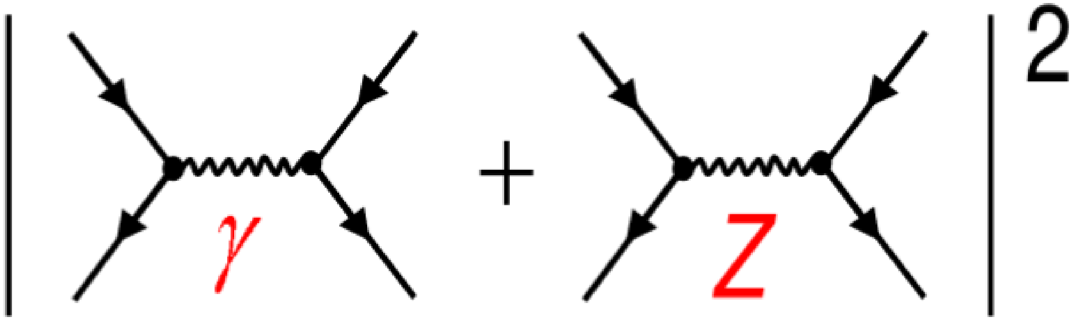
Cross section for $e^+ e^- \rightarrow \gamma / Z \rightarrow f\bar{f}$

$|M|^2 =$



for $e^+ e^- \rightarrow \mu^+ \mu^-$

$$M_\gamma = -ie^2 (\bar{u}_\mu \gamma^\nu v_\mu) \frac{g_{\rho\nu}}{q^2} (\bar{v}_e \gamma^\rho u_e)$$

$$M_Z = -i \frac{g^2}{\cos^2\theta_W} \left[\bar{u}_\mu \gamma^\nu \frac{1}{2} (g_V^\mu - g_A^\mu \gamma^5) v_\mu \right] \underbrace{\frac{g_{\rho\nu} - q_\rho q_\nu / M_Z^2}{(q^2 - M_Z^2) + iM_Z \Gamma_Z}} \left[\bar{v}_e \gamma^\rho \frac{1}{2} (g_V^e - g_A^e \gamma^5) u_e \right]$$

Z propagator considering a finite Z width (real particle)

Breit-Wigner Resonance is very general described:

$$\sigma(s) = 12\pi \frac{\Gamma_e \Gamma_\mu}{M_Z^2} \frac{s}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}$$

Cross sections and width can be calculated within the Standard Model

$$\Gamma_f = \frac{\alpha M_Z}{12\sin^2\theta_W \cos^2\theta_W}[(g_V^f)^2 + (g_A^f)^2]$$

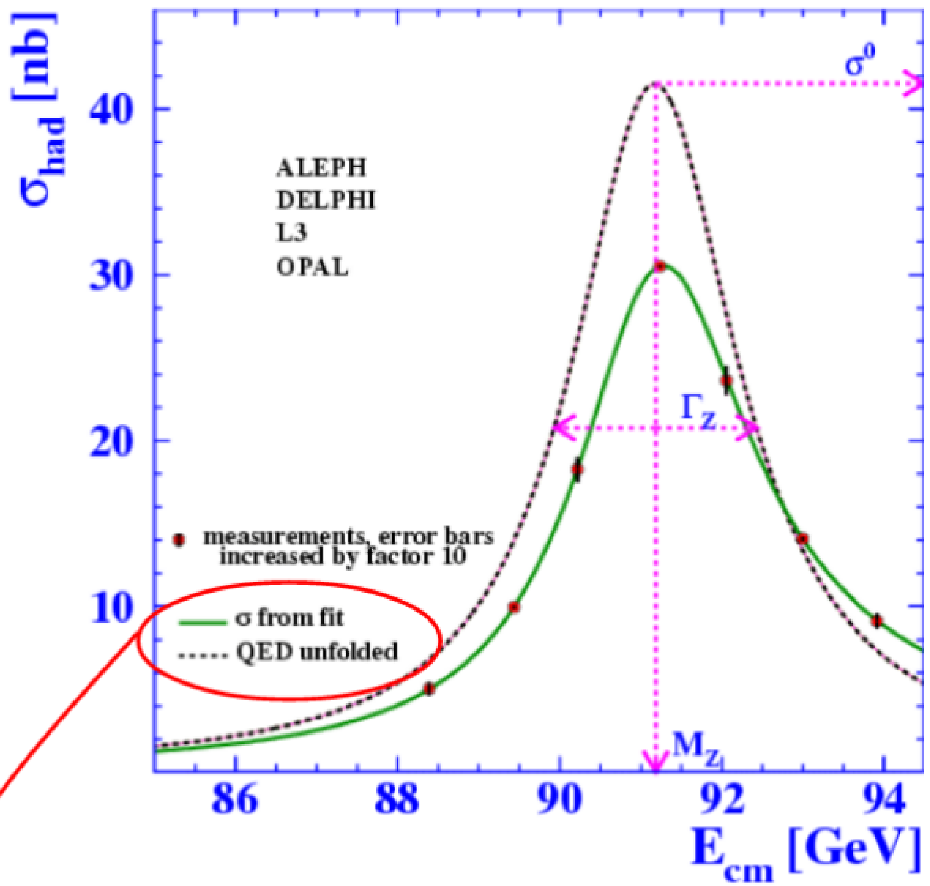
$$\Gamma_Z = \sum_i \Gamma_i$$

$$BR(Z \rightarrow ii) = \frac{\Gamma_i}{\Gamma_Z}$$

	g_V	g_A
ν	$\frac{1}{2}$	$\frac{1}{2}$
ℓ^-	$-\frac{1}{2}+2\sin^2\theta_W$	$-\frac{1}{2}$
$u-quark$	$+\frac{1}{2}-\frac{4}{3}\sin^2\theta_W$	$\frac{1}{2}$
$d-quark$	$-\frac{1}{2}+\frac{2}{3}\sin^2\theta_W$	$-\frac{1}{2}$

$$\sin^2\theta_W \sim 0.22$$

2.2 Measurement of the Z lineshape



Z Resonance curve:

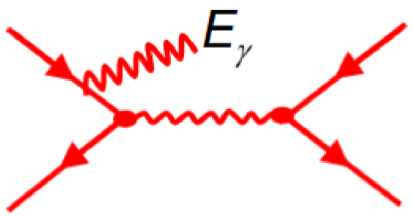
$$\sigma(s) = 12\pi \frac{\Gamma_e \Gamma_\mu}{M_Z^2} \cdot \frac{s}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

Peak: $$\sigma_0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_\mu}{\Gamma_Z^2}$$

- Resonance position → M_Z
- Height → $\Gamma_e \Gamma_\mu$
- Width → Γ_Z

Initial state Bremsstrahlung corrections

$$\sigma_{ff(\gamma)} = \int_{4m_f^2/s}^{1} G(z) \sigma_{ff}^0(zs)\, dz \qquad z = 1 - \frac{2E_\gamma}{\sqrt{s}}$$



Leads to a deformation of the resonance: large (30%) effect !

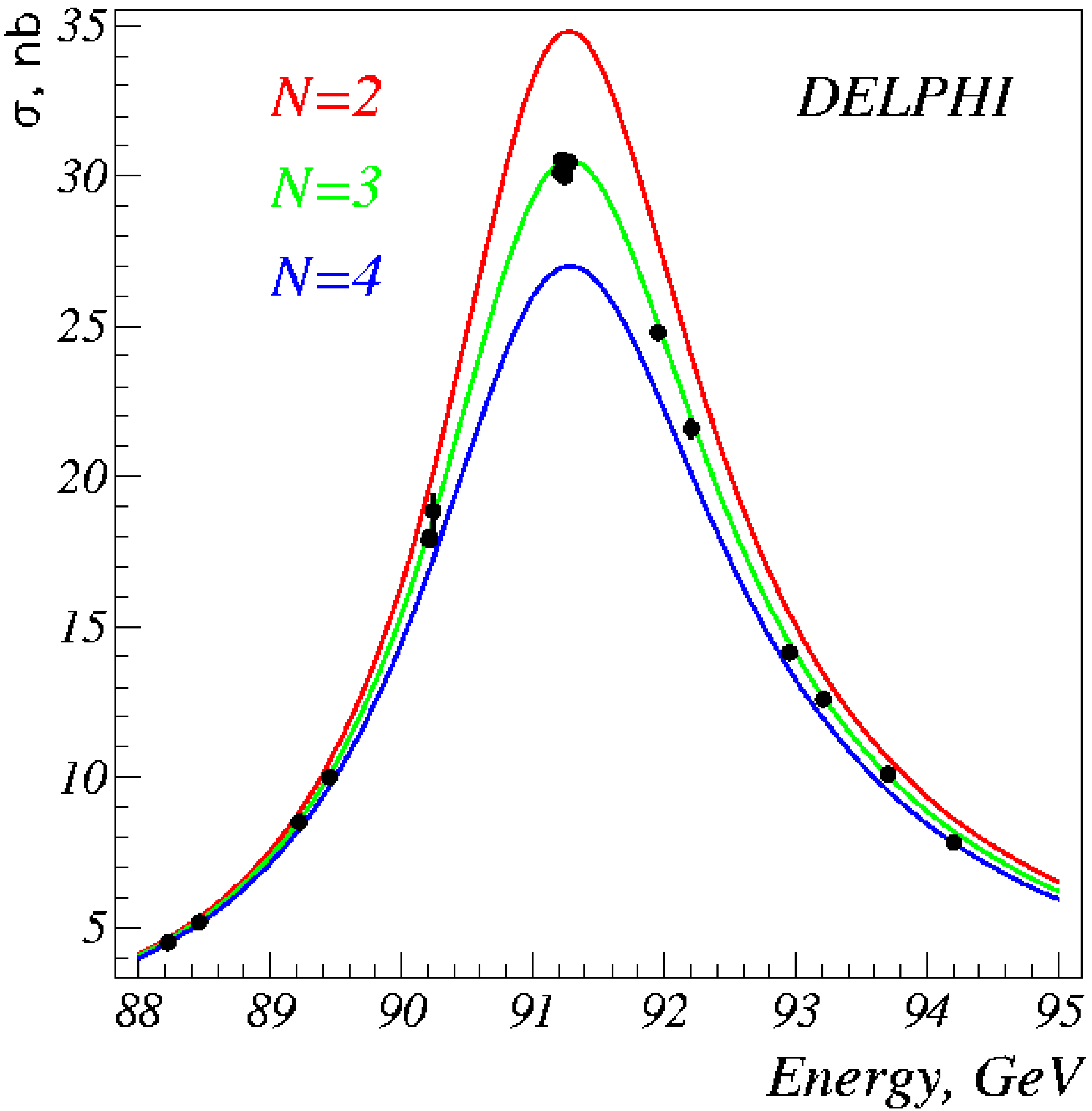
There are only three light neutrino generations!

In the Standard Model:

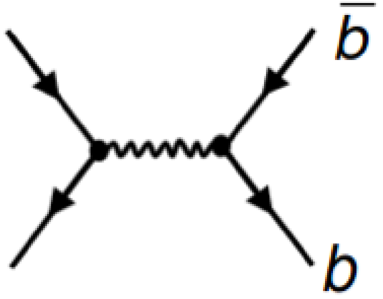
$$\Gamma_Z = \Gamma_{had} + 3\cdot\Gamma_\ell + \underbrace{N_\nu \cdot \Gamma_\nu}_{\text{invisible }:\Gamma_{inv}} \rightarrow \begin{cases} e^+e^- \to Z \to \nu_e\bar{\nu}_e \\ e^+e^- \to Z \to \nu_\mu\bar{\nu}_\mu \\ e^+e^- \to Z \to \nu_\tau\bar{\nu}_\tau \end{cases}$$

$N_\nu = 2.9840 \pm 0.0082$

No room for new physics: $Z \to$ new



Heavy Quark production

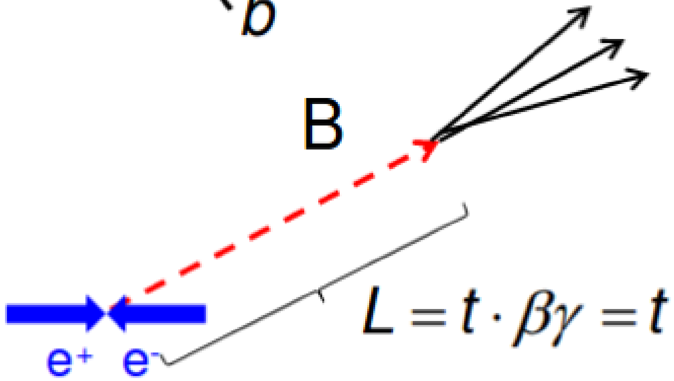


$$R_b = \frac{\Gamma_b}{\Gamma_{had}}$$

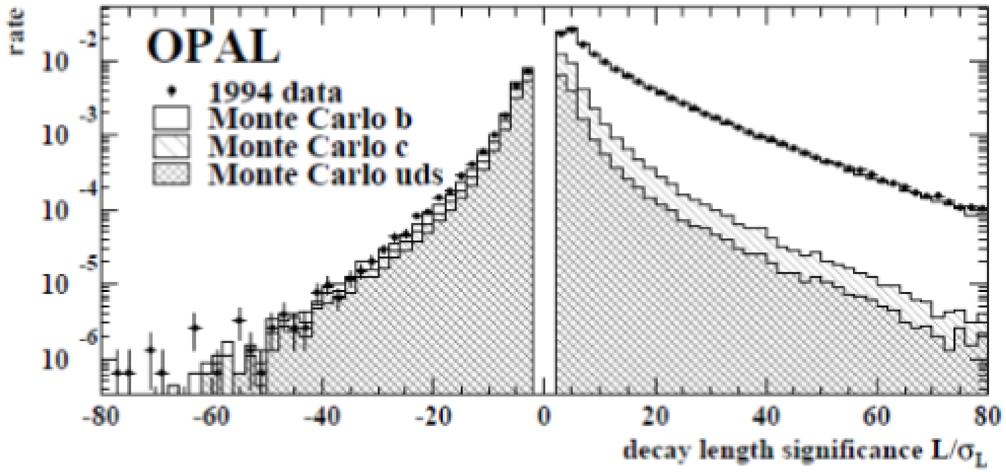
Identification of b-Quark events:

b-quarks hadronize to b-hadrons (B's, Λ_b) with typical lifetime of ~ 1 ps → decay length

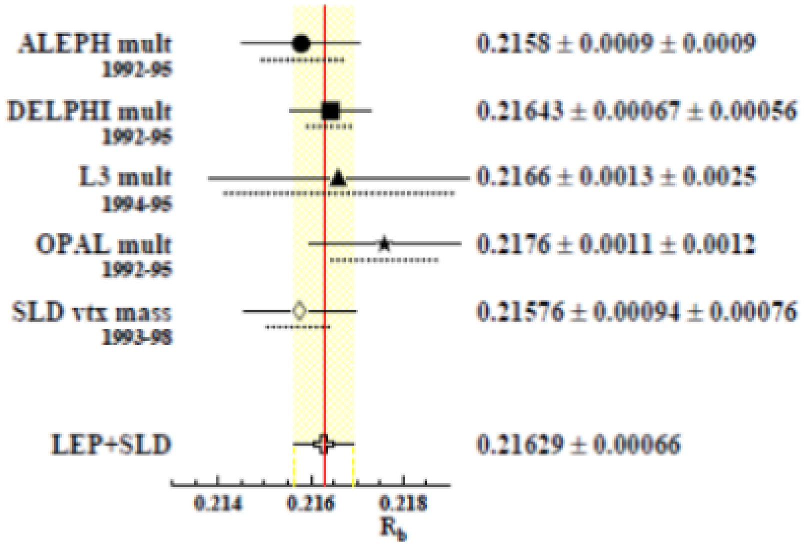
Use displaced "2nd" B decay vertex as signature.



$$L = t \cdot \beta\gamma = t \cdot \frac{p_B}{m_B}$$



Significance = L / error



$$\frac{d\sigma}{d\cos\theta} = \underbrace{\frac{\pi\alpha^2}{2s}}_{\text{known}} \left[\underbrace{F_\gamma(\cos\theta)}_{\gamma} + \underbrace{F_{\gamma Z}(\cos\theta)\frac{s(s-M_Z^2)}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}}_{\gamma/Z\ \text{interference}} + \underbrace{F_Z(\cos\theta)\frac{s^2}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}}_{Z} \right]$$

$$F_{\gamma Z}(\cos\theta) = \frac{Q_e Q_\mu}{4\sin^2\theta_W\cos^2\theta_W}\left[2g_V^e g_V^\mu(1+\cos^2\theta) + 4g_A^e g_A^\mu\cos\theta\right]$$

$$F_Z(\cos\theta) = \frac{1}{16\sin^4\theta_W\cos^4\theta_W}\left[(g_V^{e2}+g_A^{e2})(g_V^{\mu 2}+g_A^{\mu 2})(1+\cos^2\theta) + 8g_V^e g_A^e g_V^\mu g_A^\mu\cos\theta\right]$$

Forward-backward asymmetry A_{FB}

- Away from the resonance large → interference term dominates

$$A_{FB} \sim g_A^e g_A^f \cdot \frac{s(s-M_Z^2)}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2} \qquad \to \text{large}$$

- At the Z pole: Interference = 0 (see energy dependence of interference term)

$$A_{FB} = 3\cdot\frac{g_V^e g_A^e}{(g_V^e)^2+(g_A^e)^2}\cdot\frac{g_V^\mu g_A^\mu}{(g_V^\mu)^2+(g_A^\mu)^2}$$

→ very small because g_V^l small in SM

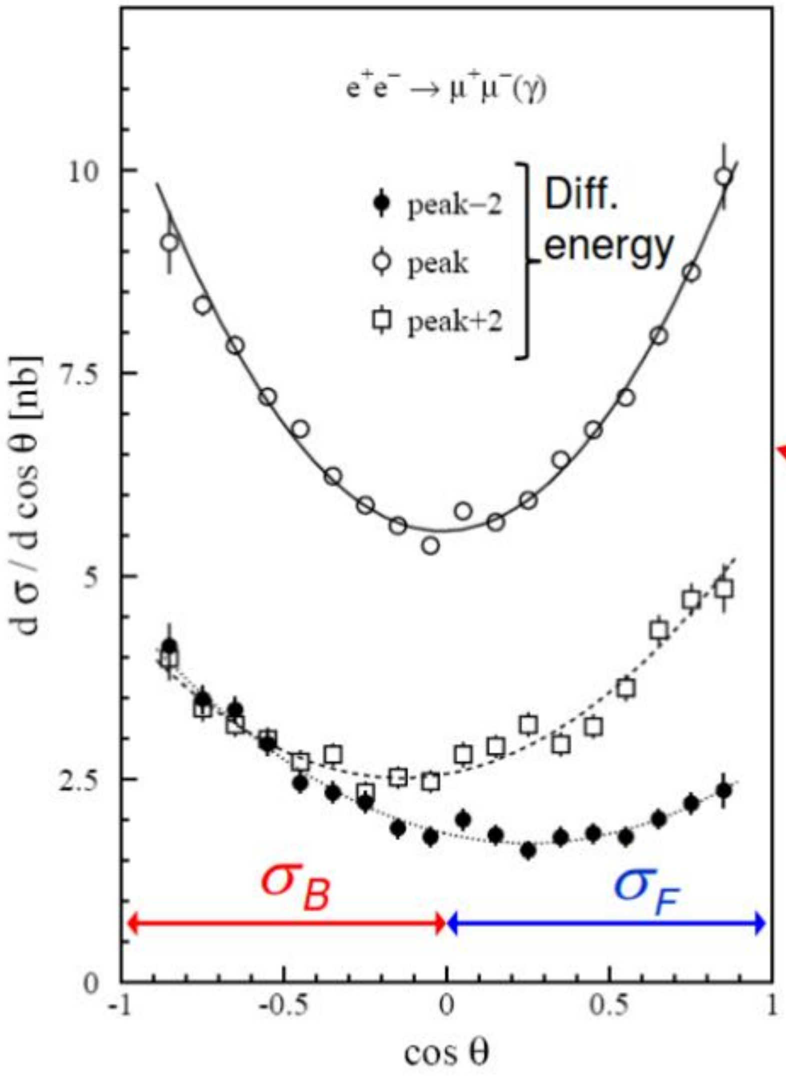
Forward-backward asymmetry and fermion couplings to Z

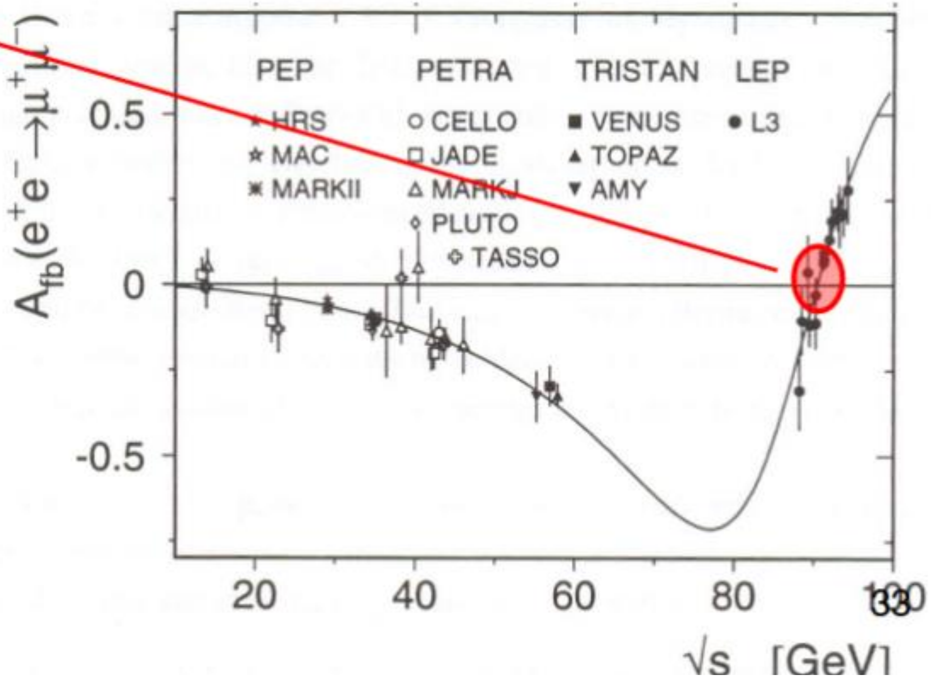
$$e^+ e^- \rightarrow Z \rightarrow \mu^+ \mu^-$$

$$\frac{d\sigma}{d\cos\theta} \sim (1+\cos^2\theta) + \frac{8}{3} A_{FB} \cos\theta$$

with $$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$$\sigma_{F(B)} = \int_{0\,(-1)}^{1\,(0)} \frac{d\sigma}{d\cos\theta} d\cos\theta$$





Asymmetrie at the Z pole

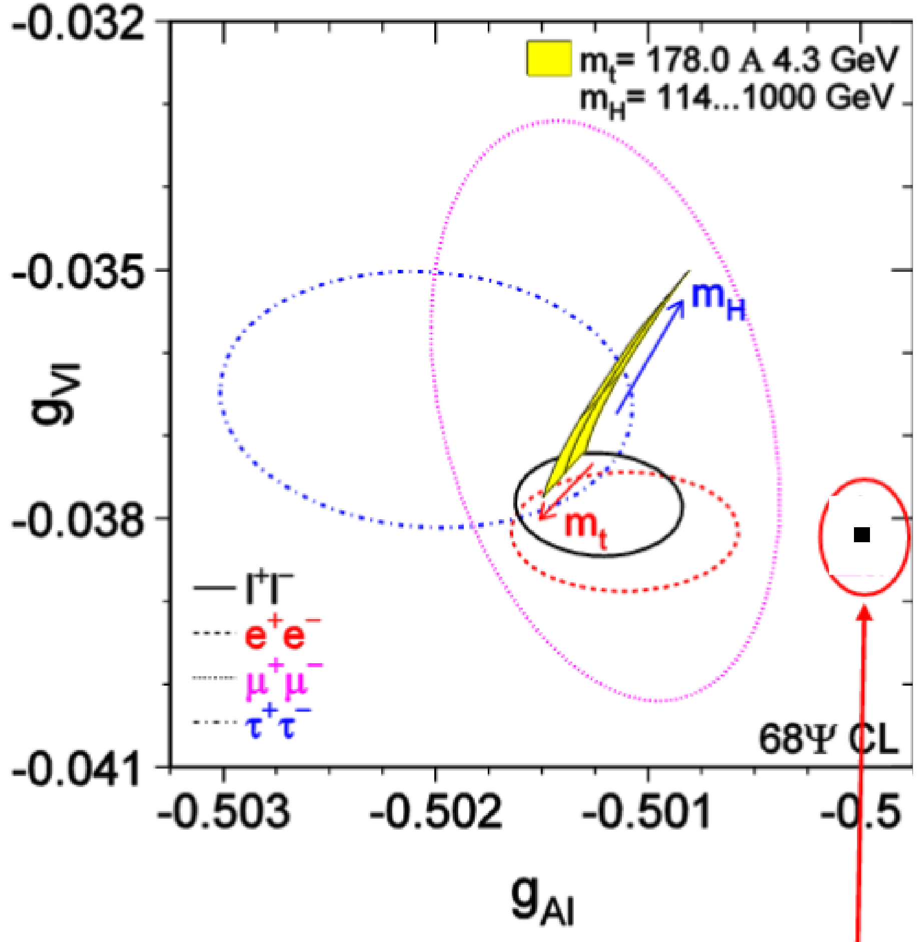
$$A_{FB} \sim g_A^e g_V^e g_A^f g_V^f$$

Cross section at the Z pole

$$\sigma_Z \sim \left[(g_V^e)^2 + (g_A^e)^2\right]\left[(g_V^\mu)^2 + (g_A^\mu)^2\right]$$

Lepton asymmetries together with lepton pair cross sections allow the determination of the lepton couplings g_A and g_V.

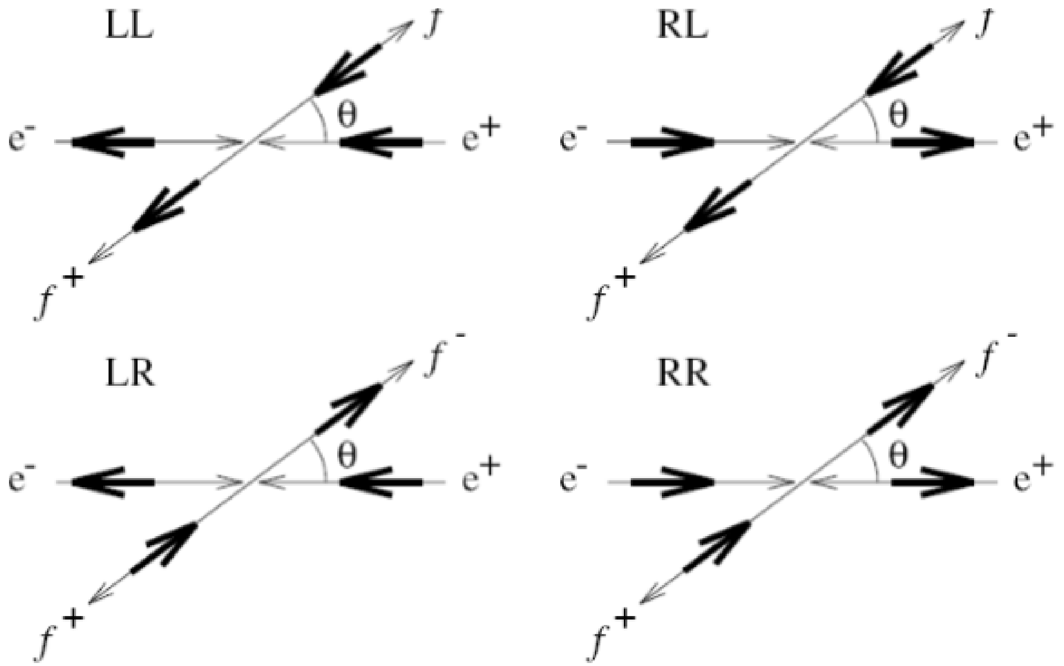
Good agreement between the 3 lepton species confirms “lepton universality”



Lowest order SM prediction:

$$g_V = T_3 - 2q\sin^2\theta_W \quad g_A = T_3$$

Deviation from lowest order SM prediction is an effect of higher-order electroweak corrections.



Observables:

$$\sigma_{F(B)} = \int_{0\,(-1)}^{1\,(0)} \frac{d\sigma}{d\cos\theta} d\cos\theta$$

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

Forward-backward asym. (final)

$\sigma_L = \sigma_{LL} + \sigma_{LR}$ $\sigma_R = \sigma_{RL} + \sigma_{RR}$

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

Left right asym. (initial)

$\sigma_- = \sigma_{LL} + \sigma_{RL}$ $\sigma_+ = \sigma_{RR} + \sigma_{LR}$

$$\mathcal{P}_f = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}$$

fermion polarization (final)

Polarization of final state leptons: tau pol.

$$\mathcal{P}_f(\cos\theta) = \frac{\dfrac{d\sigma_+}{d\cos\theta} - \dfrac{\sigma_-}{d\cos\theta}}{\dfrac{d\sigma_+}{d\cos\theta} + \dfrac{\sigma_-}{d\cos\theta}}$$

$\sigma_{L(R)}$= LH/RH fermion pol.

$$\mathcal{P}_f(\cos\theta) = \frac{\mathcal{A}_f(1+\cos^2\theta) + 2\mathcal{A}_e\cos\theta}{(1+\cos^2\theta) + 8/3\, A_{FB}\cos\theta}$$

with

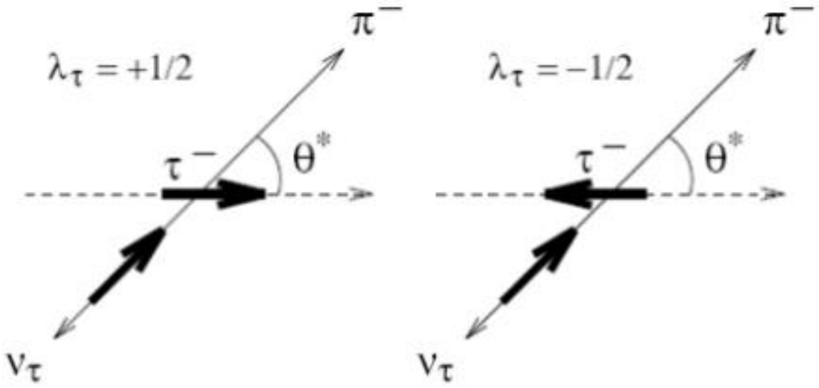
$$\mathcal{A}_i = \frac{2g_V^i g_A^i}{(g_V^i)^2 + (g_A^i)^2}$$

$$\mathcal{P}_\ell \approx -2\frac{2g_V^\ell}{g_A^\ell} = -2(1-4\sin^2\theta_w)$$

Lepton polarization measures directly $\sin^2\theta_w$.
The only lepton for which polarization can be measured at LEP is the tau!

Experimental Method to measure tau polarization:

$\tau^- \rightarrow \pi^- \nu_\tau$ Spin ½ →Spin ½ + Spin 0

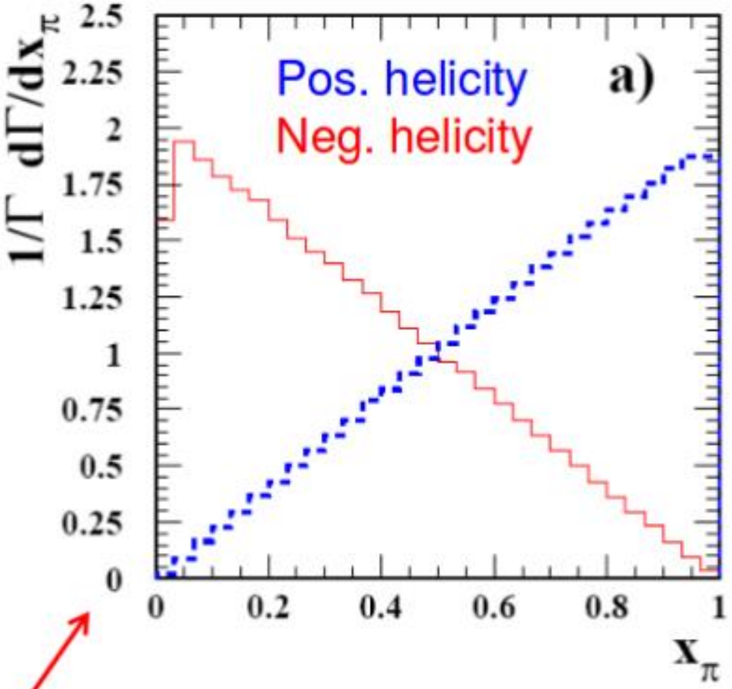


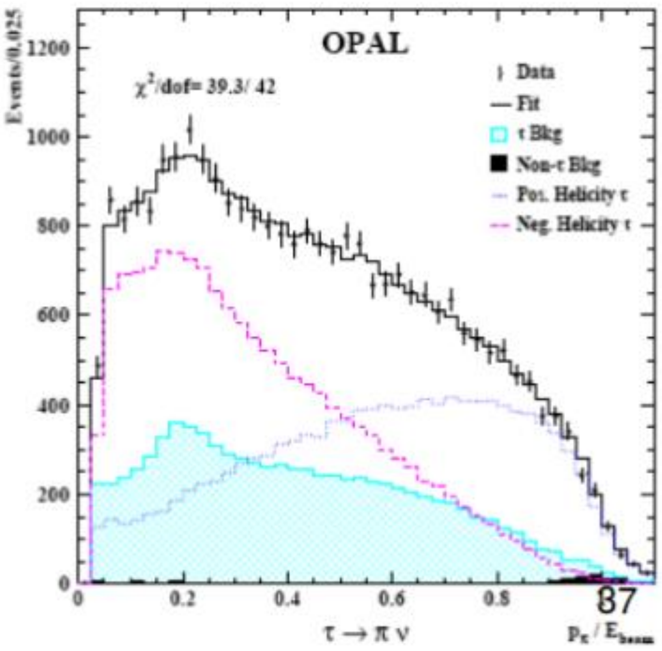
$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta^*} = \frac{1}{2}(1+\mathcal{P}_\tau \cos\theta^*)$$

Boost into lab frame

$$\frac{1}{\Gamma}\frac{d\Gamma}{dx_\pi} = 1+\mathcal{P}_\tau(2x_\pi - 1) \qquad x_\pi = E_\pi / E_\tau$$

Fit of the two theoretical distribution to data yields the polarization: ~ 0.15





Left-Right Asymmetry at SLC

Measure cross section σ_L (σ_R) for LH (RH) initial state electrons:

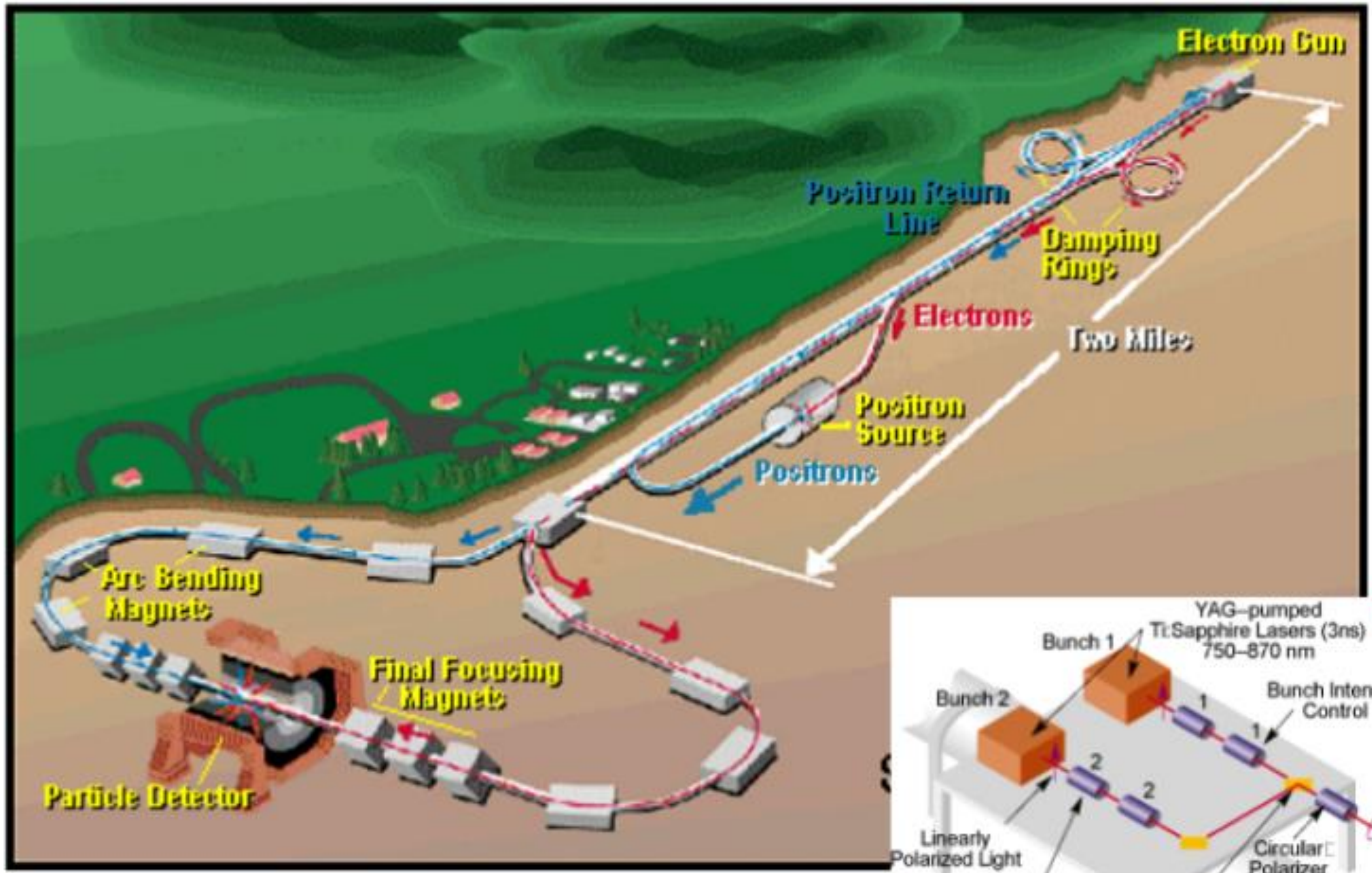
Polarization of electron beam: P~70 – 80%

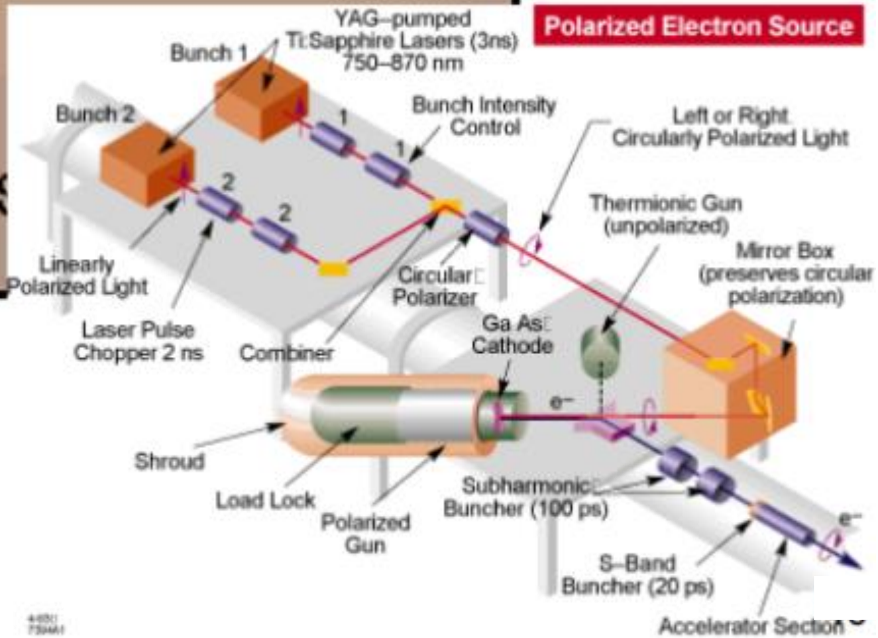
$$A_{LR} = \frac{1}{\mathcal{P}_e} \frac{\sigma_L^f - \sigma_R^f}{\sigma_L^f + \sigma_R^f}$$

$$A_{LR} = \frac{2 g_V^e g_A^e}{\left(g_V^e\right)^2 + \left(g_A^e\right)^2} = \frac{2(1 - 4\sin^2\theta_w)}{1 + (1 - 4\sin^2\theta_w)^2}$$

Powerful determination of $\sin^2\theta_w$.
Requires longitudinal polarization of colliding beams

SLAC Linear Collider





Typical beam polarization of 70%.

Precise determination of beam polarization using a Compton Polarimeter

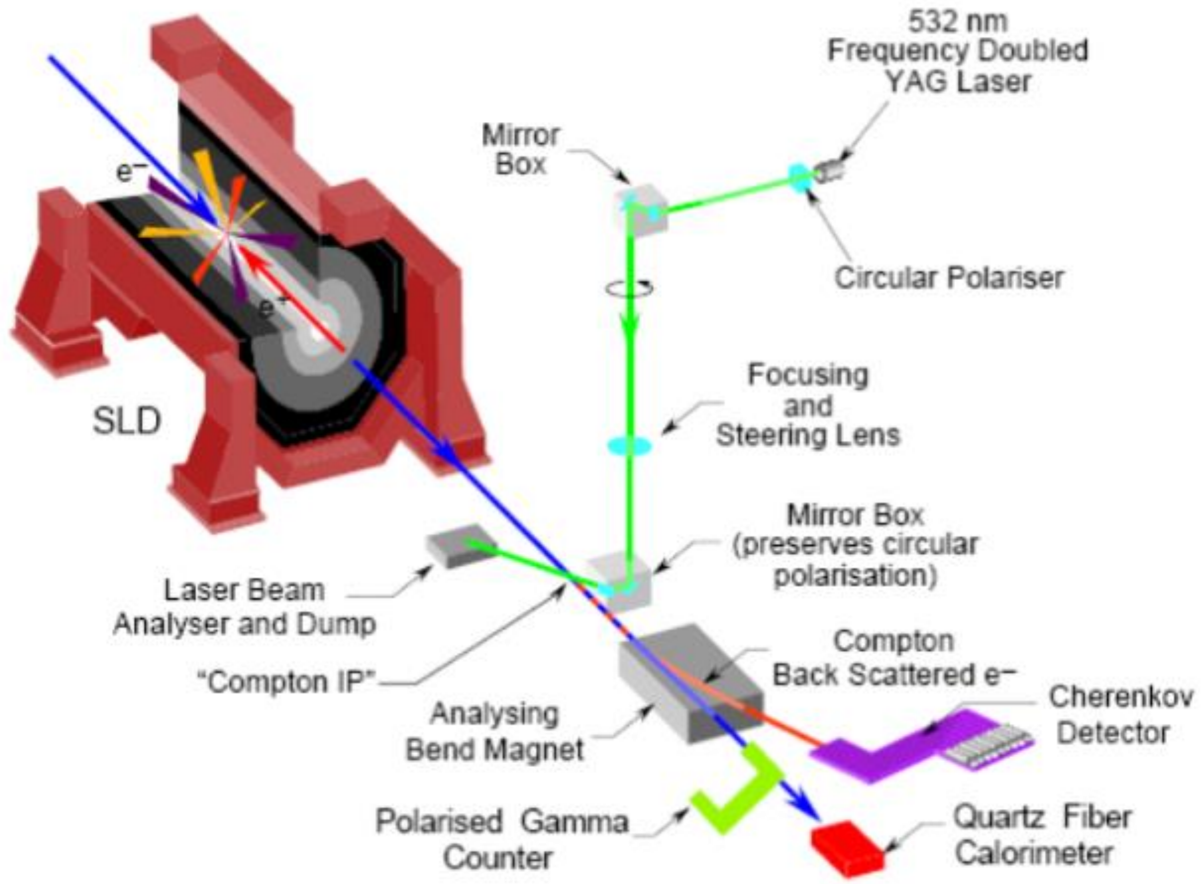
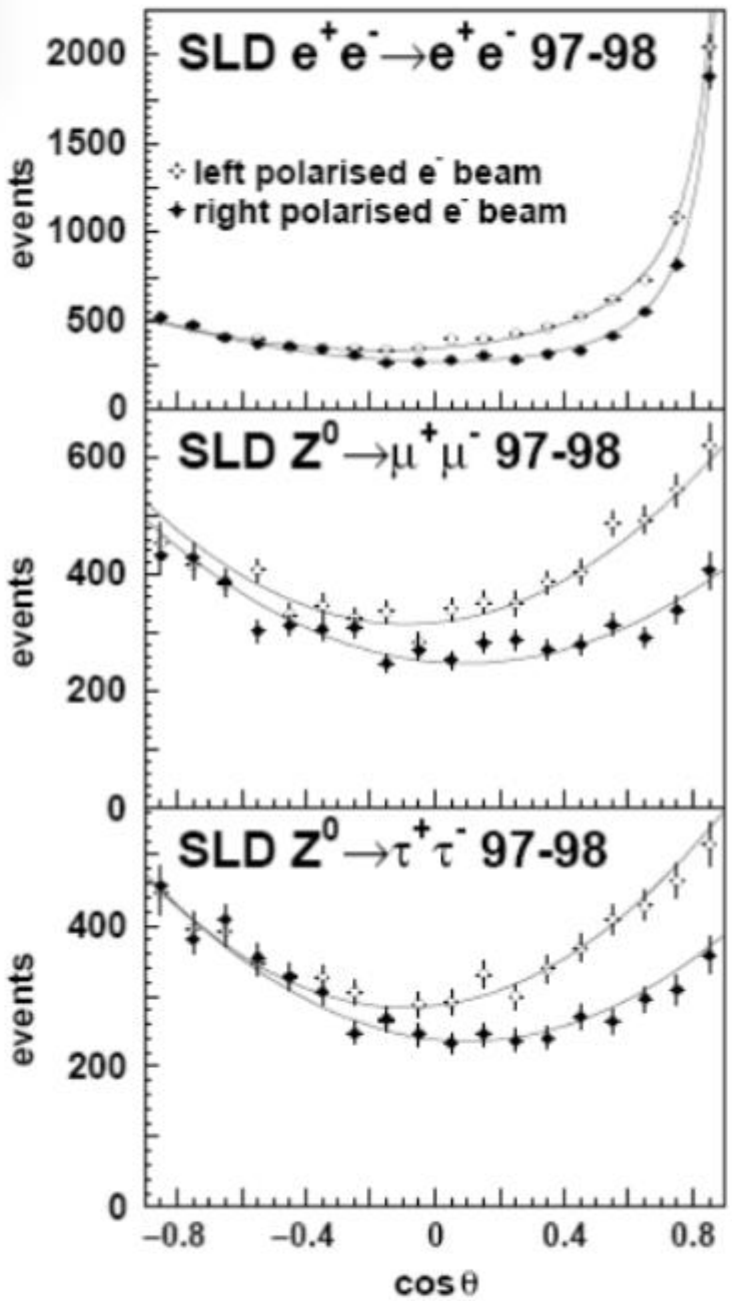


Figure 3.1: A conceptual diagram of the SLD Compton Polarimeter. The laser beam, consisting of 532 nm wavelength 8 ns pulses produced at 17 Hz and a peak power of typically 25 MW, were circularly polarised and transported into collision with the electron beam at a crossing angle of 10 mrad approximately 30 meters from the IP. Following the laser/electron-beam collision, the electrons and Compton-scattered photons, which are strongly boosted along the electron beam direction, continue downstream until analysing bend magnets deflect the Compton-scattered electrons into a transversely-segmented Cherenkov detector. The photons continue undeflected and are detected by a gamma counter (PGC) and a calorimeter (QFC) which are used to cross-check the polarimeter calibration.

Leptonic final states:



SLD

Asymmetry clearly seen for LH and RH cross section.

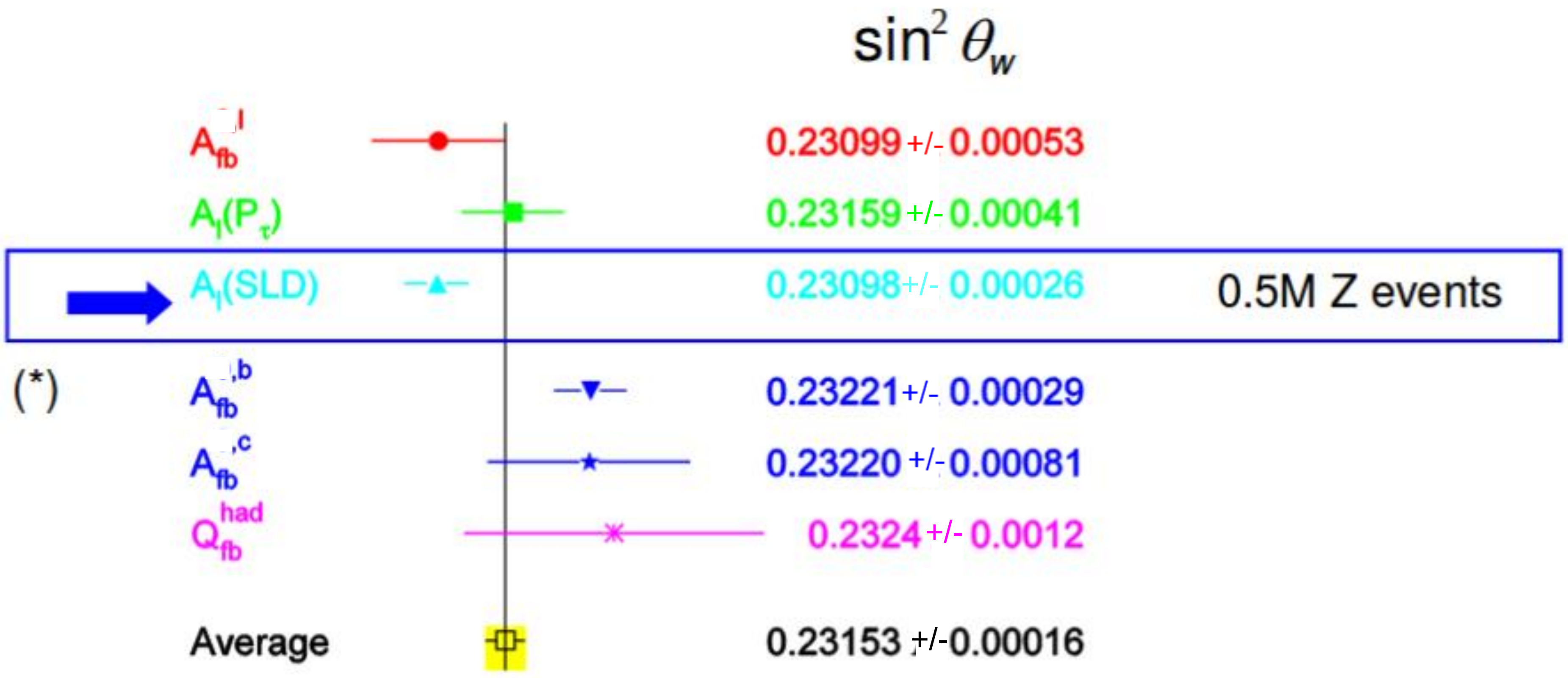
SLD All data:

$$A_{LR} = 0.1513 \pm 0.0021$$

$$\sin^2 \theta_w = 0.23098 \pm 0.00026$$

With 0.5×10^6 Z-decays

SLD versus 4×4.5 ×10^6 Z-decays at LEP



(*) similar to R_b one can also determine the forward-backward asymmetry for bb-events.