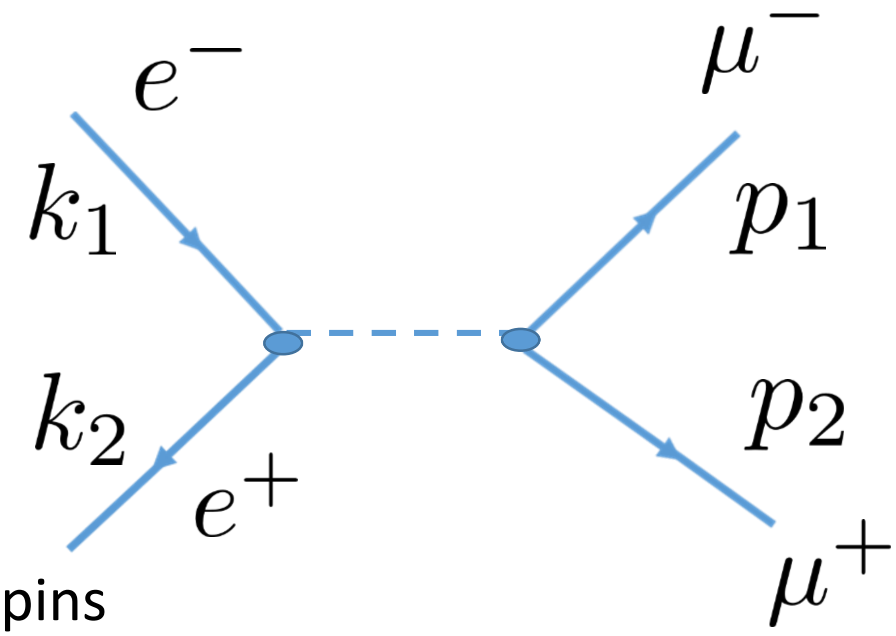
Reminder of QED results for transition amplitudes

$$-iM_{fi} = [\overline{v_2}(ieQ_e\gamma^\mu)u_1]\frac{-g_{\mu\nu}}{q^2}[\overline{u_3}(ieQ_\mu\gamma^\nu)v_4]$$

Spinors describe a specific spin state of the fermions



Spin averaged matrix element :

Unpolarize initial state and non-observation of final state spins
→ Average of possible initial state spins, sum over all final states:

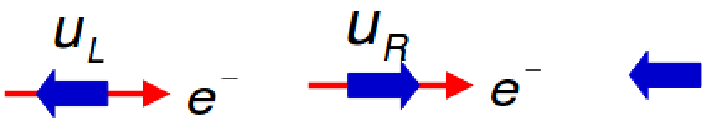
$$|\overline{M_{fi}}|^2 = \frac{1}{4}\sum_{spin_i}\sum_{spin_f}|M_{fi}|^2$$

$$= 2e^4 Q_e^2 Q_\mu^2 \frac{t^2+u^2}{s^2}$$

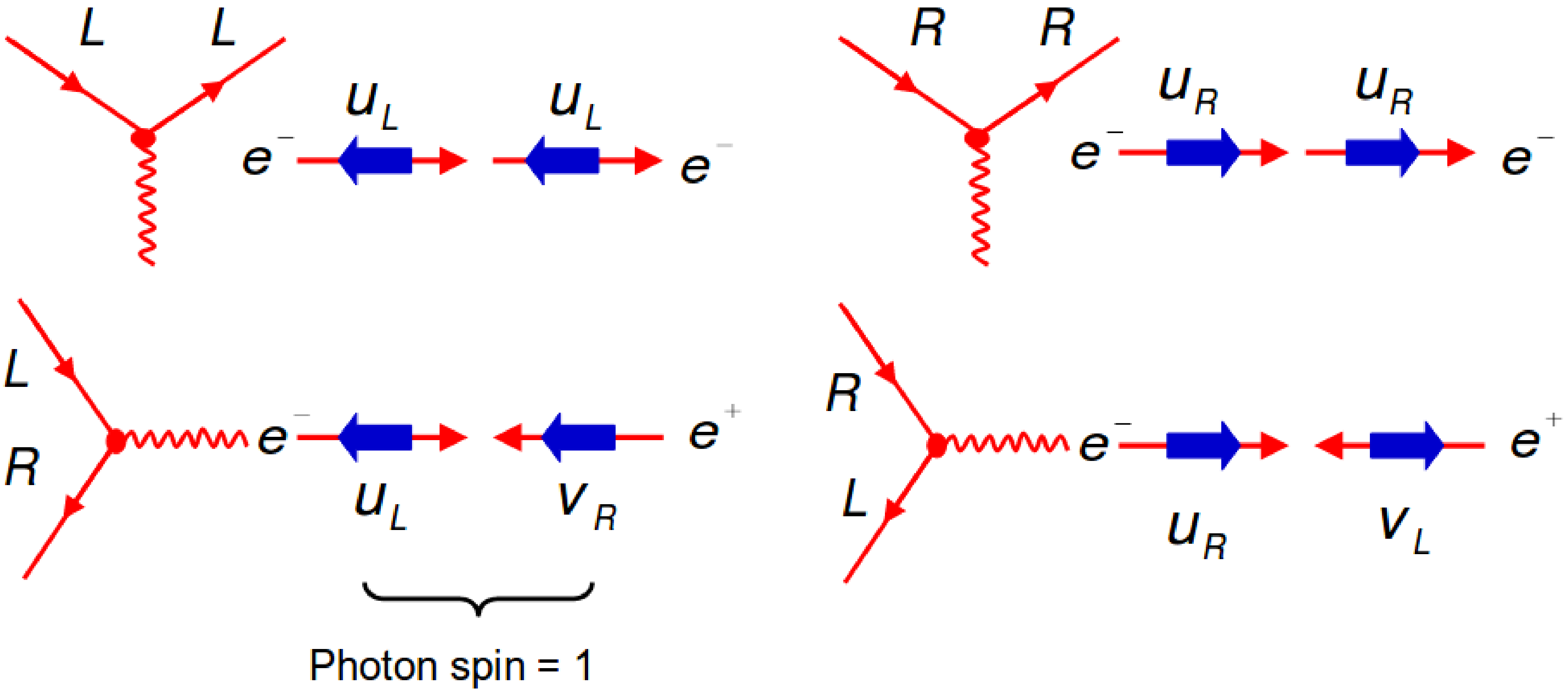
Mandelstamm variables

$$s = (k_1 + k_2)^2$$
$$t = (k_1 - p_1)^2$$
$$u = (k_1 - p_2)^2 s$$

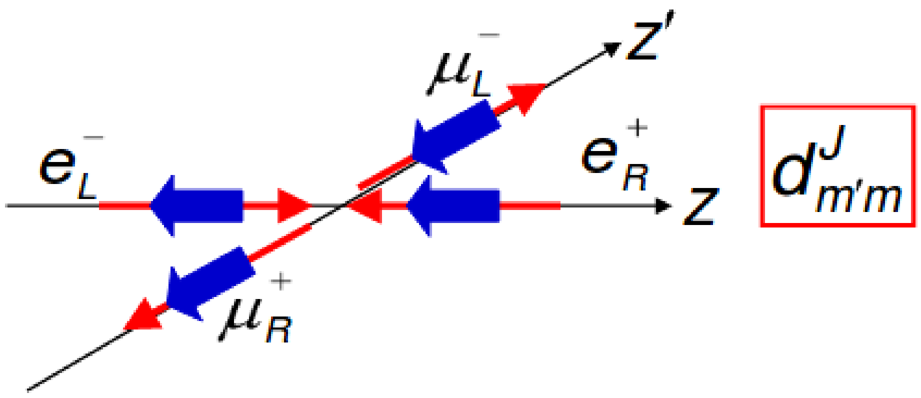
For illustration:

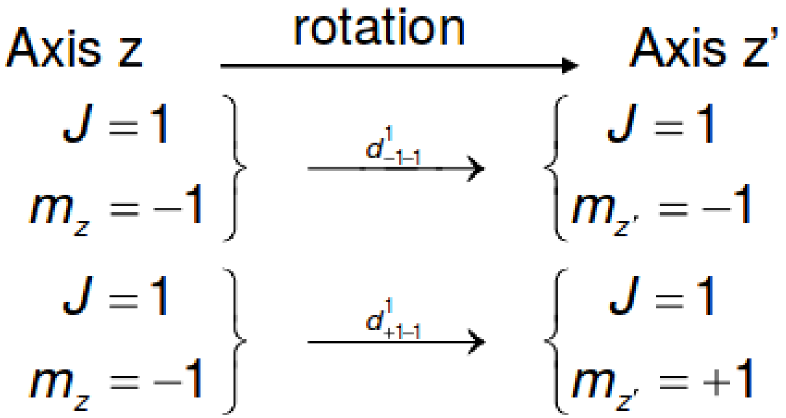


Vector current: $ie\overline{u}\gamma^\mu u$



Angular distribution: $e^+e^- \to \mu^+\mu^-$





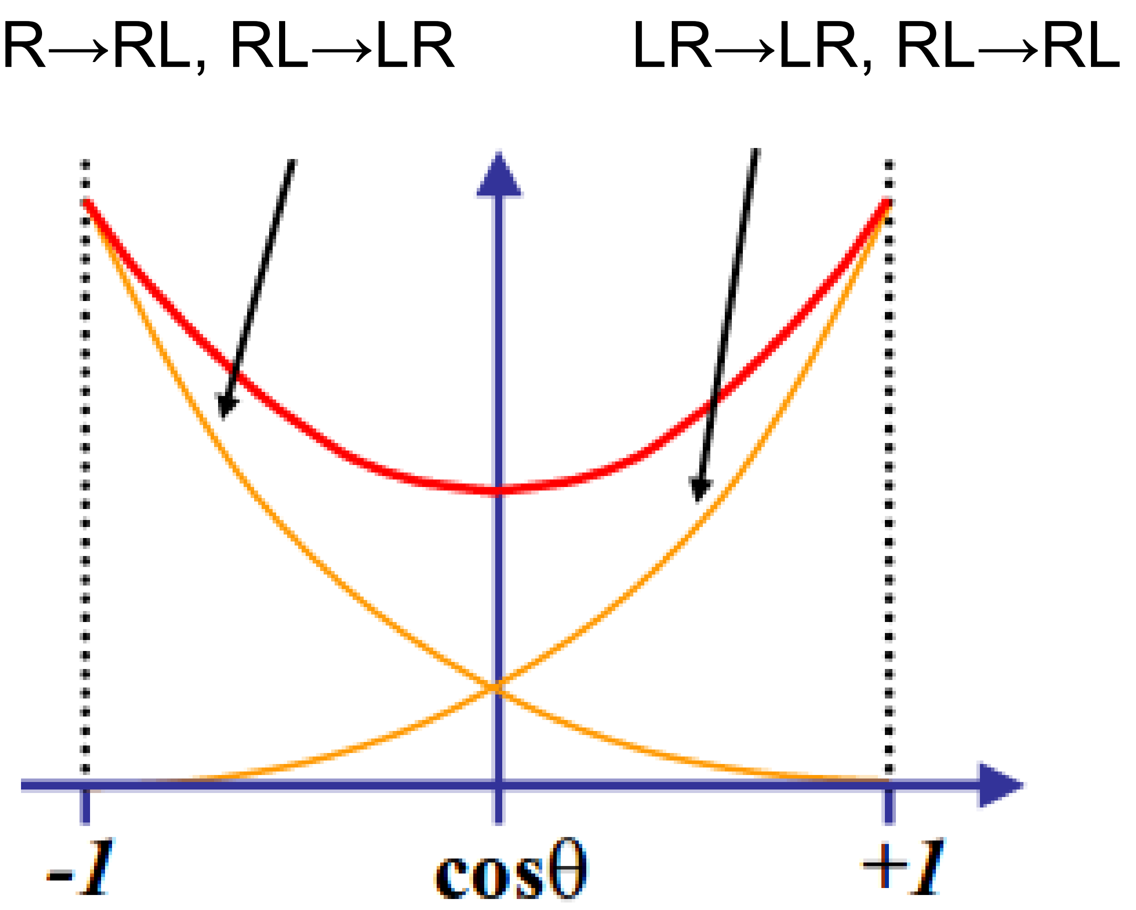
Scattering can be treated as a change of the quantization axis.

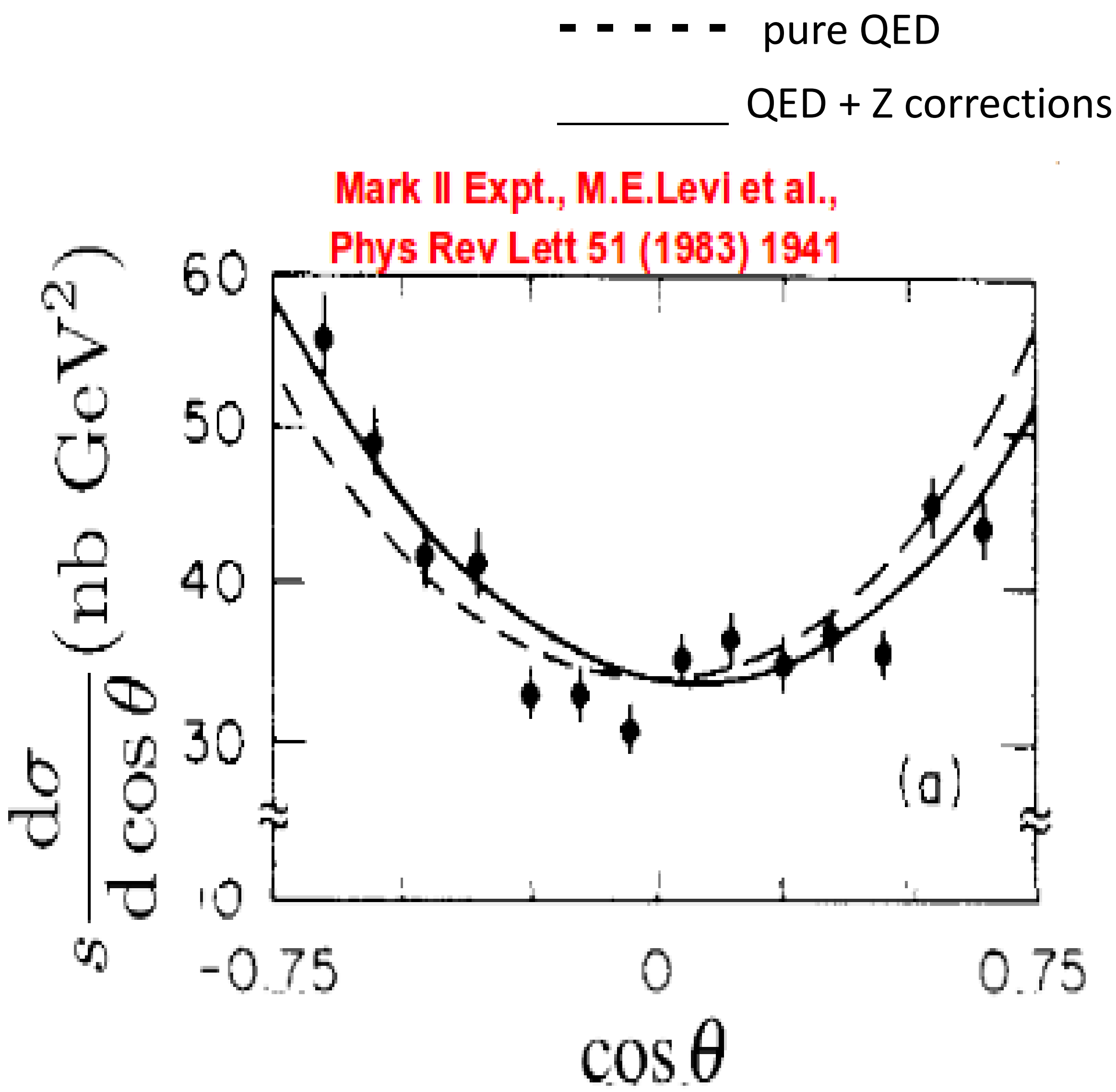
$$d^1_{1,1} = d^1_{-1,-1} = \frac{1}{2}(1+\cos(\theta))\ [\mathrm{LR} \to \mathrm{LR},\ \mathrm{RL} \to \mathrm{RL}]$$
$$d^1_{1,-1} = d^1_{-1,1} = \frac{1}{2}(1-\cos(\theta))\ [\mathrm{LR} \to \mathrm{RL},\ \mathrm{RL} \to \mathrm{LR}]$$

$$\frac{d\sigma}{d\Omega} \sim \frac{1}{4}(1+\cos\theta)^2 + \frac{1}{4}(1-\cos\theta)^2 \sim 1+\cos^2\theta$$

Angular distribution is an effect of vector coupling $ie\gamma^\mu$

$$e^+e^- \to \mu^+\mu^-$$





angular distribution becomes slightly asymmetric
In higher order QED or when Z contribution is included

Test of QED

≡ measurement of the electromagnetic fine structure constant α in differnt systems

1) High energy range, accessible with particle colliders

2) Low energy range, accessible with small experiments
(magnetic moment of the electron → most precise test of QED)

3) Condensed matter systems (quantum Hall effect, Josephson effect)

Test of QED at colliders: experimental methods

Reminder: $\dot{N}_s = \sigma \mathcal{L}$

$\mathcal{L}_{int} = \int \mathcal{L} dt$

e+ e- accelerators (a selection)

Accelerator	Lab	Experiment	√s	$\mathcal{L}_{int}$/Experiment	
SPEAR	SLAC	SPEAR	2-8 GeV		J/Ψ discovery
PEP	SLAC	MARK …	29 GeV	200-300 pb-1	τ discovery
PETRA	DESY	PLUTO, TASSO			
		JADE, CELLO, MARK J	12-47 GeV	~20 pb-1	gluon discovery
TRISTAN	KEK	TRISTAN	50-60 GeV	~20 pb-1	Aim: discovery of 3rd quark family
LEP	CERN	DELPHI, L3,			Z physics
		OPAL, ALEPH	90 GeV	~200 pb-1	

Cross section (experimental definition)

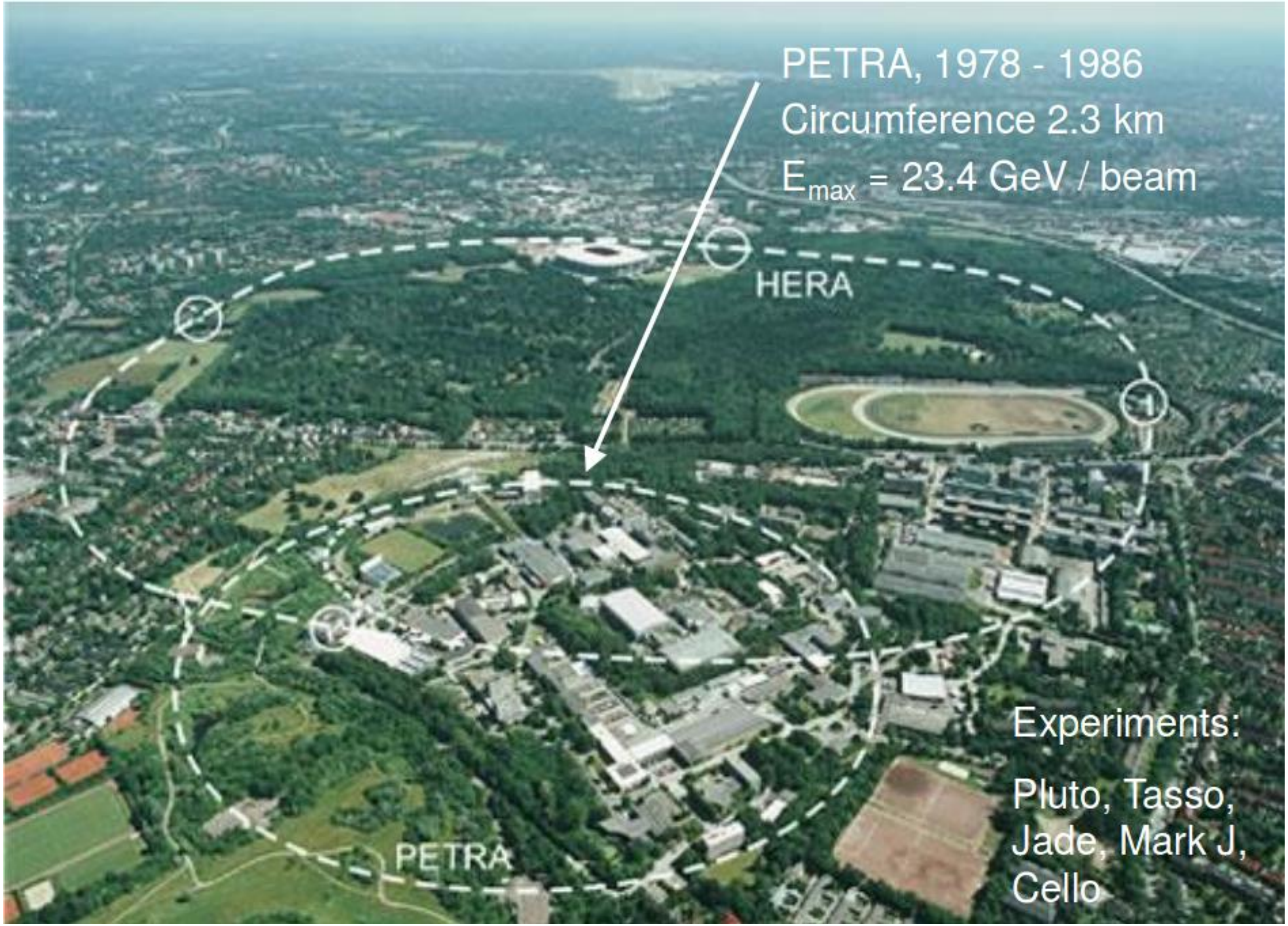
$$\sigma(e^+e^- \to f\overline{f}) = \frac{N_{ff}(1-b)}{\epsilon \mathcal{L}_{int}}$$

N_{ff}: number of detected $e^+e^- \to ff$ events

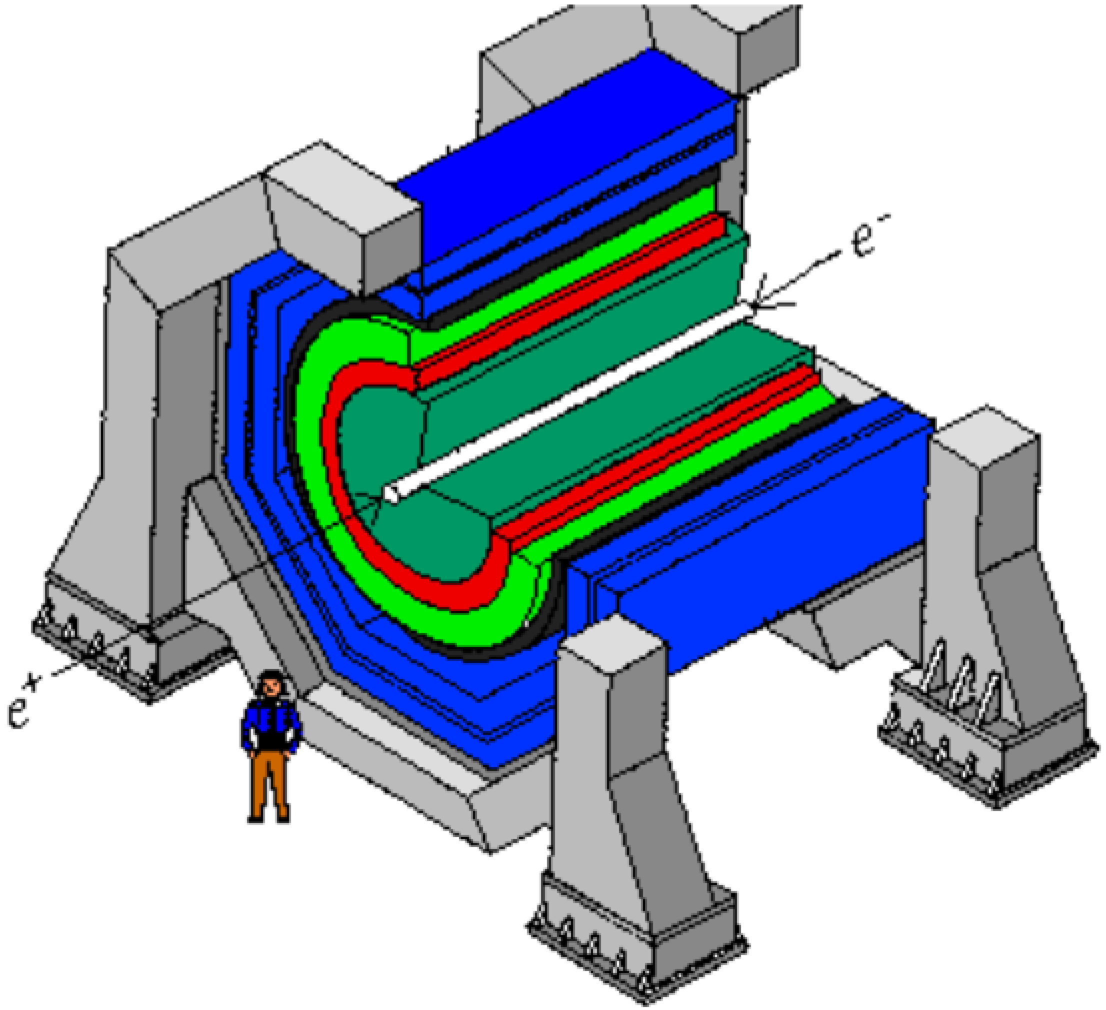
b : background fraction

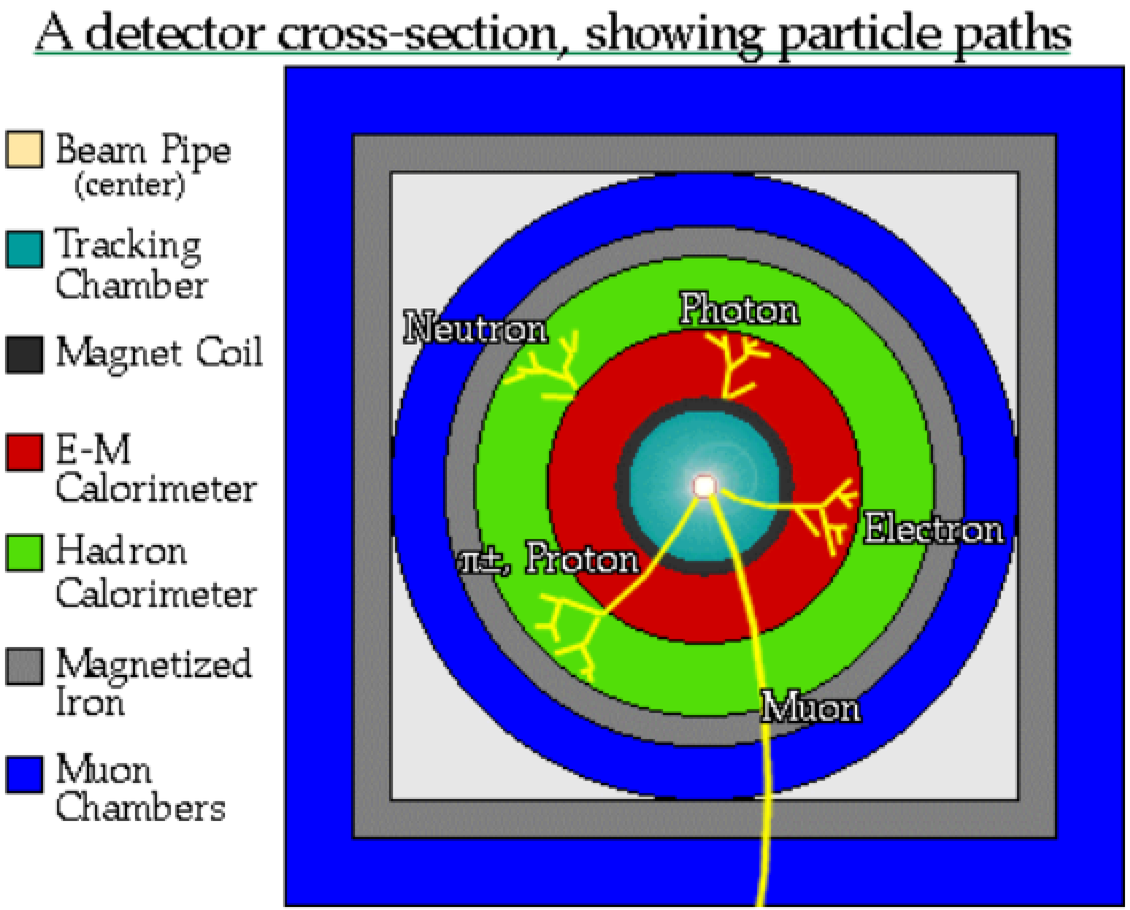
ϵ : acceptance/efficiency

$\mathcal{L}_{int}$: integrated luminostiy of collider

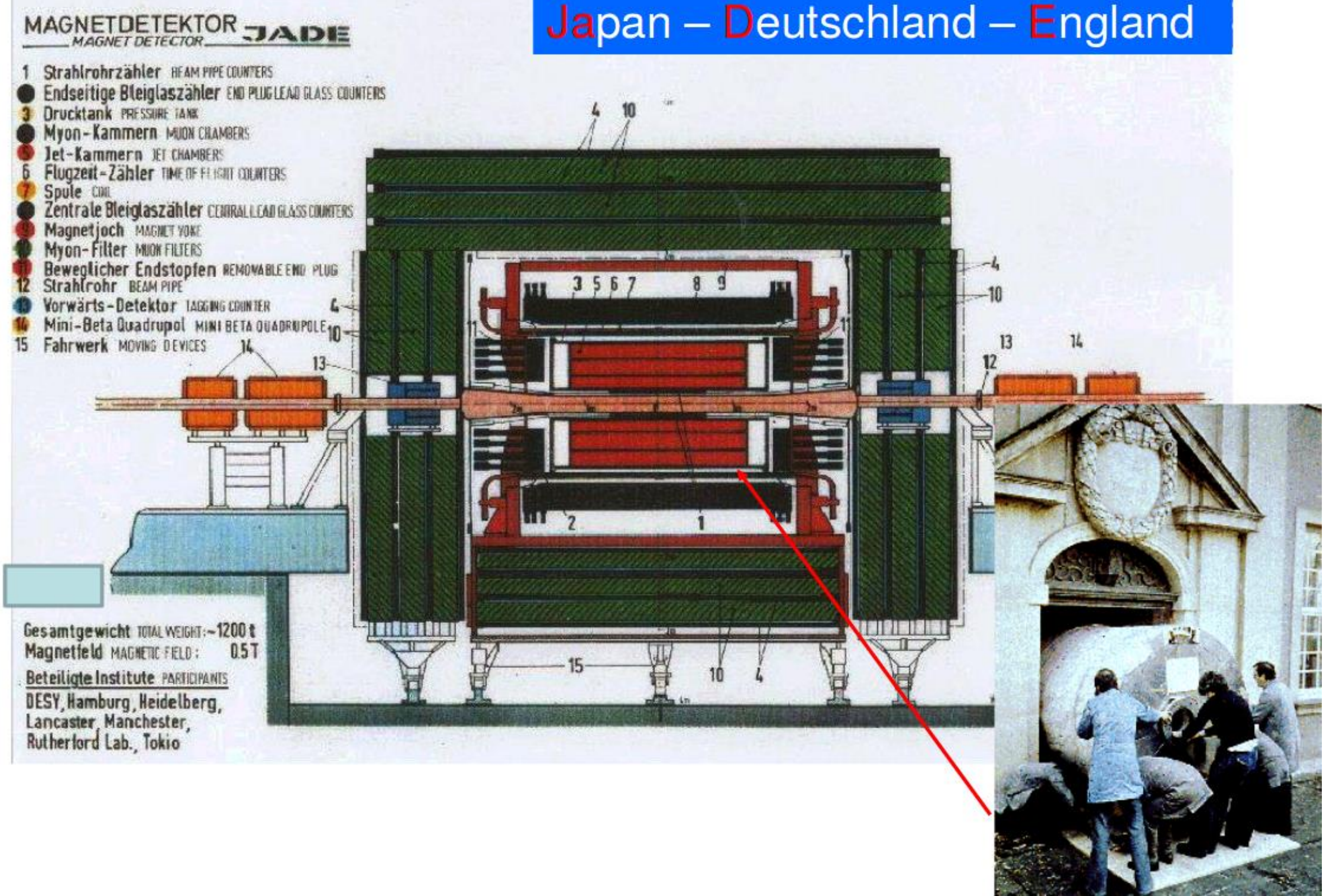

PETRA, 1978 - 1986
Circumference 2.3 km
E_{max} = 23.4 GeV / beam
HERA
Experiments:
Pluto, Tasso, Jade, Mark J, Cello
PETRA

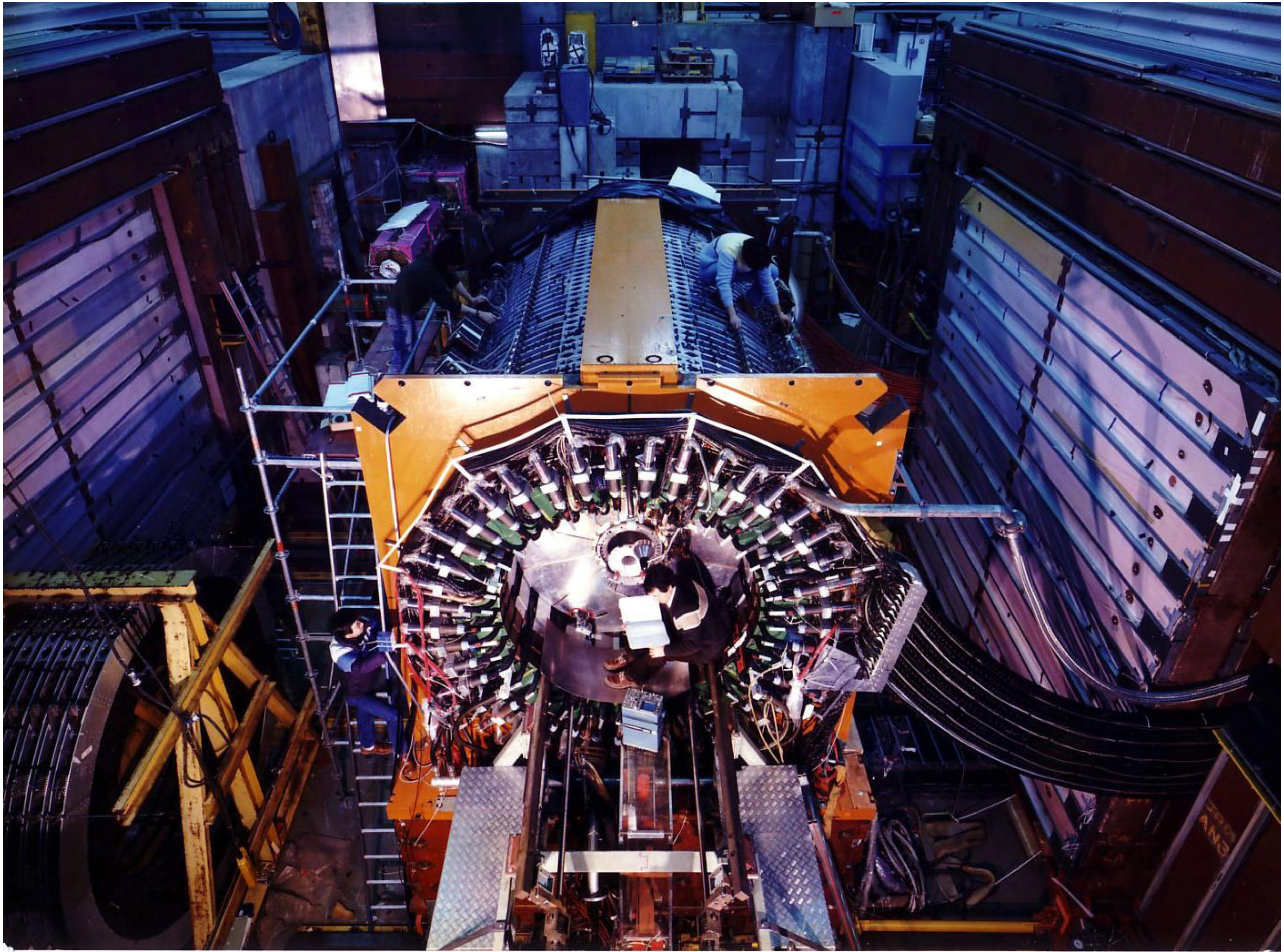
Particle detectors

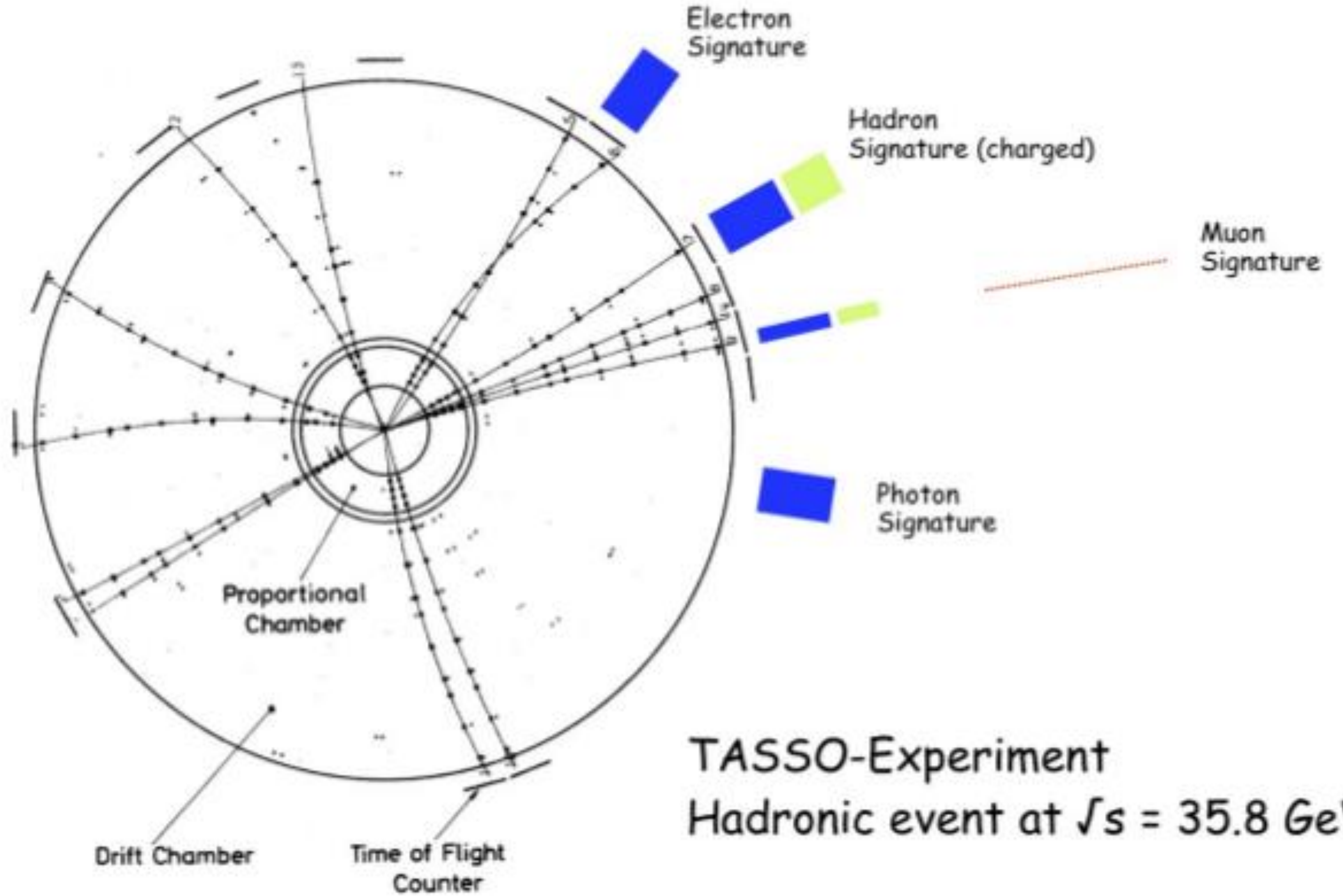




Japan – Deutschland – England





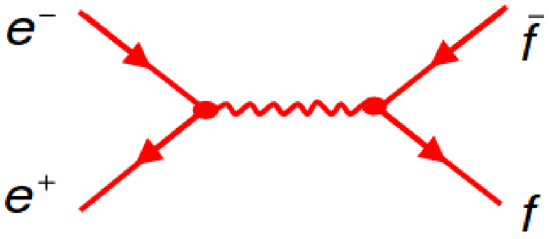


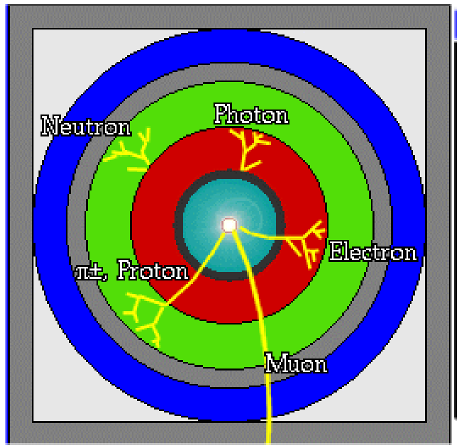
TASSO-Experiment

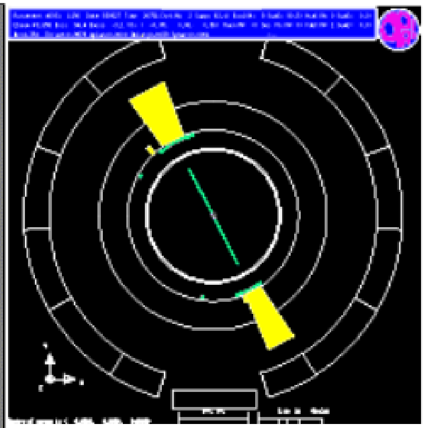
Hadronic event at $\sqrt{s}$ = 35.8 GeV

Experimental signatures

OPAL / LEP



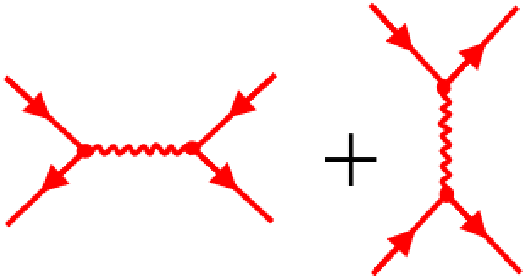




e^-e^+

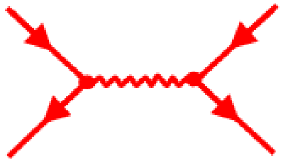
$f\,\bar{f}$ =

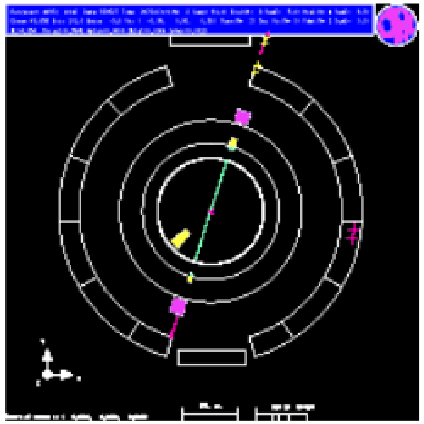
e^-e^+



$\mu^-\mu^+$

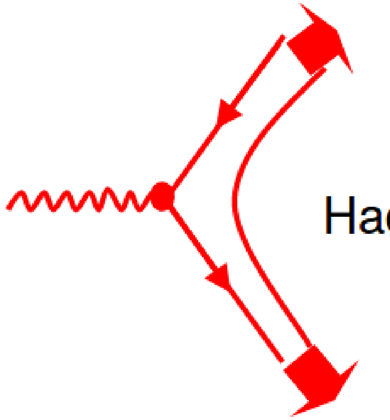
$\tau^-\tau^+$





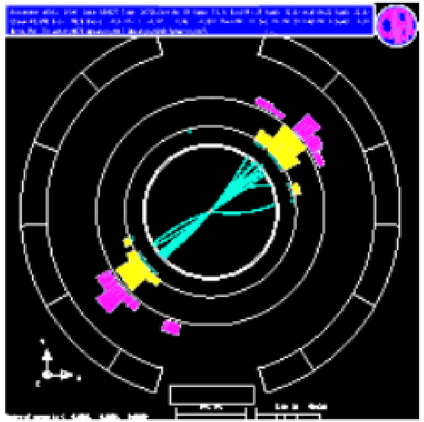
$\mu^-\mu^+$

$q\,\bar{q}$ mit
$q = u, d, s, c, b, (t)$

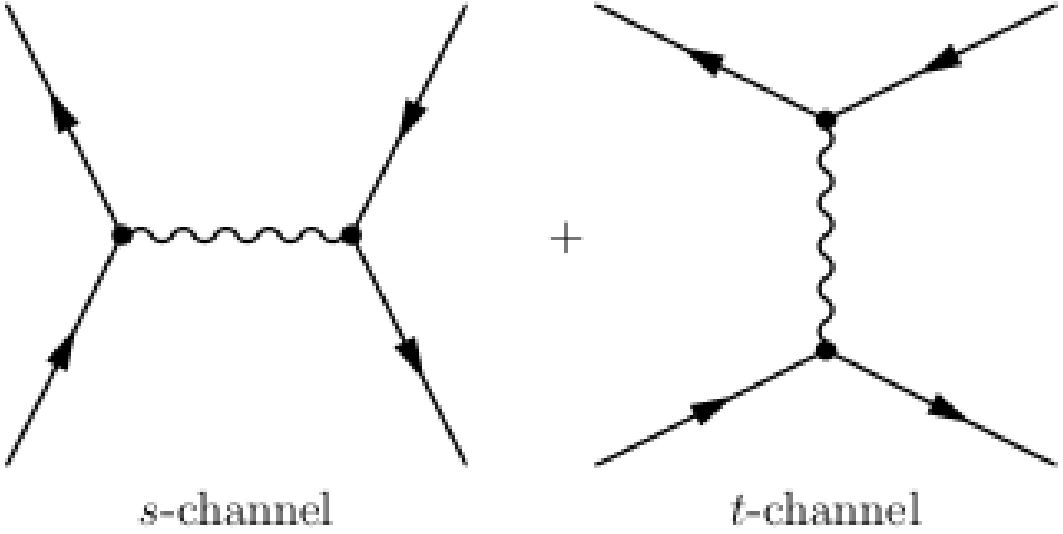


Hadron jets

$q\,\bar{q}$



Bhabha scattering: e+e- → e+e-

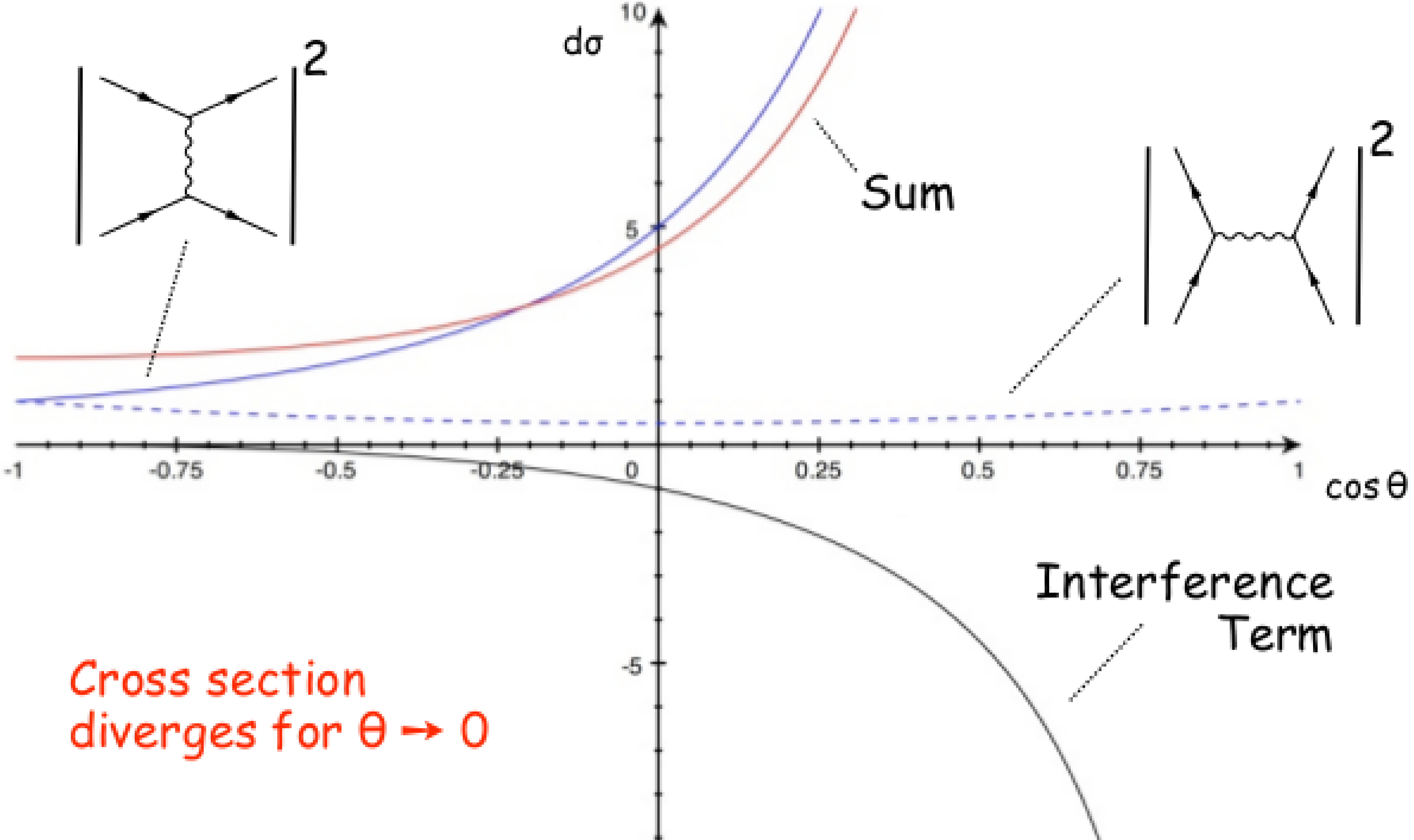


$$\frac{d\sigma_0}{d\Omega} = \frac{\alpha^2}{4s}\left(\underbrace{\frac{t^2+s^2}{u^2}}_{t\text{-channel}} + \underbrace{\frac{2t^2}{us}}_{\text{interference}} + \underbrace{\frac{t^2+u^2}{s^2}}_{s\text{-channel}}\right)$$

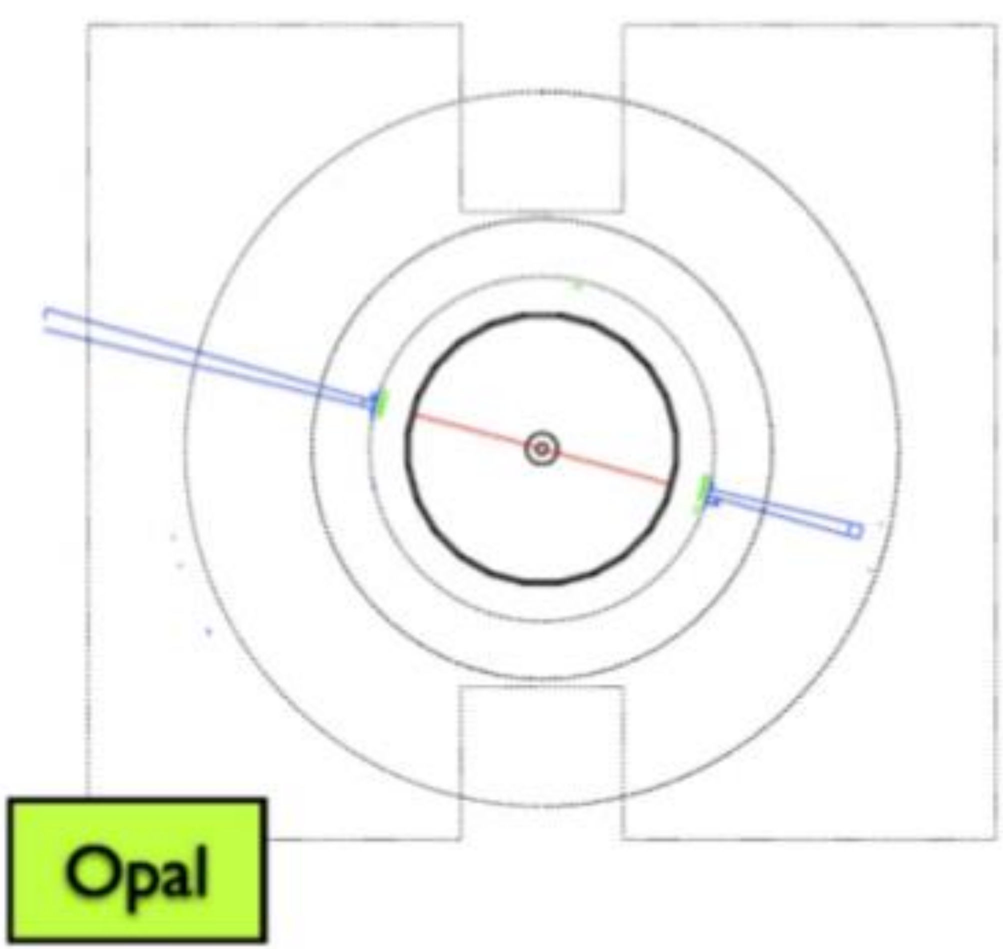
$$= \frac{\alpha^2}{4s}\left(\frac{3+\cos^2\theta}{1-\cos\theta}\right)^2$$

$$\frac{d\sigma}{d\Omega}_{CMS} = \frac{\alpha^2}{4s}(1+\cos^2\theta)$$

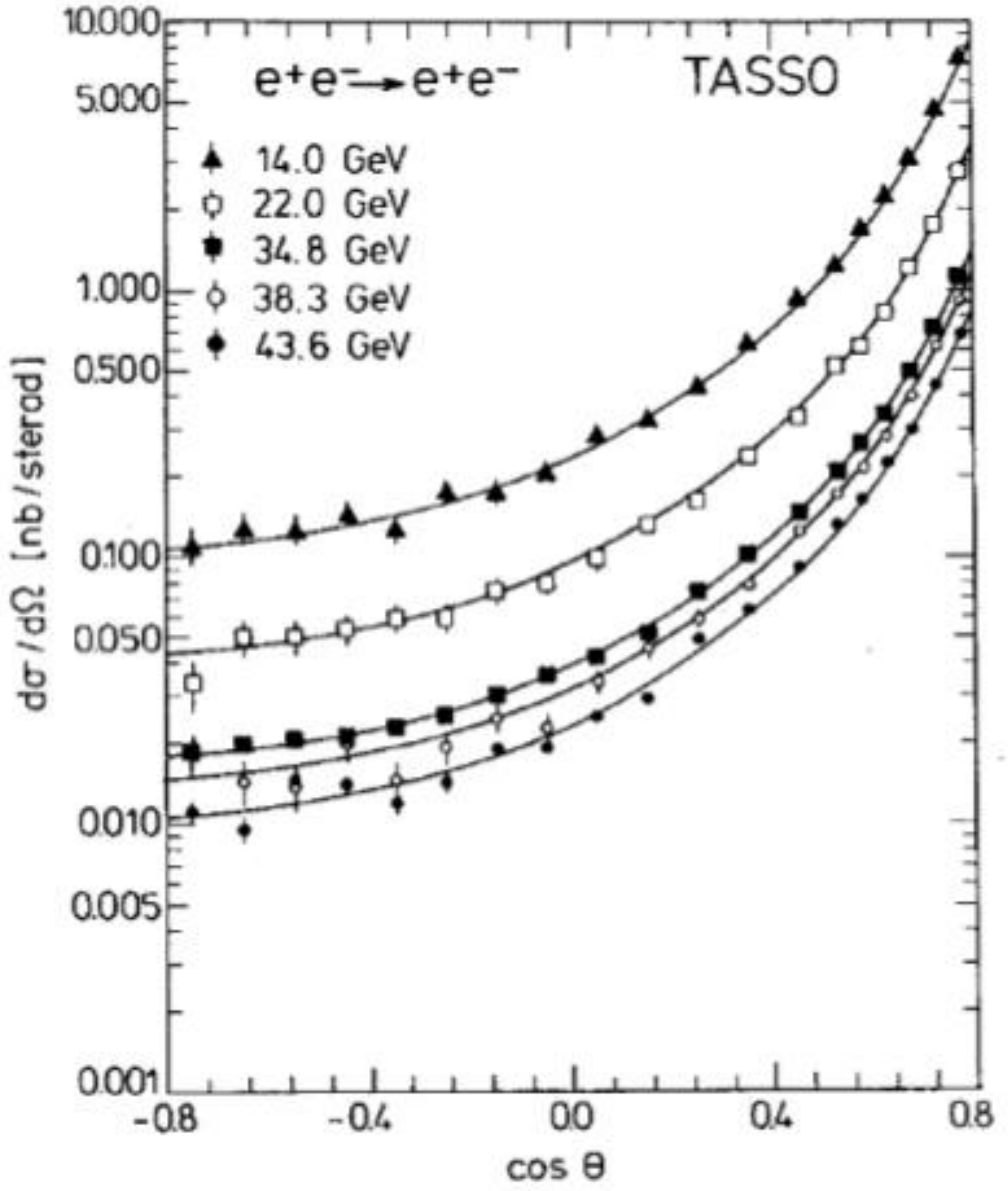
$$\frac{d\sigma}{d\Omega_{CMS}} = \frac{2\alpha^2}{s}\,\frac{1+0.25(1+\cos\theta)^2}{(1-\cos\theta)^2}$$



dσ
10
5
-5
-1
-0.75
-0.5
-0.25
0
0.25
0.5
0.75
1
cos θ
2
2
Sum
Interference
Term
Cross section
diverges for θ → 0



Impressive agreement for first order computation!



Higher order loop and radiative corrections

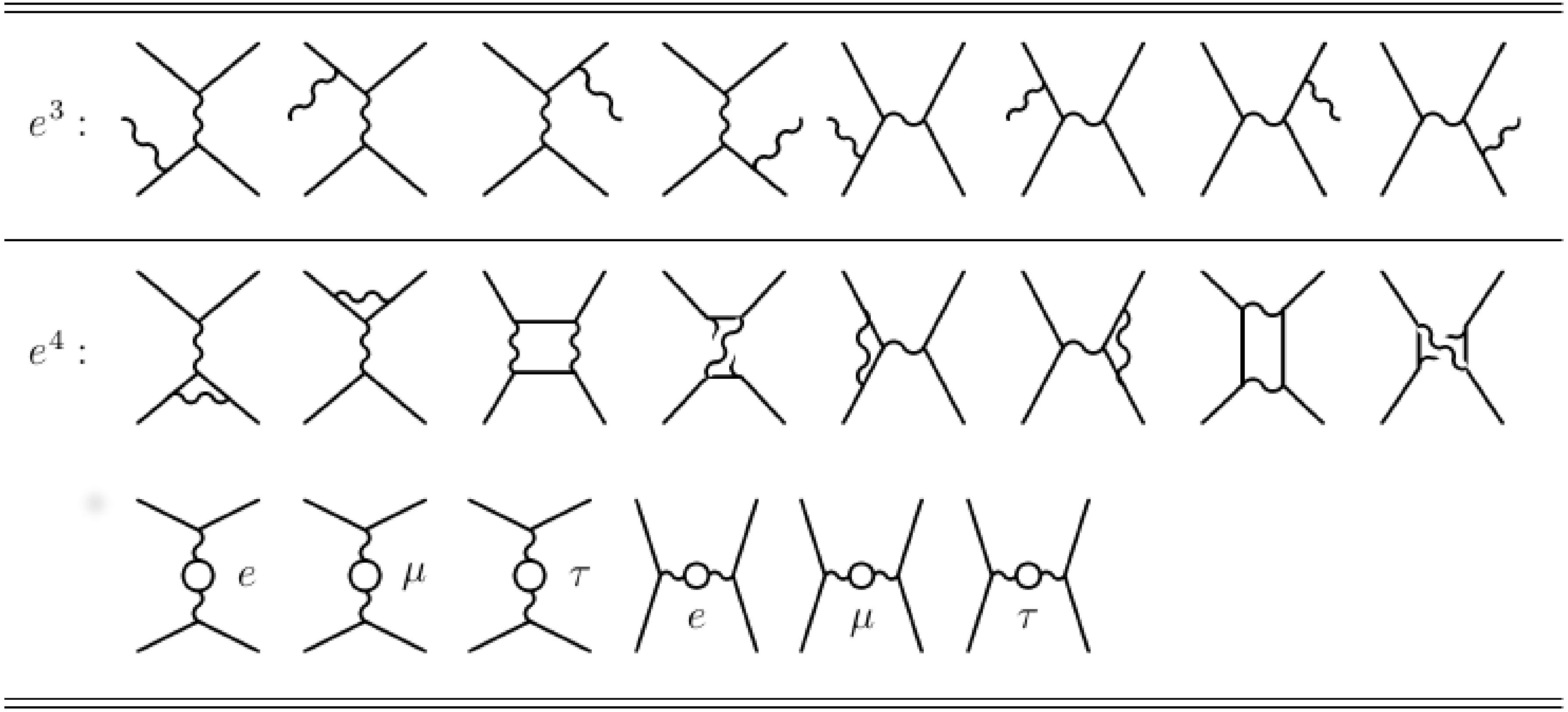


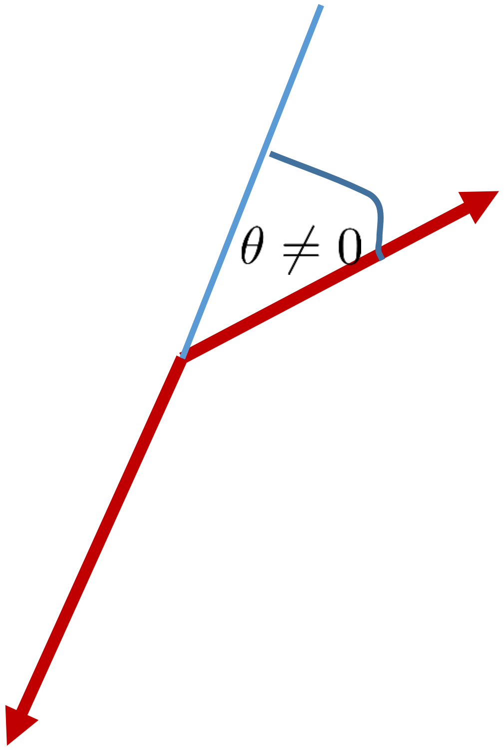
Table 6.2: Diagrams of radiative and loop corrections up to e^4

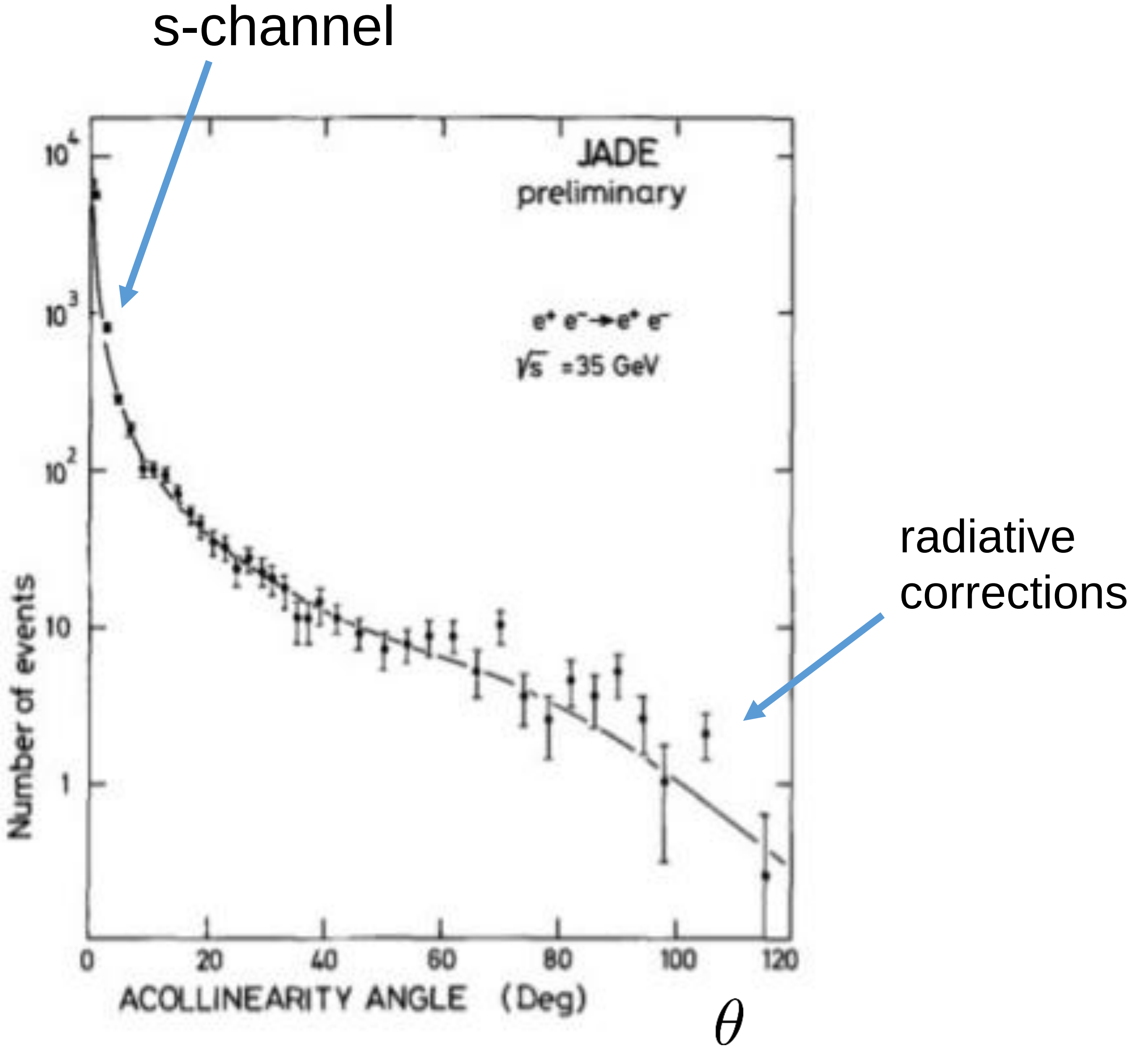
radiative corrections

loop corrections

Test of radiative corrections

Emission of photon in final state
→ final state particle are not
anymore back-to-back





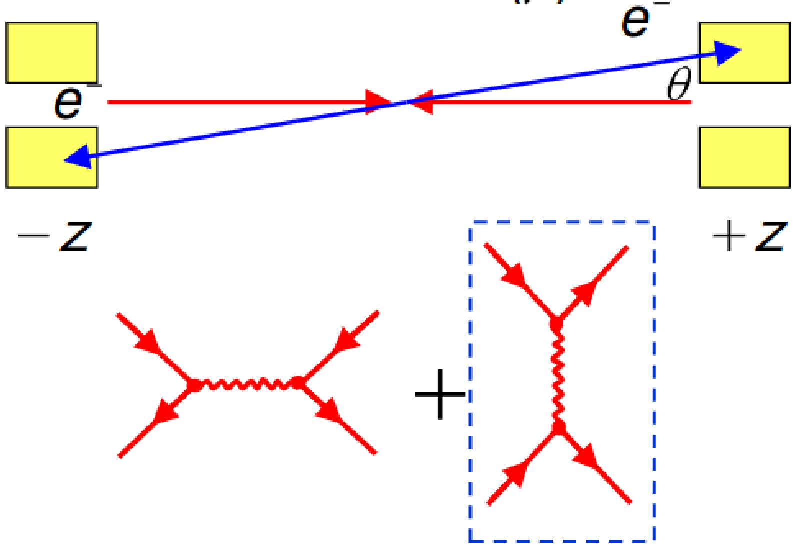
Very good agreement as well in higher order corrections.

Interlute: determination of integrated luminosity

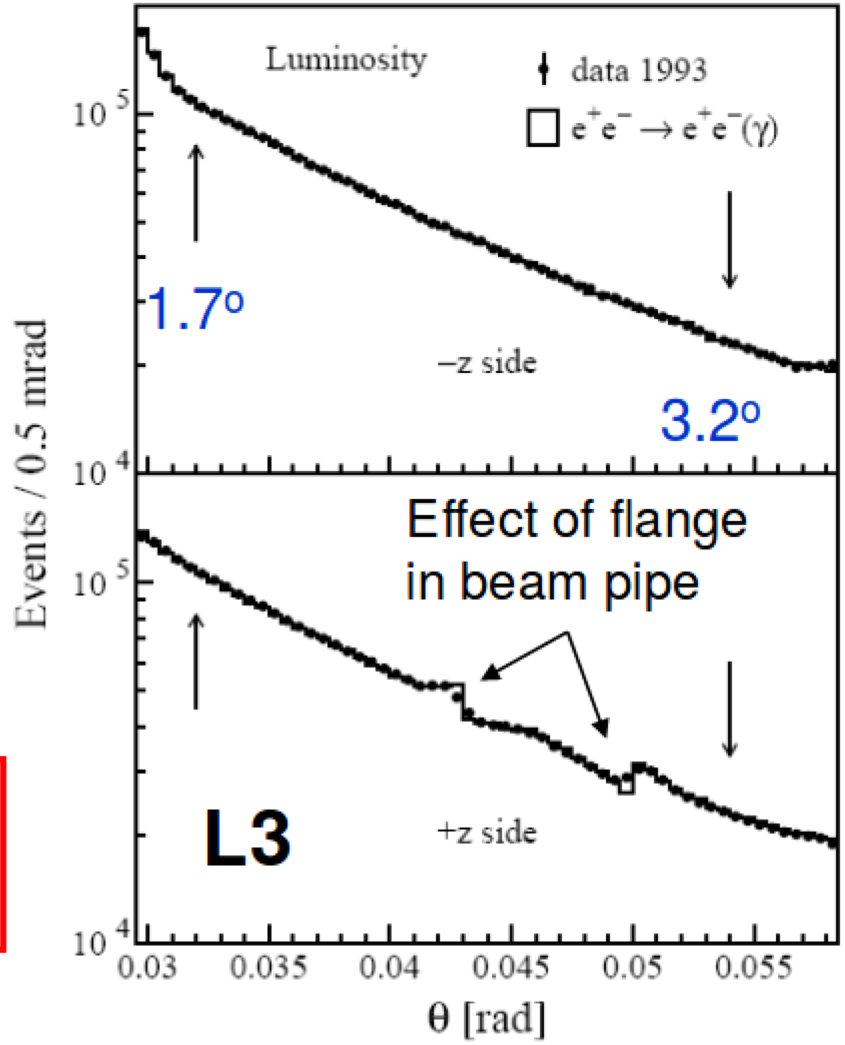
$$\mathcal{L}_{int} = \mathcal{L}_{ee}(t)dt$$

Use reference process to determine $\mathcal{L}_{int}$:
small angel Bhabha scattering (low momentum transfer)

$e^+e^- \rightarrow e^+e^-(\gamma)$



$$\mathcal{L}_{int} = \frac{N_{ee}(1-b)}{\sigma_{theo}\epsilon}$$



Small angle Bhabha scattering is **t channel** dominated;
theoretical cross section well known, σ_{theo}
background fraction b extremly low, ε (acceptance effects, efficiencies)

at LEP typical errors < 0.5%

Luminosity determination a the LHC

Problem: Cross section for nearly all reference processes at LHC
rely on the knowledge of the proton structure functions
and cross sections for well understood processes are small!
→ **Need process independent luminosity determination!**

Luminosity from first principle:

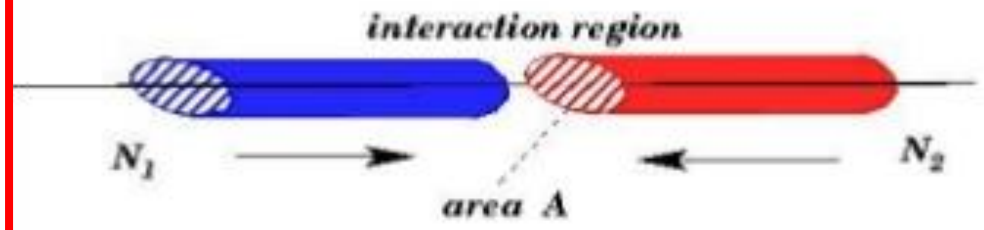
Remember: $\dot{N}_s = \sigma \mathcal{L} = \sigma \frac{\dot{N}_i N_t}{A}$

$$\mathcal{L} = \frac{N_1 N_2 f N_b}{A} = \frac{N_1 N_2 f N_b}{4\pi \sigma_x \sigma_y}$$

$A_{eff} = \int g_1(x,y) g_2(x,y) dxdy$

with equal gaussians:

$g_1 = g_2 = \frac{1}{2\pi\sigma_x\sigma_y} exp[-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}]$

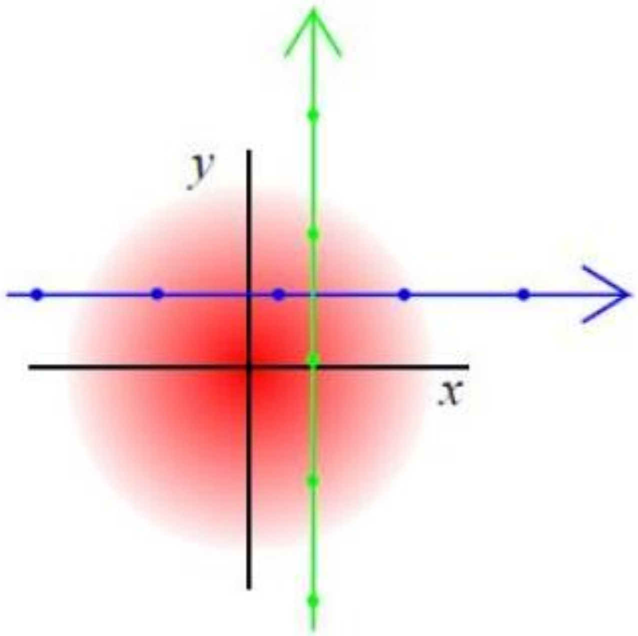


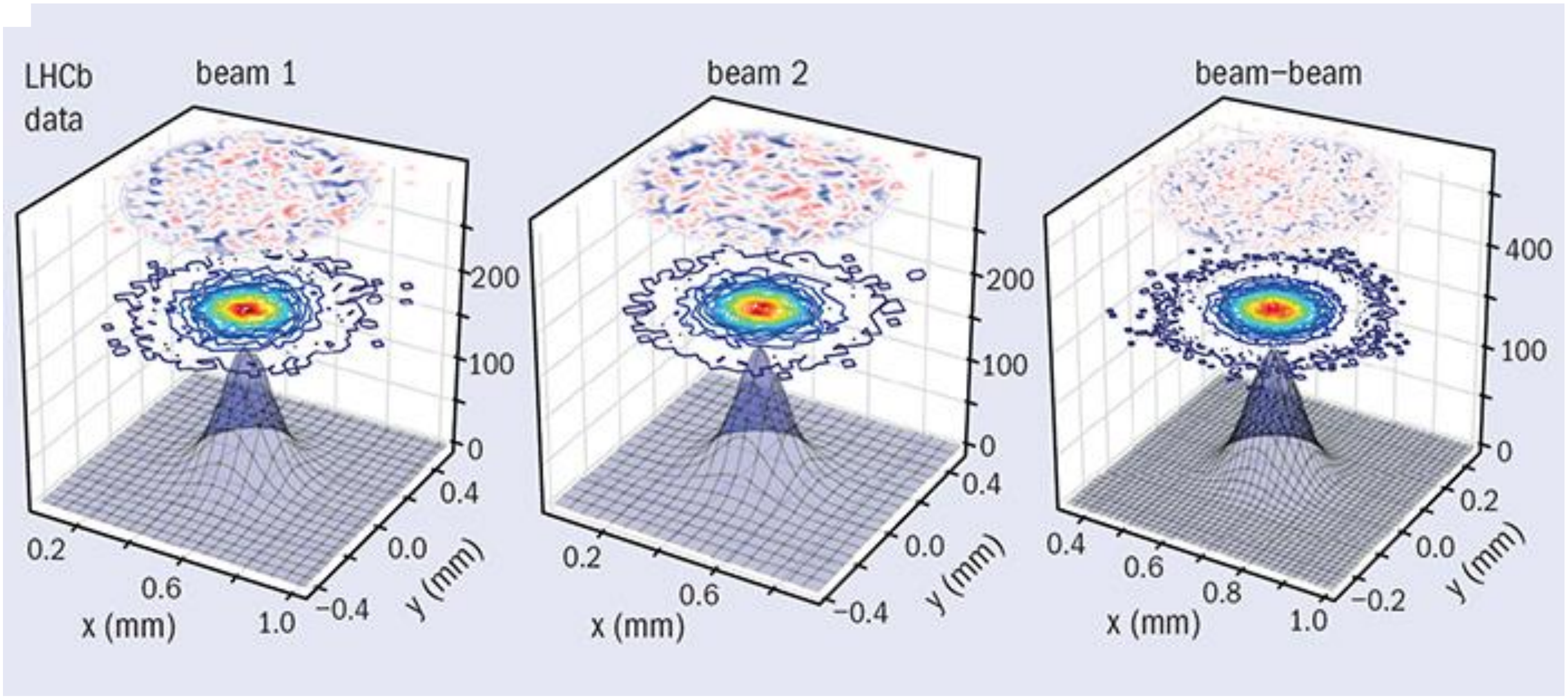
$N_{1,2}$: number of protons in bunches
N_b : number of colliding bunches in the ring
f : revolution frequency
A : effective beam transverse A

$N_{1,2}, N_b, f$ known by the operations crew.
only unknown: beam profiles at the collision points.

Van der Meer scan:

Separate beams by knwon amount and measure the change of particle rate

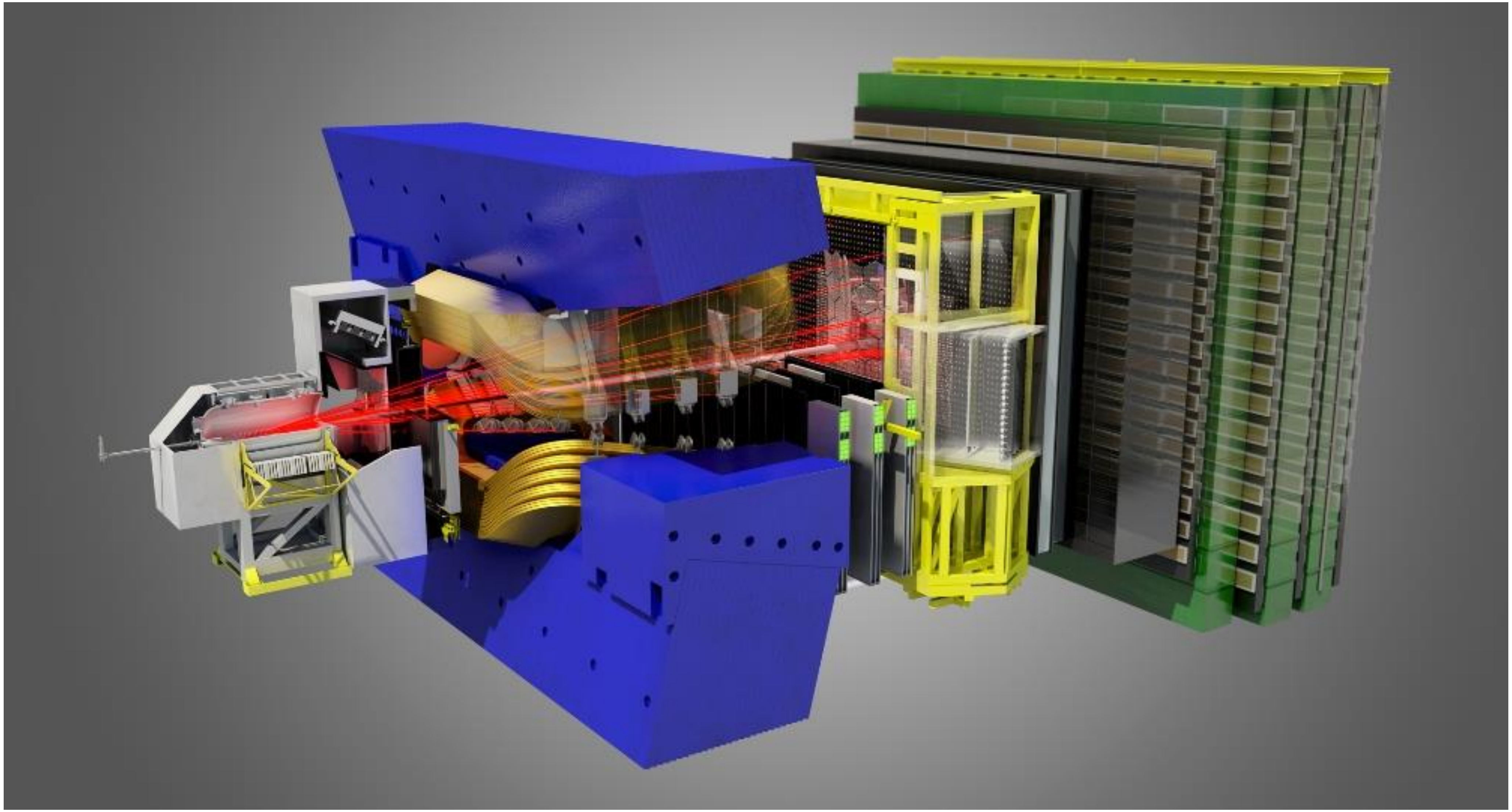




Luminosity uncertainty at the LHC about 1.5%.

Beam gas method:

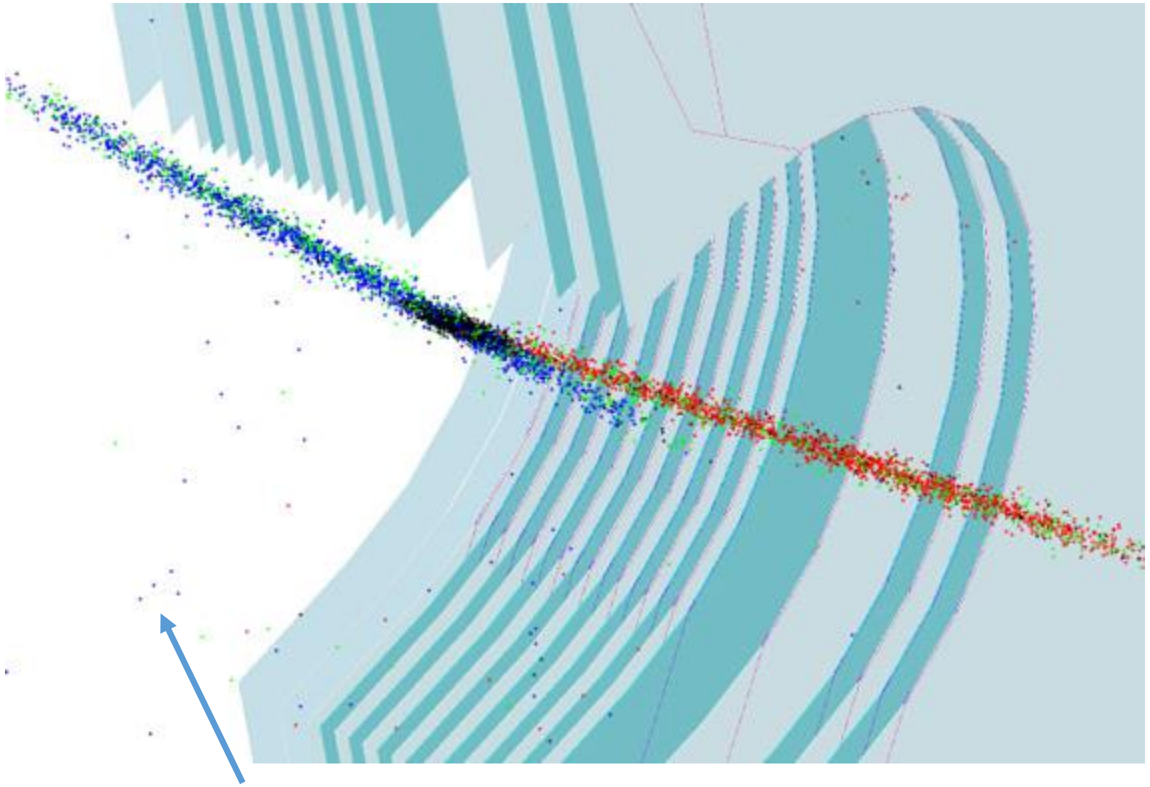
measure beam profile by reconstruction the primary vertex position of events from scattering the beam on gas in the beampipe



Special features of LHCb:
high precision vertex resolution and equipped closed to the beam (8mm) an in high eta range (forward region)

vertex detector of LHCb experiment

reconstructed vertices of beam gas events



LHCb Beam-Gas-Method: ~ 1.5% resolution

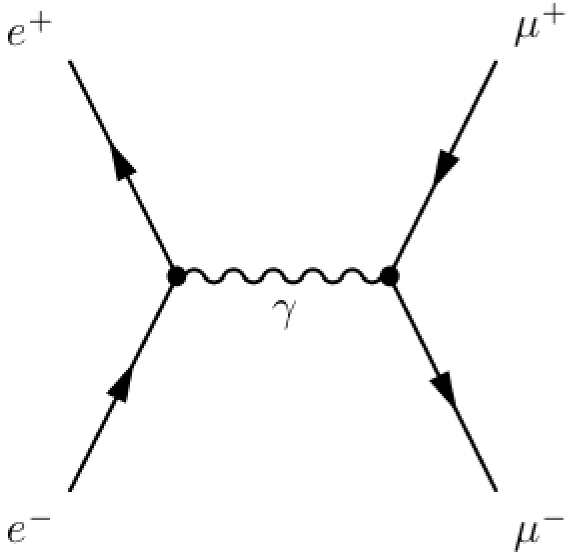
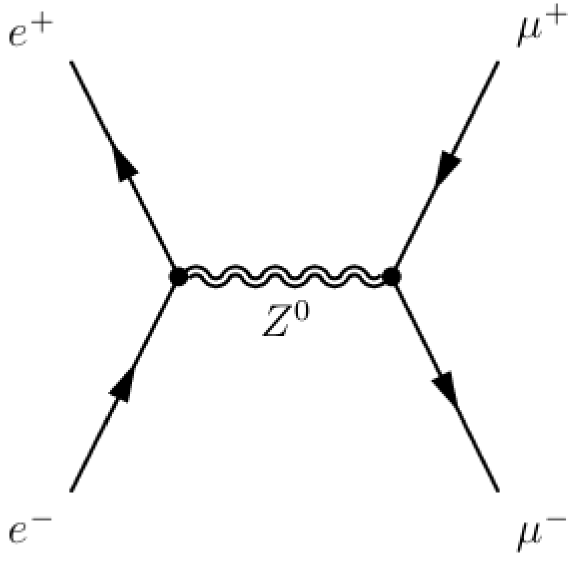
→ average Van-der-Meet and BGM

Special BGM are about to be installed infront and behind GPDs
(General purpose detectors: ATLAS, CMS)

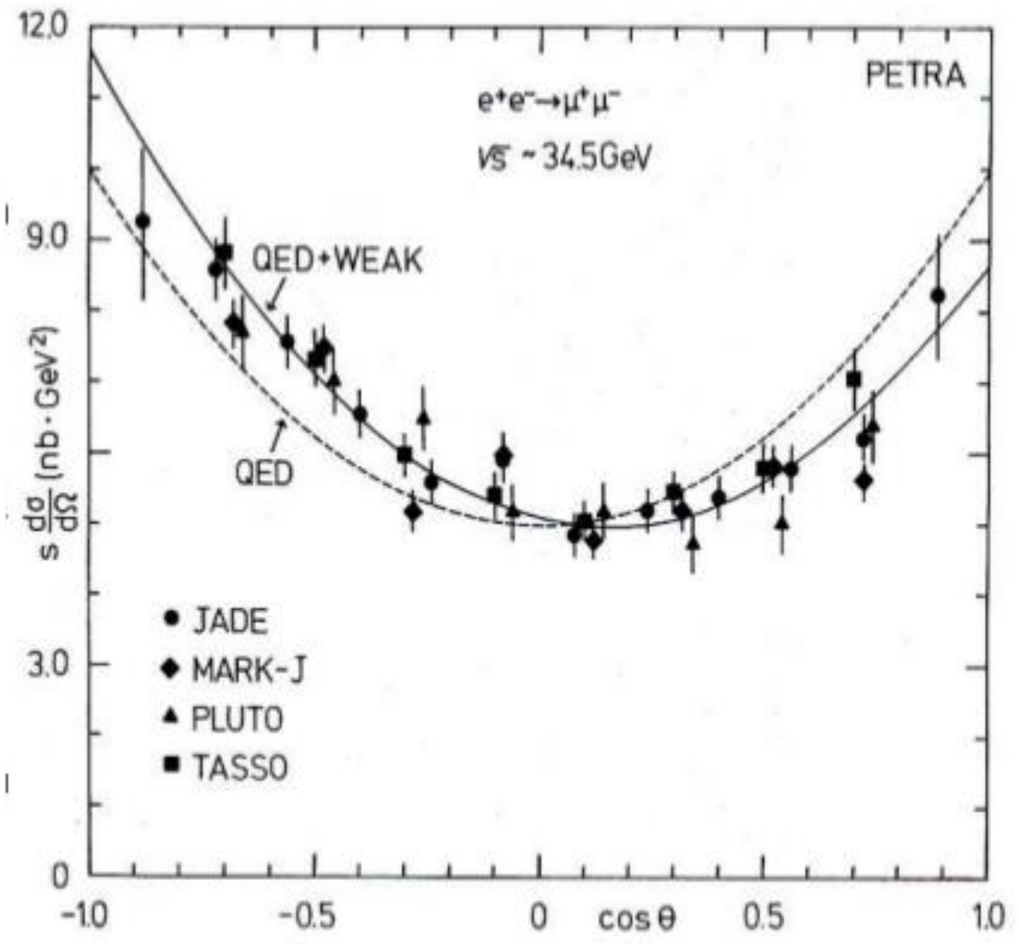
Beam gas method revealed proton spread of the beam
→ correct for this systematical effect in Van der Meer scan

$$e^+e^- \rightarrow \mu^+\mu^-$$

Only s-channel possible, thus need to take interference with Z into account

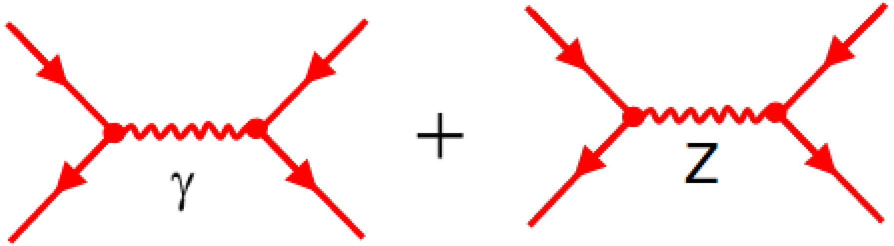
$\Bigg|$  $\Bigg|^2$ + interference + $\Bigg|$  $\Bigg|^2$

Effect of Z boson exchange



Clear deviation from QED
effect of electro-weak γ/Z inteference

„heavy photon w/ different couplings“



$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} \cdot (1 + \cos^2\theta)\Big|_{QED}$$

Vector coupling!

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} \cdot (1 + \cos^2\theta + A\cos\theta)\Big|_{\gamma+Z}$$

The effect of the “heavy” Z boson is already seen at low energies!

Limits of QED

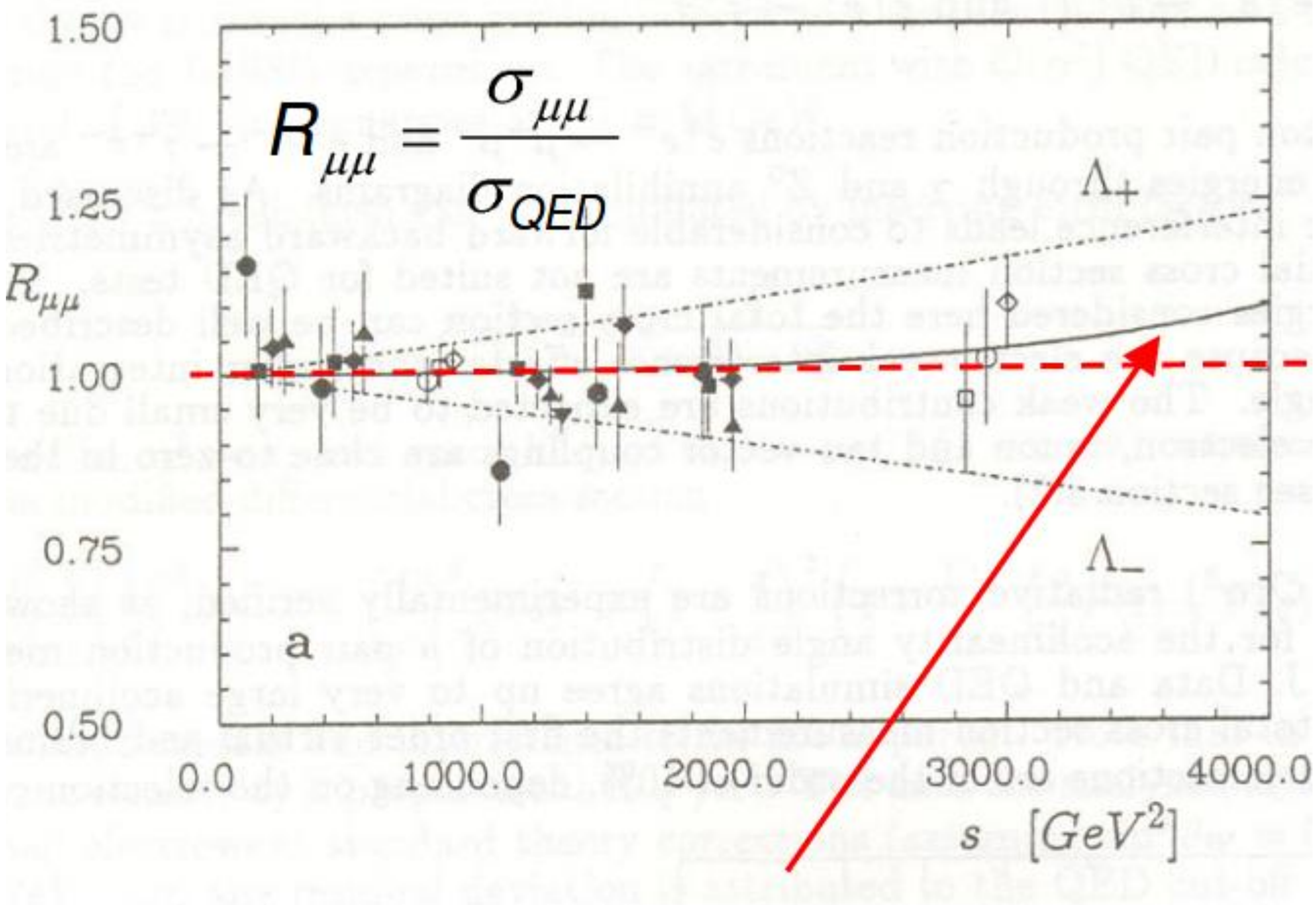
Possible deviation from QED:
Additonal heavy photon

$$\frac{1}{r} \rightarrow \frac{1}{r}(1 - e^{-\Lambda r})$$

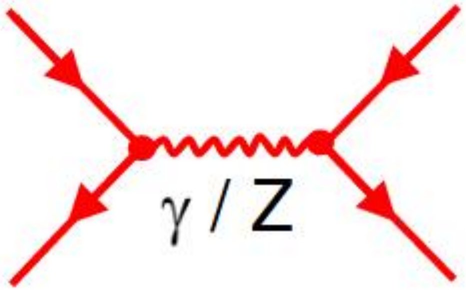
$$\frac{1}{q^2} \rightarrow \frac{1}{q^2}(1 + \frac{q^2}{\Lambda^2}) = \frac{1}{q^2}F(q^2)$$

$$\sigma^{e^+e^- \rightarrow \mu^+\mu^-} \rightarrow \frac{4\pi\alpha^2}{3s}(1 \pm \frac{s}{\Lambda^2 - s})^2$$

Λ corresponds to the mass of the new photon



$$\frac{\sigma_{\mu\mu}^{\gamma+Z}}{\sigma_{QED}} \leftarrow$$



Λ > 200 GeV
→ confirms „Coulomb law“
& point-like nature of electron down to 10^{-18} m

Limits of QED

Similar tests have been performed in Bhabha scattering

Form factor modifies differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\left(\frac{u^2+s^2}{t^2}\left|F(t)\right|^2 + \frac{2u^2}{ts}\left|F(t)F(s)\right| + \frac{u^2+t^2}{s^2}\left|F(s)\right|^2\right)$$

s-channel interference t-channel

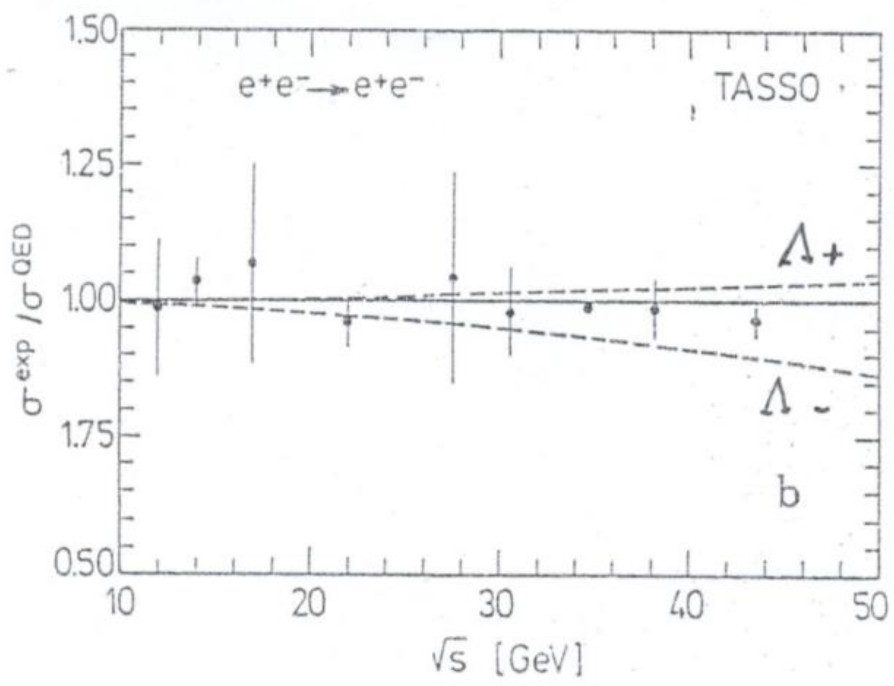
Combined PETRA result:

$\Lambda^+ > 435$ GeV at 95% CL

$\Lambda^- > 590$ GeV at 95% CL

Tasso: $\Lambda_+ > 370$ GeV

$\Lambda_- > 190$ GeV



Discovery of the Tau Lepton

MARK I (SLAC), 1975 M. Pearl et al.
Nobel Prize 1995 for M. Pearl

Evidence for Anomalous Lepton Production in e^+-e^- Annihilation*

M. L. Perl, G. S. Abrams, A. M. Boyarski, M. Breidenbach, D. D. Briggs, F. Bulos, W. Chinowsky, J. T. Dakin,† G. J. Feldman, C. E. Friedberg, D. Fryberger, G. Goldhaber, G. Hanson, F. B. Heile, B. Jean-Marie, J. A. Kadyk, R. R. Larsen, A. M. Litke, D. Lüke,‡ B. A. Lulu, V. Lüth, D. Lyon, C. C. Morehouse, J. M. Paterson, F. M. Pierre,§ T. P. Pun, P. A. Rapidis, B. Richter, B. Sadoulet, R. F. Schwitters, W. Tanenbaum, G. H. Trilling, F. Vannucci,‖ J. S. Whitaker, F. C. Winkelmann, and J. E. Wiss

Lawrence Berkeley Laboratory and Department of Physics, University of California, Berkeley, California 94720, and Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

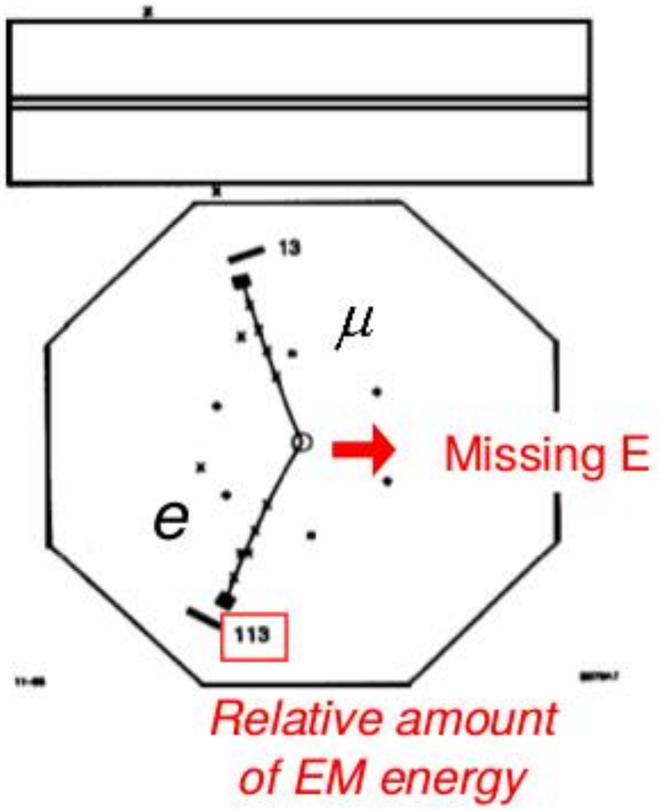
(Received 18 August 1975)

We have found events of the form $e^+ + e^- \to e^\pm + \mu^\mp$ + missing energy, in which no other charged particles or photons are detected. Most of these events are detected at or above a center-of-mass energy of 4 GeV. The missing-energy and missing-momentum spectra require that at least two additional particles be produced in each event. We have no conventional explanation for these events.

We have found 64 events of the form

$$e^+ + e^- \to e^\pm + \mu^\mp + \geq 2 \text{ undetected particles} \quad (1)$$

for which we have no conventional explanation. The undetected particles are charged particles or photons which escape the 2.6π sr solid angle of the detector, or particles very difficult to detect such as neutrons, $K_L{}^0$ mesons, or neutrinos. Most of these events are observed at center-of-mass energies at, or above, 4 GeV. These events were found using the Stanford Linear Accelerator Center–Lawrence Berkeley Laboratory (SLAC-



Explanation:

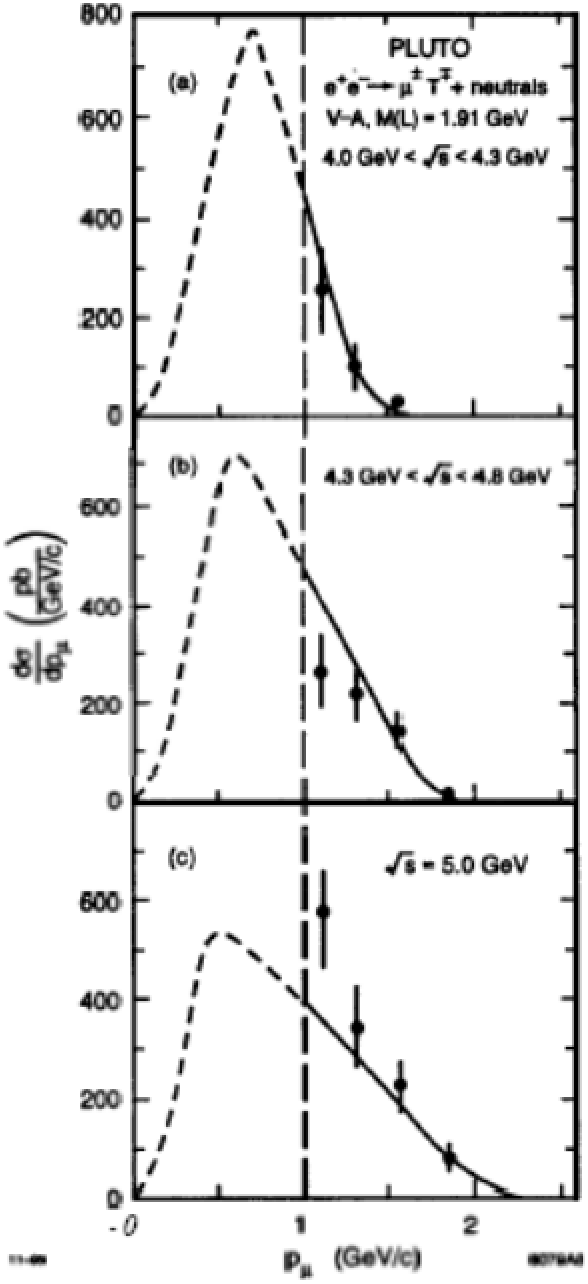
$$e^+ e^- \to \tau^+ \tau^-$$

$$\tau^- \to e^- \bar{\nu}_e \nu_\tau$$

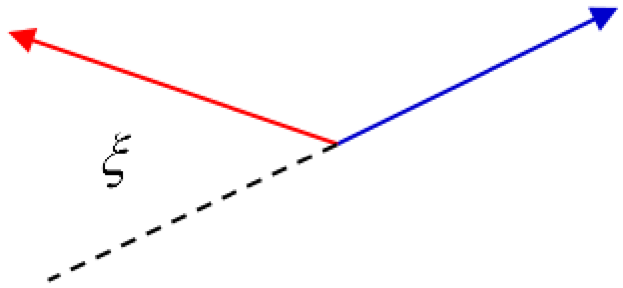
$$\tau^+ \to \mu^+ \nu_\mu \bar{\nu}_\tau$$

A lot of Discussions in 1975:

Are these events really decays of a new 3rd generation of heavy lepton ?



1) Large acollinearity confirms tau hypothesis



2) Anomalous "single muon events" predicted:

Expectation: $BR(\tau \to e(\mu)\nu\bar{\nu}) \approx 20\%$

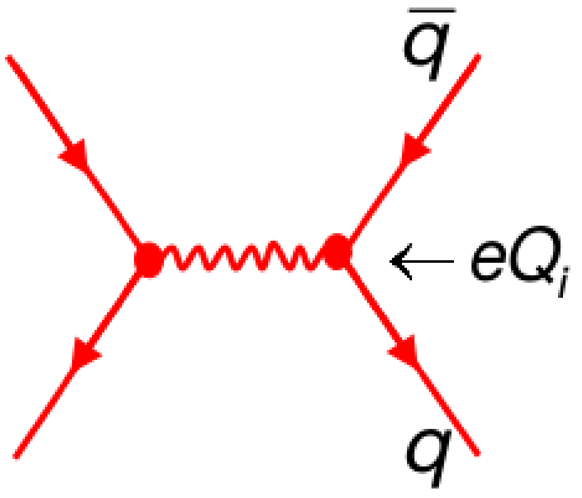
$BR(\tau \to h + \nu) \approx 60\%$

➡ $e^+ + e^- \to \mu^\pm + h^\mp + \text{missing E}$

⬅ PLUTO (DESY, 1976) confirms the anomalous "single muon events". Muon spectrum consistent with 3-body tau decay.

$$e^+e^- \rightarrow q\bar{q}$$

e^+e^- annihilation to a pair of quarks with subsequent hadronization.



$$Q_i = \begin{cases} +\frac{2}{3} \\ -\frac{1}{3} \end{cases}$$

Quarks have fractional charges and carry "color" as additional quantum number.

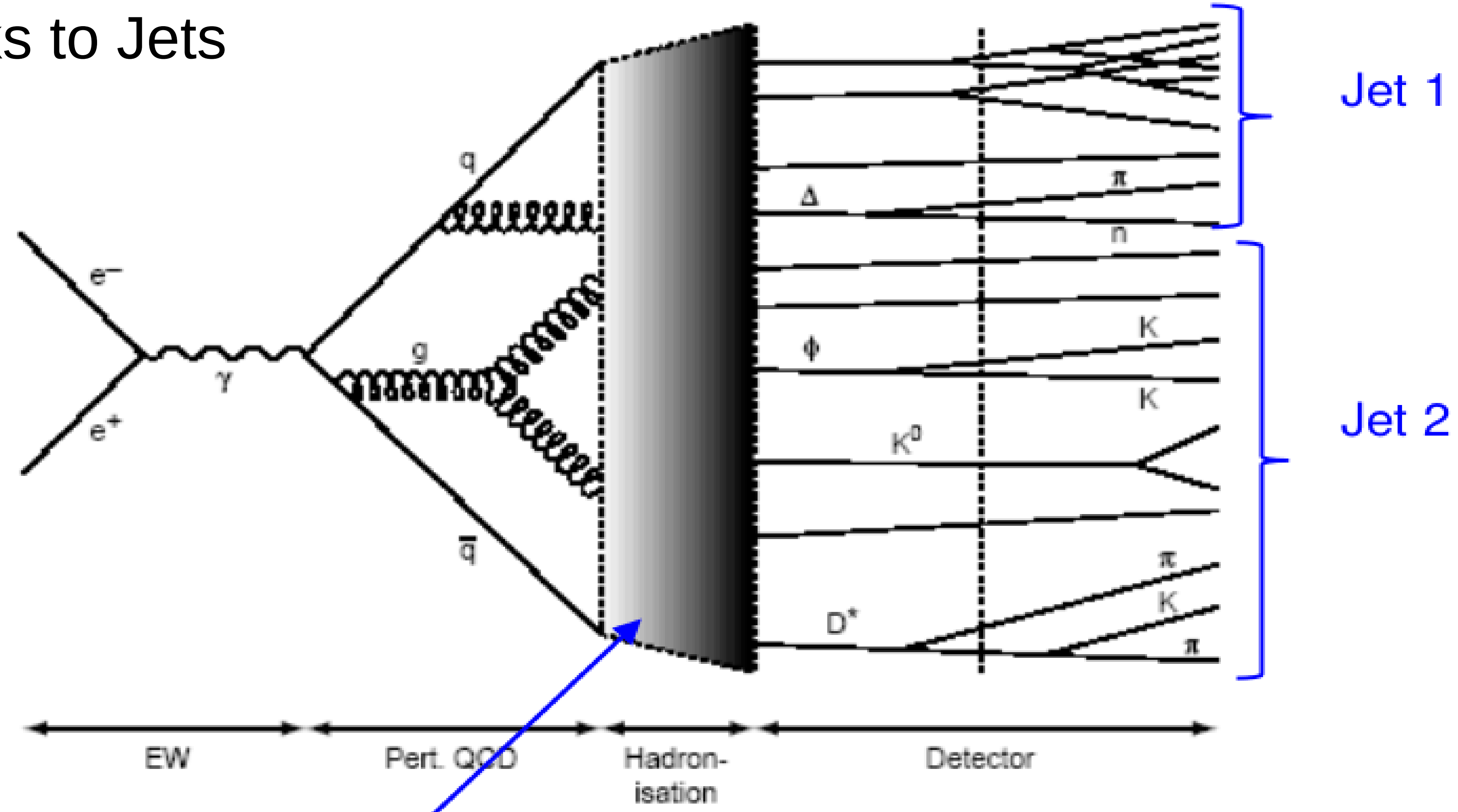
Additional color factor N_C

$$\left.\frac{d\sigma}{d\Omega}\right|_{ee\rightarrow hadrons} = \frac{\alpha^2}{4s} \cdot N_C \cdot \sum_{quarks\,i} Q_i^2 (1+\cos^2\theta)$$

Sum over kinematically possible quark flavors: $4m_q^2 < s$

$\sqrt{s}$	Quarks
< ~3 GeV	uds
< ~10 GeV	udsc
< ~350 GeV	udscb
> ~350 GeV	udscbt

From Quarks to Jets

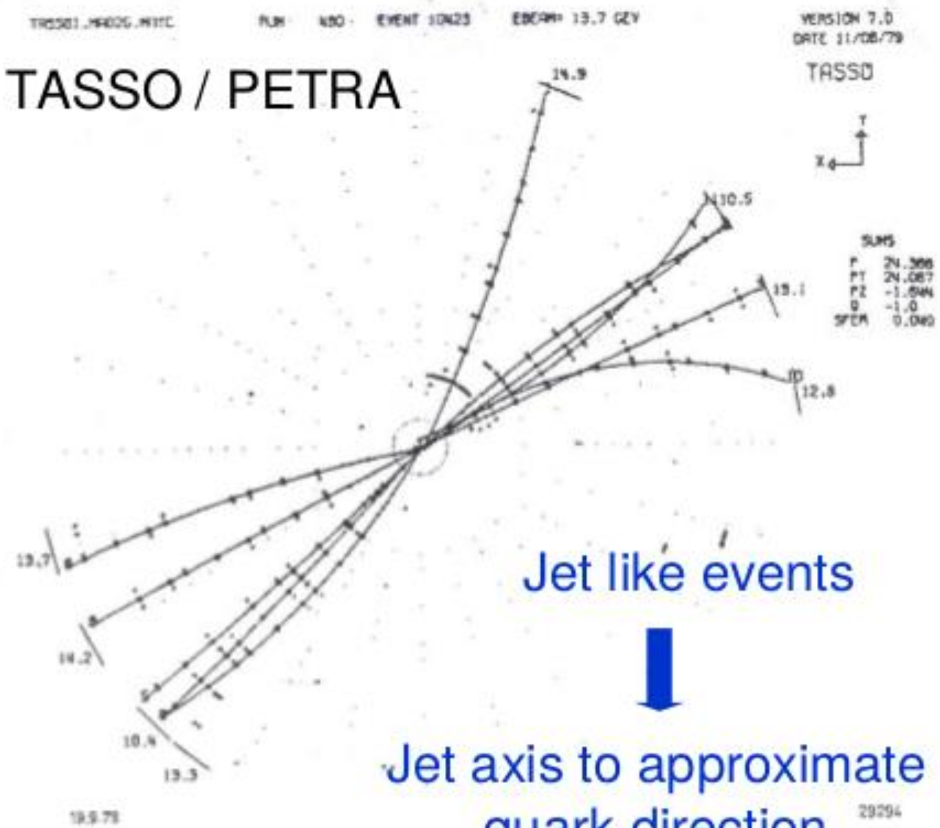


Described successfully by different phenomenological fragmentation models realized as Monte Carlo programs:
PHYTIA, HERWIG, SHERPA

~ 20 particles at 90 GeV

Quark jets and angular distribution

TASSO / PETRA

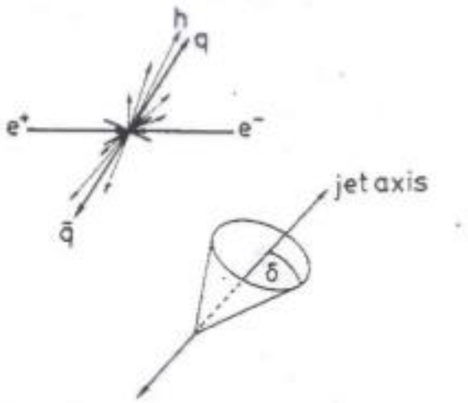


Jet like events

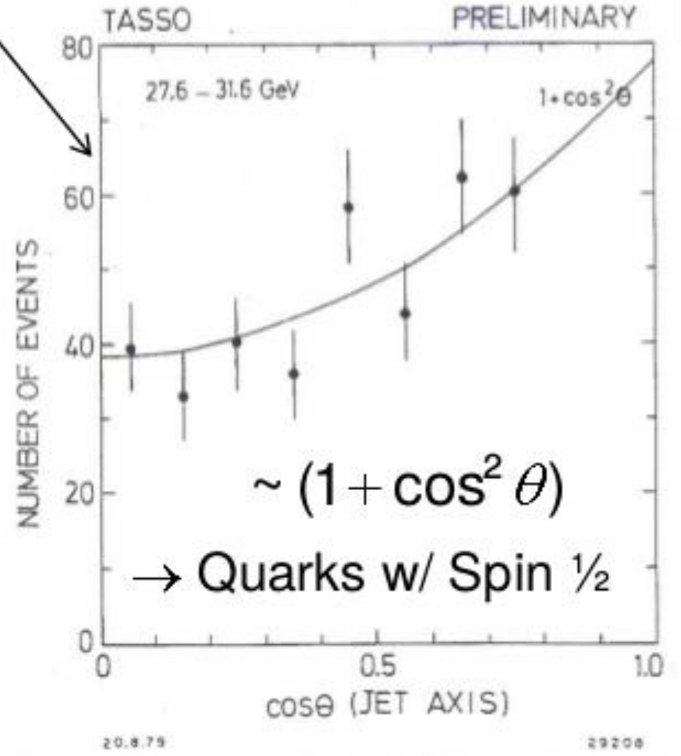
⬇

Jet axis to approximate quark direction

Fig.2 A typical multihadron event at 27.4 GeV recorded in the central detector. The inner 4 layers belong to the proportional chamber, the following 9 are zero degree layers of the drift chamber. The solid bars at the periphery mark time-of-flight counters.



Not very convincing



$\sim (1+\cos^2\theta)$

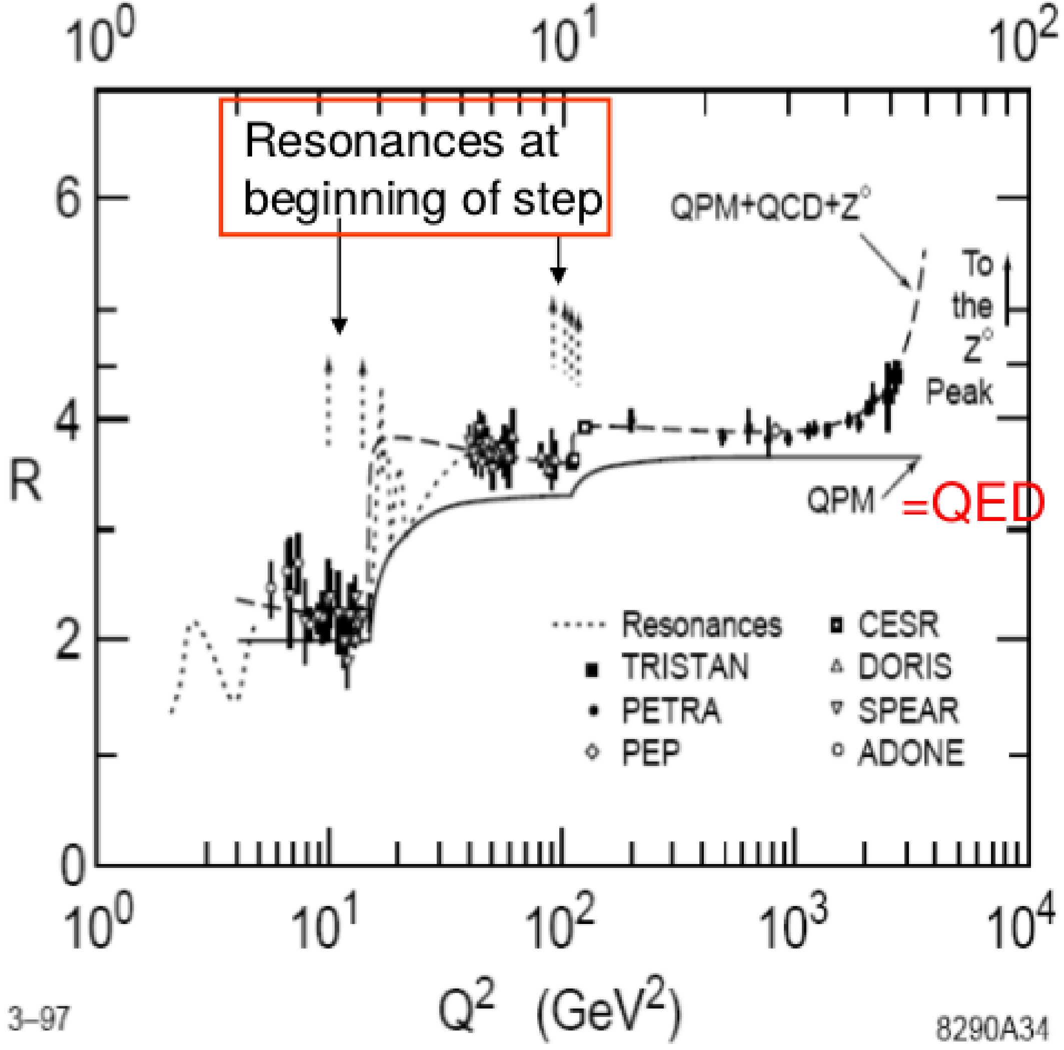
$\rightarrow$ Quarks w/ Spin ½

Fig.7 Angular distribution of the jet axis with respect to the beam.

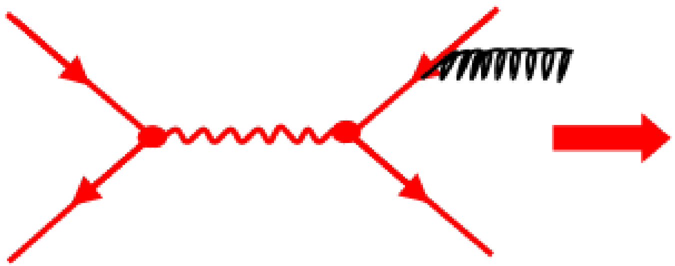
Definition:

$$R_{had} = \frac{\sigma(ee \to hadrons)}{\sigma(ee \to \mu\mu)} = 3 \cdot \sum_i Q_i^2$$

$\sqrt{s}$	Quarks	$R_{had} = 3 \cdot \sum_i Q_i^2$
< ~3 GeV	uds	3·6/9=2.00
< ~10 GeV	udsc	3·10/9=3.33
< ~350 GeV	udscb	3·11/9=3.67
> ~350 GeV	udscbt	3·15/9=5.00



Data lies systematically higher than the prediction from Quark Parton Model (QPM) → gluon bremsstrhl.



$$\sigma^{qq}(s) = \sigma^{qq}_{QED}(s)\left[1 + \underbrace{\frac{\alpha_s(s)}{\pi} + 1.411 \cdot \frac{\alpha_s(s)^2}{\pi^2} + \ldots}_{\sim 7\%}\right]$$