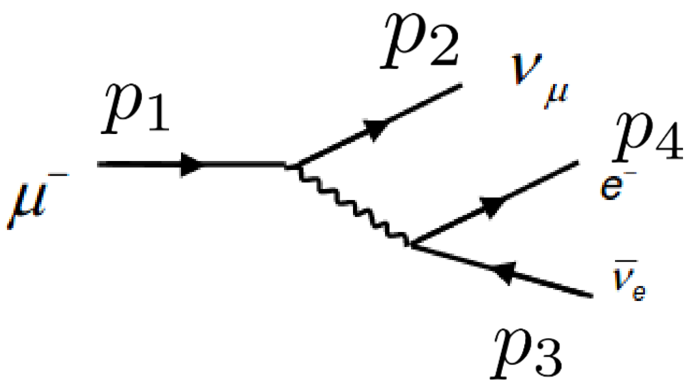
μ decay

$$q^2 < m_\mu^2 << m_W^2$$

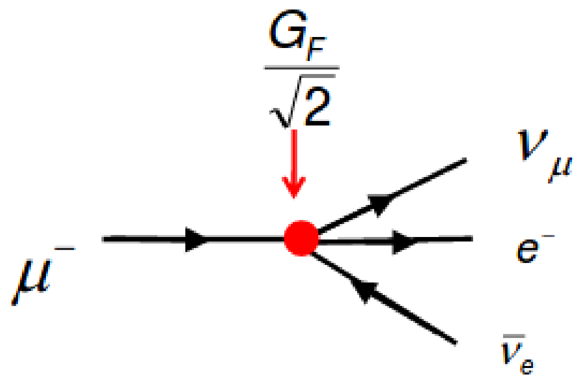
effective approach
of Fermi IA valid!

$$\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8m_W^2}$$

Non local current – current coupling



Point-like 4-fermion interaction



Fermi coupling constant,
dimension = $(1/M)^2$

$$M = \frac{g_w^2}{8M_W^2}[\overline{u}_\nu \gamma^\mu (1-\gamma^5) u_\mu][\overline{u}_e \gamma^\mu (1-\gamma^5) v_\nu]$$

Spin averaged matrix element:

$$M^2 = \frac{1}{2}\sum_{spin} |M^2| = 64 G_F^2 (p_2 p_4)(p_3 p_1)$$

$$\Gamma(\mu^- \to e^- \nu_\mu \overline{\nu}_e) = \frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

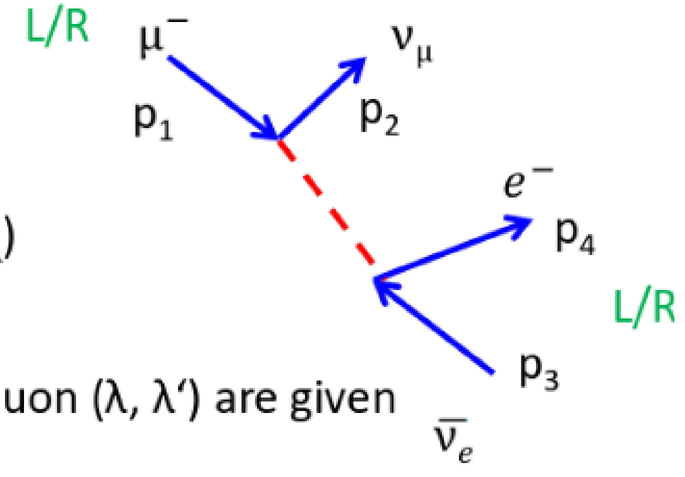
from experiment

$$\tau_\mu = 2.19610^{-6}\ \text{s}$$

$$G_F = 1.16638 \cdot 10^{-5} \quad GeV^{-2}$$

Experimental Probe of V-A structure

Most general form of matrix element, include scalar (S), vector (V) and tensor (T) currents.



$$M = \frac{G_F}{\sqrt{2}} \sum_{\substack{i=S,V,T \\ \lambda,\lambda'=R,L}} g^i_{\lambda\lambda'} \left(\overline{u(p_4)_{\lambda'}} \;\; \Gamma^i \, v(p_3)_m\right) \left(\overline{u(p_2)_n} \;\; \Gamma^i \, u(p_1)_\lambda\right)$$

n,m = R/L given if coupling i and handiness of electron and muon (λ, λ‘) are given

Possible current-current couplings

i / λλ‘	RR	RL	LR	LL
S	x	x	x	x
V	x	x	x	x
T		x	x	

There are in general 10 complex amplitudes $g^i_{\lambda\lambda'}$

pure V-A couling: $g^V_{LL} = 1$, all others 0

Experimental idea: measure polarization of electron for a given polarization of initial state.
Determine energy and angular distribution of electron.

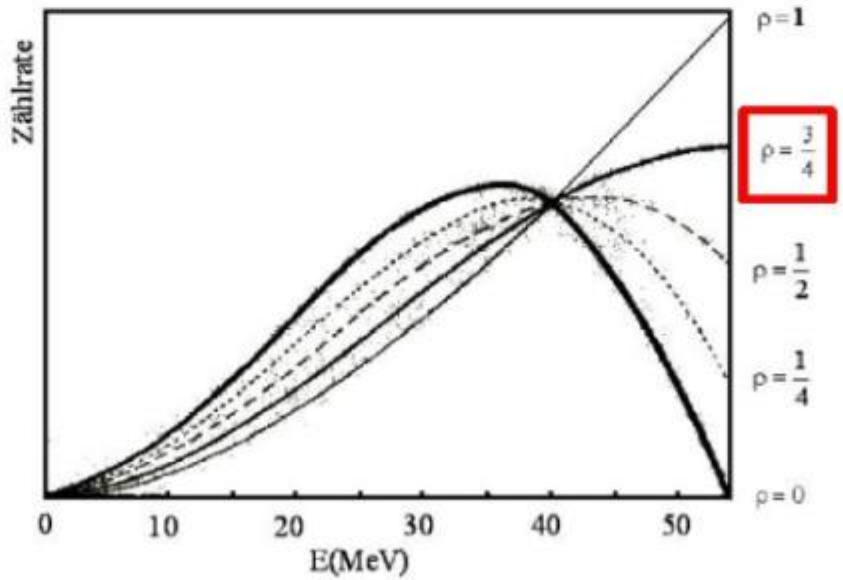
Note: in the notation of the Experimental data on the next Slide V stands for vector and axial vector coupling and accordingly S for scalar an pseudo-scalar

Experimental Probe of V-A structure: Muon Decay

Energy spectrum of emitted electron:

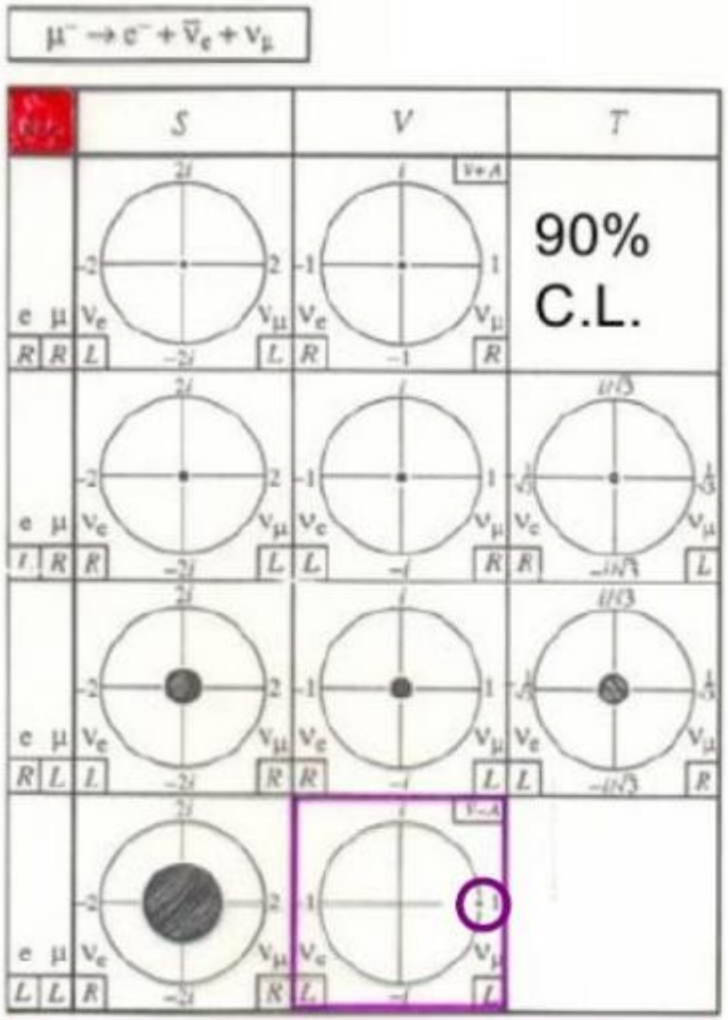
$$dN(E) = \frac{4E^2 dE}{\tau_\mu}\left[3(1-E) + \frac{2}{3}\rho\,(4E-3)\right]$$

Michelparameter: ρ



V-A theory: ρ = 0.75

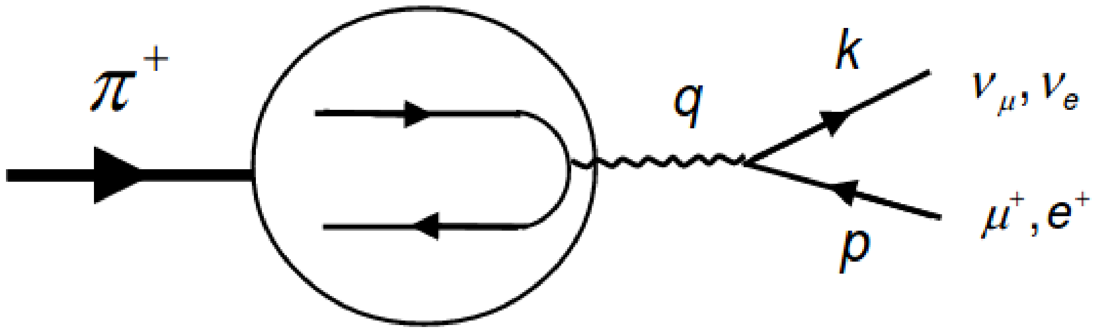
Couplings in muon decay



V-A theory is confirmed

Sindrum experiment

Determination of decay rates:



Quarks in pion are bound

Effective interaction – ignore propagator

$$M = \frac{G_F}{\sqrt{2}} \cdot (\ \pi\ )_\mu \cdot \left[\bar{u}_\nu \gamma^\mu (1-\gamma^5) v_\mu\right]$$

As the pion spin s_π=0, q is the only relevant 4-vector:

$$q^\mu = p^\mu + k^\mu$$

$$(\ \pi\ )_\mu = q_\mu f_\pi(q^2)$$

Pion form factor:

$$q^2 = m_\pi^2: \quad f_\pi(q^2) = f_\pi(m_\pi^2) = f_\pi$$

Must be measured !

scalar particles

$$M = \frac{G_F}{\sqrt{2}} \cdot (p_\mu + k_\mu) \cdot f_\pi \cdot \left[\bar{u}_\nu \gamma^\mu (1-\gamma^5) v_\mu\right]$$

$$\Gamma(\pi^+ \to \mu^+ \nu_\mu) = \frac{G_F^2}{8\pi} \cdot f_\pi^2 \cdot m_\pi m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)$$

$$\Gamma(\pi^+ \to e^+ \nu_\mu) = \frac{G_F^2}{8\pi} \cdot f_\pi^2 \cdot m_\pi m_e^2 \left(1 - \frac{m_e^2}{m_\pi^2}\right)$$

$$\frac{\Gamma(\pi^+ \to e^+ \nu_\mu)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)} = \left(\frac{m_e^2}{m_\mu^2}\right)\left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right) = 1.275 \cdot 10^{-4}$$

V-A or V+A predictions
(first order computation)

$(1.230 \pm 0.004) \cdot 10^{-4}{}_{PDG}$

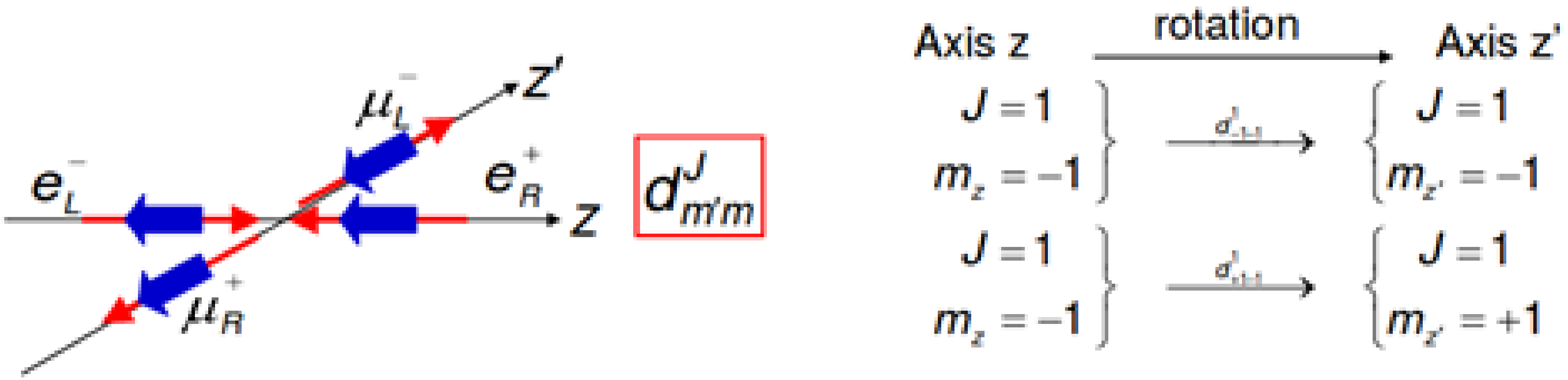
experimental result

for scalar $\overline{\Psi}\Phi$ and pseudoscalar coupling $\overline{\Psi}\gamma^5\Phi$: $\frac{\Gamma(\pi^+ \to e^+ + \nu_e)}{\Gamma(\pi^+ \to \mu^+ + \nu_\mu)} = 5.5$

The predicted ratio from V-A coupling confirmed in experiment!

Reminder:

Angular distribution: $e^+e^- \to \mu^+\mu^-$



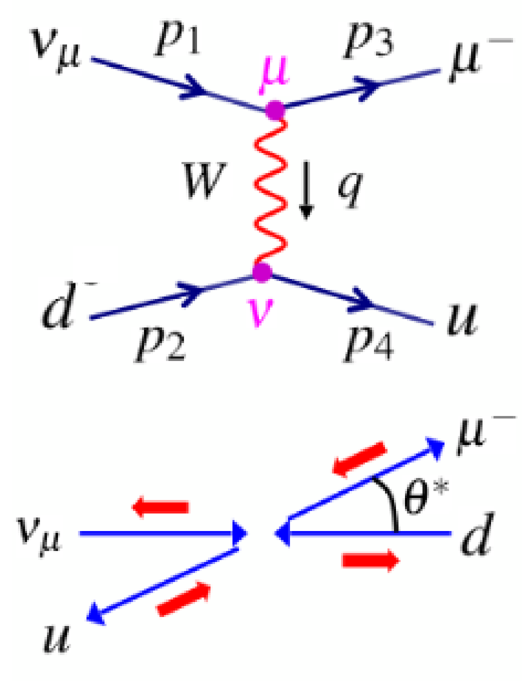
Scattering can be treated as a change of the quantization axis.

$$d^1_{1,1} = d^1_{-1,-1} = \tfrac{1}{2}(1+\cos(\theta))\ [\mathrm{LR} \to \mathrm{LR},\ \mathrm{RL} \to \mathrm{RL}]$$
$$d^1_{1,-1} = d^1_{-1,1} = \tfrac{1}{2}(1-\cos(\theta))\ [\mathrm{LR} \to \mathrm{RL},\ \mathrm{RL} \to \mathrm{LR}]$$

$$\frac{d\sigma}{d\Omega} \sim \frac{1}{4}(1+\cos\theta)^2 + \frac{1}{4}(1-\cos\theta)^2 \sim 1+\cos^2\theta$$

Angular distribution is an effect of vector coupling $ie\gamma^\mu$

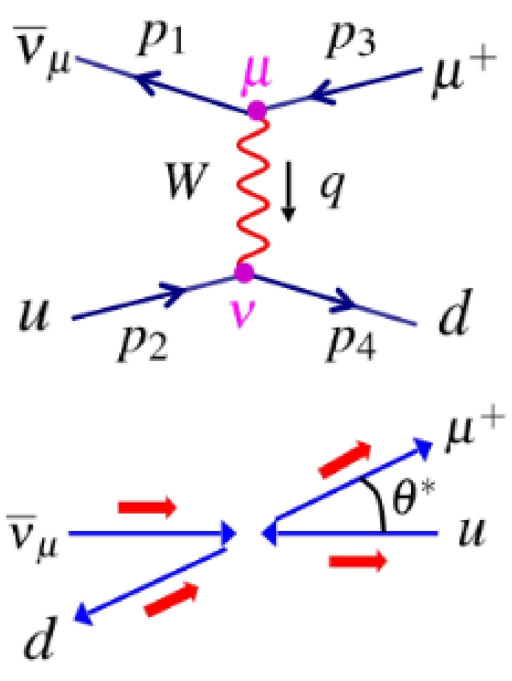
(Anti-) Neutrino – Quark-Scattering



$S_z = 0$

$$\frac{d\sigma_{\nu q}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$$

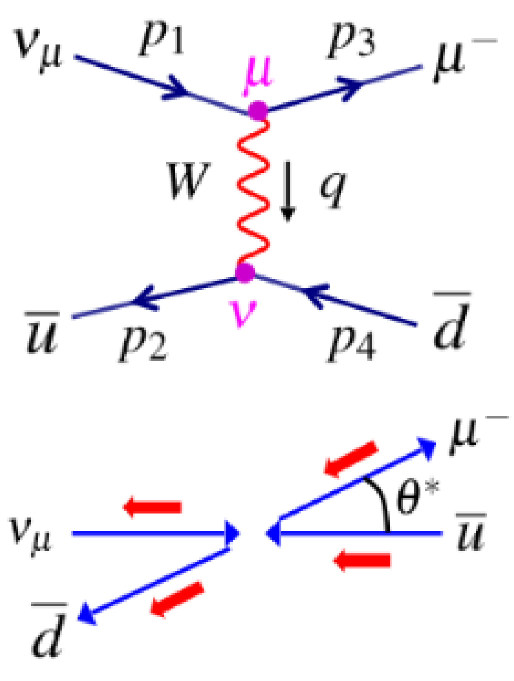
$$\sigma_{\nu q} = \frac{G_F^2 \hat{s}}{\pi}$$



$S_z = +1$

$$\frac{d\sigma_{\overline{\nu} q}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$$

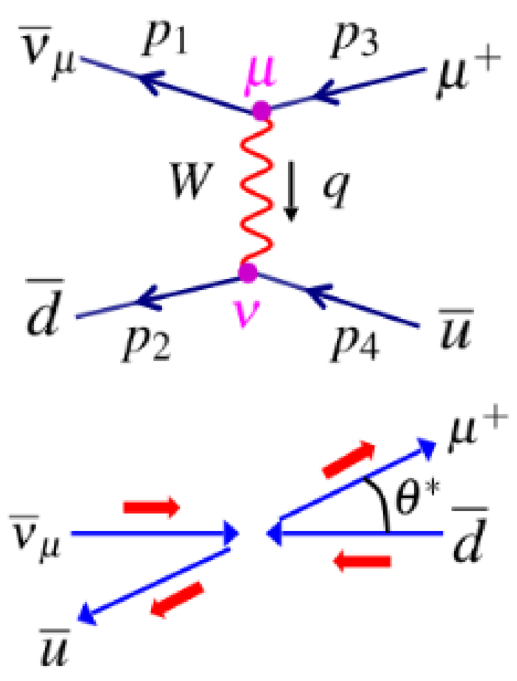
$$\sigma_{\overline{\nu} q} = \frac{G_F^2 \hat{s}}{3\pi}$$



$S_z = -1$

$$\frac{d\sigma_{\nu \overline{q}}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$$

$$\sigma_{\nu \overline{q}} = \frac{G_F^2 \hat{s}}{3\pi}$$



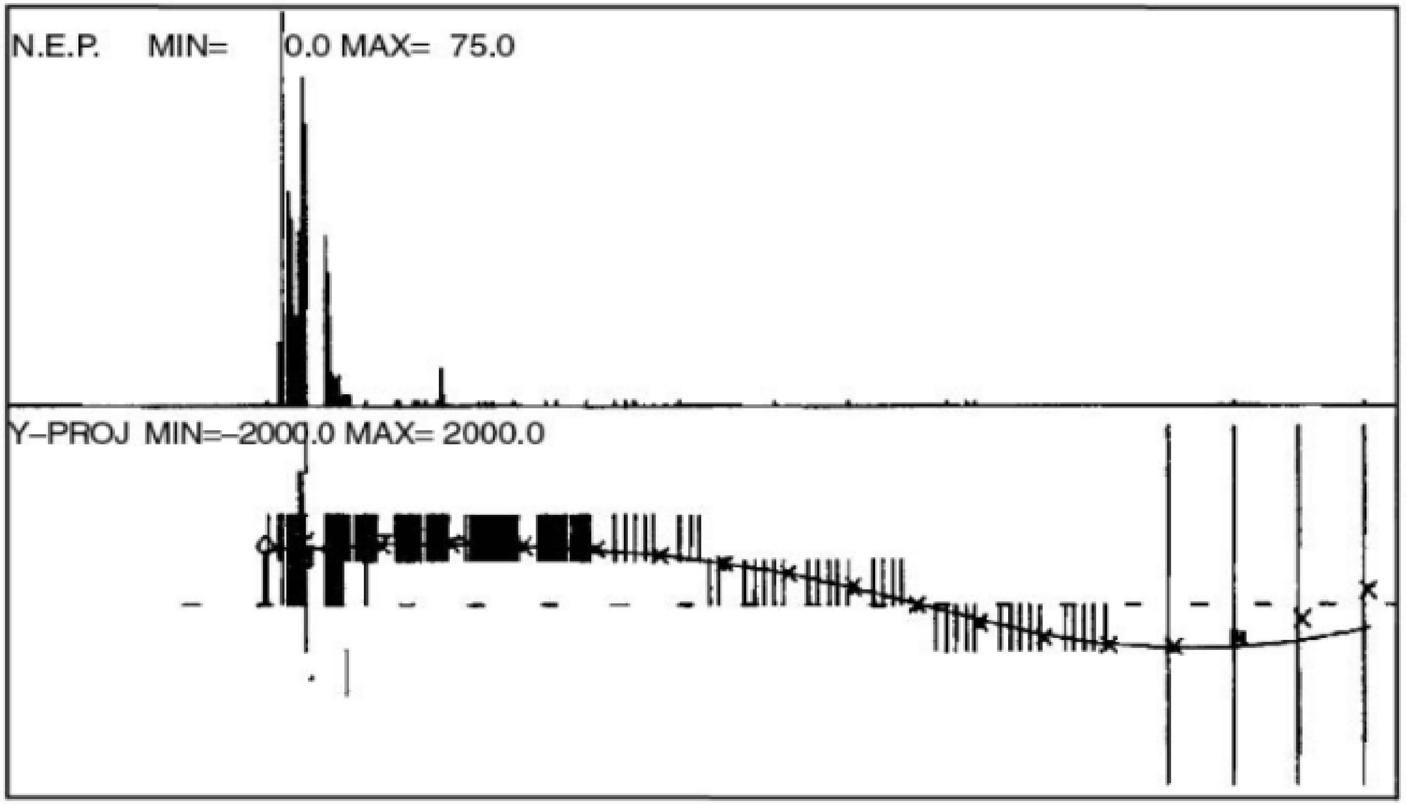
$S_z = 0$

$$\frac{d\sigma_{\overline{\nu} \overline{q}}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$$

$$\sigma_{\overline{\nu} \overline{q}} = \frac{G_F^2 \hat{s}}{\pi}$$

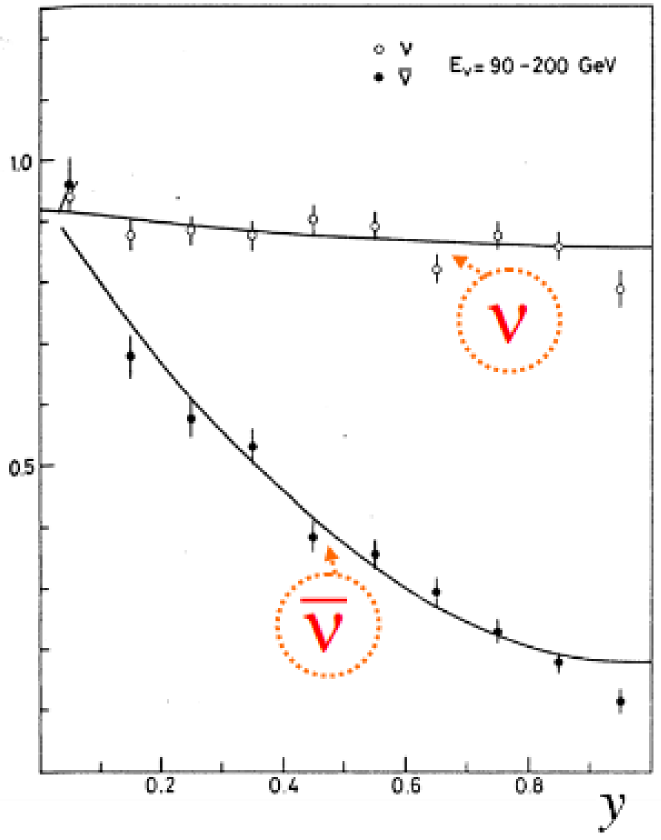
CDHS – CERN-Dortmund-Heidelberg Experiment

Deep-Inelastic Neutrino Interaction in CDHS



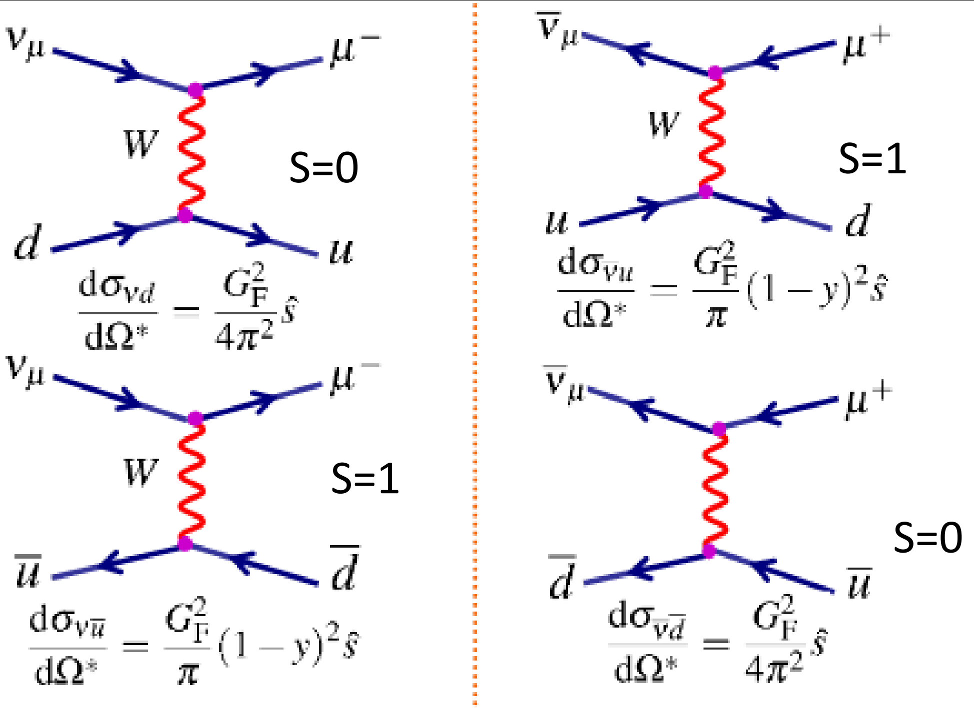
Measured y Distribution

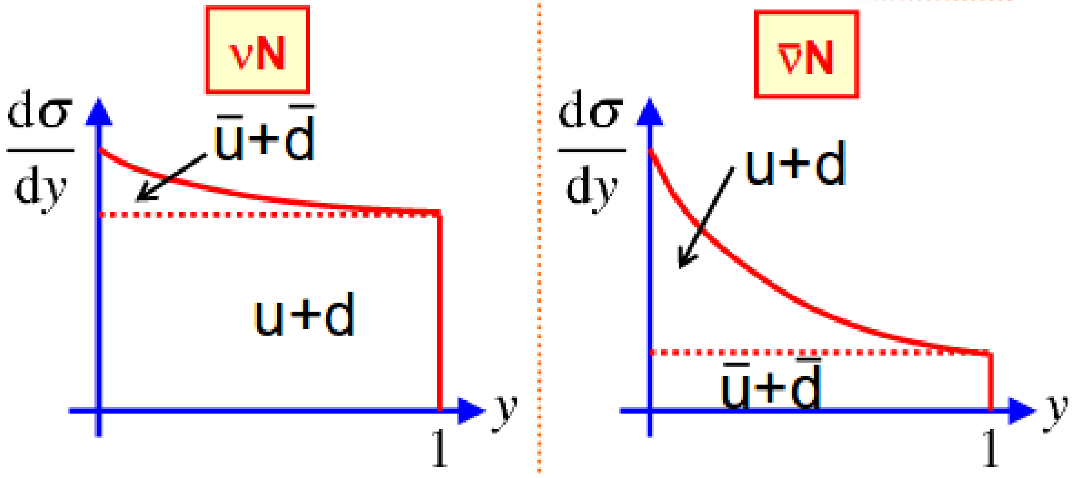
- CDHS measured y distribution



J. de Groot et al., Z.Phys. C1 (1979) 143

- Shapes can be understood in terms of (anti)neutrino – (anti)quark scattering





Neutrino-nucleon N scattering

$$\sigma(\nu N) = \frac{G_F^2 M E_\nu}{2\pi} \cdot \left[Q_l + \frac{1}{3}\overline{Q}_l \right]$$

$$\sigma(\overline{\nu} N) = \frac{G_F^2 M E_\nu}{2\pi} \cdot \left[\overline{Q}_l + \frac{1}{3} Q_l \right]$$

with $Q_l = \int x Q(x) dx$

(integral of quark / anti-quark distribution)

$$R = \frac{\sigma_{\overline{\nu}N}}{\sigma_{\nu N}} = \frac{1 + 3\overline{Q}_l / Q_l}{3 + \overline{Q}_l / Q_l}$$

If nucleon consists only of valence quarks ($\overline{Q}=0$): R=1/3 , because of V-A structure

Measurement: $R = \frac{0.34}{0.67} \Rightarrow \overline{Q}_l / Q_l \approx 0.15$

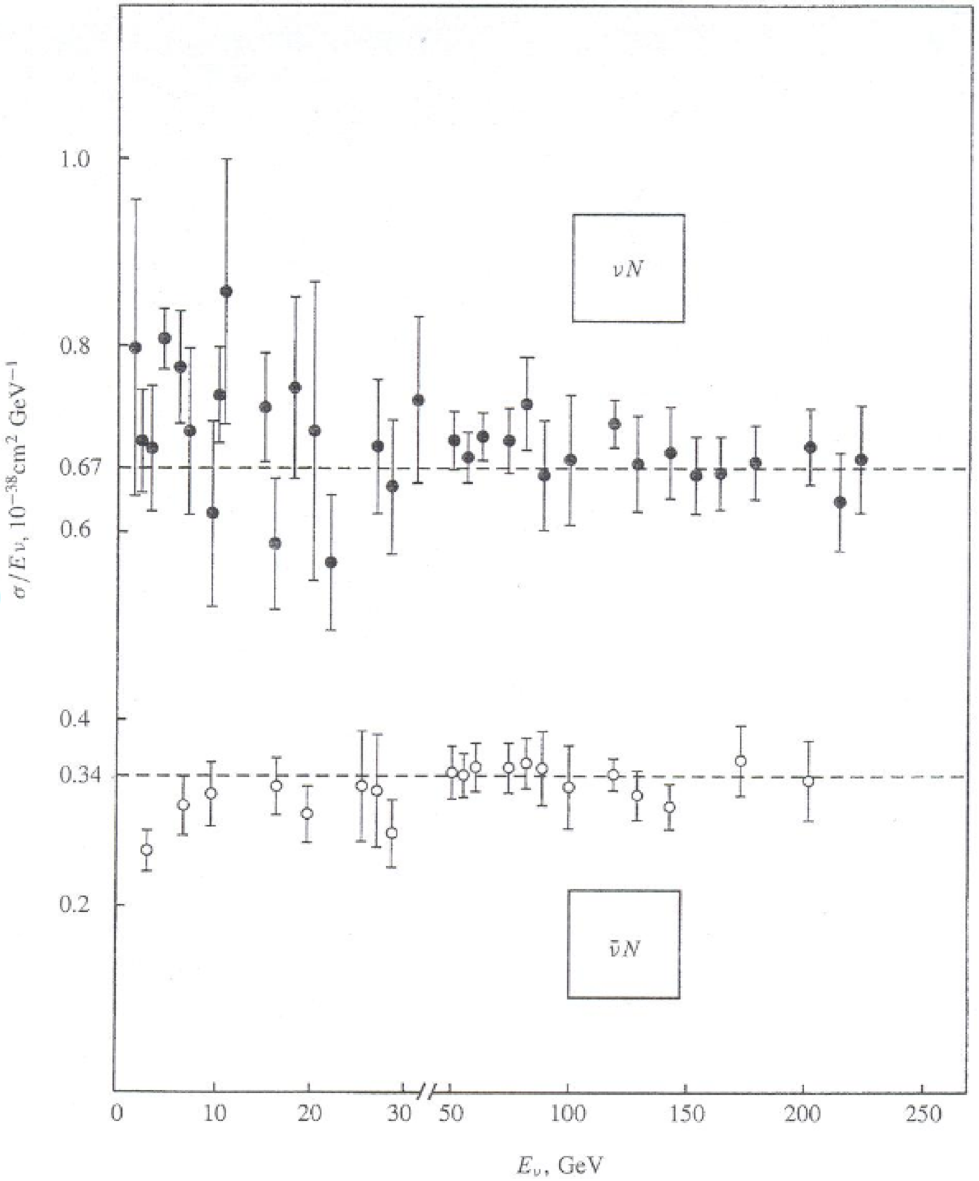


Fig. 5.13. Neutrino and antineutrino cross-sections on nucleons. The ratio σ/E_ν is plotted as a function of energy and is indeed a constant, as predicted in (5.45) and (5.46).

⇒V-A theory confirmed, there are sea quarks

Problems with pure V-A (4-fermion) theory

- Cross section for νq in 4-fermion ansatz: i.e. cross section goes to infinity if s→∞: violates unitarity

$$\sigma(\nu q) = \frac{G_F^2 s}{\pi}$$

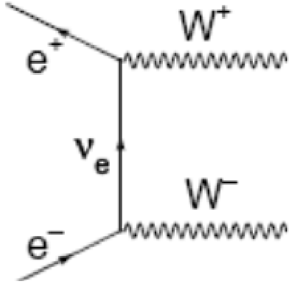
- Lee and Wu (1965) introduced a massive exchange boson. Effect of propagator:

$$\frac{G_F}{\sqrt{2}} \mapsto \frac{G_F}{\sqrt{2}} \frac{1}{1 - q^2/M_W^2} \qquad \sigma(\nu q) \mapsto const$$

Not trivial, see e.g.:
C.Quigg, Gauge Theory of Strong and Weak interactio

This fix leads to a new problem, namely the violation of unitarity of the predicted W pair production !

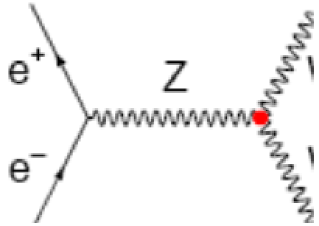
W pair production:



Violates unitarity

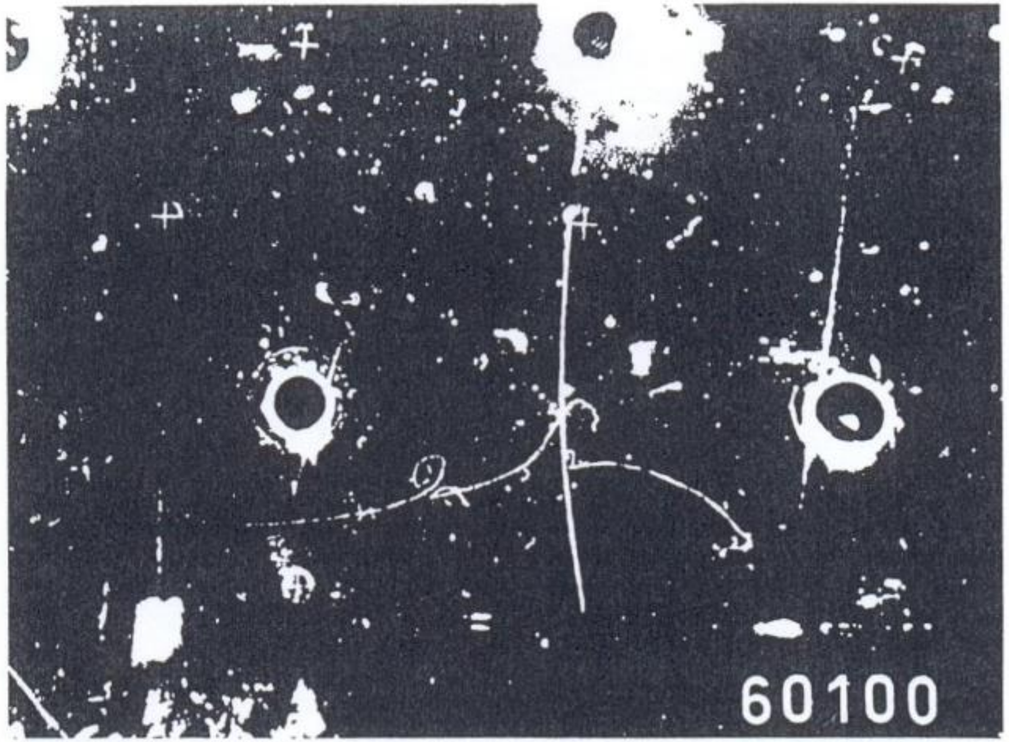
Can be cures by adding a Z boson:

$$e^+e^- \to Z \to WW$$

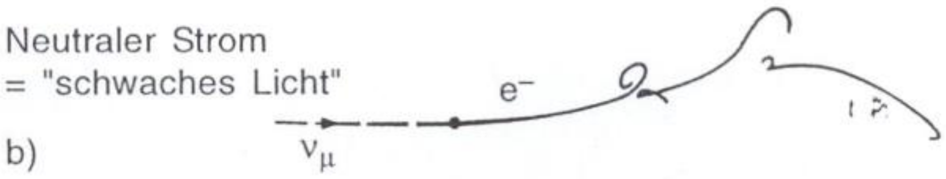


➡ **Standard Model**

Gargamelle Bubble Chamber



a)

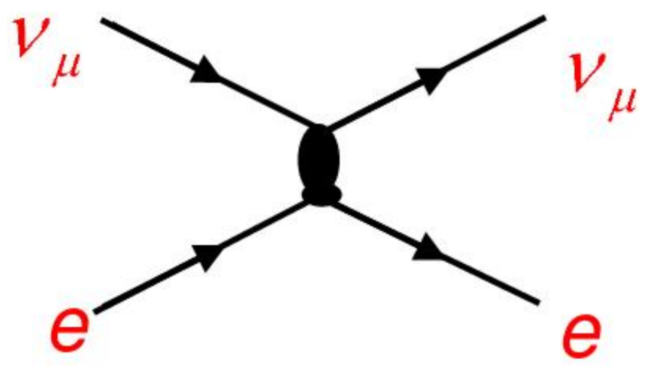


b)

Abb. 9. Dieses erste Ereignis mit einem neutralen schwachen Strom wurde in Aachen entdeckt. Ein Neutrino dringt von links in die Blasenkammer ein (auf dem Bild nicht sichtbar) und wird elastisch an einem Elektron gestreut. Das Elektron ist als rechte Spurkaskade (Bremsstrahlung) zu erkennen. Dieses Bild ist in die Geschichte des CERN eingegangen

One out of three $\nu e \rightarrow \nu e$ events

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$



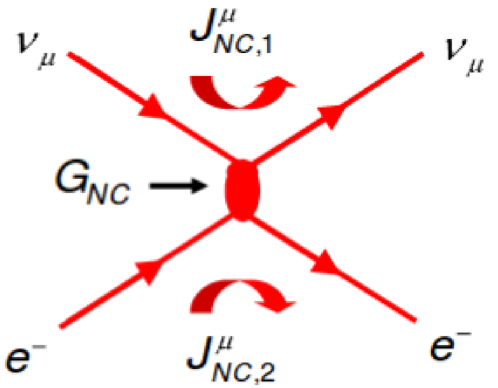
Neutral current νN events appear with a significant rate:

$$R_\nu = \frac{\sigma_{NC}(\nu N \rightarrow \nu X)}{\sigma_{CC}(\nu N \rightarrow \mu X)} = 0.307 \pm 0.008$$

i.e. approx.1/3 of the νN interactions are neutral current interactions.

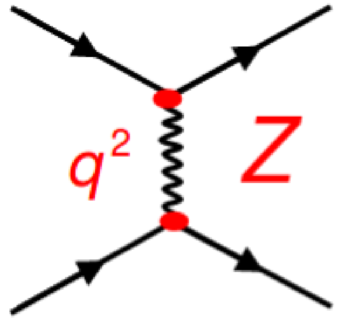
Structure of Neutral currents

Ansatz: four-fermion interaction



$$M = \frac{8 G_{NC}}{\sqrt{2}} \cdot J_{NC,1,\mu} \cdot J^\mu_{NC,2}$$

as $q^2 \rightarrow 0$ approximation of:



Experimental determination of the structure of the weak neutral currents:

$$J^\mu_{NC} = \bar{u}\,\gamma^\mu \frac{1}{2}(g_V - g_A \gamma^5) u$$

→ Neutral weak interaction couples to left- and right-handed chiral fermion currents differently:

$$g_L = \frac{1}{2}(g_V + g_A) \qquad g_R = \frac{1}{2}(g_V - g_A)$$

$$J^\mu_{NC} = \bar{u}\gamma^\mu (g_R \frac{1+\gamma^5}{2} + g_L \frac{1-\gamma^5}{2}) u$$