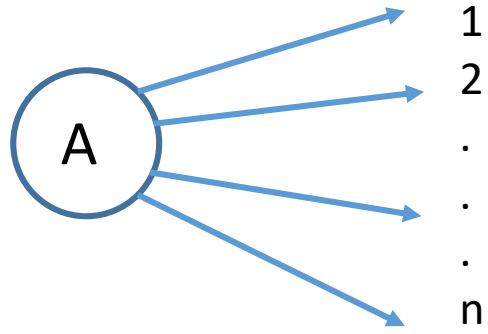


Decay Width



$$\tau = \frac{1}{\Gamma} \quad \Gamma = \sum_f \Gamma_f$$

Differential decay with (rate):

$$d\Gamma_f(A \rightarrow 1 + 2 + \dots + n) = \frac{|M_{fi}|^2}{2E_A} (2\pi)^4 \delta^4(p_A - p_1 - p_2 - \dots - p_n) \frac{d^3 p_1}{2E_1 (2\pi)^3} \frac{d^3 p_2}{2E_2 (2\pi)^3} \cdot \dots \cdot \frac{d^3 p_n}{2E_n (2\pi)^3}$$

Two-body decay:

$$A \rightarrow 1 + 2$$

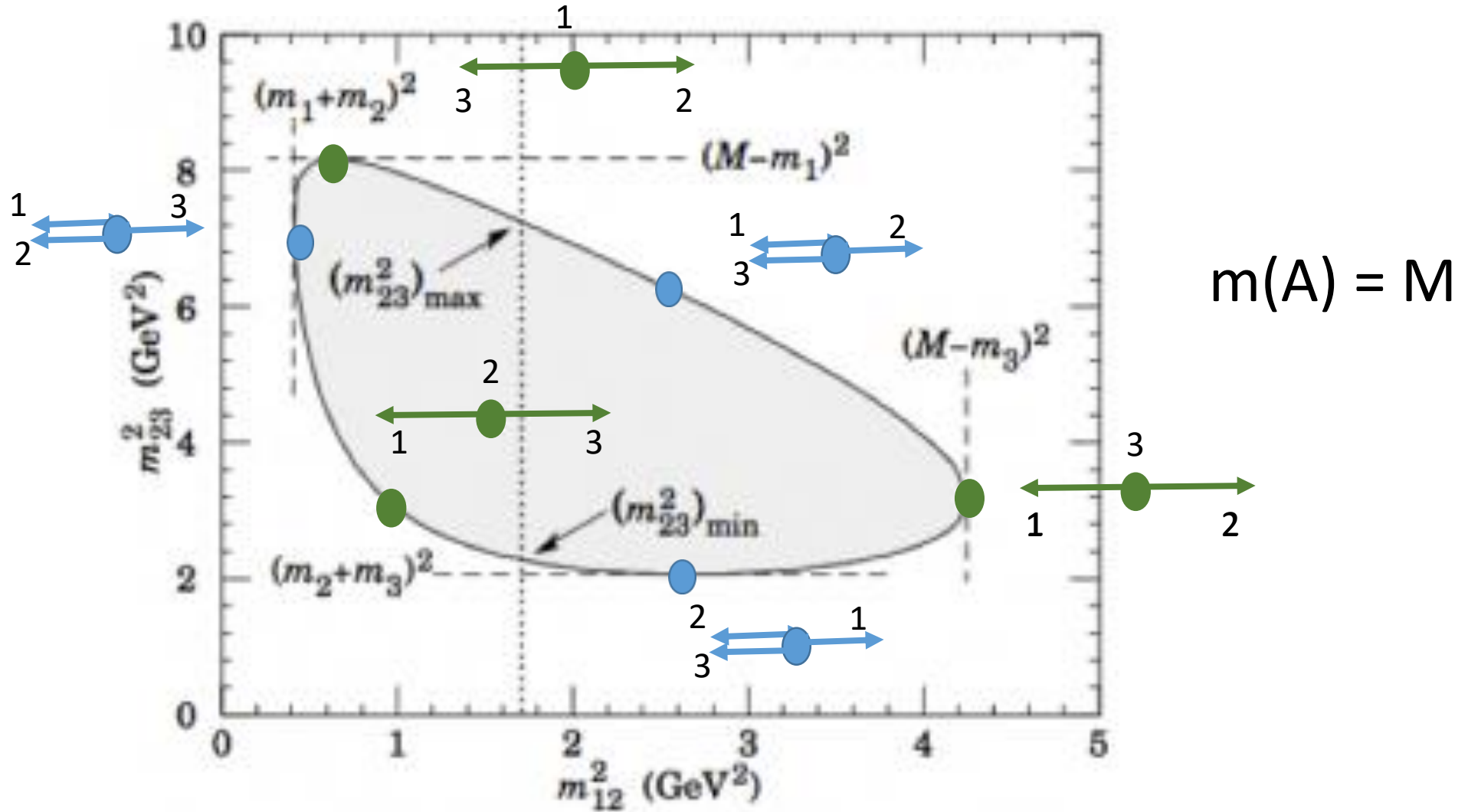
$$d\Gamma_f(A \rightarrow 1 + 2) = \frac{|M_{fi}|^2}{2E_A} dLIPS_2 = \frac{|M_{fi}|^2}{2E_A} \frac{1}{16\pi^2} \frac{|\vec{p}_f|}{\sqrt{s}} d\Omega_f$$

$$\text{CMS: } dLIPS_2 = \frac{1}{16\pi^2} \frac{|\vec{p}_f|}{\sqrt{s}} d\Omega_f$$

$$\sqrt{s} = E_A = m_A$$

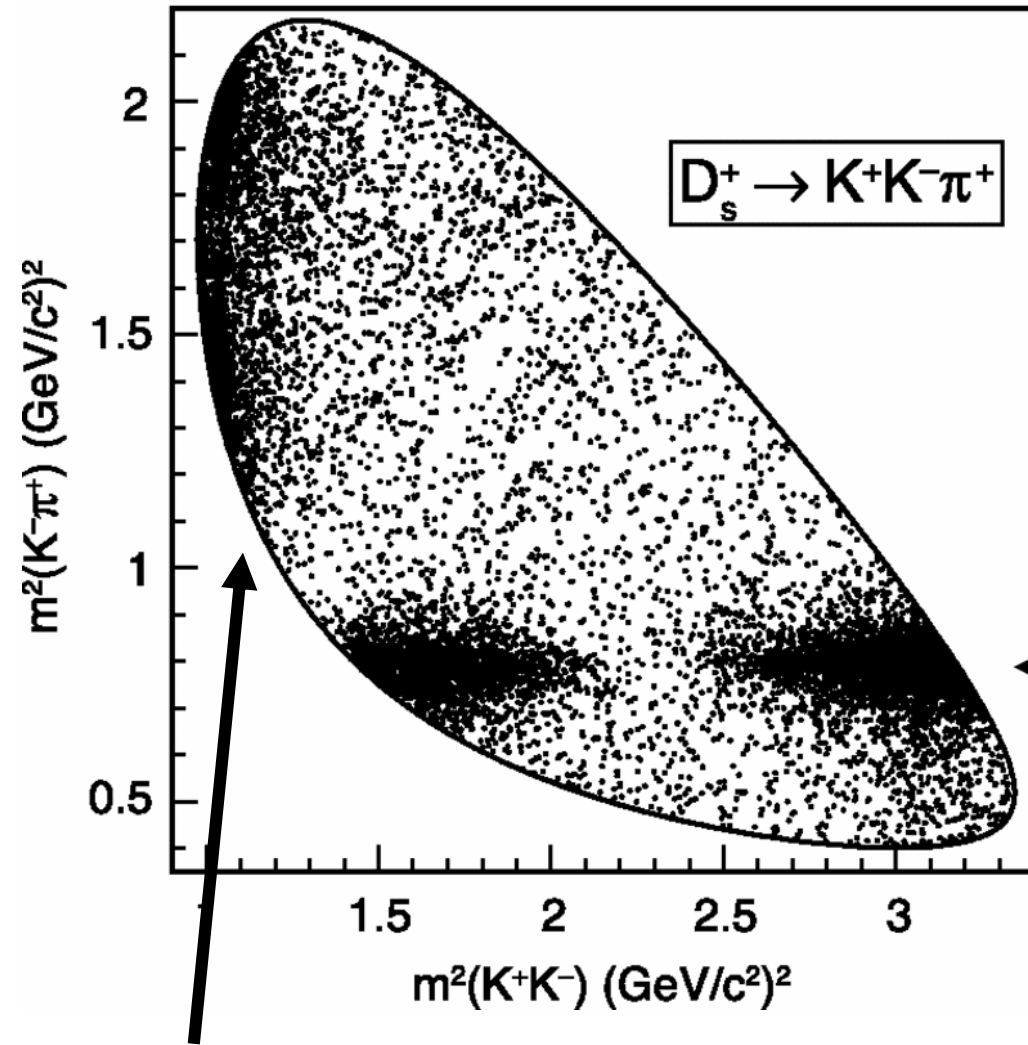
$$\Gamma_f(A \rightarrow 1 + 2) = \frac{|\vec{p}_f|}{32\pi^2 m_A^2} |M_{fi}|^2 d\Omega_f$$

Dalitz Plot: $A \rightarrow 1 + 2 + 3$



$$m(A) = M$$

$$d\Gamma(m_{12}^2, m_{23}^2) = \frac{1}{256\pi^3} \frac{1}{M^3} |M_{fi}|^2 dm_{12}^2 dm_{23}^2$$



$$\phi \rightarrow K^+ K^-$$

Intermediate resonances

→ non flat Dalitz plot distribution

→ non uniform $|M_{fi}|^2$

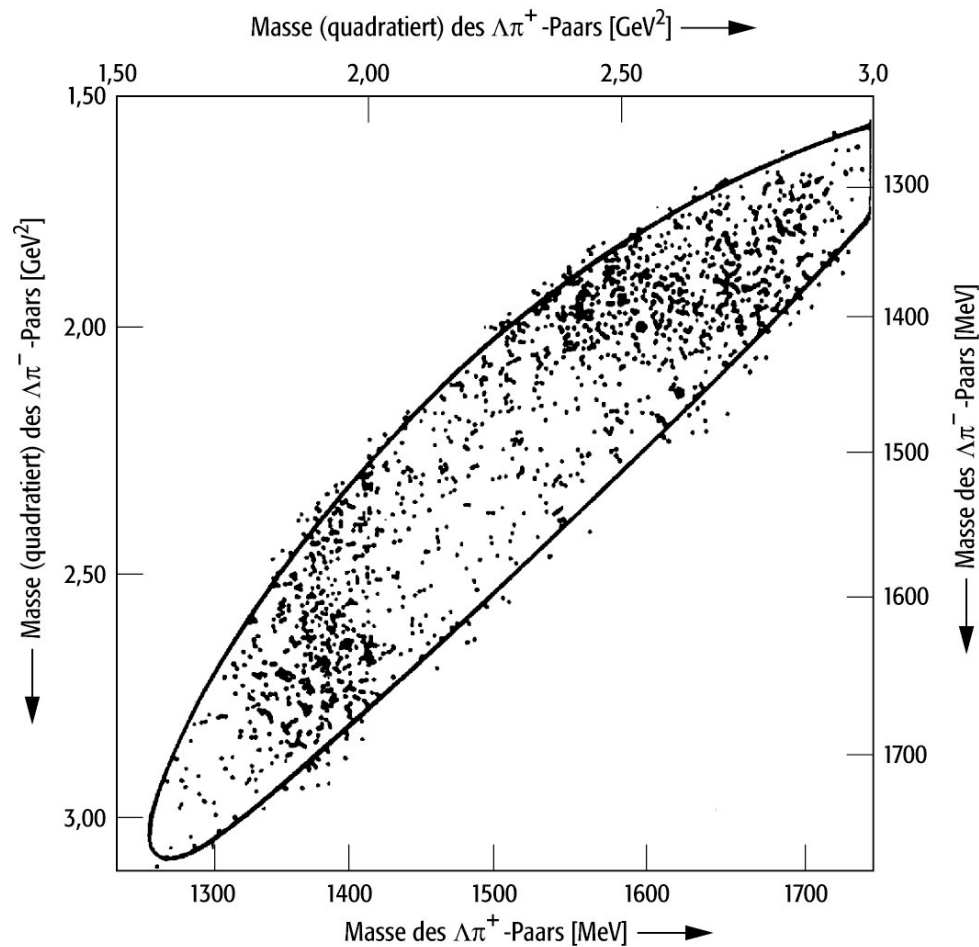
$$K^* \rightarrow K^- \pi^+$$

$$S(K^*) = 1$$

$$S(D_s^-) = S(K^-) = S(K^+) = S(\pi^+) = 0$$

→ relative angular momentum between
 $K^- \pi^+$

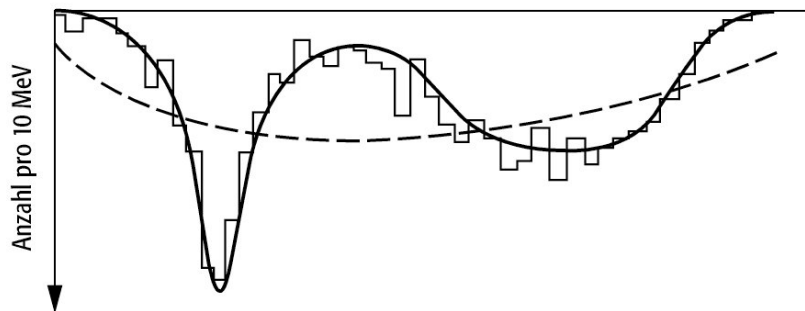
→ preferred relative orientation



Warning:

1D projection can easily give you
Fake signals

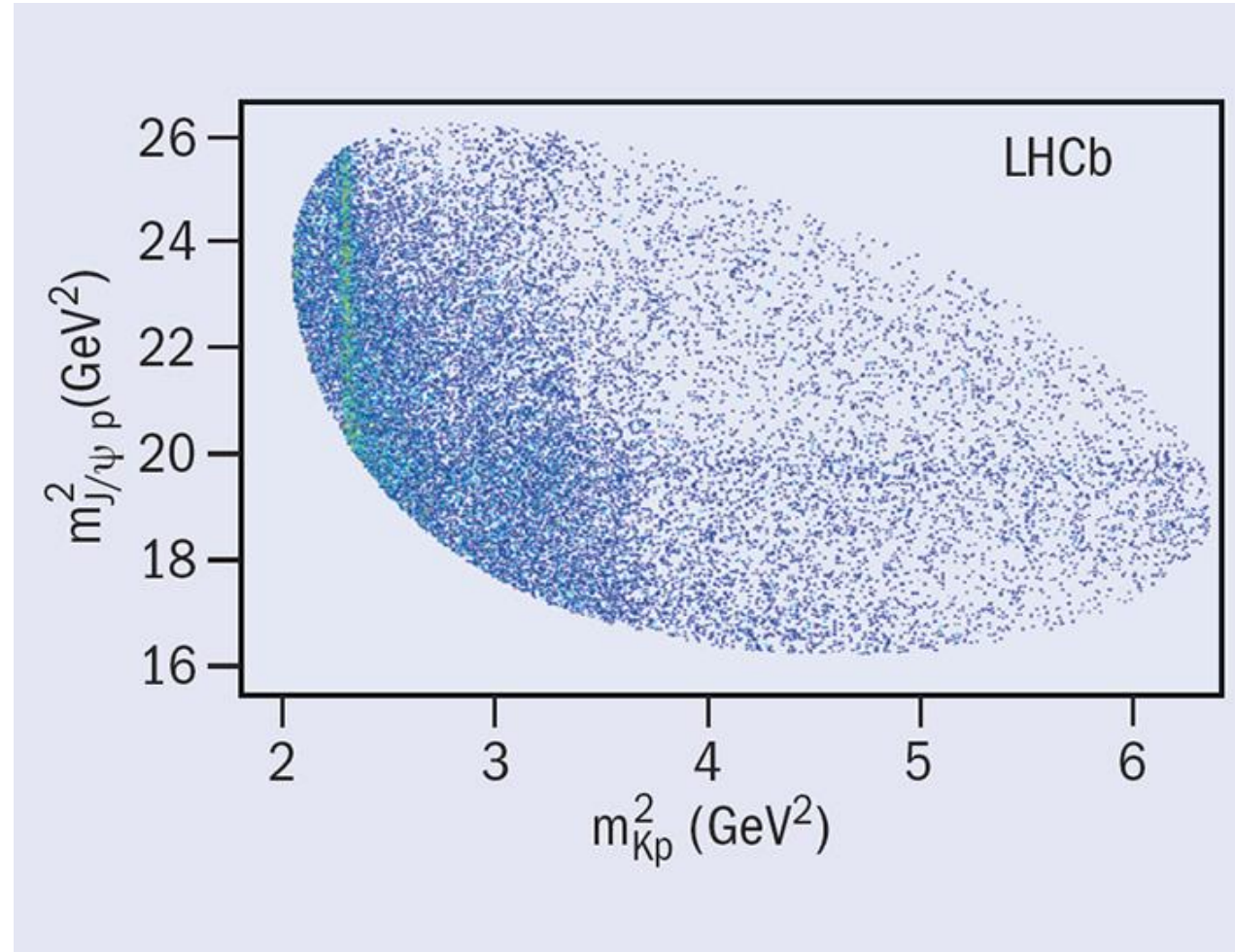
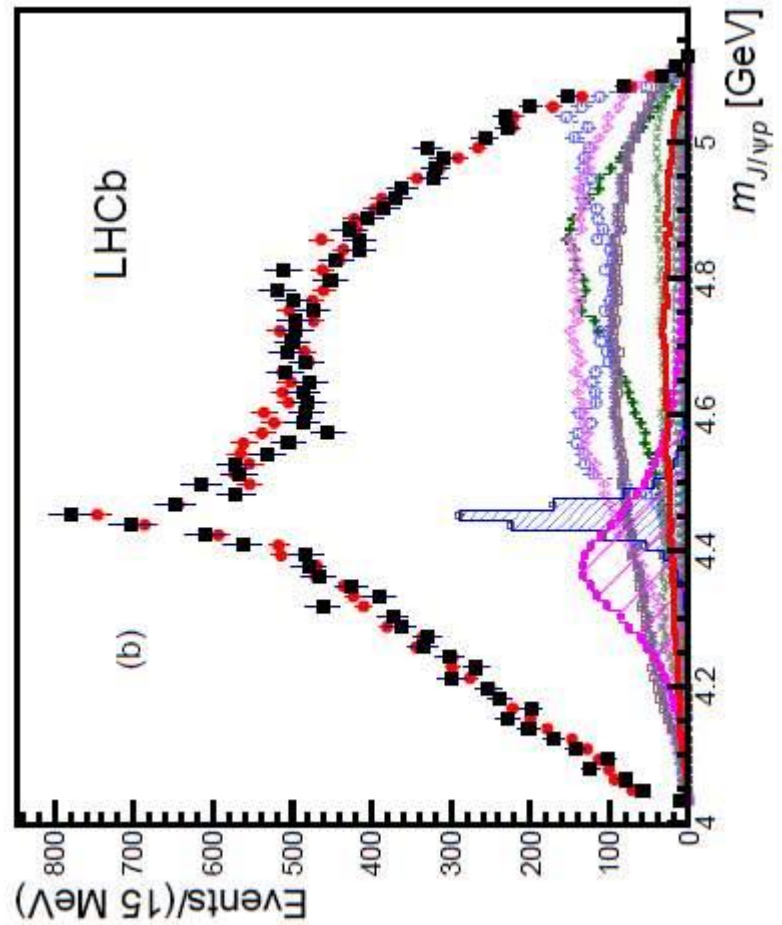
Full Daltiz analysis (amplitude
analysis) needed to claim discoveries.



Pentaquark Discovery at LHCb (2015)



$$\Lambda_b \rightarrow \underbrace{J/\Psi p}_{\text{pentaquark}} \underbrace{K}_{\lambda \text{ resonances}}$$



$$m(K_S, \pi^-)^2$$

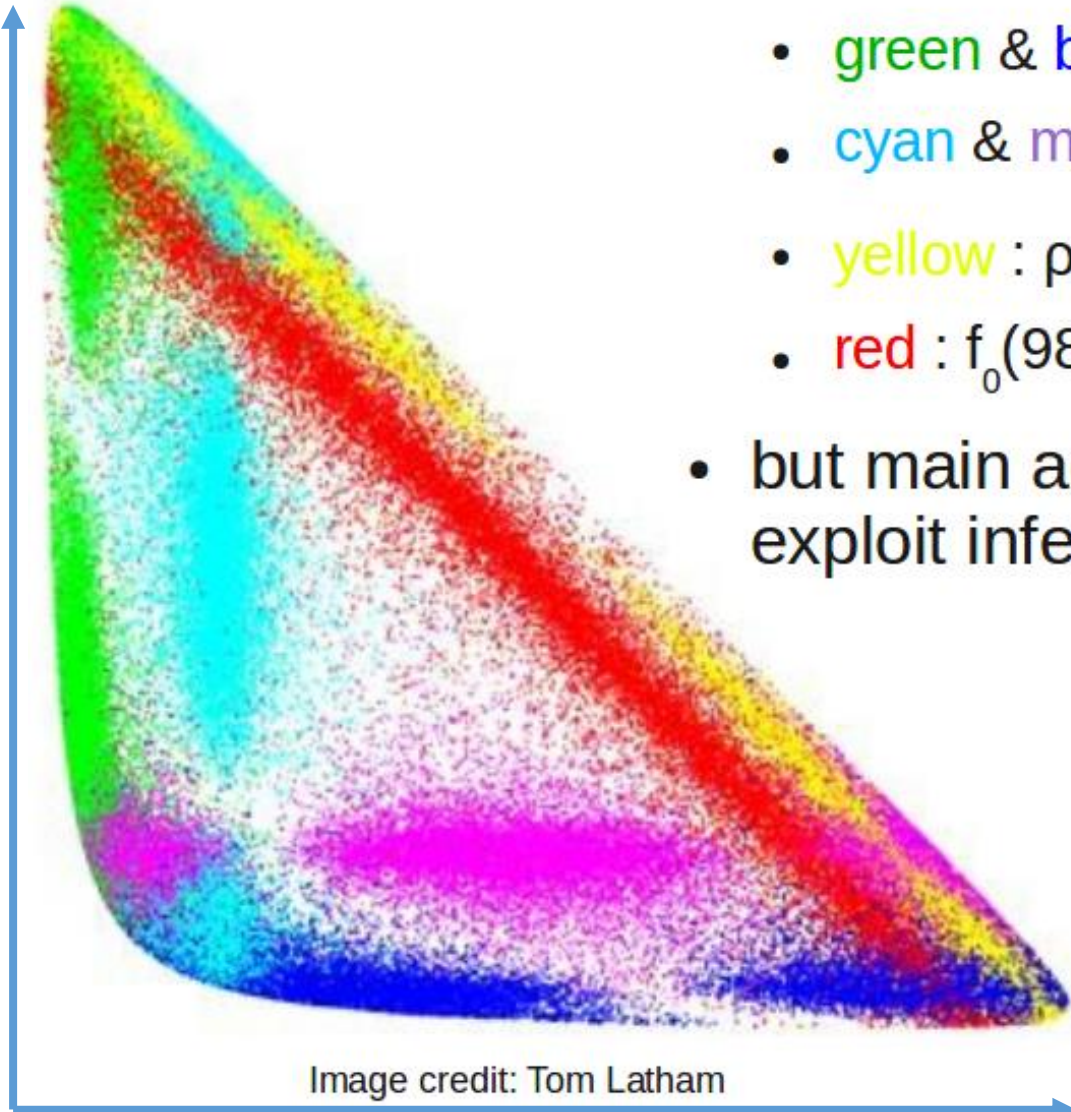
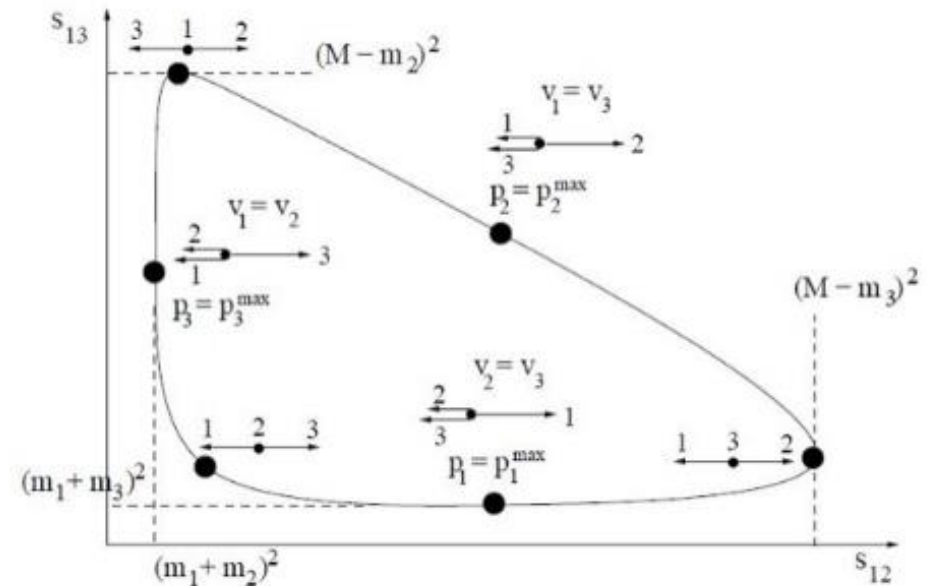


Image credit: Tom Latham

$$m(K_S, \pi^+)^2$$

- Illustration for $D \rightarrow K_S \pi^+ \pi^-$

- green & blue: $K^*(892)$ (vector)
- cyan & magenta : $K_2^*(1430)$ (tensor)
- yellow : $\rho(770)$ (vector)
- red : $f_0(980)$ (scalar)
- but main advantage of Dalitz plots is ability to exploit inference between different resonance



Reminder of QED results for transition amplitudes

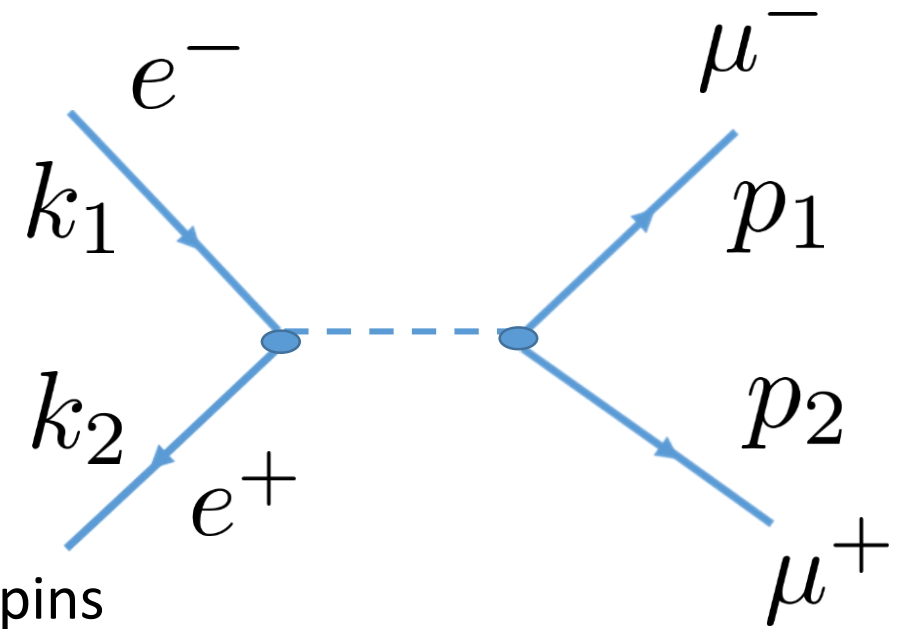
$$-iM_{fi} = \underbrace{[\bar{v}_2(ieQ_e\gamma^\mu)u_1] \frac{-g_{\mu\nu}}{q^2} [\bar{u}_3(ieQ_\mu\gamma^\nu)v_4]}$$

Spinors describe a specific spin state of the fermions

Spin averaged matrix element :

Unpolarize initial state and non-observation of final state spins
→ Average of possible initial state spins, sum over all final states:

$$\begin{aligned} |\overline{M_{fi}}|^2 &= \frac{1}{4} \sum_{spin_i} \sum_{spin_f} |M_{fi}|^2 \\ &= 2e^4 Q_e^2 Q_\mu^2 \frac{t^2 + u^2}{s^2} \end{aligned}$$



Mandelstamm variables

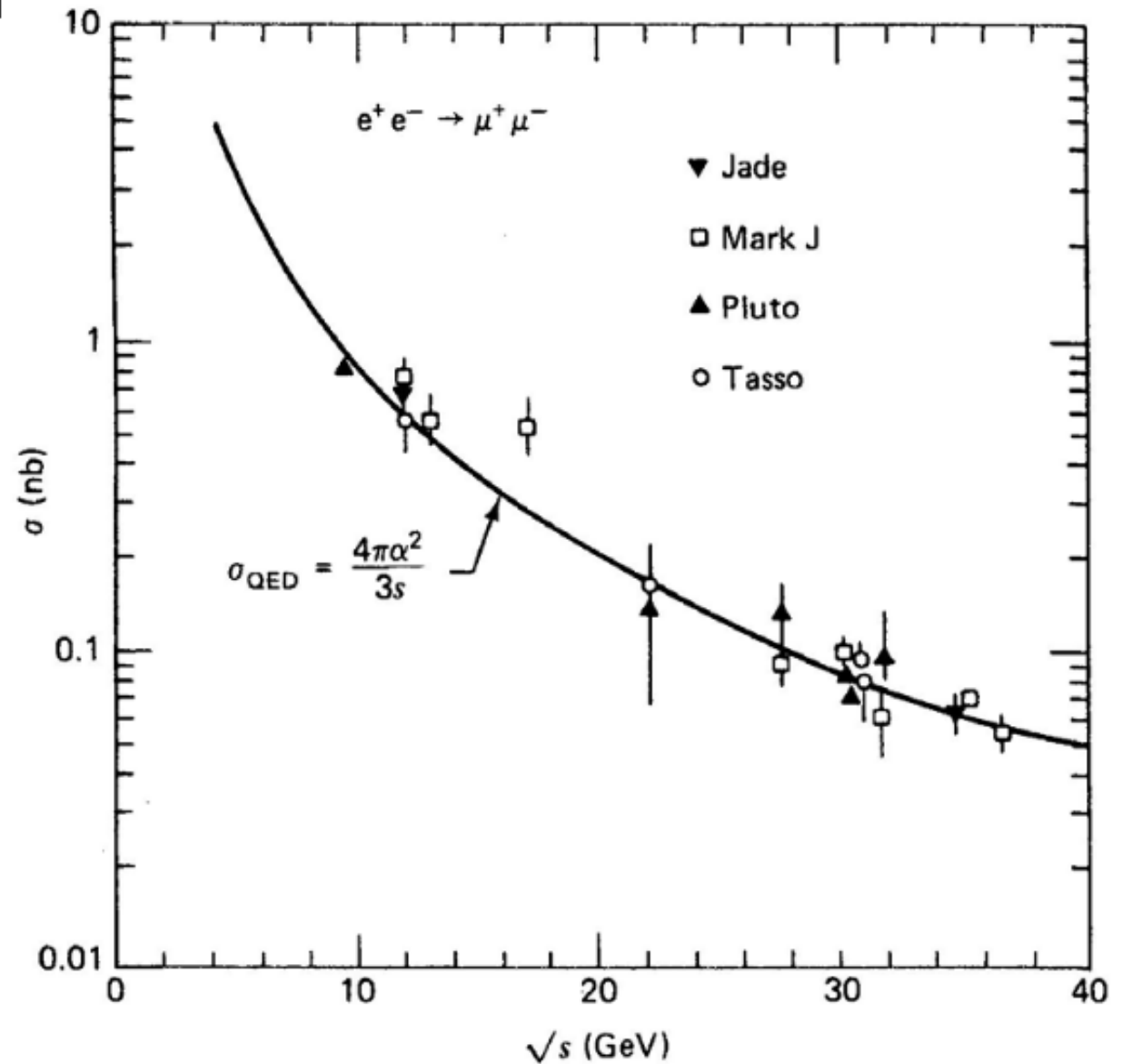
$$\begin{aligned} s &= (k_1 + k_2)^2 \\ t &= (k_1 - p_1)^2 \\ u &= (k_1 - p_2)^2 \end{aligned}$$

$$e^+e^- \rightarrow \mu^+\mu^-$$

$$\frac{d\sigma}{d\Omega}_{CMS} = \frac{\alpha^2}{4s} (1 + \cos^2 \theta)$$

$$\sigma_{tot} = \frac{4\pi\alpha^2}{3s} \sim \frac{87nb\,GeV^2}{s}$$

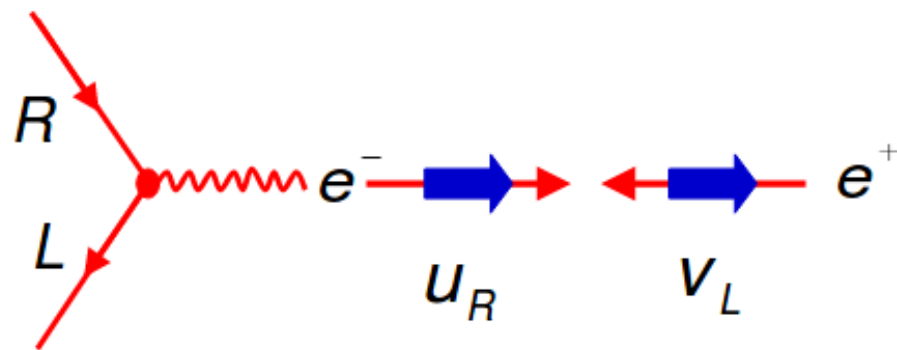
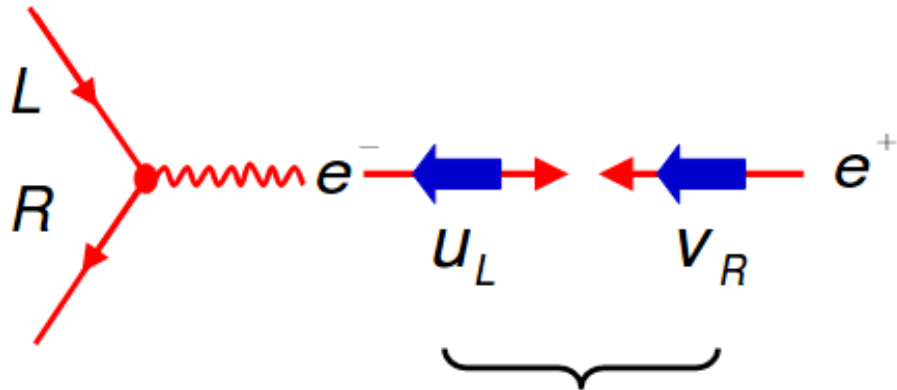
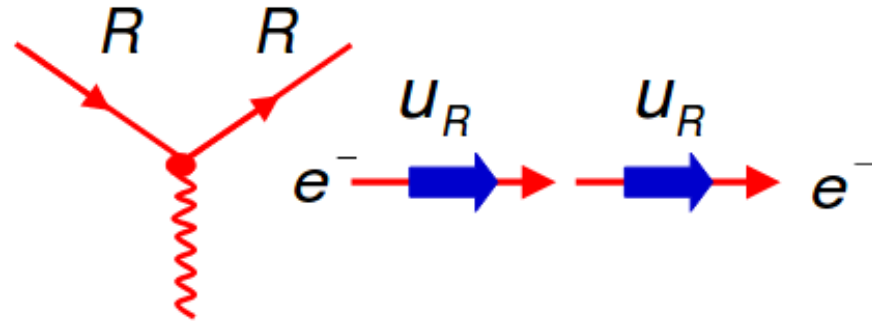
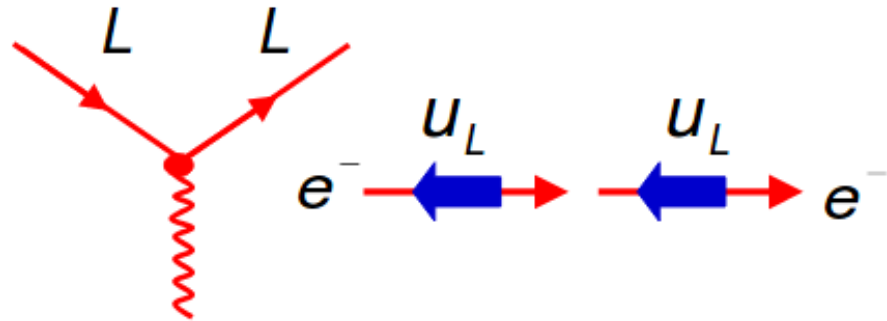
Impressive agreement (1% level)
for lowest order QED computation



For illustration:

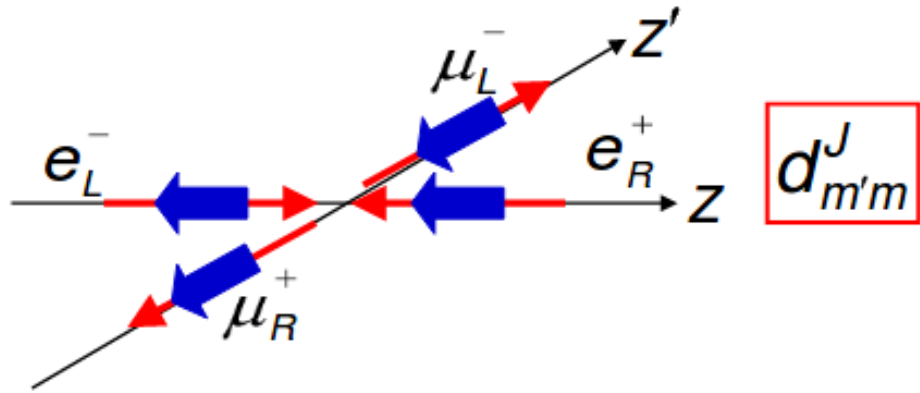


Vector current: $ie\bar{u}\gamma^\mu u$



Photon spin = 1

Angular distribution: $e^+e^- \rightarrow \mu^+\mu^-$



Axis z	rotation	Axis z'
$\left. \begin{array}{l} J=1 \\ m_z = -1 \end{array} \right\}$	$\xrightarrow{d_{-1-1}^1}$	$\left\{ \begin{array}{l} J=1 \\ m_{z'} = -1 \end{array} \right.$
$\left. \begin{array}{l} J=1 \\ m_z = -1 \end{array} \right\}$	$\xrightarrow{d_{+1-1}^1}$	$\left\{ \begin{array}{l} J=1 \\ m_{z'} = +1 \end{array} \right.$

Scattering can be treated as a change of the quantization axis.

$$d_{1,1}^1 = d_{-1,-1}^1 = \frac{1}{2}(1 + \cos(\theta)) \quad [\text{LR} \rightarrow \text{LR}, \text{RL} \rightarrow \text{RL}]$$

$$d_{1,-1}^1 = d_{-1,1}^1 = \frac{1}{2}(1 - \cos(\theta)) \quad [\text{LR} \rightarrow \text{RL}, \text{RL} \rightarrow \text{LR}]$$

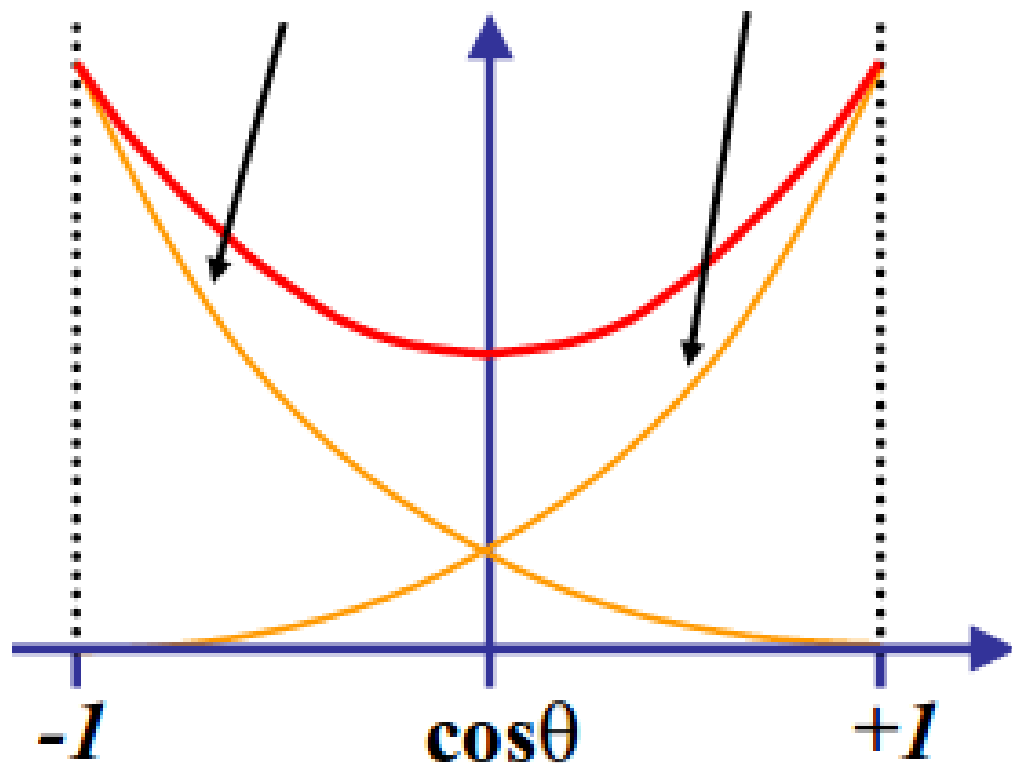
$$\frac{d\sigma}{d\Omega} = \frac{1}{4}(1 + \cos \theta)^2 + \frac{1}{4}(1 - \cos \theta)^2 \sim 1 + \cos^2 \theta$$

Angular distribution is an effect of vector coupling $ie\gamma^\mu$

$$e^+e^- \rightarrow \mu^+\mu^-$$

LR→RL, RL→LR

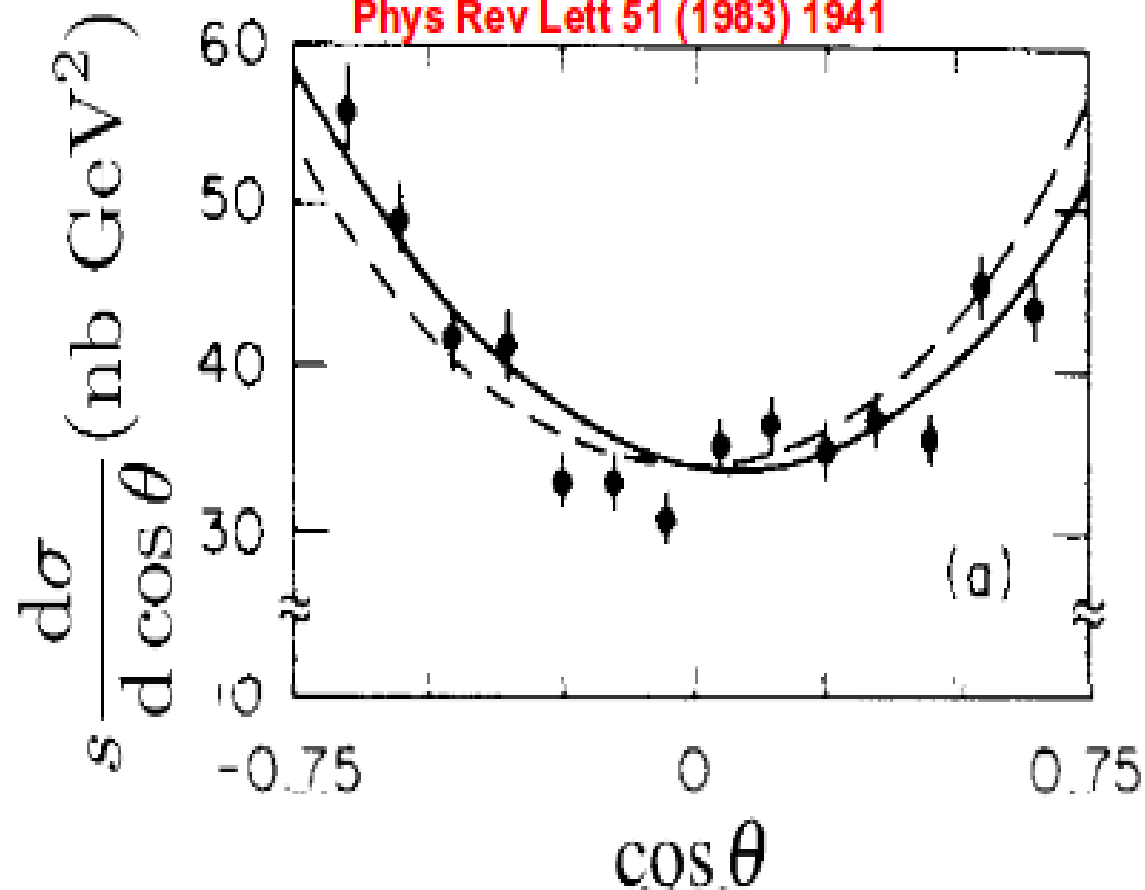
LR→LR, RL→RL



--- pure QED

— QED + Z corrections

Mark II Expt., M.E. Levi et al.,
Phys Rev Lett 51 (1983) 1941



angular distribution becomes slightly asymmetric
In higher order QED or when Z contribution is included