Statistical Methods in Particle Physics / WS 13

Lecture X
Confidence Intervals & Limits

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Part XI:

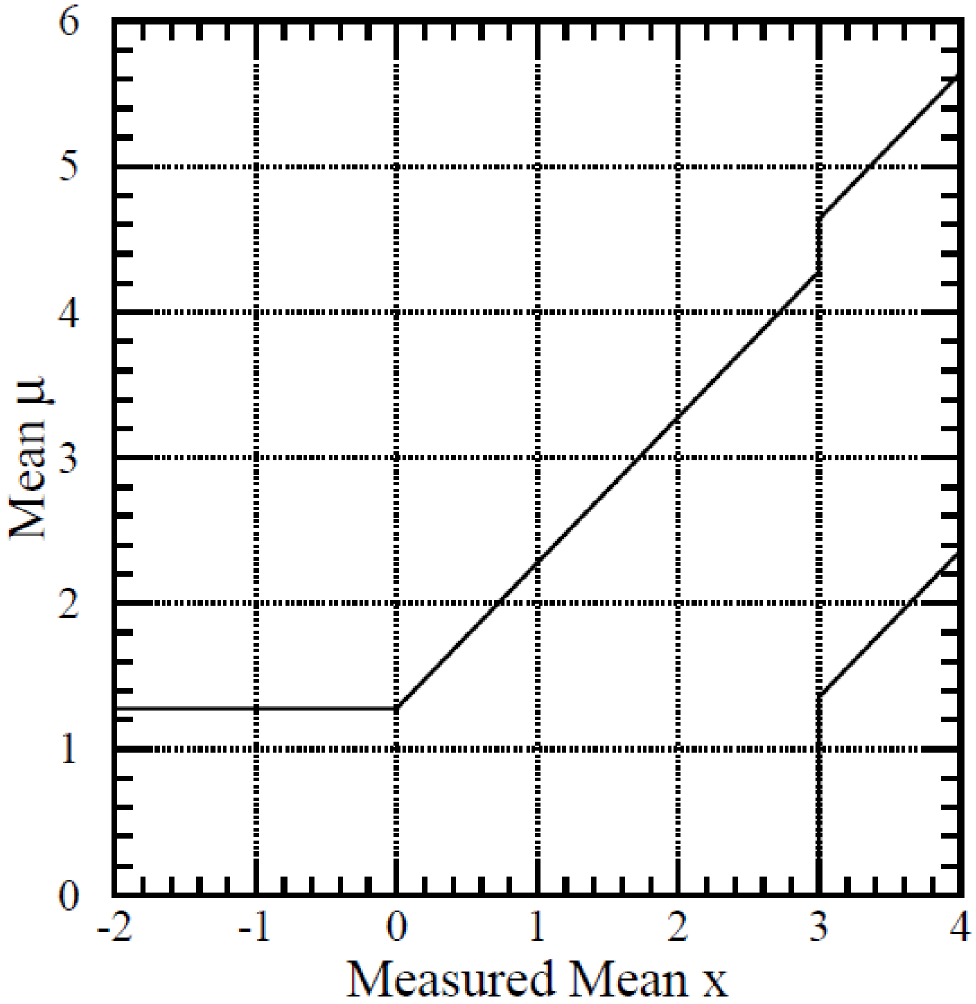
Confidence Intervals and Limits

11.7. Issues with confidence intervals

- What to do close to a boundary?
- When should I switch from one- to two sided? (if based on data, this messes up confidence intervals; flip-flopping)
- If Bayesian, what about the prior?

- Frequentist solution: Feldman-Cousins

  Read: Feldman & Cousins, arxiv:physics/9711021



90% confidence limits close to 0
(From Feldman & Cousins, arxiv:physics/9711021)

11.8. Feldman-Cousins Confidence Intervals

Example:

- Poisson variables
- Expected background $\nu_B = 3.0$
- True signal μ_T, signal hypothesis μ
- Probability (likelihood) for n observed events given μ: $P(n|\mu)$
- μ_{best} is the physically allowed μ which maximizes $P(n|\mu)$, here

$$\mu_{best} = \max(0, n\text{-}b)$$

Feldman-Cousins Confidence Intervals

Example:

- Poisson variables
- Expected background $\nu_B = 3.0$
- True signal μ_T, signal hypothesis $\mu = 0.5$
- Probability (likelihood) for n observed events given μ: $P(n|\mu)$
- μ_{best} is the physically allowed μ which maximizes $P(n|\mu)$, here

$$\mu_{best} = \max(0, n-b)$$

Ordering principle:

- Sort n's by the ratio

$$R = P(n|\mu)/\,P(n|\mu_{best})$$

- Keep adding n's to the confidence interval in decreasing order of R until the sum of the $P(n|\mu)$ exceeds the desired C.L.
- Repeat for "all" μ's - obtain confidence band for a measured n

Feldman-Cousins Confidence Intervals

TABLE I. Illustrative calculations in the confidence belt construction for signal mean μ in the presence of known mean background $b = 3.0$. Here we find the acceptance interval for $\mu = 0.5$.

n	$P(n\|\mu)$	μ_{best}	$P(n\|\mu_{\mathrm{best}})$	R	rank	U.L.	central
0	0.030	0.	0.050	0.607	6		
1	0.106	0.	0.149	0.708	5	√	√
2	0.185	0.	0.224	0.826	3	√	√
3	0.216	0.	0.224	0.963	2	√	√
4	0.189	1.	0.195	0.966	1	√	√
5	0.132	2.	0.175	0.753	4	√	√
6	0.077	3.	0.161	0.480	7	√	√
7	0.039	4.	0.149	0.259		√	√
8	0.017	5.	0.140	0.121		√	
9	0.007	6.	0.132	0.050		√	
10	0.002	7.	0.125	0.018		√	
11	0.001	8.	0.119	0.006		√	

(From Feldman & Cousins, arxiv:physics/9711021)

Feldman-Cousins Confidence Intervals

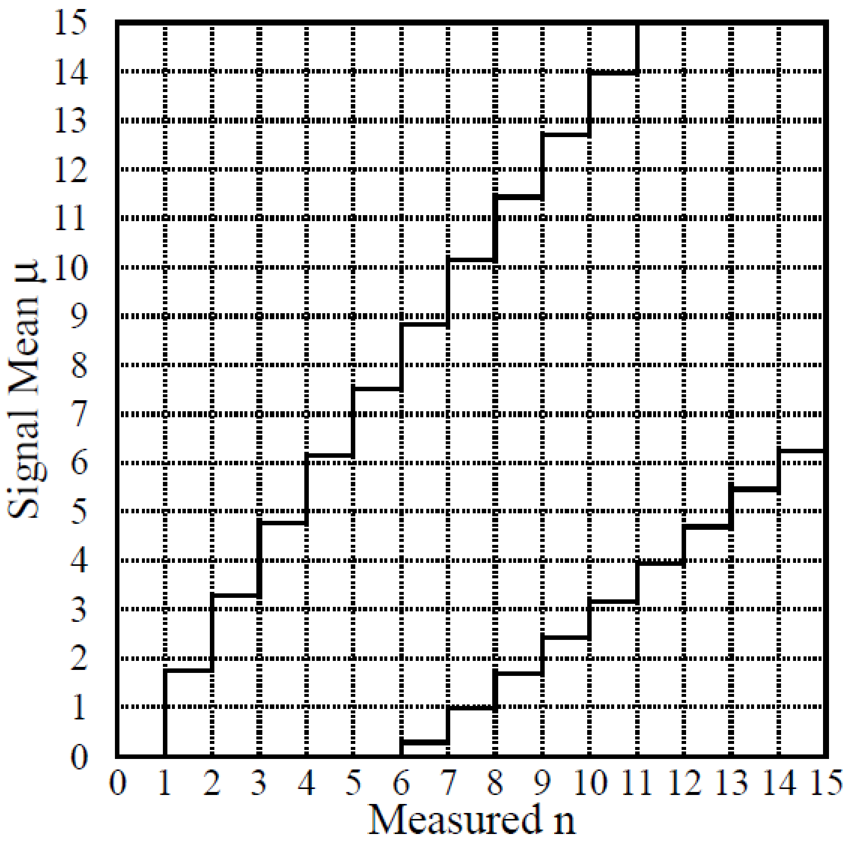


FIG. 6. Standard confidence belt for 90% C.L. central confidence intervals, for unknown Poisson signal mean μ in the presence of Poisson background with known mean $b = 3.0$.

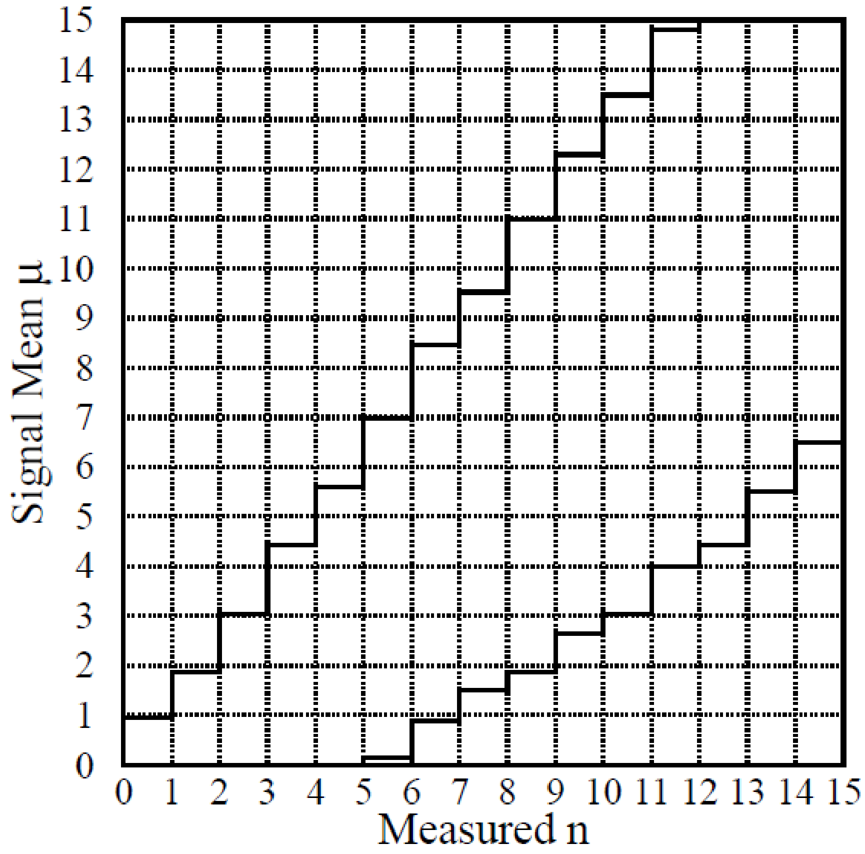


FIG. 7. Confidence belt based on our ordering principle, for 90% C.L. confidence intervals for unknown Poisson signal mean μ in the presence of Poisson background with known mean $b = 3.0$.

(From Feldman & Cousins, arxiv:physics/9711021)

… and for a Gaussian with Boundary at the Origin

B. Gaussian with Boundary at Origin

It is straightforward to apply our ordering principle to the other troublesome example of Sec. III, the case of a Gaussian resolution function (Eq. 3.1) for μ, when μ is physically bounded to non-negative values. In analogy with the Poisson case, for a particular x, we let μ_{best} be the physically allowed value of μ for which $P(x|\mu)$ is maximum. Then $\mu_{\text{best}} = \max(0, x)$, and

$$P(x|\mu_{\text{best}}) = \begin{cases} 1/\sqrt{2\pi}, & x \geq 0 \\ \exp(-x^2/2)/\sqrt{2\pi}, & x < 0. \end{cases} \tag{4.2}$$

We then compute R in analogy to Eq. 4.1, using Eqs. 3.1 and 4.2:

$$R(x) = \frac{P(x|\mu)}{P(x|\mu_{\text{best}})} = \begin{cases} \exp(-(x-\mu)^2/2), & x \geq 0 \\ \exp(x\mu - \mu^2/2), & x < 0. \end{cases} \tag{4.3}$$

During our Neyman construction of confidence intervals, R determines the order in which values of x are added to the acceptance region at a particular value of μ. In practice, this means that for a given value of μ, one finds the interval $[x_1, x_2]$ such that $R(x_1) = R(x_2)$ and

$$\int_{x_1}^{x_2} P(x|\mu)dx = \alpha. \tag{4.4}$$

(From Feldman & Cousins, arxiv:physics/9711021)

Problem solved?

Yes…

- … the interval covers only physical values
- … we have coverage
- … there is no flip-flopping
- … no (subjective) prior is needed

but…

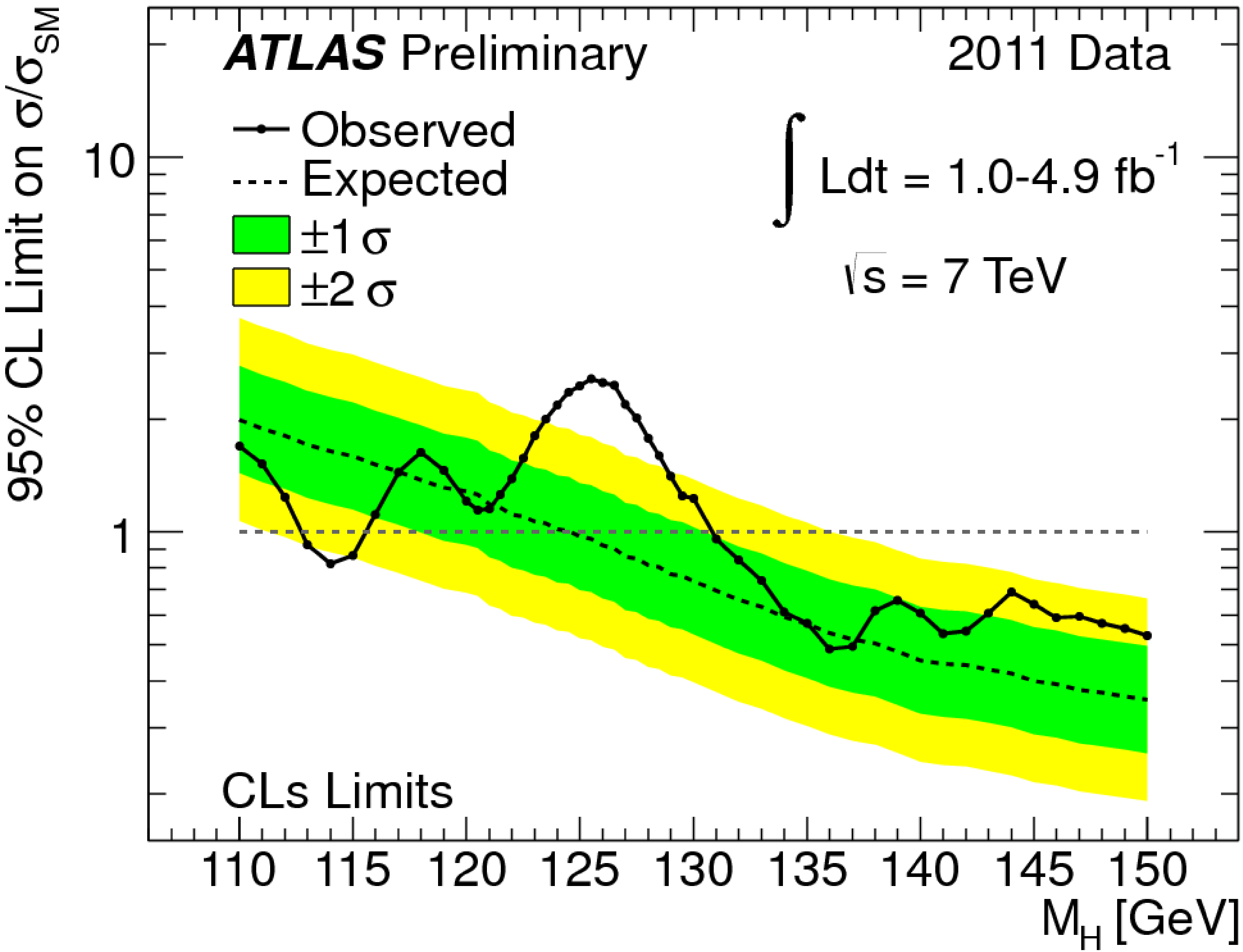
VI. THE PROBLEM OF FEWER EVENTS THAN EXPECTED BACKGROUND

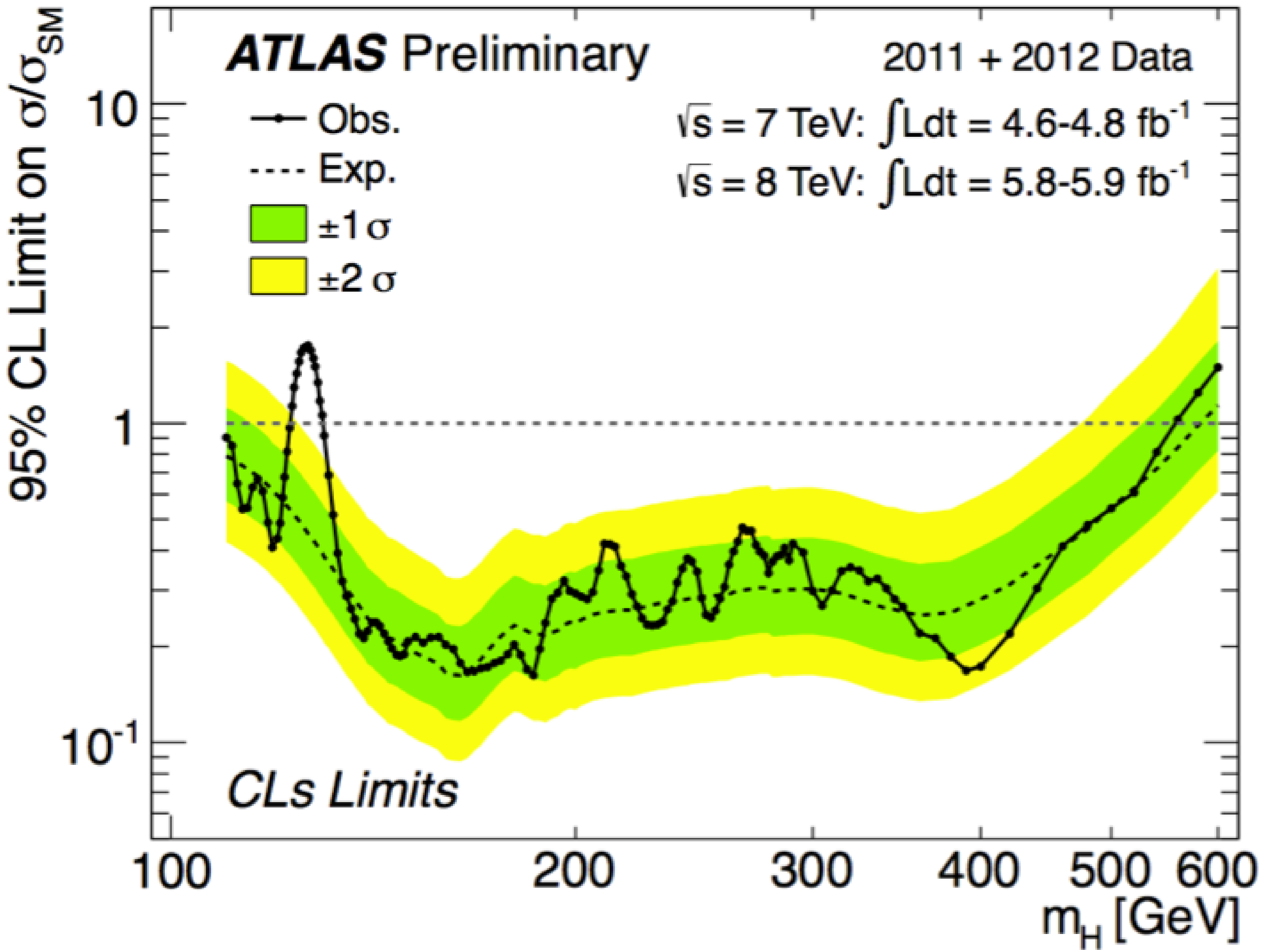
We started this investigation to solve the problem in classical statistics in which an experiment which measures significantly fewer events than are expected from backgrounds will report a meaningless or unphysical result. While we have solved that problem, our solution still yields results that are bothersome to some in that an experiment that measures fewer events than expected from backgrounds will report a lower upper limit than an identical experiment that measures a number of events equal to that expected from background. This seems particularly troublesome in the case in which the experiment has no observed events. Why should an experiment claim credit for expected backgrounds, when it is clear, in that particular experiment, there were none? Or why should a well designed experiment which has no background and observes no events be forced to report a higher upper limit than a less well designed experiment which expects backgrounds, but, by chance, observes none?

11.9. The CL_S method

Suggested reading:

A.L. Read: Modified Frequentist Analysis of Search Results (The CL_s Method), J. Phys. G: Nucl. Part. Phys. 28 2693


ATLAS Preliminary
2011 Data
Observed
Expected
±1 σ
±2 σ
∫ Ldt = 1.0-4.9 fb^-1
√s = 7 TeV
CLs Limits
95% CL Limit on σ/σ_SM
10
1
110
115
120
125
130
135
140
145
150
M_H [GeV]


ATLAS Preliminary
2011 + 2012 Data
√s = 7 TeV: ∫Ldt = 4.6-4.8 fb-1
√s = 8 TeV: ∫Ldt = 5.8-5.9 fb-1
Obs.
Exp.
±1 σ
±2 σ
CLs Limits
95% CL Limit on σ/σSM
10
1
10-1
100
200
300
400
500
600
mH [GeV]

Part XII:

Merry Christmas!