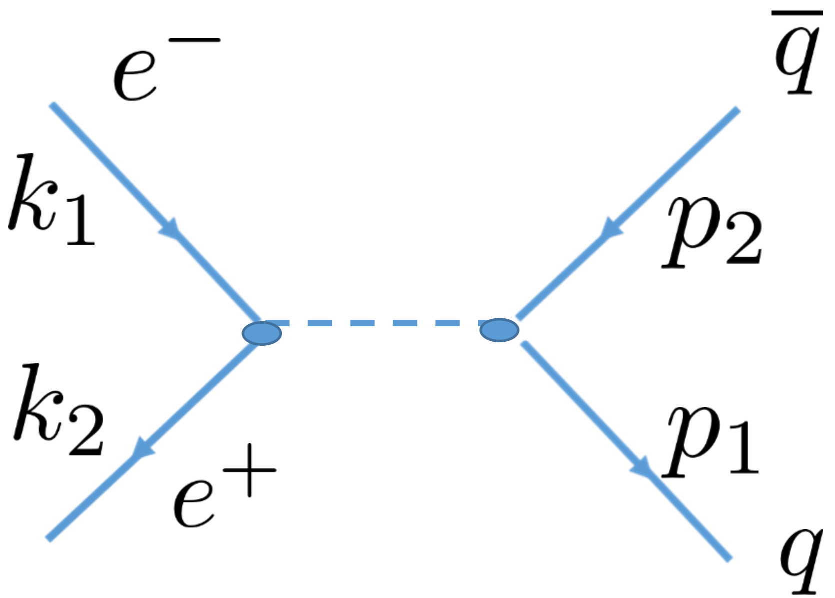
From the matrix element to the measurement

Reminder of QED results for transition amplitudes



Particle Spinor $u(p, s)$
Antiparticle Spinor $v(p, s)$

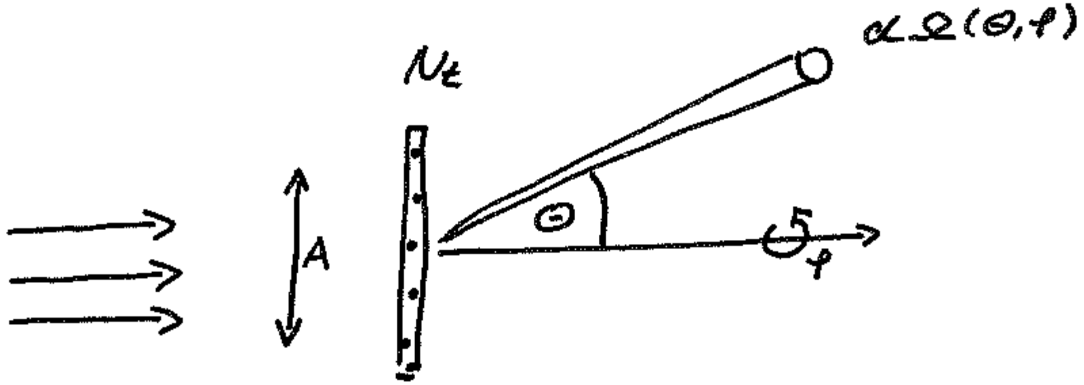
p, s: 4-momentum and spin

$$|\mathcal{M}|^2 = \sum_{spin,color} e^4 Q_e^2 Q_q^2 \frac{1}{(k_1+k_2)^4} (\overline{v_4}\gamma_\nu u_3)(\overline{u_3}\gamma_\mu v_4)(\overline{u_1}\gamma^\nu v_2)(\overline{v_2}\gamma^\mu u_1)$$

$$= 32 e^4 Q_e^2 Q_q^2 N_c \frac{1}{(k_1+k_2)^4} [(k_1 p_1)(k_2 p_2) + (k_2 p_2)(k_2 p_1)]$$

What is the (differential) cross section for the reaction?

How to measure a cross-section ?



flux Φ

area A

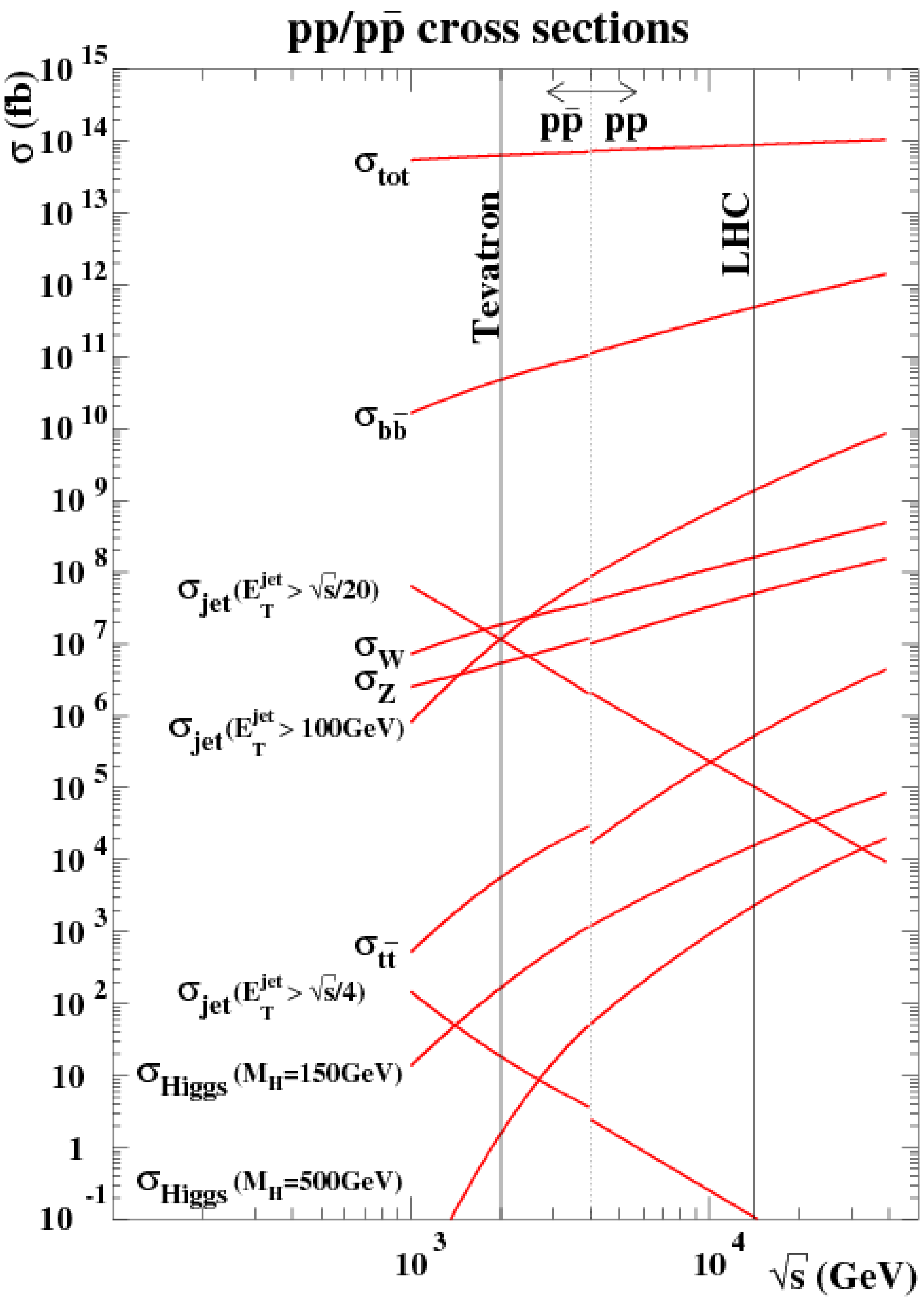
scattering rate: $\frac{dN_s}{dt} = \dot{N}_s$

incomming flux: $\Phi = \frac{\dot{N}_i}{A} = \frac{dN_i}{dt}\,\frac{dx}{A\,dx}$

$= \frac{dN_i}{A\,dx}\,\frac{dx}{dt} = n_i \cdot v_i$

Φ: incomming particle flux $[\frac{1}{sm^2}]$
$\dot{N}_i$: rate of incoming particles on surface A $[\frac{1}{s}]$
n_i: particle density in the beam $[\frac{1}{m^3}]$
v_i: velocity of incoming particles $[\frac{m}{s}]$
N_t: number of target particles
N_s: number of scattered particles
$\dot{N}_s$: rate of scattered particles $[\frac{1}{s}]$

$$\boxed{\dot{N}_s = \sigma\ \Phi\ N_t}$$



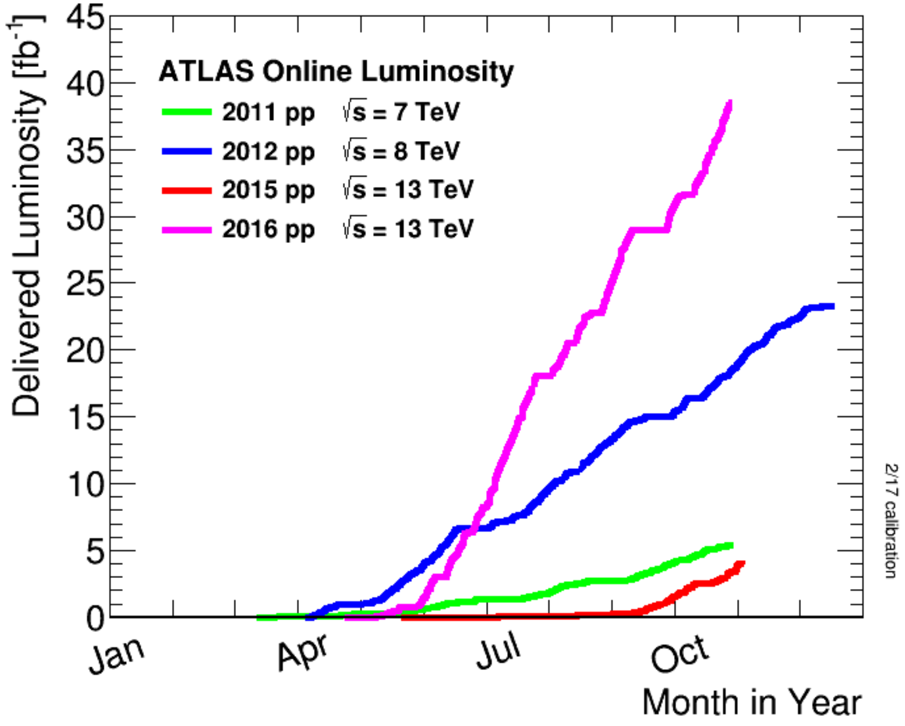
production cross-section

typical unit for cross-section: barn $= 10^{-28}\text{m}^2$

$$\boxed{\dot{N} = \mathcal{L} \cdot \sigma}$$

$\mathcal{L}$: luminosity $[\frac{fb^{-1}}{s}]$

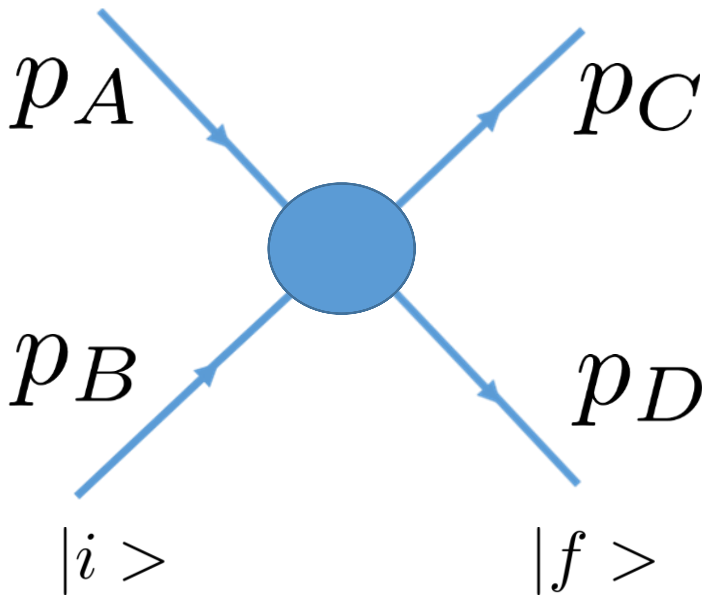
integrated luminosity $\mathcal{L}_{int} = \int \mathcal{L} dt$



$$A + B \to C + D$$

Scattering operator S (≡ S matrix)

$$|f>= \lim |t>_{t\to\inf} = S|i>$$



Measurement selects specific final state |f>

$$<f|t>=<f|S|i>= S_{fi}$$ ← Transition amplitude

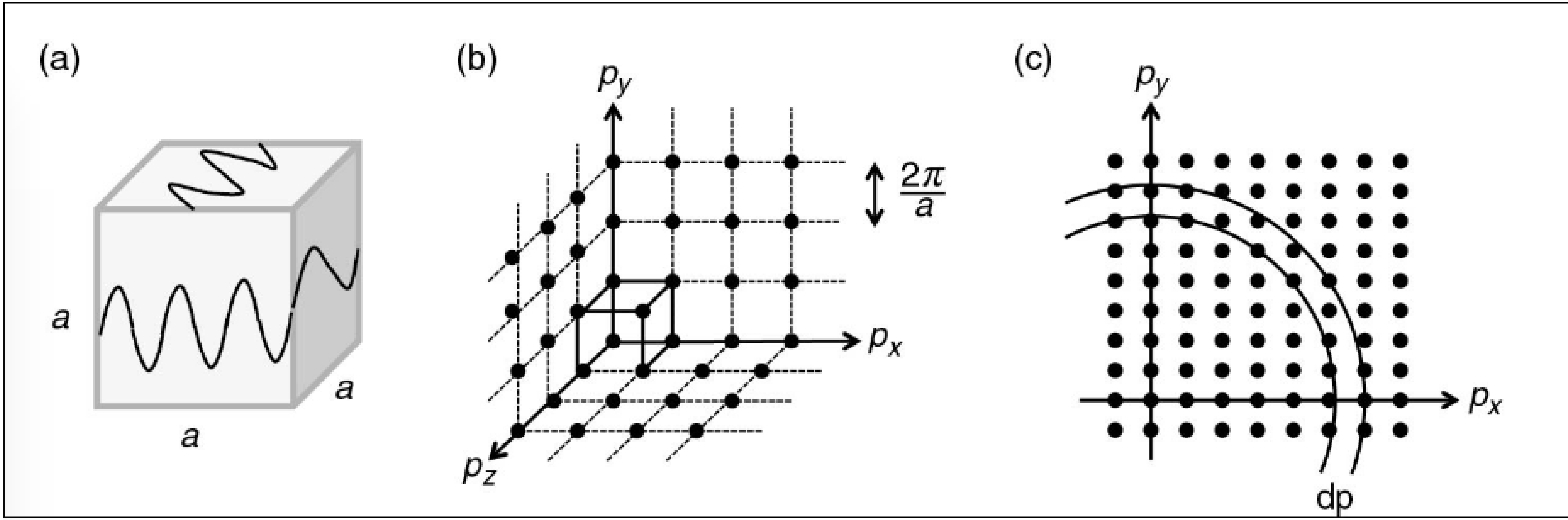
$$S_{fi} = \delta_{fi} + i(2\pi)^4\delta(p_f - p_i)M_{fi}$$

Probability density: $$P_{fi} = |S_{fi}|^2 = (2\pi)^8[\delta^4(p_f - p_i)]^2|M_{fi}|^2$$

The calculation of the transition probability has to consider the number of possible states for each of the outgoing particles -> phase space

LI form of phase space: $$dN_f = \frac{d^3p_C}{2E_C(2\pi)^3}\frac{d^3p_D}{2E_D(2\pi)^3}$$

Definition of phase space element



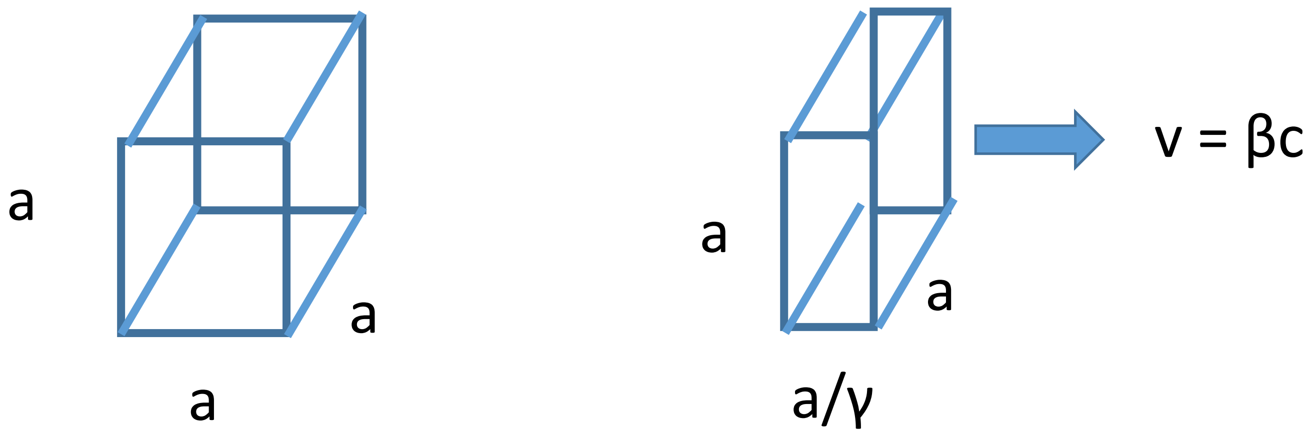
position space
normalisation of wave function
to 1 particle per V = a^3

momentum space
1 state per
$(2\pi)^3/V = (2\pi/a)^3$

phase space element
(V=1)

$$dN = \frac{d^3p}{(2\pi)^3} = \frac{p^2dp}{(2\pi)^3} * 4\pi$$

Lorentz Invariant Phase Space and Matrix Element



particle density increases with γ

Thus LI normalisation needs to be proportional to E particles per volume.

Fix by choice of normalisation $\int d^3x \Psi^\dagger \Psi = 2E$

$$|V_{fi}|^2 \;\rightarrow\; |M_{fi}|^2 = 2E_A 2E_B 2E_C 2E_D |V_{fi}|^2 \qquad dN_f = \frac{d^3p}{(2\pi)^3} \;\rightarrow\; dN_f = \frac{d^3p}{2E_C 2E_D}$$

$$\Gamma_{fi} = \frac{(2\pi)^4}{2E_A 2E_D} \int_V |M_{fi}|^2 \delta(P_A + P_B - P_C - P_D) \frac{dp_C^3}{(2E_C(2\pi)^3} \frac{dp_D^3}{2E_D(2\pi)^3}$$

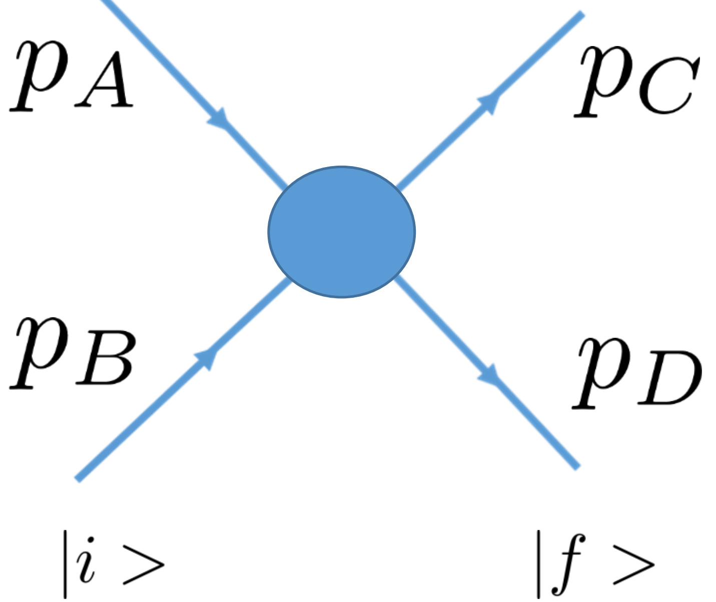
transition rate (not LI)

matrix element (LI)

phase space (LI)

Differential cross section

A+B → C+D



$$d\sigma = \frac{|M_{fi}|^2}{4[(p_A p_B)^2 - m_A^2 m_B^2]^{1/2}} \underbrace{(2\pi)^4 \delta^4(p_f - p_i) \frac{d^3 p_C}{2E_C (2\pi)^3} \frac{d^3 p_D}{2E_D (2\pi)^3}}$$

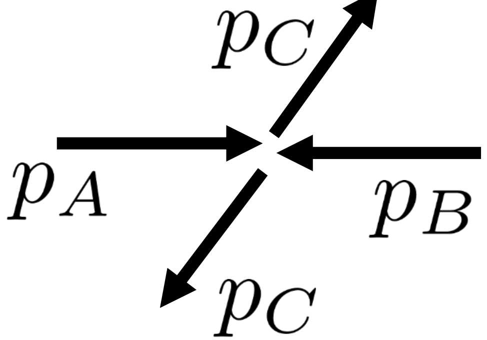
$LIPS_2$ ≡ Lorentz invariant
2-body phase space

LIPS for n particles

$$dLIPS_n(p_i, \underbrace{p_1, p_2, ..., p_n}_{\text{final state particle}}) = (2\pi)^{2n} \delta^4(p_i - (p_1 + p_2 + ... + p_n)) \Pi \frac{dp_f^3}{(2\pi)^3 2E_f}$$

product of final state particle

Differential cross section



In CMS:

$$\int dLIPS_2 = \frac{1}{4\pi^2}\int \delta^3(\vec{p_C}+\vec{p_D})\delta(E_a+E_B-E_C-E_D)\frac{d^3}{2E_C}\frac{d^3}{2E_D}$$

$$= \int d\Omega_c^* \frac{1}{16\pi^2}\int \delta(E_A+E_B-E_C-E_D)\frac{|p_C|^2 d|\vec{p}_C|}{E_C E_D}$$

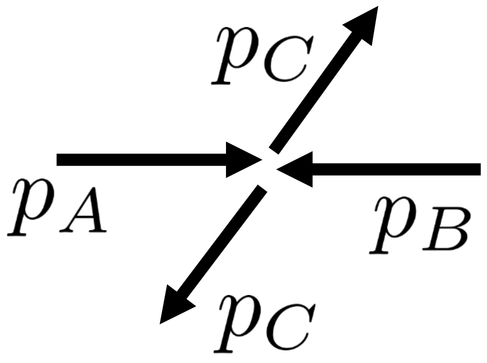
$$\left(\begin{array}{l} W = E_C + E_D = \sqrt{m_C^2+p_C^2}+\sqrt{m_D^2+p_D^2} \\ \\ \frac{dW}{dp_C} = p_C\left(\frac{1}{E_C}+\frac{1}{E_D}\right) \quad \rightarrow \quad p_c dp_c = dW\frac{E_C E_D}{E_C+E_D} \end{array}\right)$$

$$\int dLIPS_2 = \int d\Omega_C^* \frac{1}{16\pi^2}\int (E_A+E_B-W)\frac{|\vec{p}_C|}{E_C+E_D}dW$$

$$= \int d\Omega_C^* \frac{1}{16\pi^2}\frac{|p_C|}{E_C+E_D} = \int d\Omega_C^* \frac{1}{16\pi^2}\frac{|p_C|}{\sqrt{s}}$$

Differential cross section

In CMS:



$$d\sigma = \frac{|M_{fi}|^2}{4|p_i^*|\sqrt{s}} \frac{1}{16\pi^2} \frac{1}{\sqrt{2}} |p_f^*| d\Omega^*$$

$$\boxed{\frac{d\sigma}{d\Omega^*} = \frac{|p_f^*|}{|p_i^*|} \frac{|M_{fi}|^2}{s}}$$

$$|\vec{p}_A| = |\vec{p}_B| = |p_i|$$

$$|\vec{p}_C| = |\vec{p}_D| = |p_f|$$

(the * indicates that the coordinates are in the CMS)

Example: $e^+e^- \rightarrow q\bar{q}$

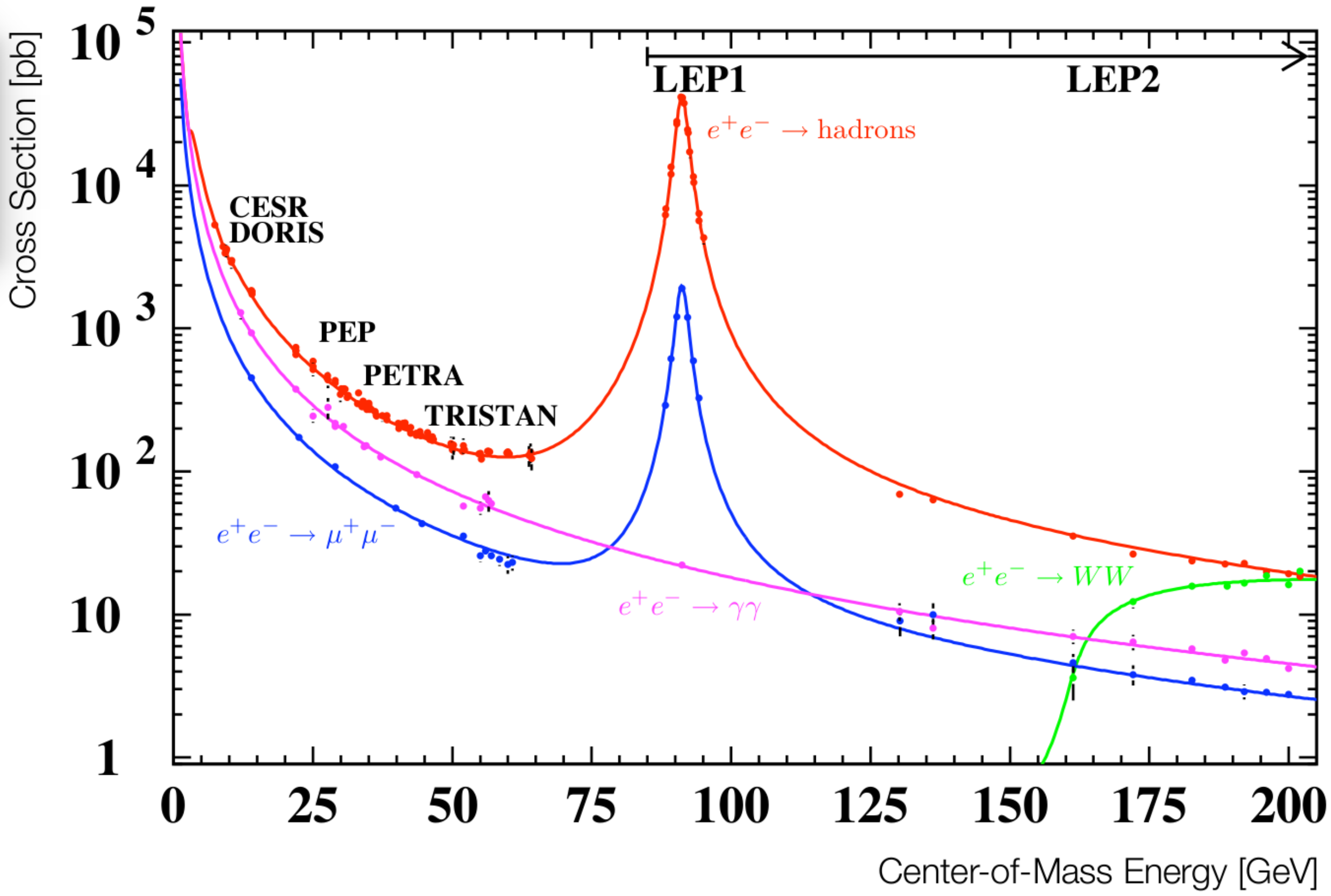
$$\frac{d\sigma}{d\Omega^*} = \frac{|p_f^*|}{|p_i^*|} \frac{|\overline{M_{fi}}|^2}{s}$$

kinematics

spin averaged matrix element

$$|\overline{M_{fi}}|^2 = \frac{1}{4} \sum_{spin_i} \sum_{spin_f} |M_{fi}|^2$$

$$= 2e^4 Q_e^2 Q_q^2 N_c \frac{t^2+u^2}{s^2}$$



Rapidity $y = 1/2 \ln(\frac{E+p_z}{E-p_z})$

$\boxed{\text{Δy is LI! → dσ/dy is LI!}}$

Pseudorapidity $\eta = -\ln \tan \theta/2 \sim y$

for highly relativisticparticles