Exercise 11: Extended likelihood fits, limits

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Please send your solutions to nberger@physi.uni-heidelberg.de until 16. 1. 2012, 12:00. Put your answers in an email (subject line *SMIPP:Exercise11*).

1. **Extended maximum likelihood** The usual maximum likelihood method does not allow for determining the absolute normalisation. This can be overcome by using the extended likelihood (for n events, coming from a Poisson distribution with mean ν):

$$L(\nu, \theta) = \frac{\nu^n}{n!} e^{-\nu} \prod_{i=1}^{n} f(x_i; \theta), \tag{1}$$

where θ are the model parameters and x_i the observed values. The logarithm of the likelihood then becomes

$$\ln L(\nu, \theta) = -\nu + \sum_{i=1}^{n} \ln(\nu f(x_i; \theta)) + C, \tag{2}$$

where ν can either be a free parameter or a function of θ. C is independent of ν and θ and can thus be ignored in an optimization.

Building on exercise 10, perform an extended maximum likelihood fit using minuit first to a Gaussian distribution and then to a broad (background) Gaussian distribution with a narrow (signal) Gaussian distribution on top.

Ensure that the fit reproduces your chosen imput values.
(Attach code)

2. **The $\mathbf{CL_s}$ method** Read *Modified frequentist analysis of search reslts* by A.L. Read, available at
http://cdsweb.cern.ch/record/451614/files/p81.ps.gz?version=1.
This is the method used to present limits in searches by both the LEP and the LHC experiments. Try to answer the following questions:

   - What is *coverage*?
   - What is *flip-flopping*?
   - In this context, what is *conservative*?
   - What happens if the background estimate is too large?
   - What if is too small?
   - Would you consider yourself a Bayesian or a Frequentist? Why?

3. **The CL_s method in root** The CL_s method is implemented in the root class `TLimit`. Assume you do a search in a simple counting experiment, where you expect 100 background events. Using `TLimit`, answer the following questions:

   - If you expect 10 signal events and observe a total of 90, 100, 110 events, what are your expected and observed confidence levels for the signal plus background hypothesis using the CL_s method?
   - If you observe 100 events with the same background expectation, what signal size can you exclude at the 95% confidence level?
   - How much would you have to reduce the background to be able to exclude a 10 event signal at the 95% confidence level?

   (Attach code)