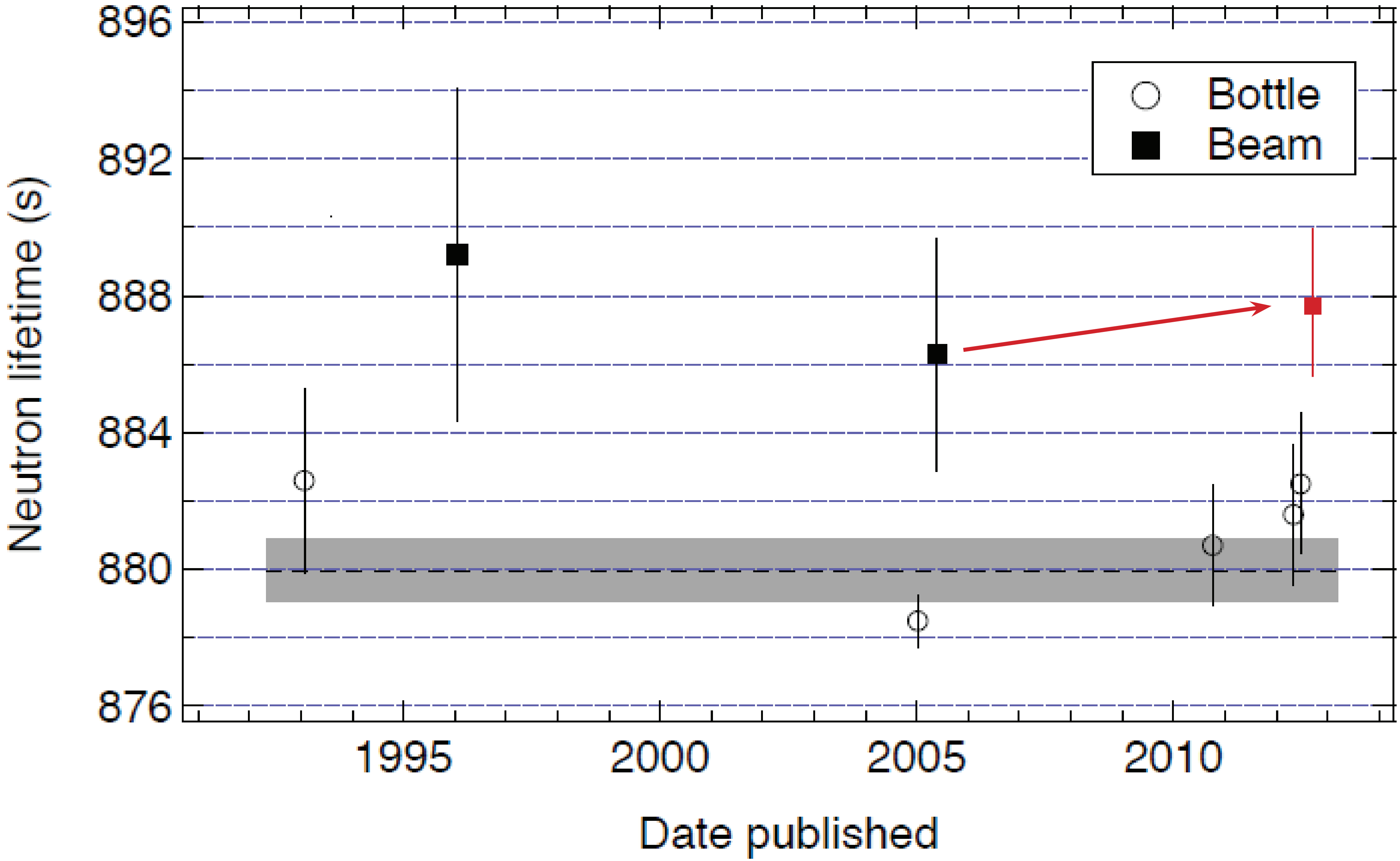

5.4. Weighted averages with correlations

Example: Suppose we measure the length of a stick twice and would like to combine the measurements

- Measurements are x_1 = 10 cm with an uncertainty σ_1 of 0.1 cm
  x_2 = 11 cm with an uncertainty σ_2 of 0.5 cm
- If the two measurements are uncorrelated, then

$$x_{mean} = (x_1/\sigma_1^2 + x_2/\sigma_2^2)/(1/\sigma_1^2 + 1/\sigma_2^2)$$, the weighted mean (10.038 ± 0.098) cm

5.4. Weighted averages with correlations

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- If not, then…
  - Anything between 10 and 11 cm
  - 10 cm, the better measurement
  - 10.5 cm, the arithmetic mean
  - Still the weighted mean 10.038 cm
  - Something else entirely

?