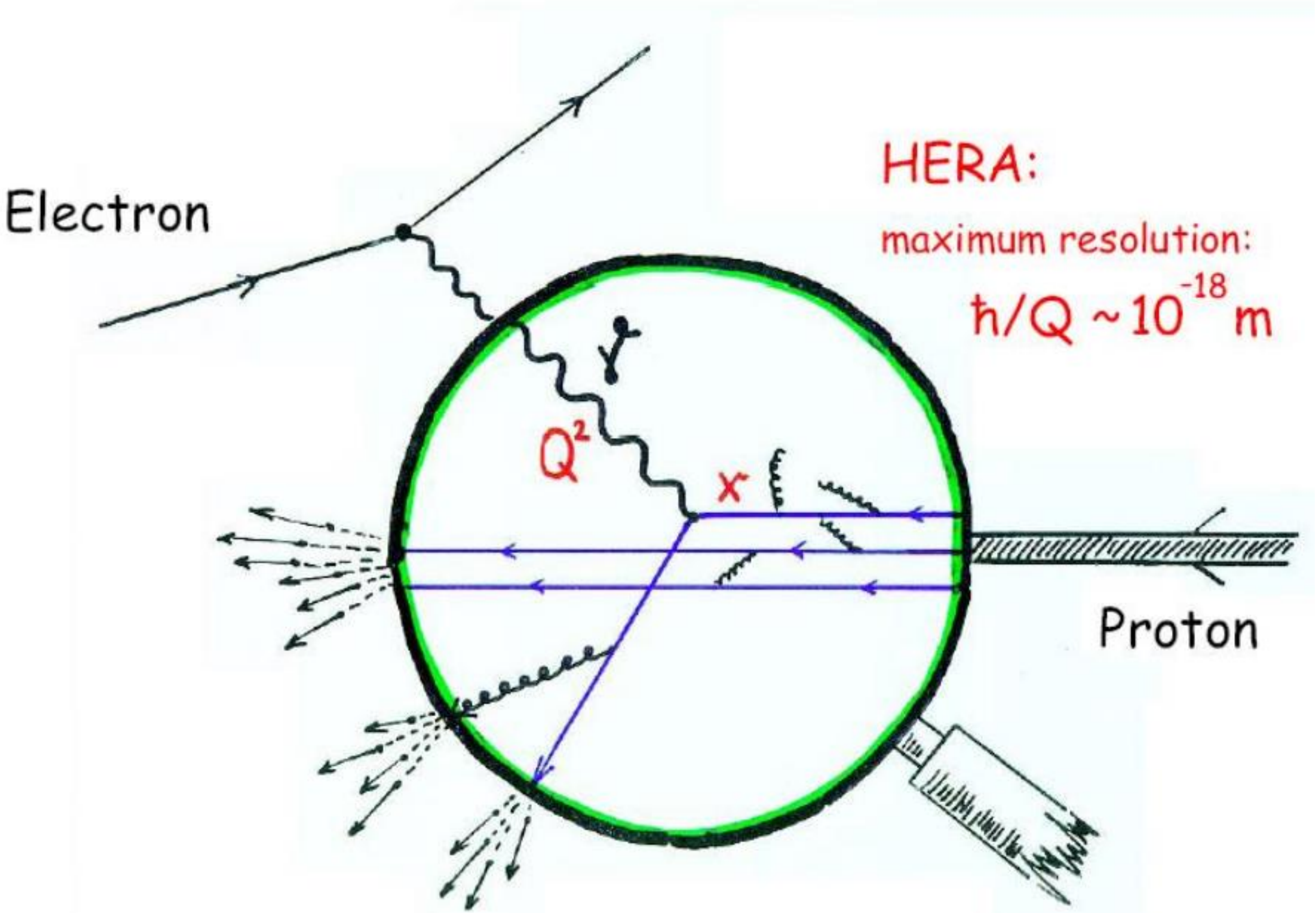
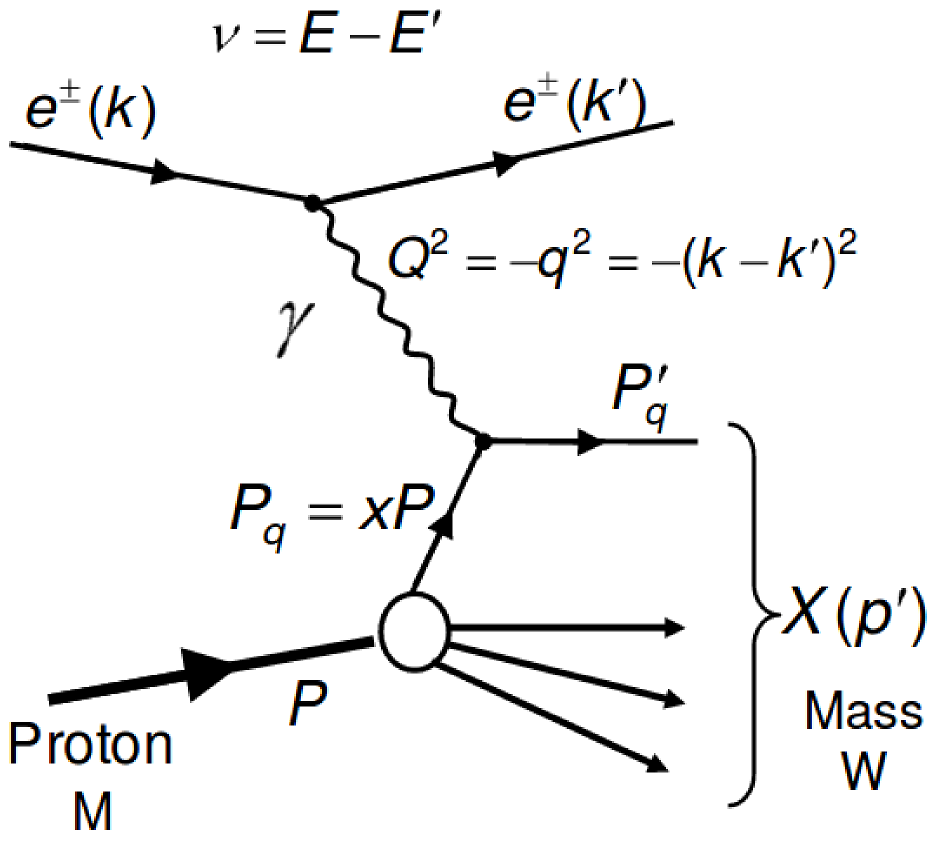
Experimental Tests of QCD

1. Test of QCD of in e+e- annihilation
2. Running of the strong coupling constant
3. Study of QCD in deep inelastic scattering
4. Hadron-Hadron collisions
5. Hadron spectroscopy (e.g. penta quark)
6. A new state of hadronic matter
   (quark gluon plasma)

3. Study of QCD in deep inelastic scattering (DIS)



DIS in the Quark Parton Model



- Elastic scattering: W = M

  ⇒ only one free variable

$$\frac{Q^2}{2M\nu} = 1$$

- Inelastic scattering: W ≠ M

  ⇒scattering described by 2 independent variables

  (E,ν), (Q^2,x), (x,y), …

x = fractional momentum of struck quark

y = Pq/Pk = elasticity, fractional energy transfer in proton rest frame

ν = E - E′ = energy transfer in lab

$$y = \frac{P \cdot q}{P \cdot k}$$

$$x = \frac{Q^2}{2M\nu} = \frac{Q^2}{2P \cdot q} \quad \text{(Bjorken x)}$$

HERA

$e^{\pm}$ → 30 GeV ← p 900 GeV

$$s = 4E_eE_p \approx 10^5\,\mathrm{GeV}^2$$

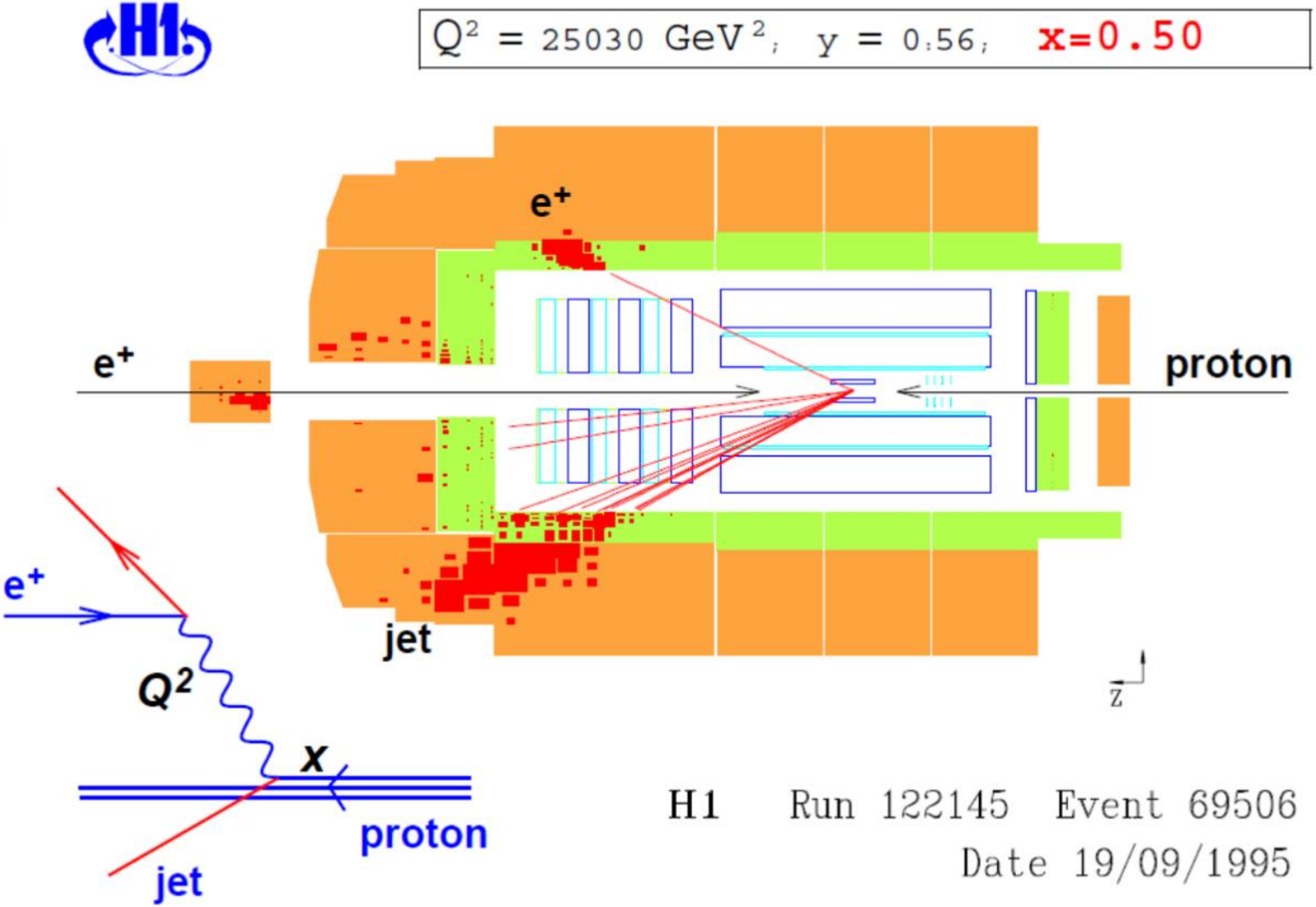
6.3km circumference

Data taking:

1992 – 2007

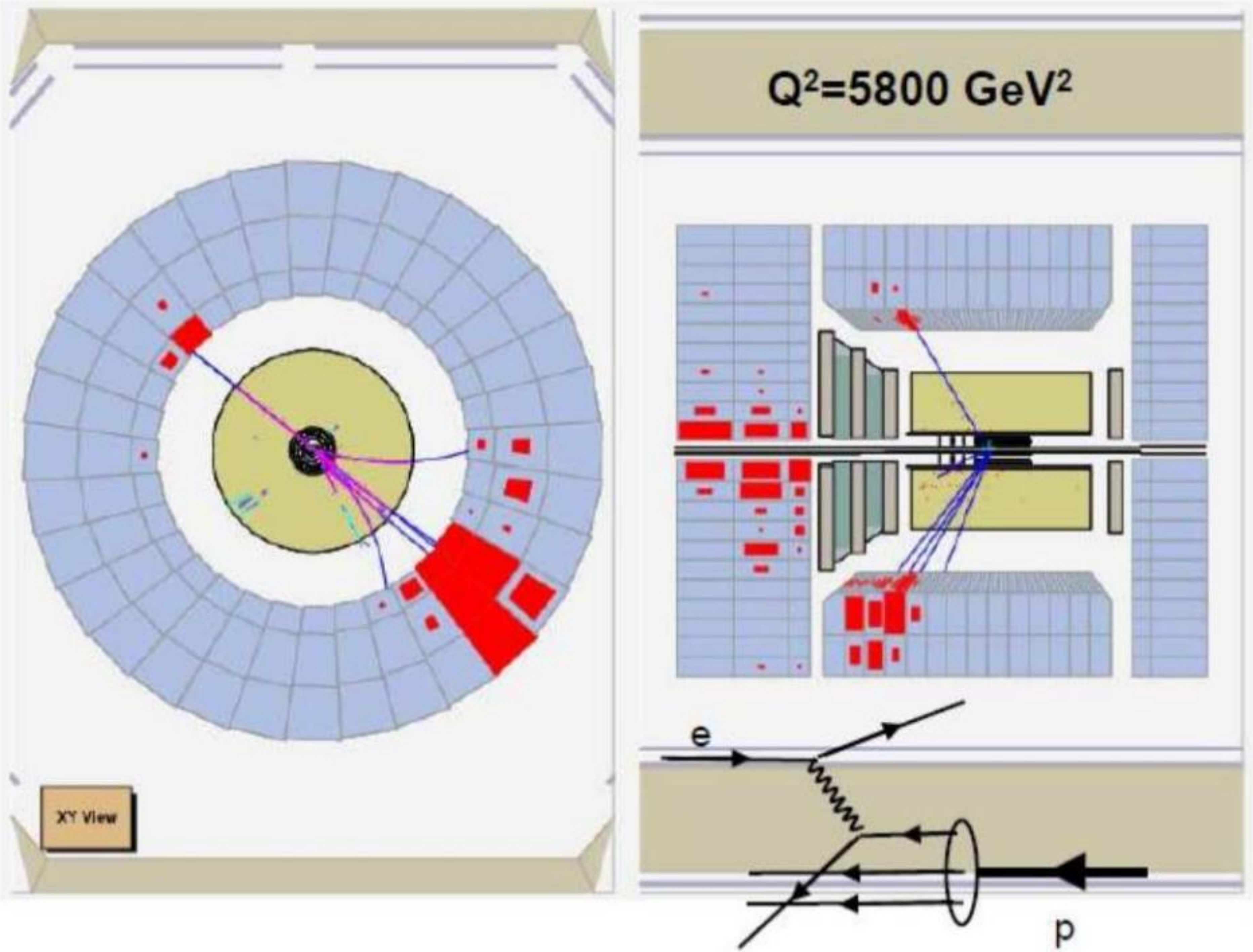
4 experiments:

ZEUS, H1, HERMES, HeraB

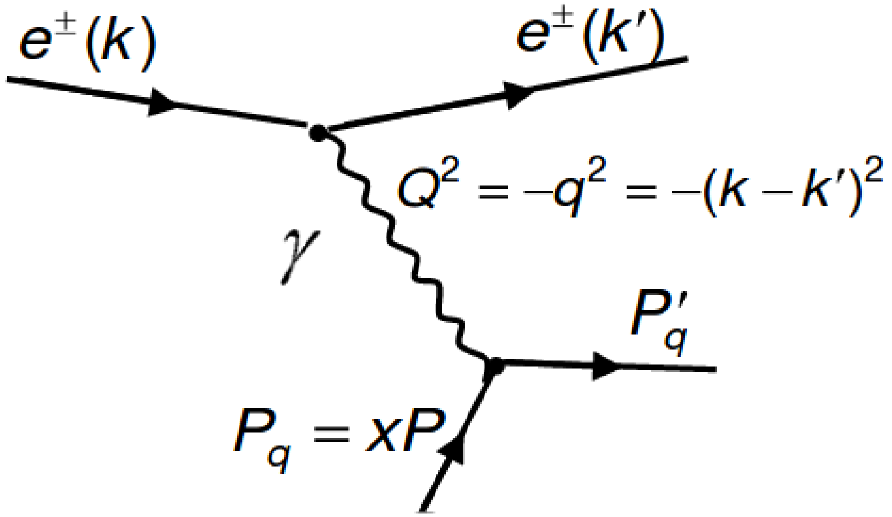

Q² = 25030 GeV², y = 0.56, x=0.50
e+
e+
proton
jet
e+
Q²
x
proton
jet
z
H1 Run 122145 Event 69506
Date 19/09/1995

ZEUS

experiment



Elastic scattering on single quark



$$\frac{d\sigma^{Mott}}{d\Omega} \Big/ \frac{d\sigma^{Ruth}}{d\Omega}$$

$$= \left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2}\sin^2\frac{\theta}{2} \right)$$

helicity conservation

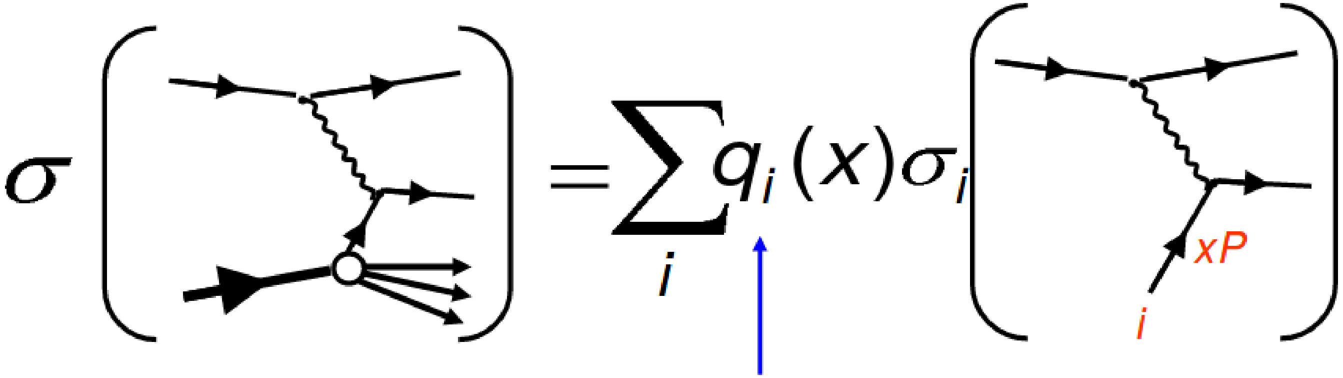
spin-spin IA

Electron-quark scattering (quark momentum fraction x):

$$\frac{d\sigma}{dQ^2} = \left(\frac{4\pi\alpha^2}{Q^4}\right)\frac{E'}{E}\cdot e_i^2 \left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2x^2M^2}\sin^2\frac{\theta}{2} \right)$$

Charge of struck quark

Rutherford x-section + recoil of scatter partner



Parton density $q_i(x)dx$: Probability to find parton i in momentum interval [x, x+dx]

$$\frac{d^2\sigma}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\sum_i e_i^2 q_i(x)(\cos^2\theta/2 + \frac{Q^2}{2x^2M^2}\sin^2\theta/2)$$

$$\frac{d^2\sigma}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}(\frac{F_2(x,Q^2)}{x}\cos^2\theta/2 + 2F_1(x,Q^2)\frac{Q^2}{2x^2M^2}\sin^2\theta/2)$$

Parton distribution function PDF:

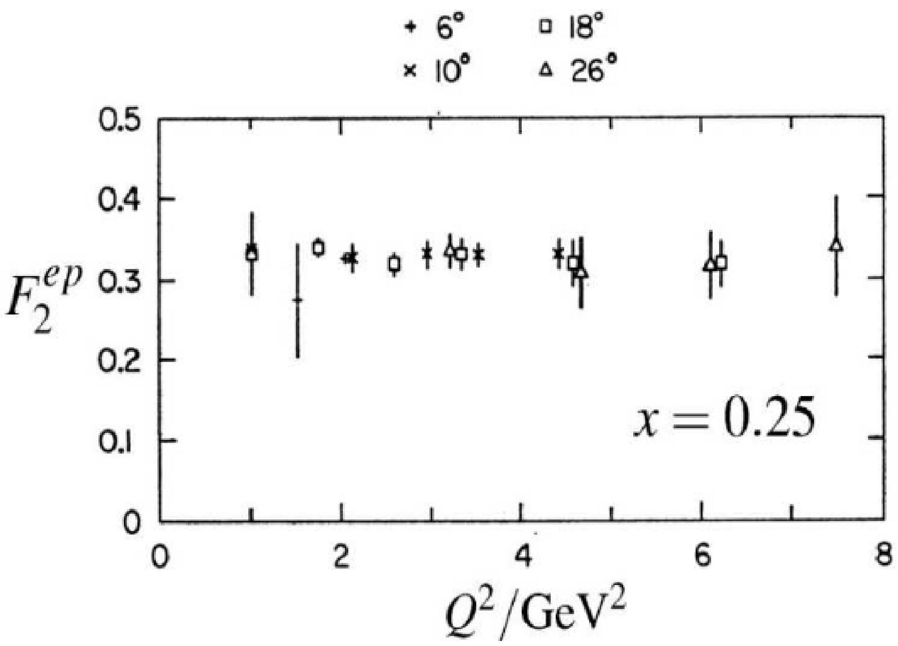
$$F_2(x,Q^2) = F_2(x) = x\sum_i e_i^2 q_i(x)$$

$$F_1(x,Q^2) = F_1(x) = \frac{1}{2}\sum_i e_i^2 q_i(x)$$

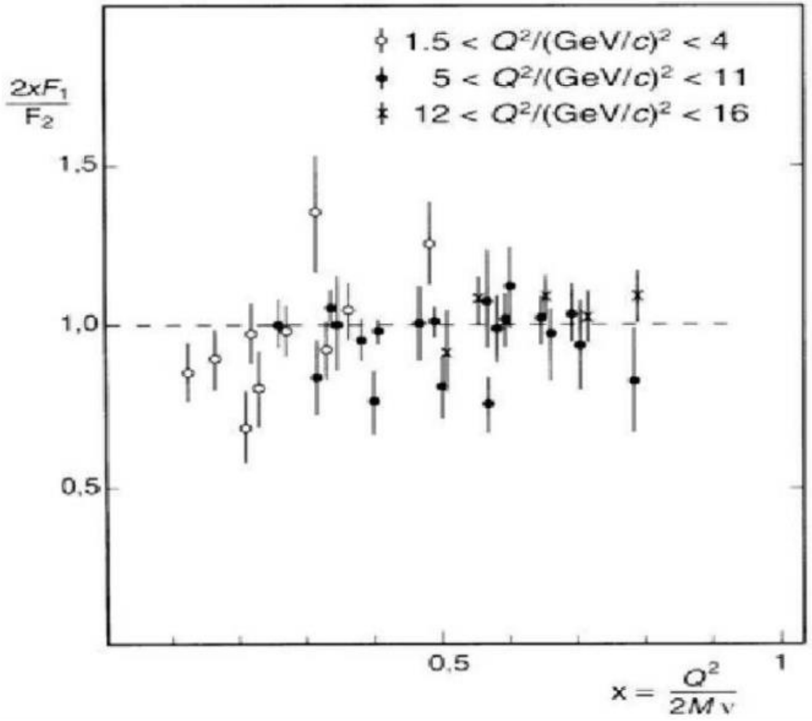
$$\frac{d^2\sigma}{dQ^2dx} = \left(\frac{4\pi\alpha^2}{Q^4}\right)\frac{E'}{E}\cdot\left(\frac{F_2(x)}{x}\cos^2\frac{\theta}{2} + 2F_1(x)\frac{Q^2}{2x^2M^2}\sin^2\frac{\theta}{2}\right)$$

Deep inelastic electron-proton scattering:

- Free partons: $F_2=F_2(x)$ ⇔ "scaling" (F_2 only function of x)
- Spin ½ partons: $2xF_1(x) = F_2(x)$ (Callan-Gross relation)



J.T.Friedman + H.W.Kendall, Ann. Rev. Nucl. Sci. 22 (1972) 203



Parton distribution functions

(ignoring see quarks)

Proton

$$F_2 = x(e_u^2 u(x) + e_d^2 d(x)) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x)\right)$$

Neutron

$$\frac{1}{x}F_2^n = \frac{4}{9}u_n(x) + \frac{1}{9}d_n(x) \simeq \frac{4}{9}d_p(x) + \frac{1}{9}u_p(x)$$

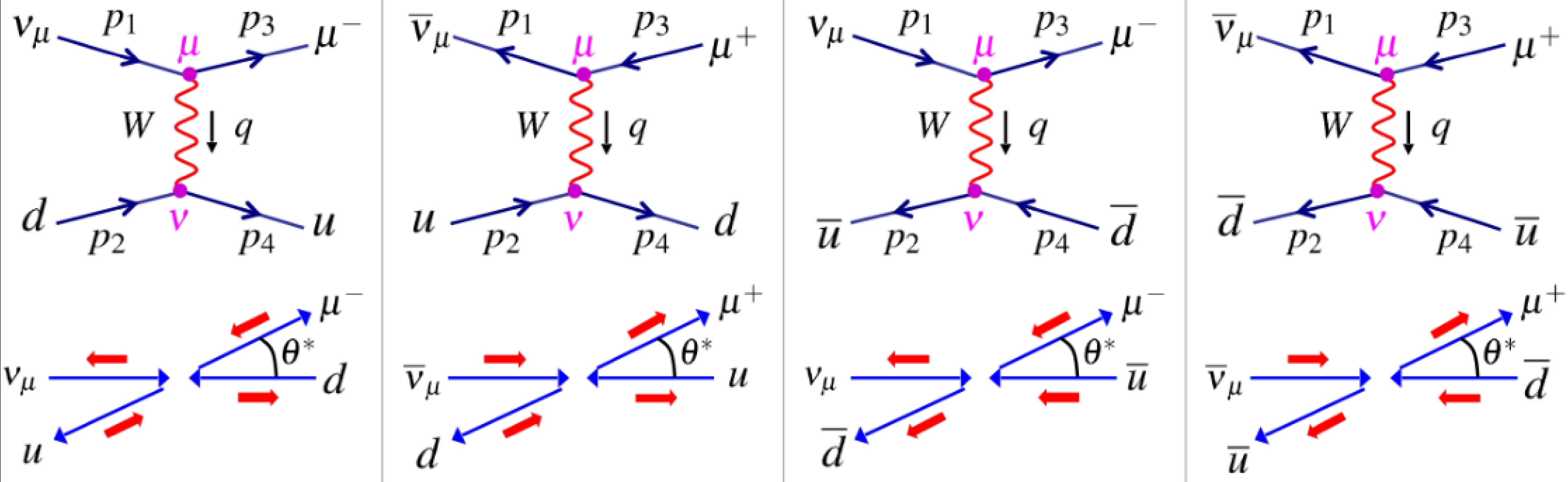
↑ isospin symmetry

Considering QCD corrections: Valence quarks + see quarks

Isoscalar Target: #n=#p

$$F_2^N = \frac{1}{2}[F_2^p + F_2^n] = \frac{5}{18}x[u + \bar{u} + d + \bar{d}] + \frac{1}{9}x[s + \bar{s}]$$

* Indicates CMS, angles are not LI



$S_z = 0$

$$\frac{d\sigma_{\nu q}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$$

$$\sigma_{\nu q} = \frac{G_F^2 \hat{s}}{\pi}$$

$S_z = +1$

$$\frac{d\sigma_{\overline{\nu} q}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$$

$$\sigma_{\overline{\nu} q} = \frac{G_F^2 \hat{s}}{3\pi}$$

$S_z = -1$

$$\frac{d\sigma_{\nu \overline{q}}}{d\Omega^*} = \frac{G_F^2}{16\pi^2}(1+\cos\theta^*)^2\hat{s}$$

$$\sigma_{\nu \overline{q}} = \frac{G_F^2 \hat{s}}{3\pi}$$

$S_z = 0$

$$\frac{d\sigma_{\overline{\nu}\overline{q}}}{d\Omega^*} = \frac{G_F^2}{4\pi^2}\hat{s}$$

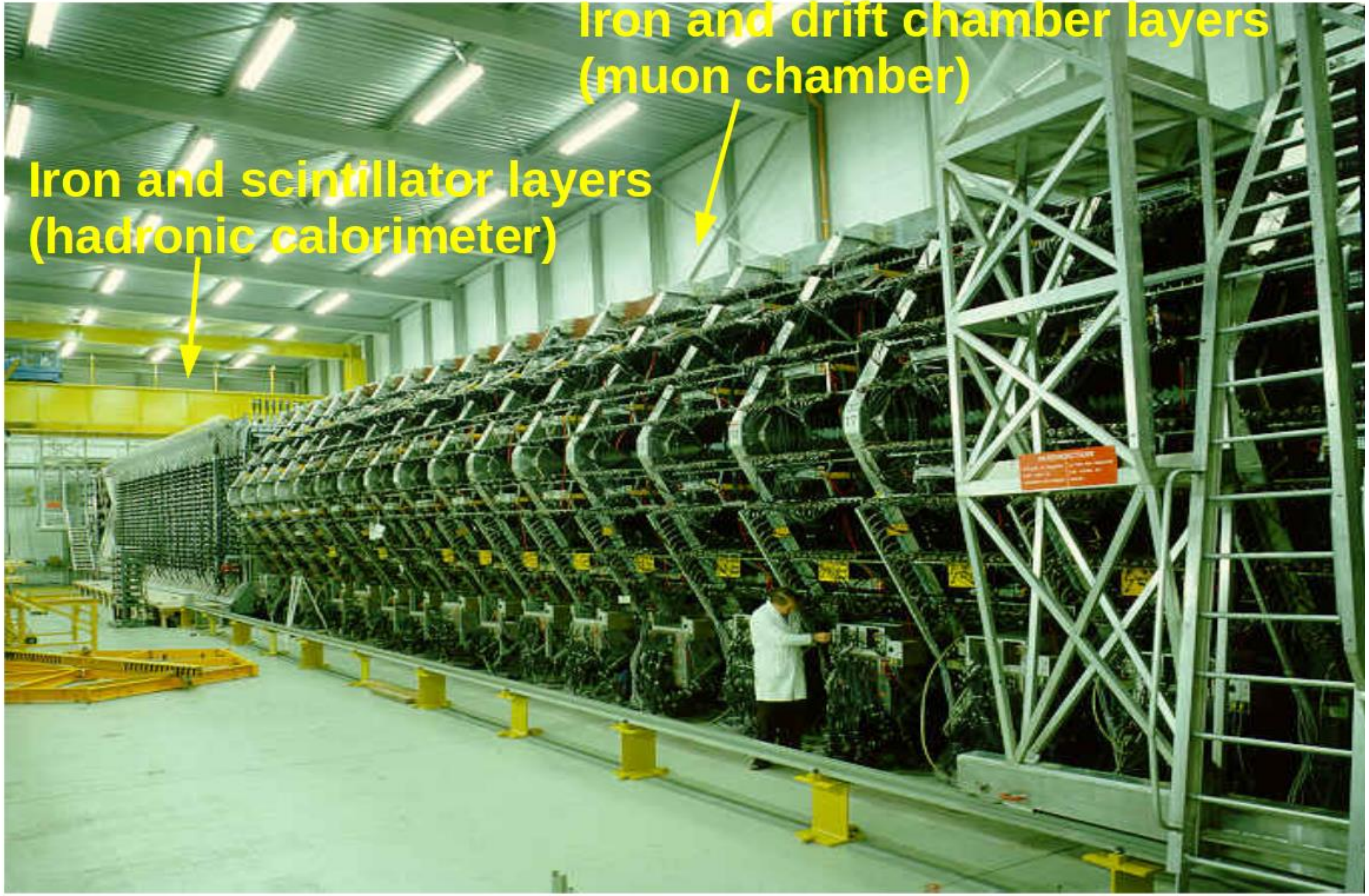
$$\sigma_{\overline{\nu}\overline{q}} = \frac{G_F^2 \hat{s}}{\pi}$$

$$\frac{d^2\sigma^{\nu p}}{dx\,dy} = \frac{G_F^2}{\pi} sx[d + (1-y)^2\overline{u}]$$

$$\frac{d^2\sigma^{\overline{\nu} p}}{dx\,dy} = \frac{G_F^2}{\pi} sx[(1-y)^2 u + \overline{d}]$$

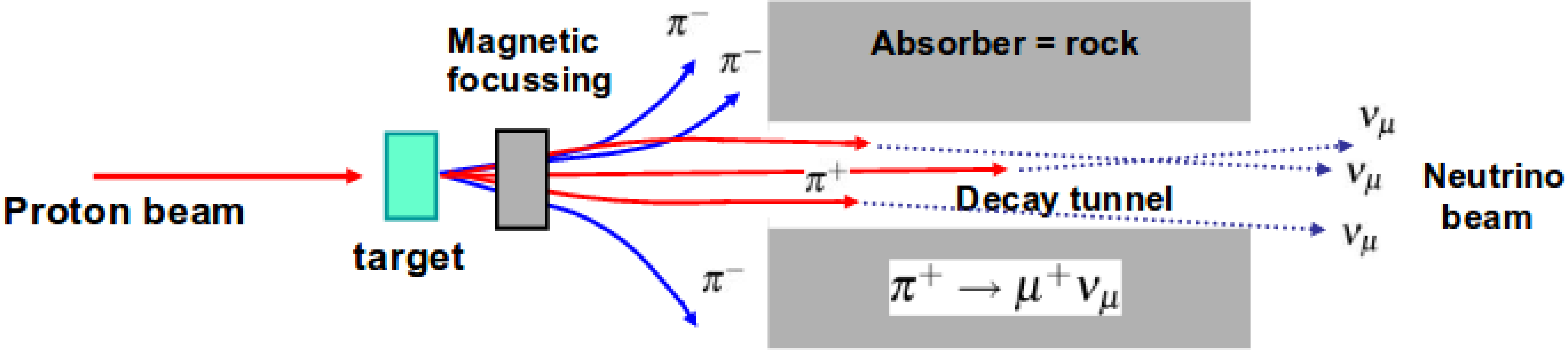
$\hat{s} = sx$

CDHS – CERN-Dortmund-Heidelberg-Saclay Experiment

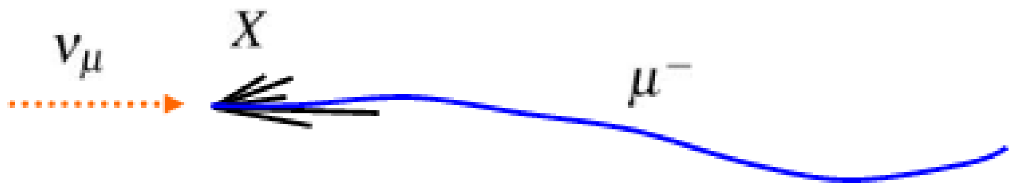


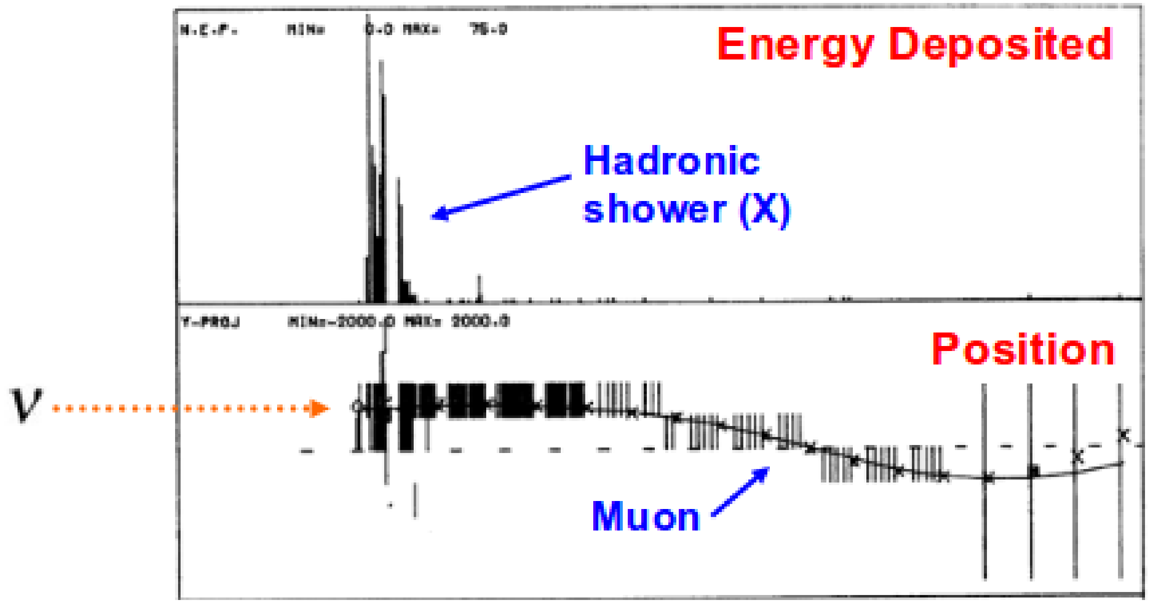
★ Neutrino Beams:

- Smash high energy protons into a fixed target ➡ hadrons
- Focus positive pions/kaons
- Allow them to decay $\pi^+ \rightarrow \mu^+ \nu_\mu$ + $K^+ \rightarrow \mu^+ \nu_\mu \; (BR \approx 64\%)$
- Gives a beam of "collimated" ν_μ
- Focus negative pions/kaons to give beam of $\overline{\nu}_\mu$



Example Event:





- Measure energy of X

$$E_X$$

- Measure muon momentum from curvature in B-field

$$E_\mu$$

★ For each event can determine neutrino energy and y !

$$E_\nu = E_X + E_\mu$$

$$E_\mu = (1-y)E_\nu \quad \longrightarrow \quad y = \left(1 - \frac{E_\mu}{E_\nu}\right)$$

$$\frac{d^2\sigma^{\nu p}}{dxdy} = \frac{G_F^2}{\pi} sx[d + (1-y)^2\overline{u}]$$

$$\frac{d^2\sigma^{\overline{\nu} p}}{dxdy} = \frac{G_F^2}{\pi} sx[(1-y)^2 u + \overline{d}]$$

Isospin-symmetry

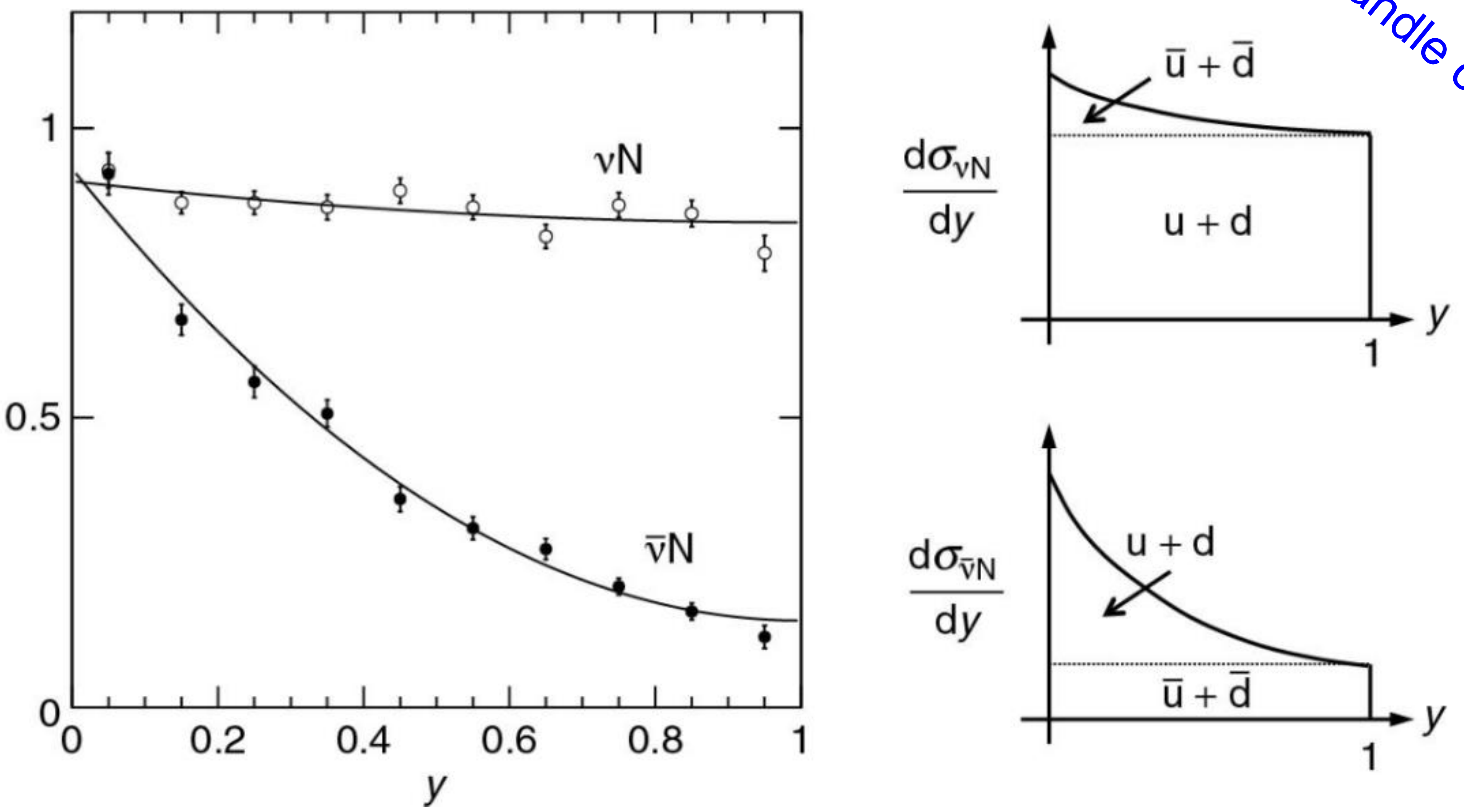
$$\frac{d^2\sigma^{\nu n}}{dxdy} = \frac{G_F^2}{\pi} sx[d^n + (1-y)^2\overline{u}^n] = \frac{G_F^2}{\pi} sx[u + (1-y)^2\overline{d}]$$

$$\frac{d^2\sigma^{\overline{\nu} n}}{dxdy} = \frac{G_F^2}{\pi} sx[(1-y)^2 u^n + \overline{d}^n] = \frac{G_F^2}{\pi} sx[(1-y)^2 d + \overline{u}]$$

$$\frac{d^2\sigma^{\nu N}}{dxdy} = \frac{G_F^2}{\pi} sx[d + u + (1-y)^2(\overline{u} + \overline{d})]$$

$$\frac{d^2\sigma^{\overline{\nu} N}}{dxdy} = \frac{G_F^2}{\pi} sx[(1-y)^2(u + d) + \overline{d} + \overline{u}]$$

Neutrino-Nukleon Differential Cross Section



direct handle on sea quarks

Cross-section for inelastic e-p scattering via EM interaction (exchange of photon):

$$\frac{d^2\sigma^{ep}}{dQ^2dx} = \frac{4\pi\alpha^2}{Q^4}\frac{E'}{E}\left(\frac{F_2^{ep}(x)}{x}\cos^2\theta/2 + 2F_1^{ep}(x)\frac{Q^2}{2x^2M^2}\sin^2\theta/2\right)$$

In LI representation of inelastic e-p scattering:

$$\frac{d^2\sigma^{ep}}{dxdy} = \frac{4\pi\alpha^2 s}{Q^4}[(1-y)F_2^{ep}(x) + y^2 xF_1^{ep}(x)]$$

← most general form of parity conserving scattering x-section

Cross-section for inelastic ν-p scattering via CC interaction (exchange of $W^\pm$):

$$\frac{d^2\sigma^{\nu p}}{dxdy} = \frac{G_F^2 s}{2\pi}[(1-y)F_2^{\nu p}(x) + y^2 xF_1^{\nu p}(x) + y(1-\frac{y}{2})xF_3^{\nu p}(x)]$$

$$\frac{d^2\sigma^{\overline{\nu} p}}{dxdy} = \frac{G_F^2 s}{2\pi}[(1-y)F_2^{\overline{\nu} p}(x) + y^2 xF_1^{\overline{\nu} p}(x) - y(1-\frac{y}{2})xF_3^{\overline{\nu} p}(x)]$$

Additional PDF to allow for partiy violation.

Charged-current ($W^{\pm}$) scattering by using neutrinos instead of electrons, allows to determine the valence quark distributions.

PDF for Neutrino Scattering

Additional PDF F_3 to account for parity violation

$$F_2^{\nu p} = 2x[d + \bar{u}] \qquad xF_3^{\nu p} = 2x[d - \bar{u}]$$

$$F_2^{\nu n} = 2x[d^n + \bar{u}^n] = 2x[u + \bar{d}] \qquad xF_3^{\nu n} = 2x[d^n - \bar{u}^n] = 2x[u - \bar{d}]$$

$$F_2^{\nu N} = x[u + \bar{u} + d + \bar{d}] \qquad xF_3^{\nu N} = x[(u + d) - (\bar{u} + \bar{d})]$$

Iso-scalar target →

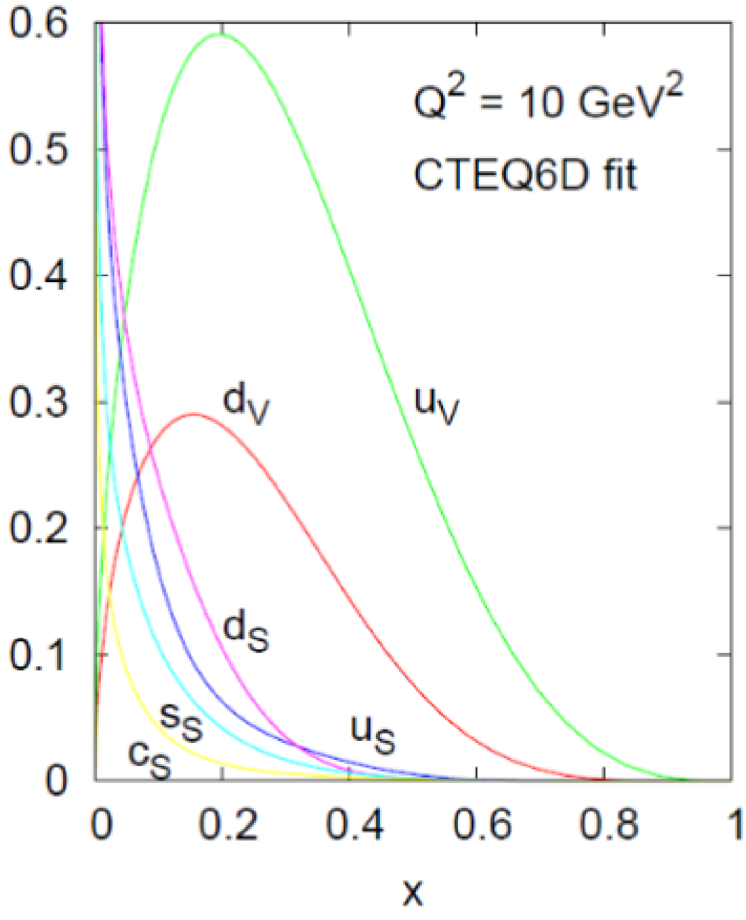
$$F_2^{\nu N} = x[Q(x) + \bar{Q}(x)] \qquad xF_3^{\nu N} = x[Q(x) - \bar{Q}(x)]$$

Measures sum of quarks and anti-quarks

Measures valence quarks

Measurement: $F_2^{\nu N} + xF_3^{\nu N} = 2xQ(x)$ → Sea and valence quarks

$F_2^{\nu N} - xF_3^{\nu N} = 2x\bar{Q}(x)$ → Sea quarks



Definition of PDFs: $\sum_i \int dx\, x q_i(x) = 1$

q_i	momentum
d_V	0.111
u_V	0.267
d_S	0.066
u_S	0.053
s_S	0.033
c_S	0.016
total	0.546

46% of nucleon momentum not carried by quarks

Gluons have been neglected so far.

Sum rules

$$\int_0^1 u(x) - \bar{u}(x)dx = \int_0^1 u_v(x)dx = 2$$

$$\int_0^1 d(x) - \bar{d}(x)dx = \int_0^1 d_v(x)dx = 1$$

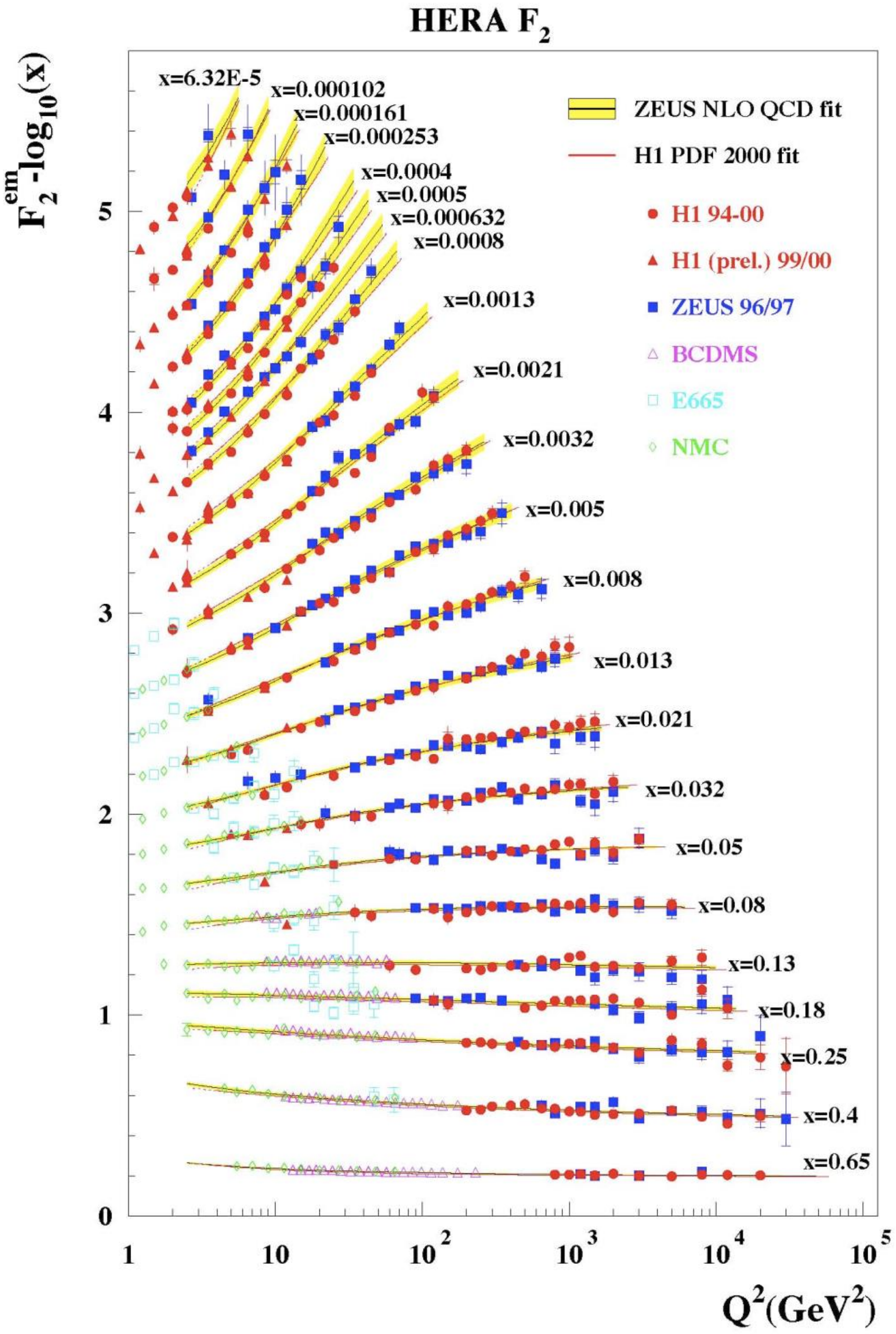
Valence quarks

$$\int_0^1 (q_s(x) - \bar{q}_s(x))dx = 0$$

Sea quarks: s, c, …

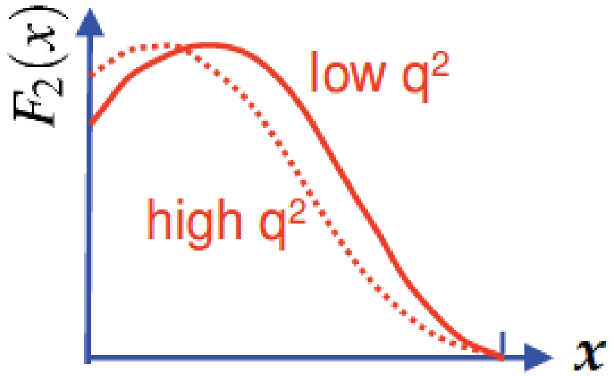
Scaling violation

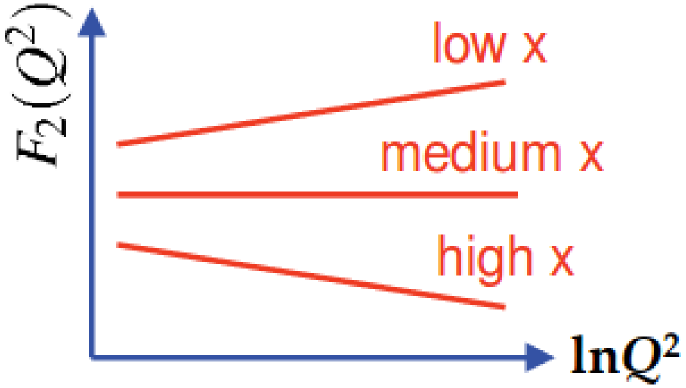
$$F_2(x) \to F_2(x, Q^2)$$



Origin of Scaling Violations

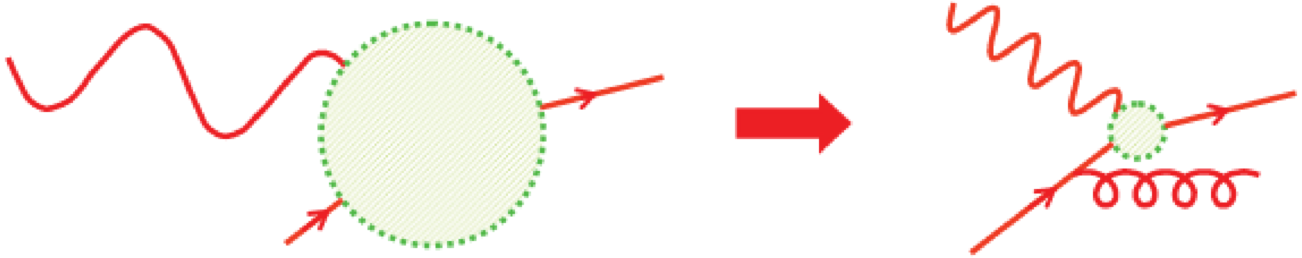
★ Observe "small" deviations from exact Bjorken scaling $F_2(x) \rightarrow F_2(x, Q^2)$





★ At high Q^2 observe more low x quarks

★ **"Explanation":** at high Q^2 (shorter wave-length) resolve finer structure: i.e. reveal quark is sharing momentum with gluons. At higher Q^2 expect to "see" more low x quarks



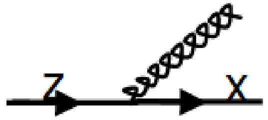
★ QCD cannot predict the x dependence of $F_2(x, Q^2)$

★ But QCD **can** predict the Q^2 dependence of $F_2(x, Q^2)$

Evolution of parton densities

DGLAP evolution equation
(**D**okshitzer, **G**ribov, **L**ipatov, **Al**tarelli, **P**arisi)

evolution of quark density with $\ln Q^2$

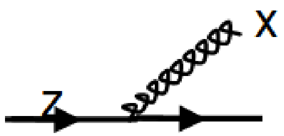


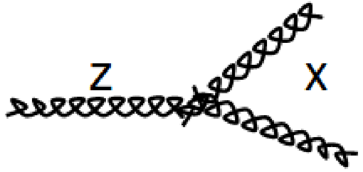
$$\frac{\partial q(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[q(z,Q^2)P_{qq}\left(\frac{x}{z}\right) + g(z,Q^2)P_{qg}\left(\frac{x}{z}\right)\right]$$

Splitting function P_{qq}:
Probability for $q(z) \rightarrow q(x) + g$

$$\frac{\partial g(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[q(z,Q^2)P_{gq}\left(\frac{x}{z}\right) + g(z,Q^2)P_{gg}\left(\frac{x}{z}\right)\right]$$

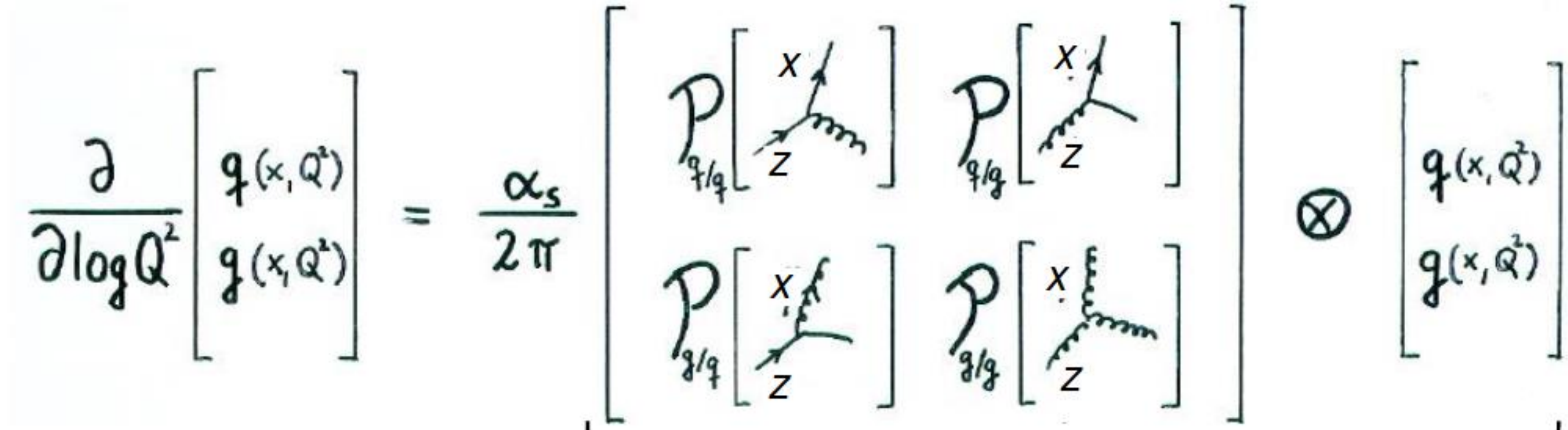
evolution of gluon density with $\ln Q^2$



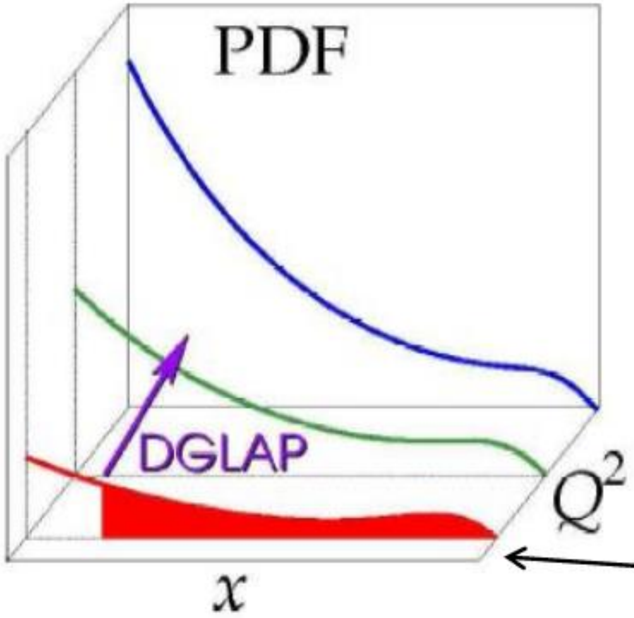


Splitting functions: Probability that a parton (quark or gluon) emits a parton **(q, g)** with momentum fraction $\varepsilon = x/z$ of the parent parton.

DGLAP Evolution ("symbolic"):



$$P \otimes f(x,Q^2) = \int_x^1 \frac{dz}{z} P\left(\frac{x}{z}\right) f(z,Q^2)$$



<u>QCD evolution:</u>

QCD predicts evolution of the PDF along the scale Q^2.

QCD cannot predict the shape of PDF, PDF must be measured!

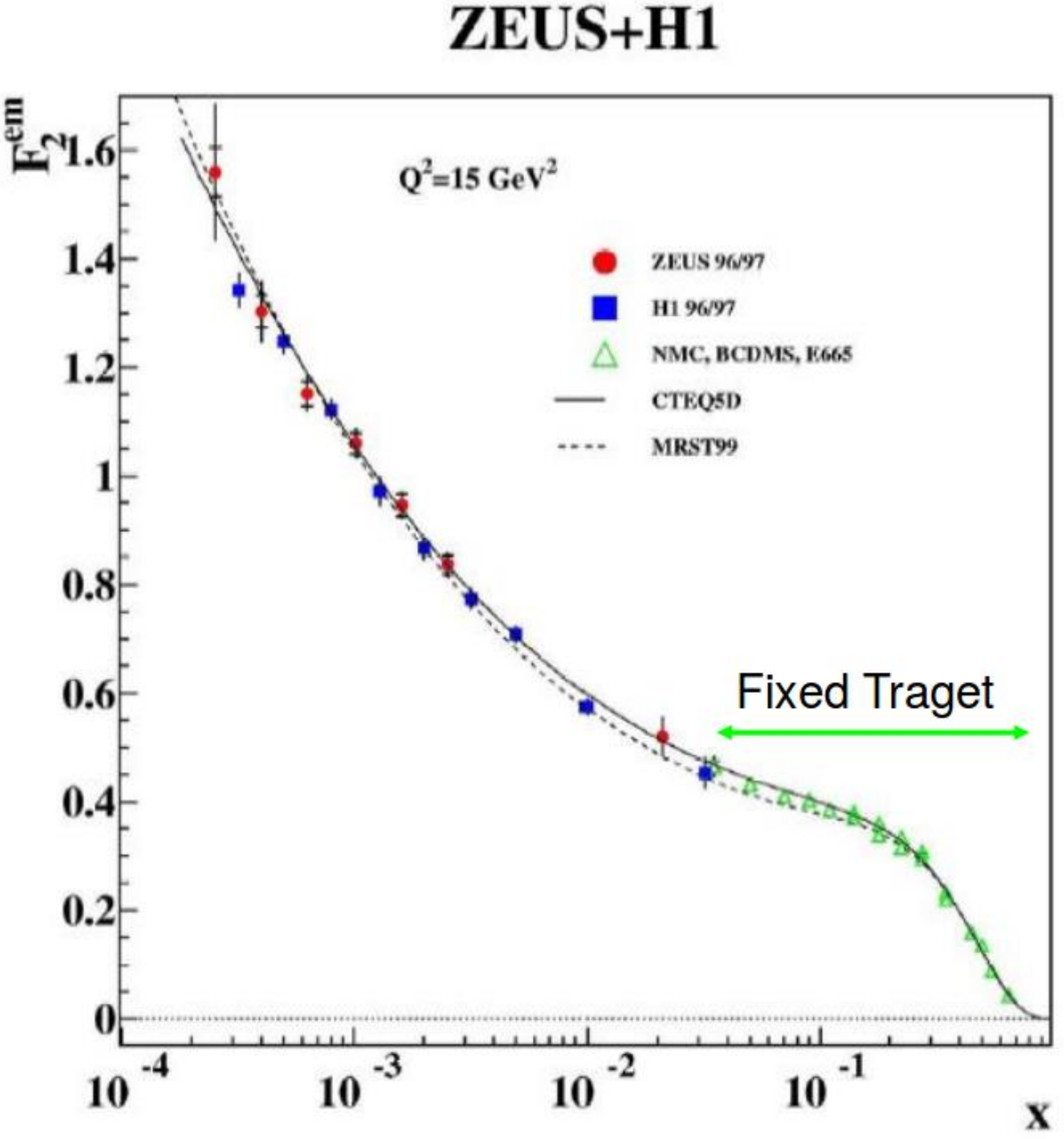
Measurement of the structure function $F_2(x, Q^2)$

$$\frac{d^2\sigma^{ep}}{dxdy} = \frac{4\pi\alpha^2 s}{xQ^4}[(1-y)F_1^{ep}(x,Q^2) + y^2 F_2^{ep}(x,Q^2)]$$

↓ e.g. for y=1

$$\frac{d^2\sigma^{ep}}{dxdy} = \frac{4\pi\alpha^2 s}{xQ^4}[F_2^{ep}(x,Q^2)]$$

$$F_2(x,Q^2) = x\sum_i e_i^2[q_i(x,Q^2) + \overline{q_i}(x,Q^2)]$$



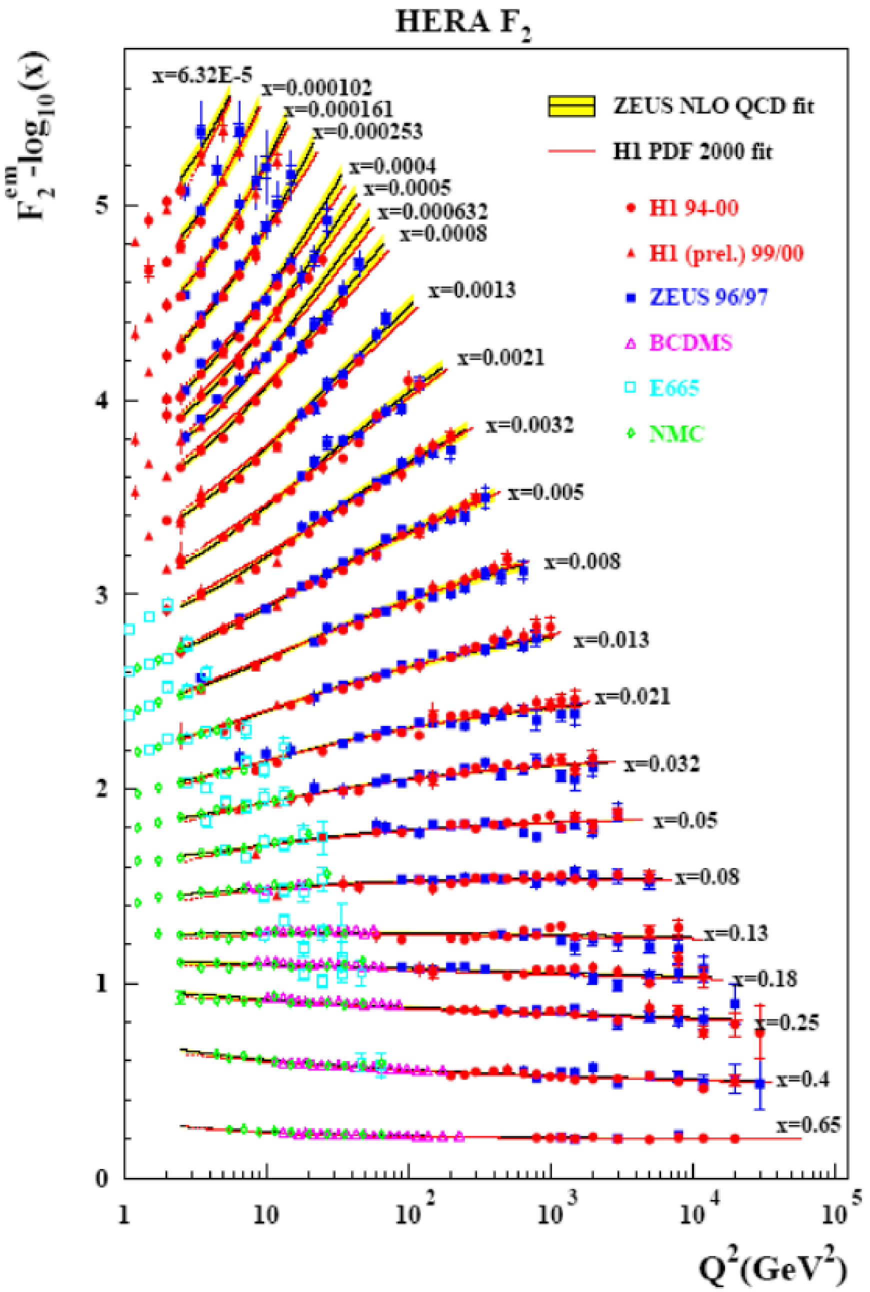
$F_2(x)$

Large increase of $F_2(x)$ for very small x - unexpected

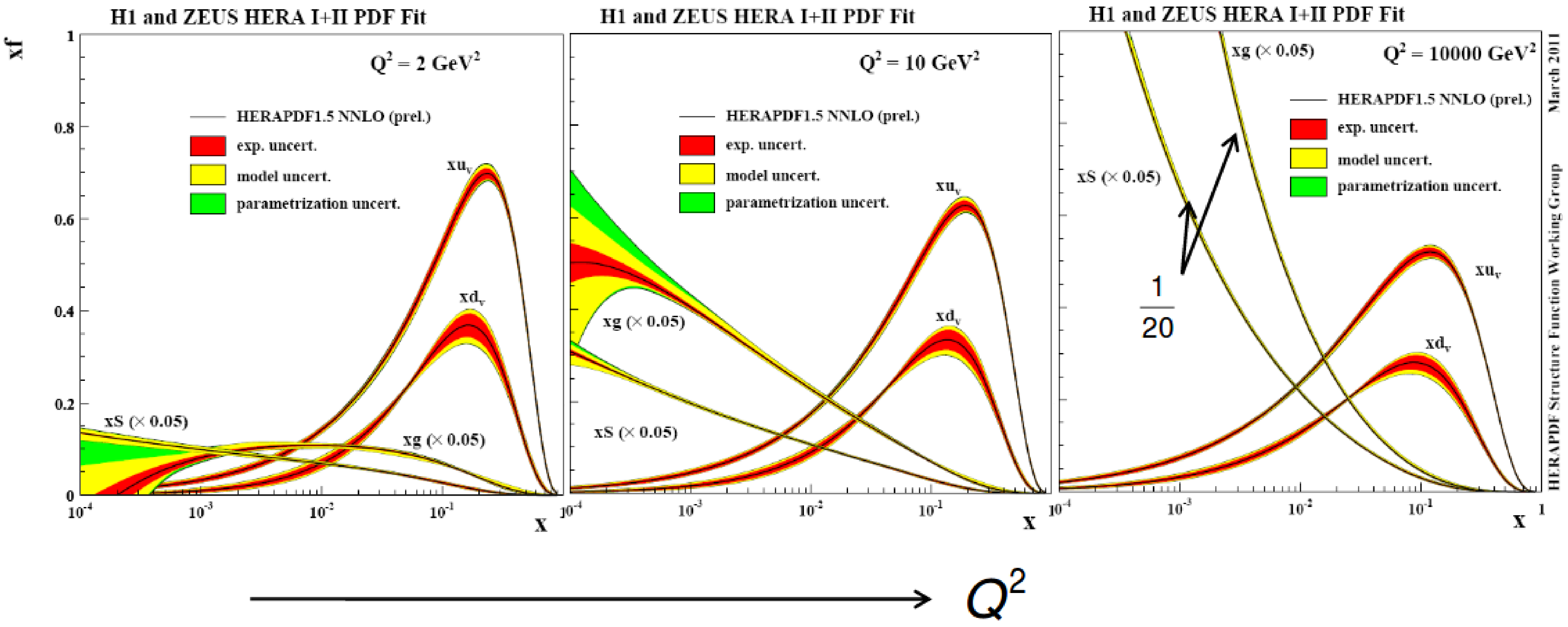
$$F_2(x,Q^2)$$

Q^2 dependence is correctly described by QCD evolution

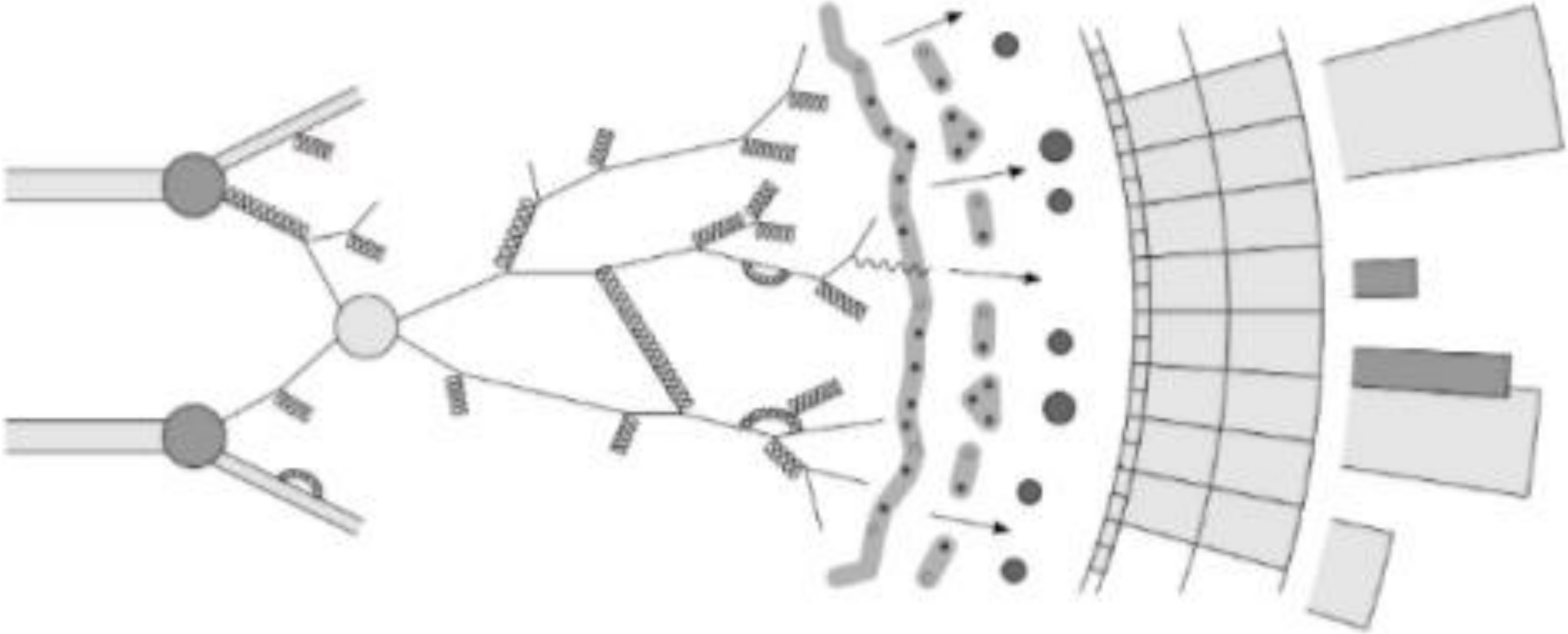
Determine the PDFs



Proton PDFs as seen by HERA



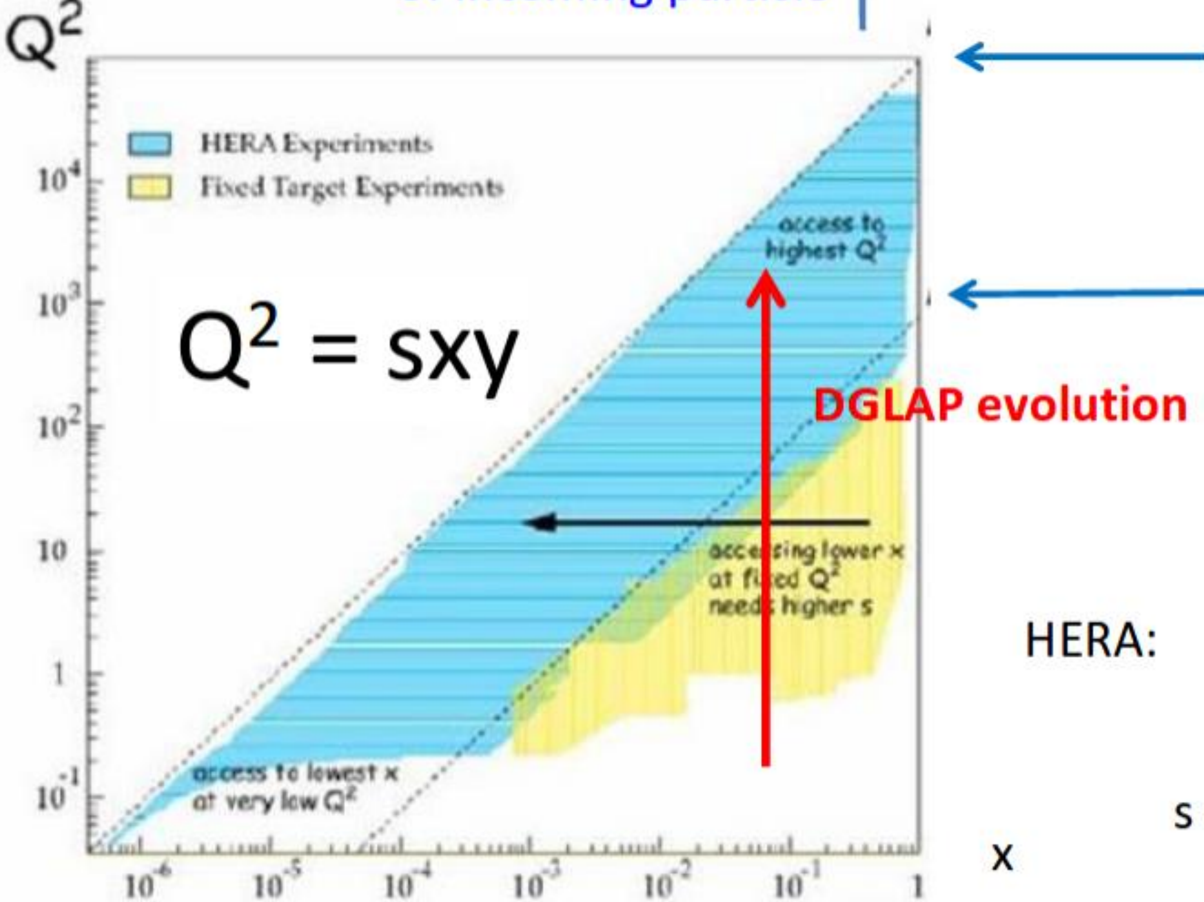
4. Hadron-hadron collisions



For all cross section estimation the knowledge of the PDF is necessary.

	Lab frame proton at rest	LI form
energy loss of incoming particle	$\nu = E_1 - E_3$	$\nu = qp_2/M$
Bjorken x	$x = \frac{-q^2}{2M\nu}$	$x = \frac{Q^2}{2qp_2}$
fraction energy loss of incoming particle	$y = (E_1 - E_3)/E_1$	$y = qp_2/p_1p_2$

$(p_1+p_2)^2 \approx 2p_1p_2$
$Q^2 = sxy$

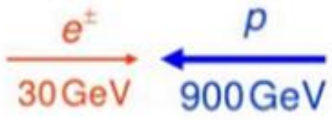


$y_{max} = 1$

HERA: $s \approx 10^5$ GeV^2

$y_{max} = 1$ for fixed target exp: $s < 10^3$ GeV^2

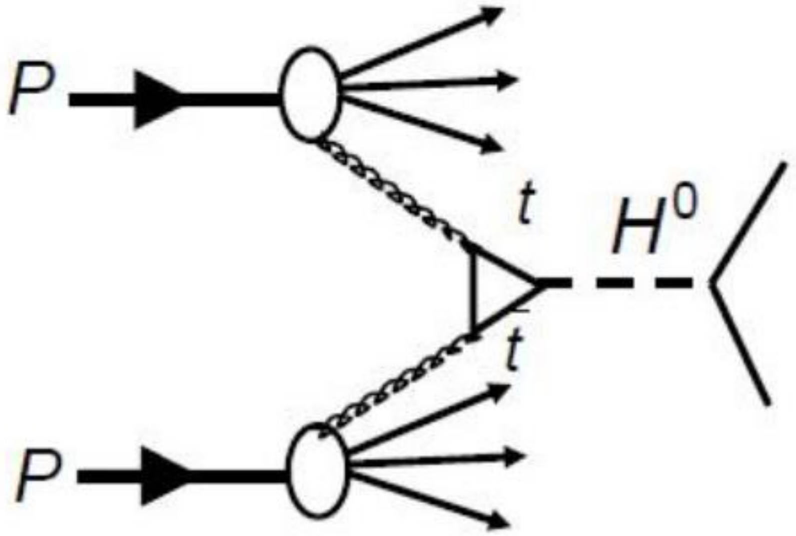
HERA:



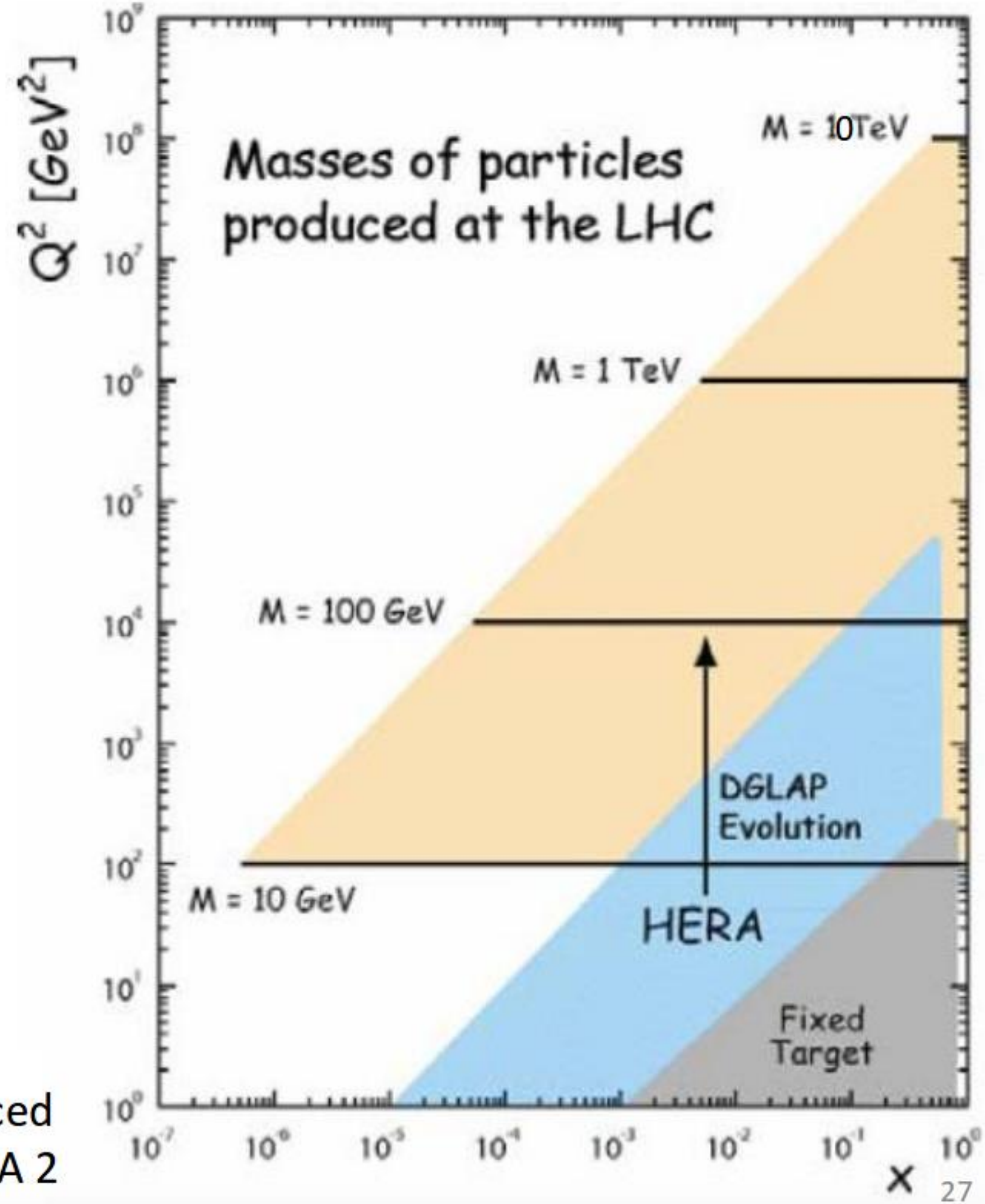
$s \approx 2p_ep_p \approx 4E_eE_p$

Parton densities important to predict signal and background at LHC:

Higgs-production at LHC



(main production process)

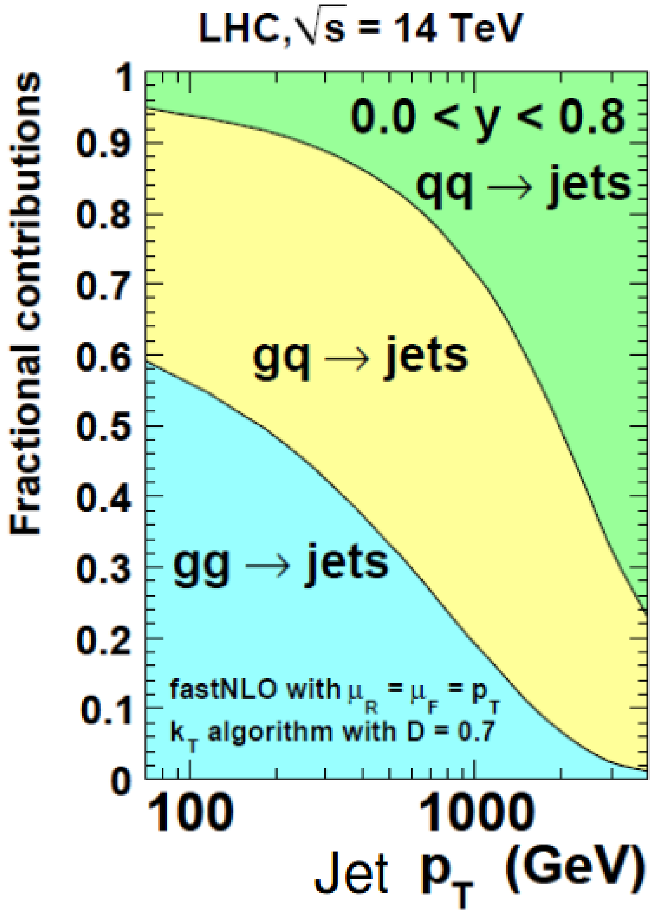


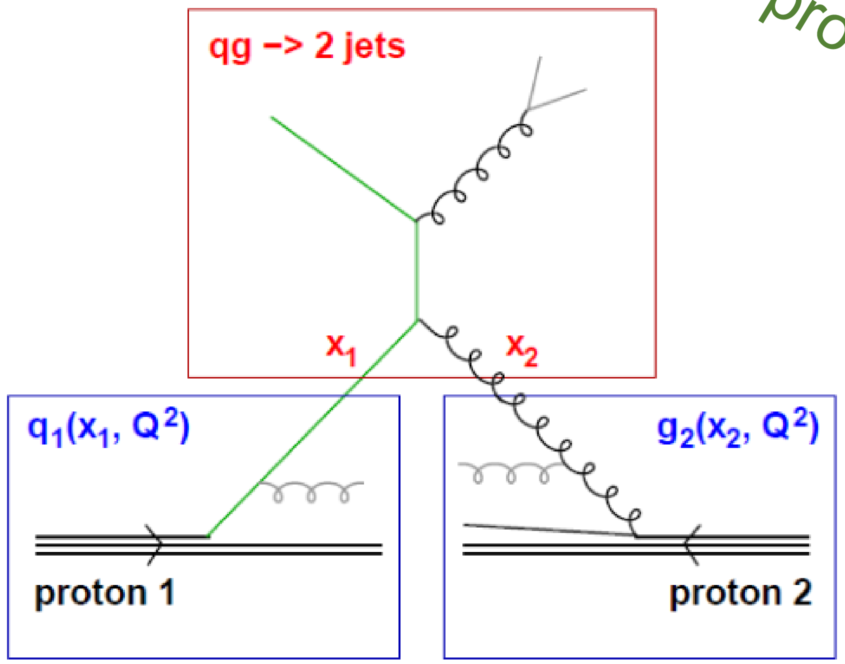
Uncertainties on cross-section were reduced from 20% to 5% by measurements at HERA 2

Example process: 2-jet production

Jet production in proton-proton collision is an excellent test of PDFs, in particular of gluon PDF, since there are large direct contributions from $gg \to gg$ and $qg \to qg$:

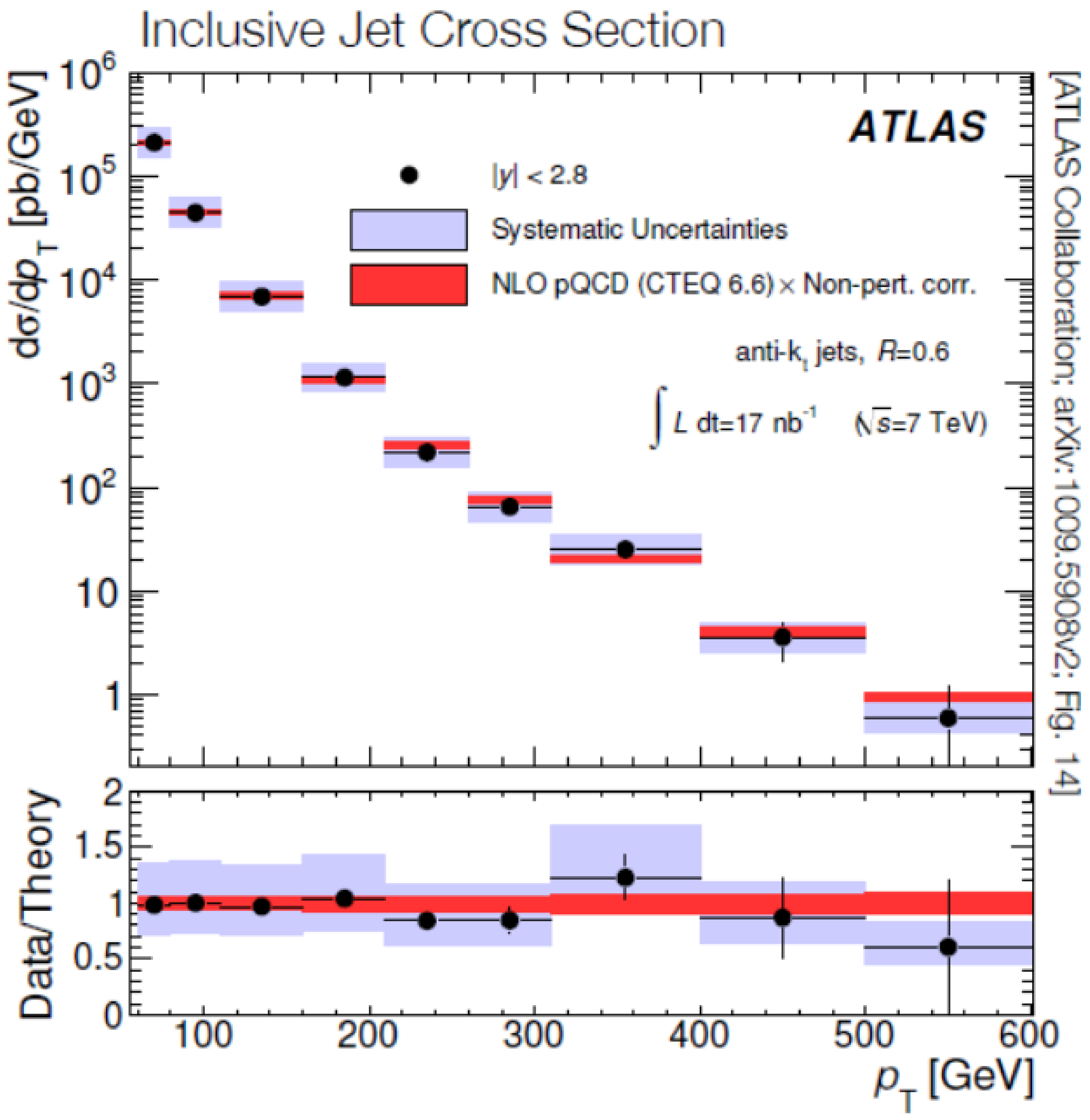
Caveat at pp:
Need to deal with
product of two pdfs



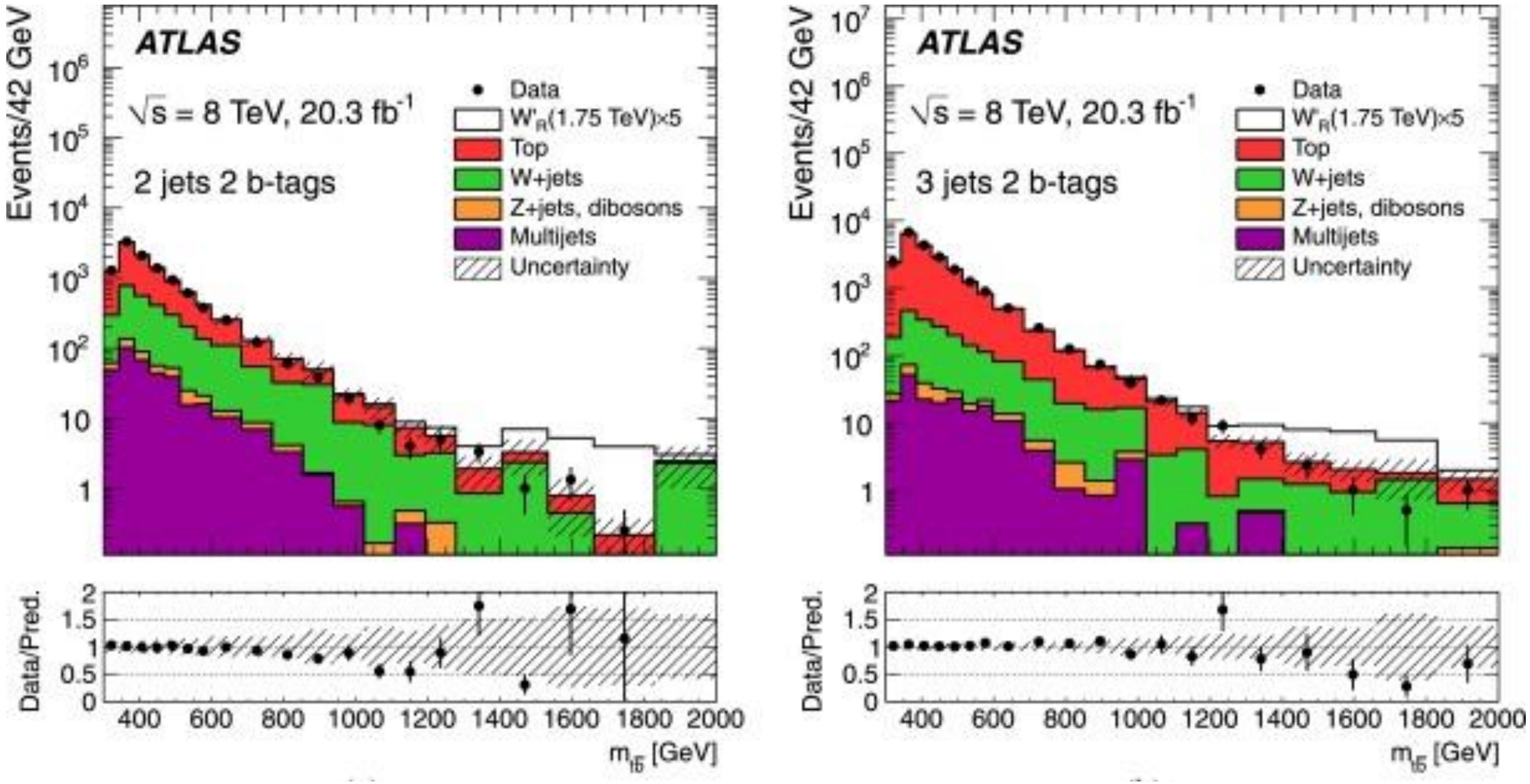


$$\sigma_{pp \to 2\,jets} = \sigma_{qg \to 2\,jets} \otimes q_1 \otimes g_2 + \cdots$$

Inclusive jet production well described with known PDF.



Example: Search for $W' \to t\bar{b}$ events



Jets are background in many analysis. A good understanding of PDFs and QCD is crucial to search for physics beyond the Standard Model.