Experimental Tests of QCD

1. Test of QCD of in e+e- annihilation
2. Running of the strong coupling constant
3. Study of QCD in deep inelastic scattering
4. Hadron-Hadron collisions
5. Hadron spectroscopy (e.g. penta quark)
6. A new state of hadronic matter
   (quark gluon plasma)

1. Test of QCD in e+e- annihilation

Evidence for Color

a) Existance of $\Delta^{++} = |uuu>$

L = 0 : ground state
$S_3 = S = 3/2$
$I_3 = I = 3/2$

$\psi_{total} = \underbrace{\psi_{space}}_{(-1)^L} \psi_{spin} \psi_{isospin}$

all three wave functions are symmetric, thus ψ_{total} as well

However combination of three fermions must have asymmetric wave function

⇒ one degree of freedom is missing!

$$\psi_{colour}(123) = \frac{1}{\sqrt{6}}(RGB + GBR + BRG - GRB - BGR - GRB)$$

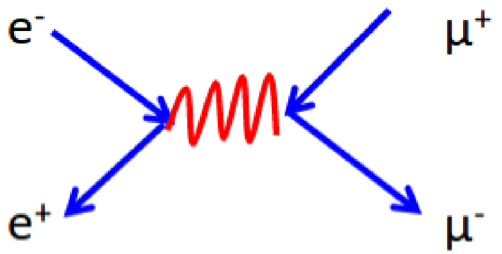
Exchange of quark 1 and 2:

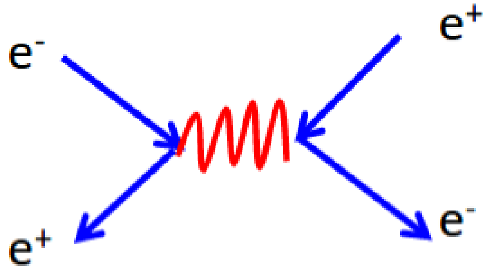
$$\psi_{colour}(213) = \frac{1}{\sqrt{6}}(GRB + BGR + RBG - RGB - GBR - RGB) = -\psi_{colour}(123)$$

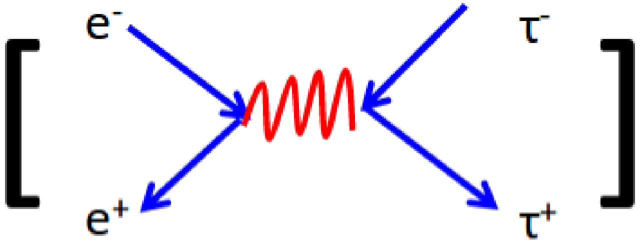
(similar for any other exchange of two quarks)

⇒ $\psi_{total} = \psi_{space}\psi_{spin}\psi_{isospin}\psi_{colour}$

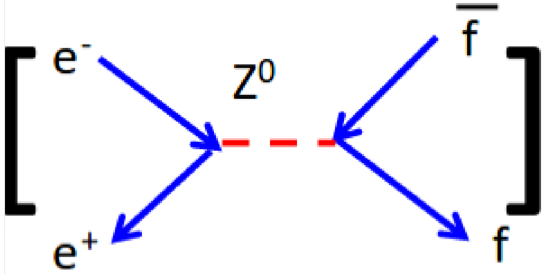
b) Non-resonant hadron production in e^+e^-







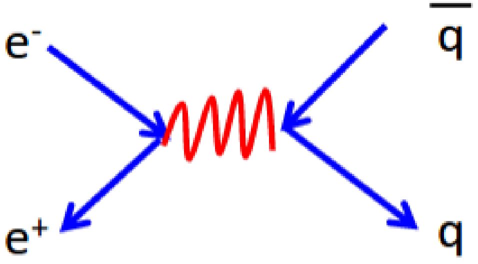
threshold dependent,
τ was not yet known at
that time (m_τ = 1.8 GeV)



weak IA negligible relative to elm IA as long as $\sqrt{s}$ away from mass of Z.

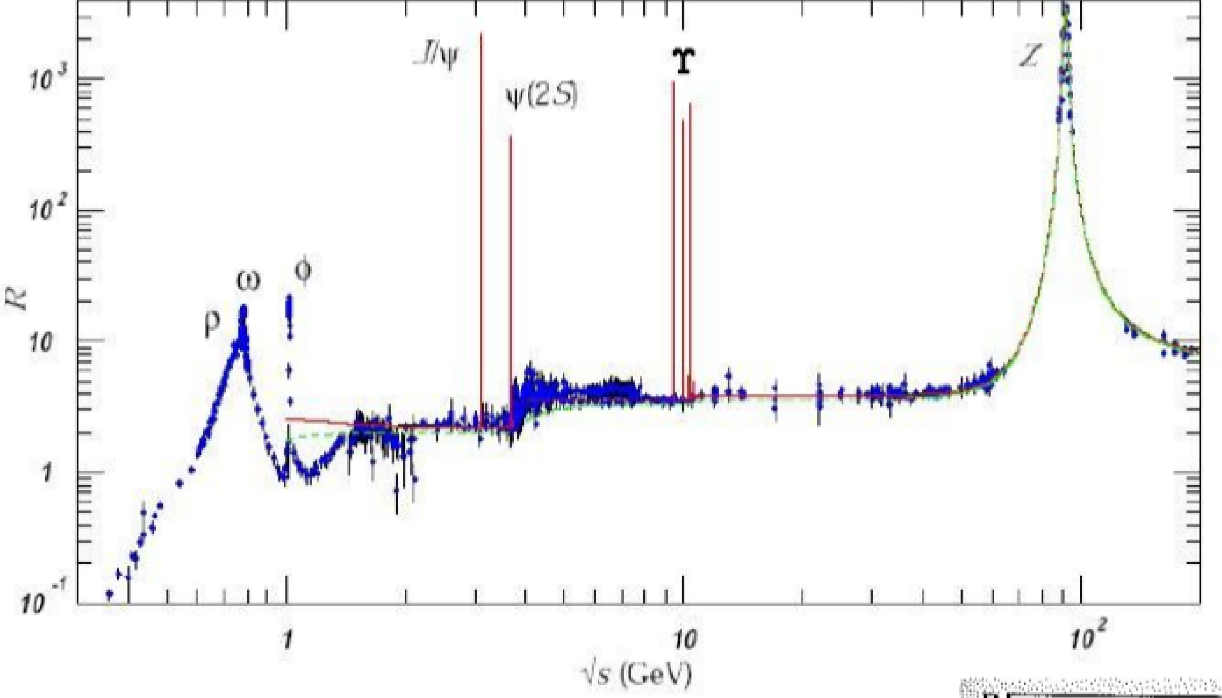
colour factor 3

$$\sigma(e^+e^- \rightarrow q\bar{q}) = N_c \sum_i^u Z_i^2 \sigma(e^+e^- \rightarrow \mu^+\mu^-)$$



Z_i: charge in elementary units

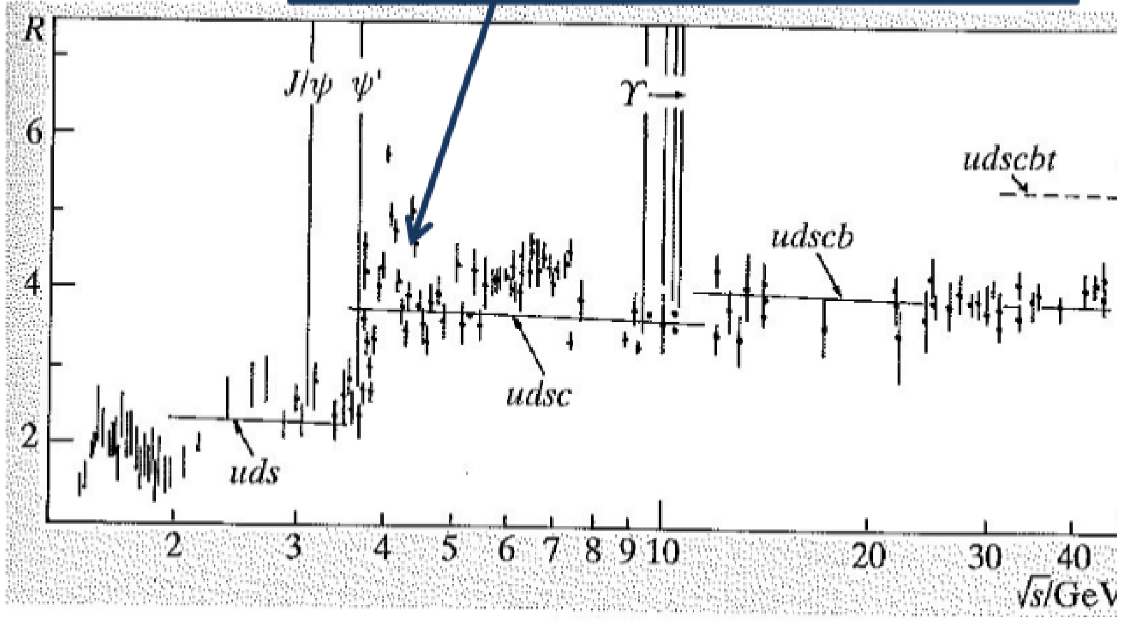
u: upper limit on quark species due to $\sqrt{s}$



$$R = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$= N_c \sum_i^u Z_i^2$$

ττ̄ treshold, however not all τ decay into hadronic jets



q	Z_i^2	$R[\sqrt{s} \le 2m(q)]$
u	4/9	4/3
d	1/9	5/3
s	1/9	2
c	4/9	10/3
b	1/9	11/3
t	4/9	5

N_c=3 „more or less“ confirmed by data!

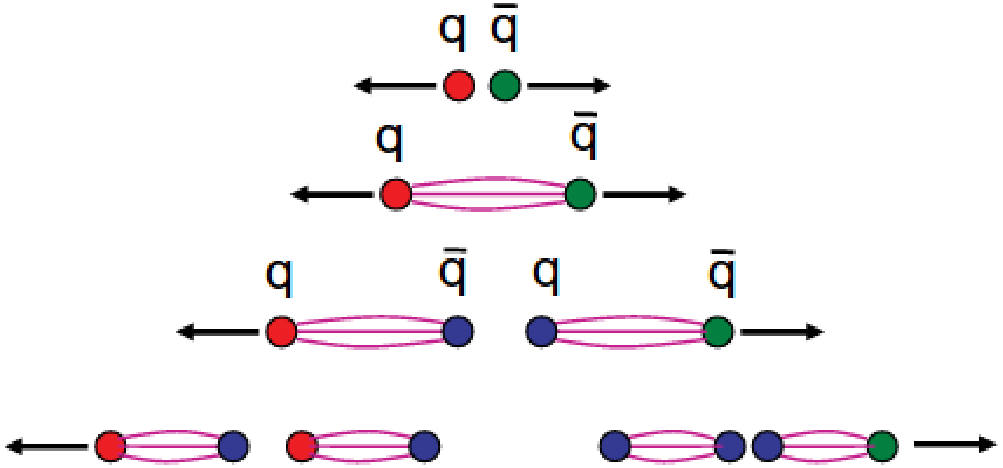
Hadronisation & Jets

★ **Consider a quark and anti-quark produced in electron positron annihilation**

i) **Initially Quarks separate at high velocity**

ii) **Colour flux tube forms between quarks**

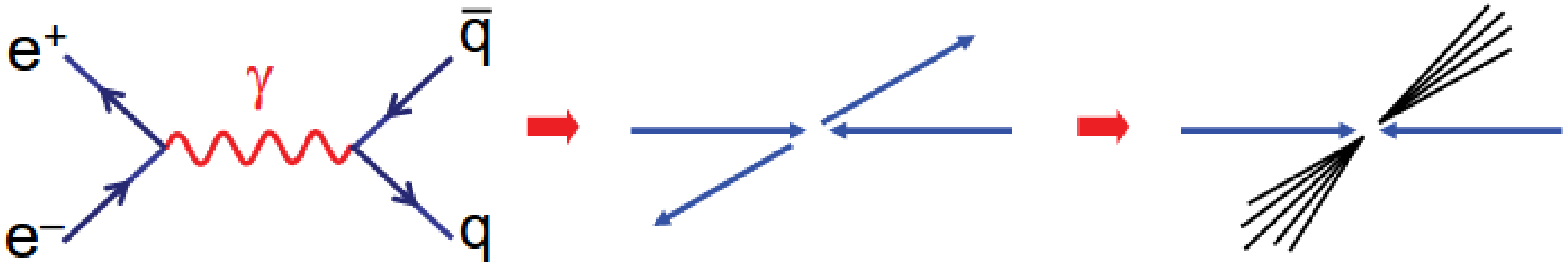
iii) **Energy stored in the flux tube sufficient to produce $q\bar{q}$ pairs**



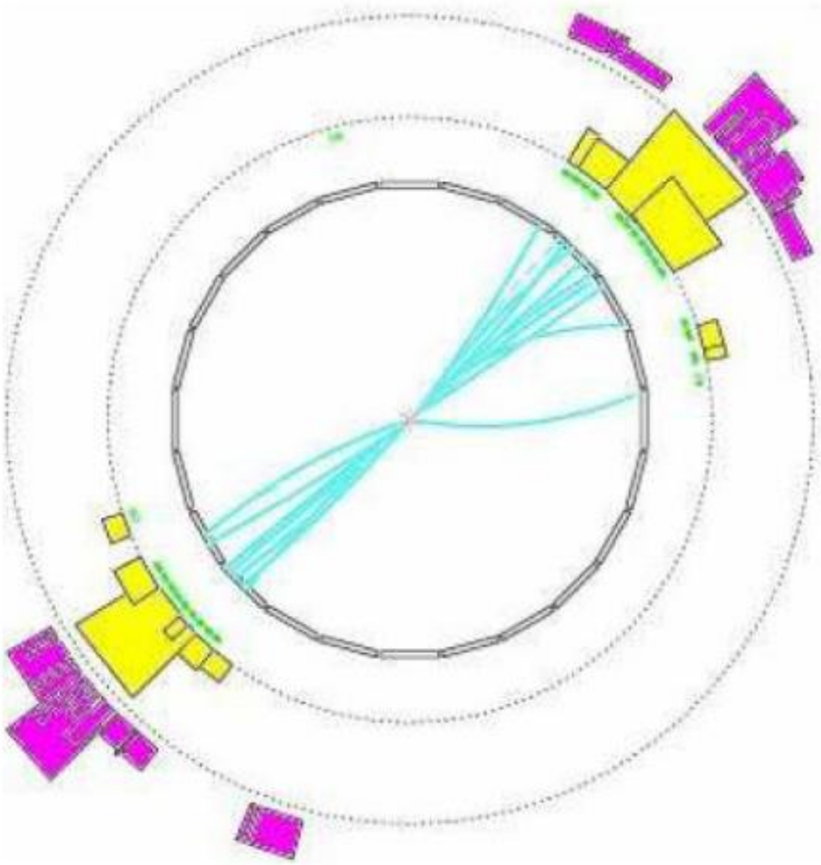
iv) **Process continues until quarks pair up into jets of colourless hadrons**

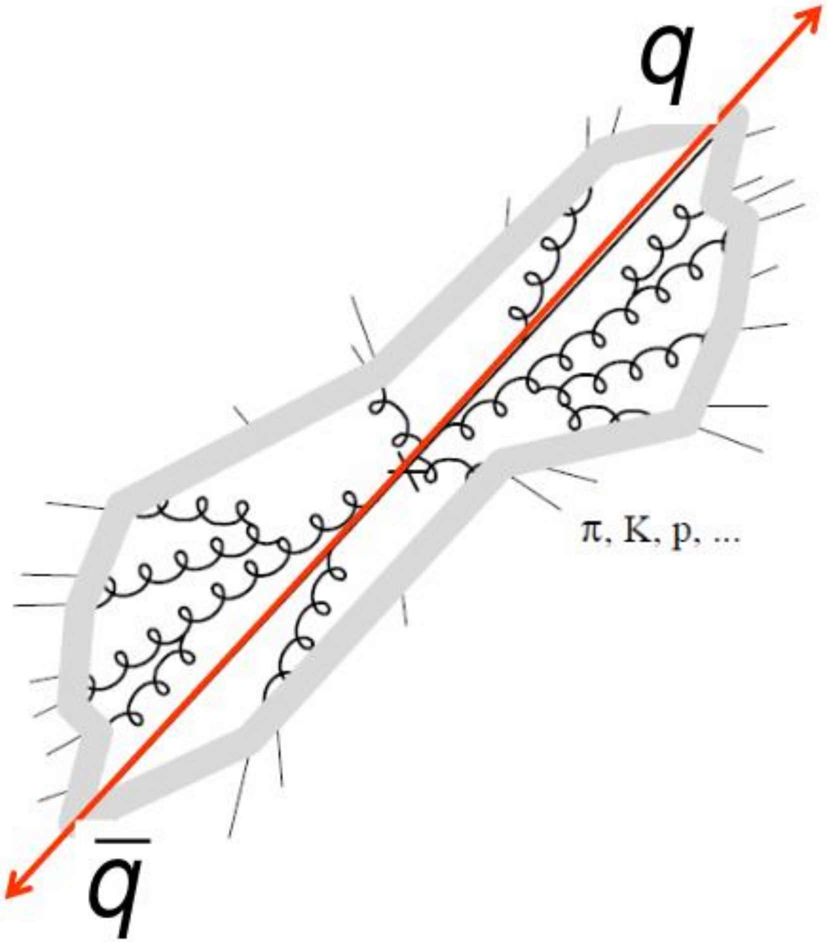
★ **This process is called hadronisation. It is not (yet) calculable.**

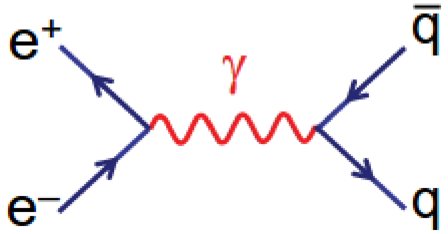
★ **The main consequence is that at collider experiments quarks and gluons observed as jets of particles**



$e^+ e^- \to q\,\bar{q}$ and hadron jets



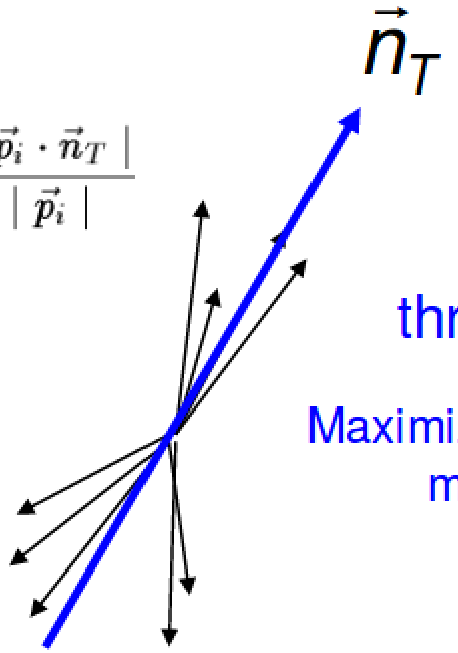




e+e- environment excellent place to study quark properties **(no knowledge of pdfs needed)**

2-Jet-likeness: Thrust

$$T = \max \frac{\sum_i |\vec{p}_i \cdot \vec{n}_T|}{\sum_i |\vec{p}_i|}$$



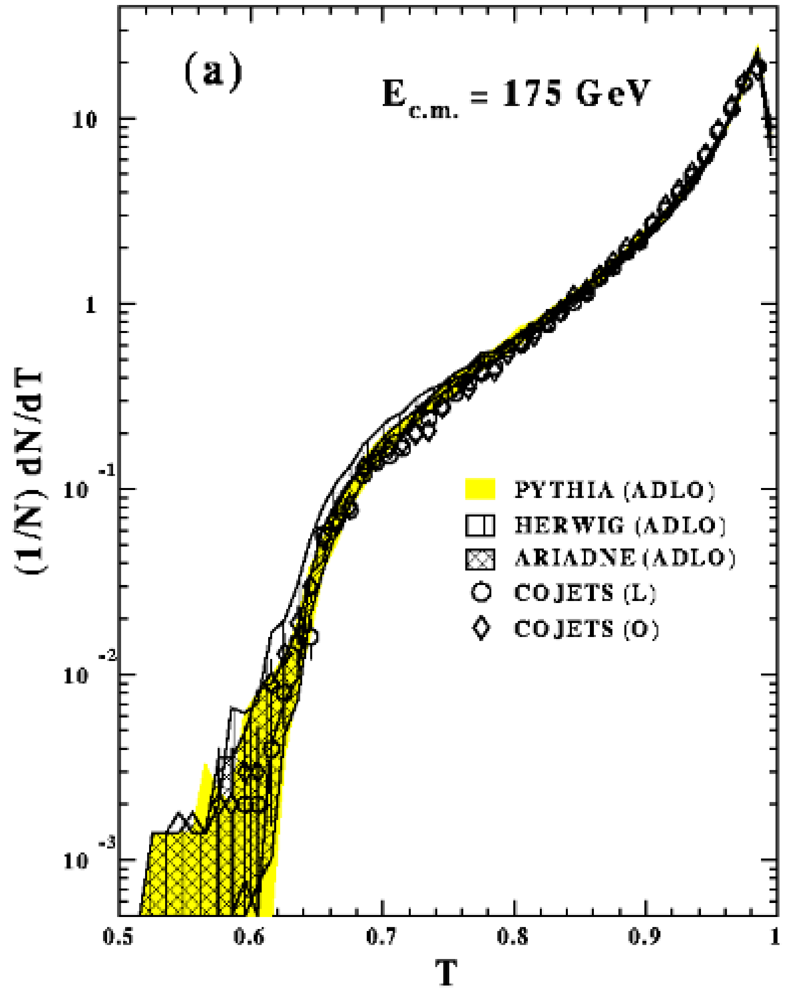
thrust axis

Maximizes longitudinal momentum

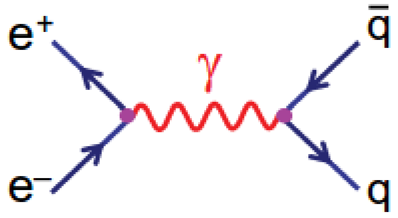
Example: →

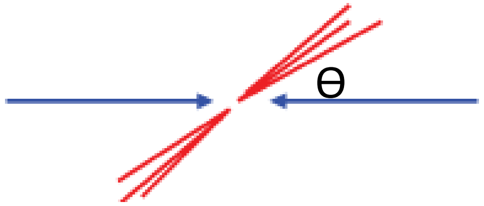
thrust distribution (LEP) in e^+e^- -events at $\sqrt{s}$ = 175 GeV: very much 2-jet like.

At early PETRA energies not that pronounced (boost effect).



Spin of quarks



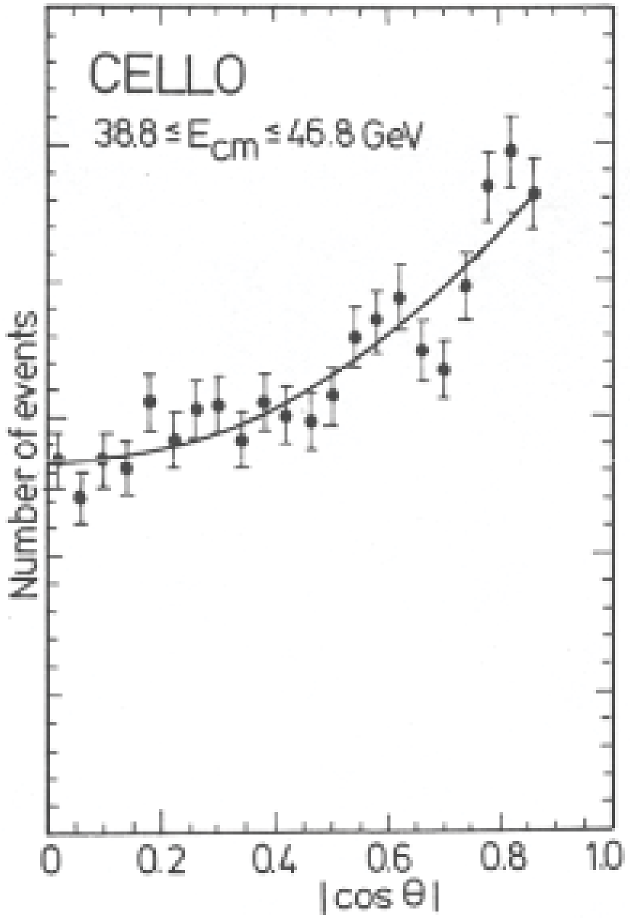


For e+e- →μ+μ- collisions derived the following form of cross-section:

$$\sigma = \frac{4\pi\alpha^2}{3s} \qquad \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left(1+\cos^2\theta\right)$$

Angular distribution fits well to describe 2-jet distribution.

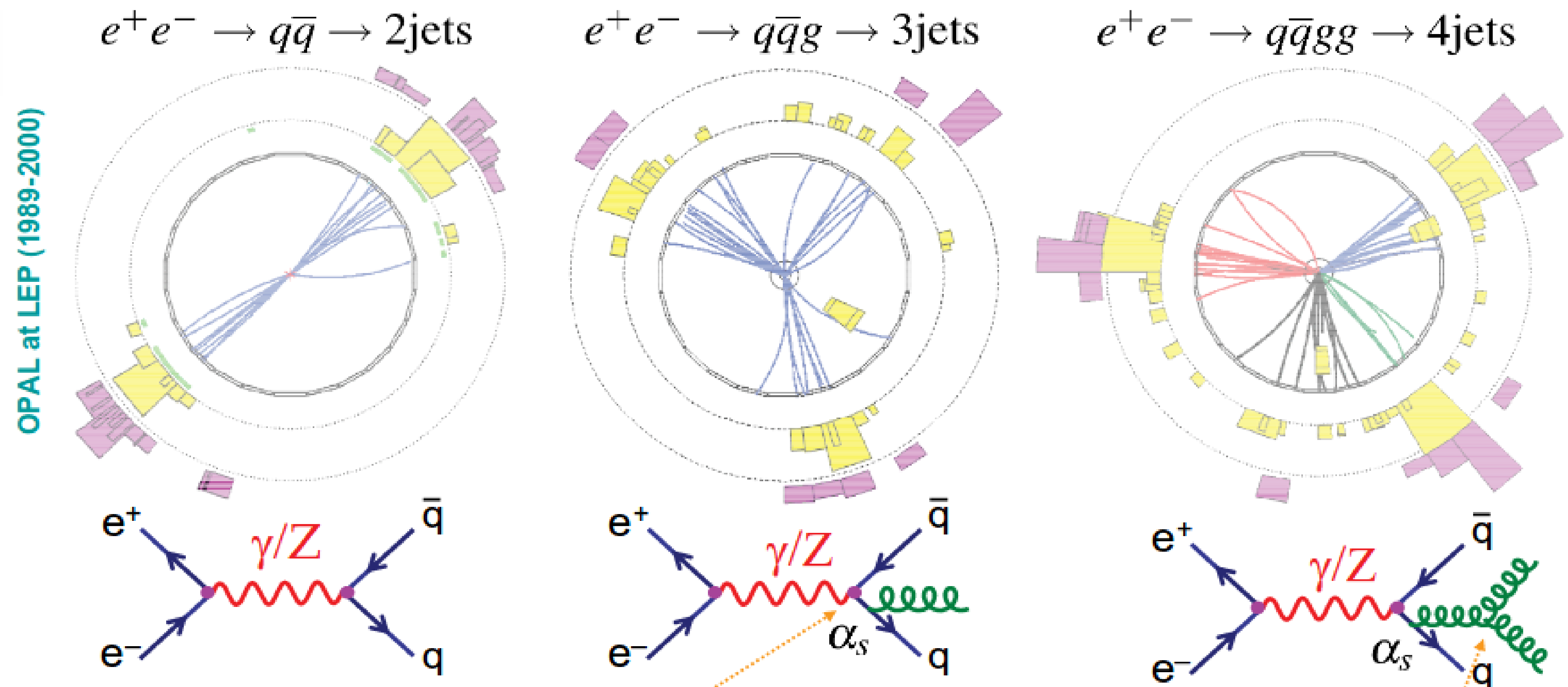
$\boxed{\text{Spin of quarks: S=1/2}}$



H.J.Behrend et al., Phys Lett 183B (1987) 400

Jet production in e+e-

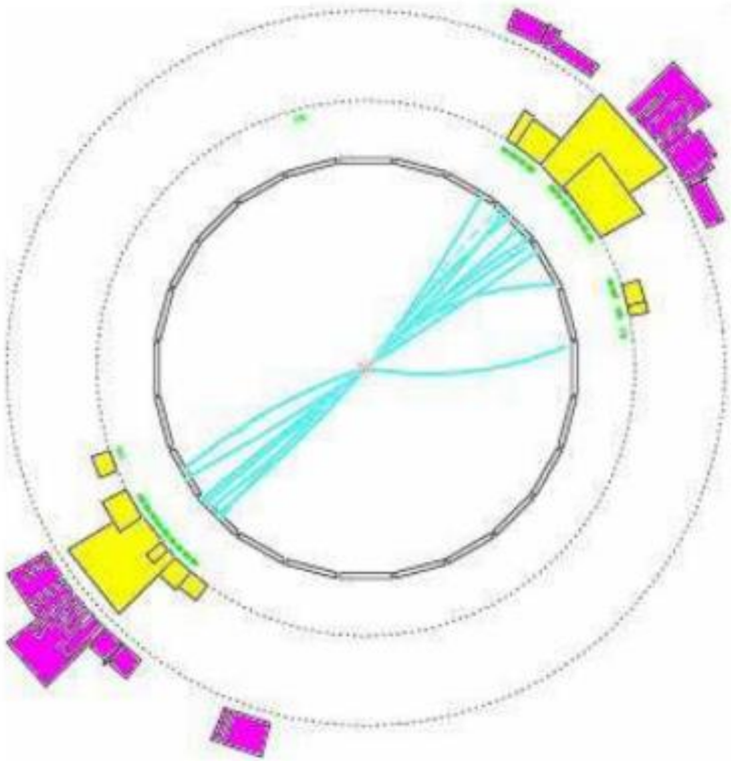
(better suited for QCD test than at hadron collider)

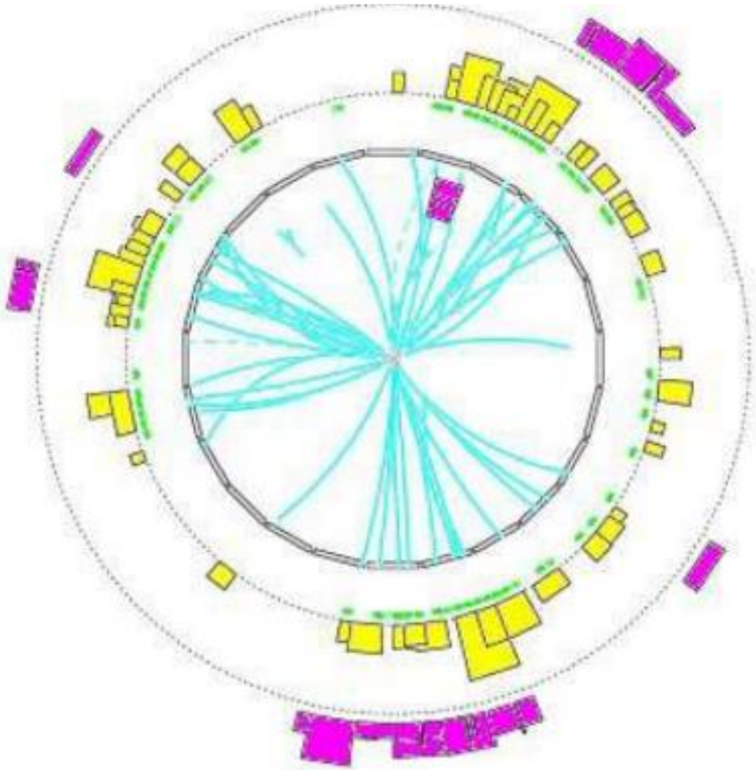


Experimentally:

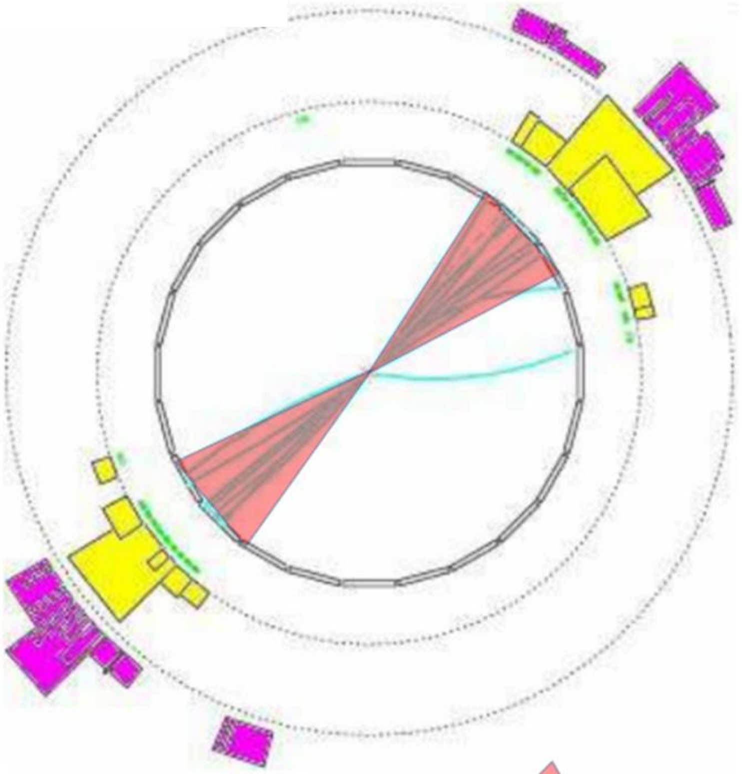
- Three jet rate ➡ measurement of α_s
- Angular distributions ➡ gluons are spin-1
- Four-jet rate and distributions ➡ QCD has an underlying SU(3) symmetry

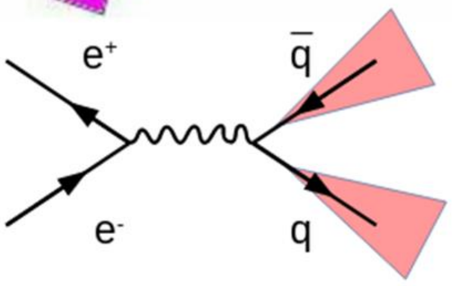
How many Jets?



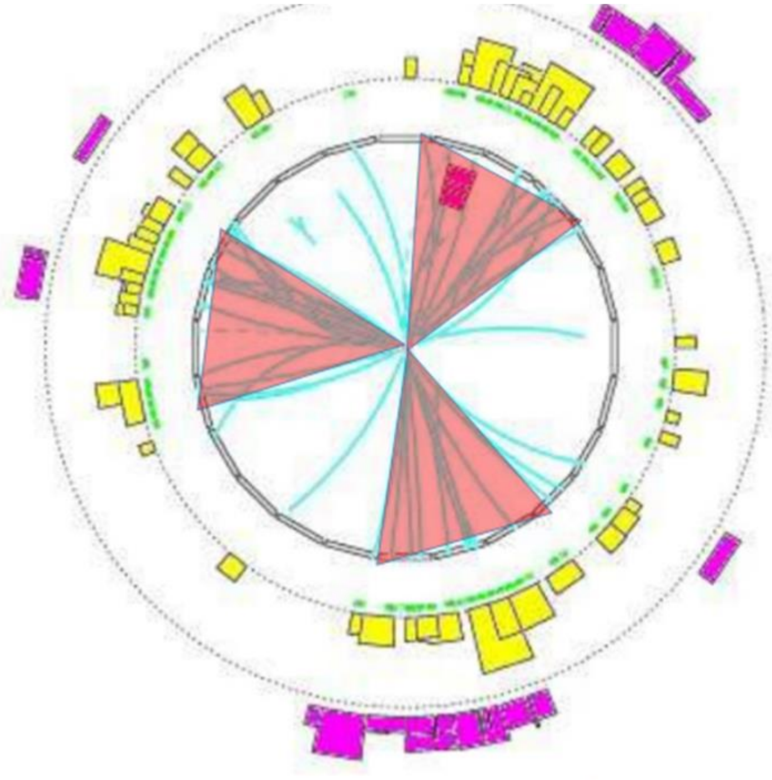


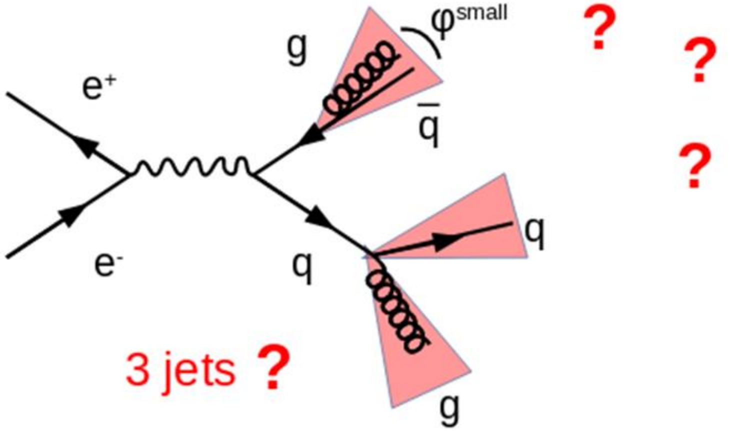
How many Jets?





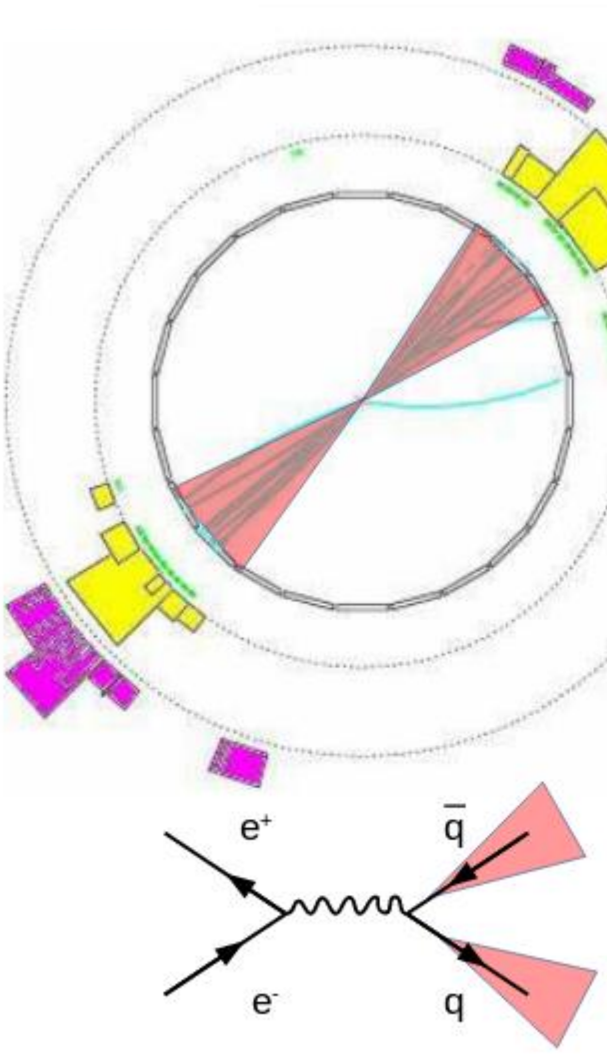
2 clear jets



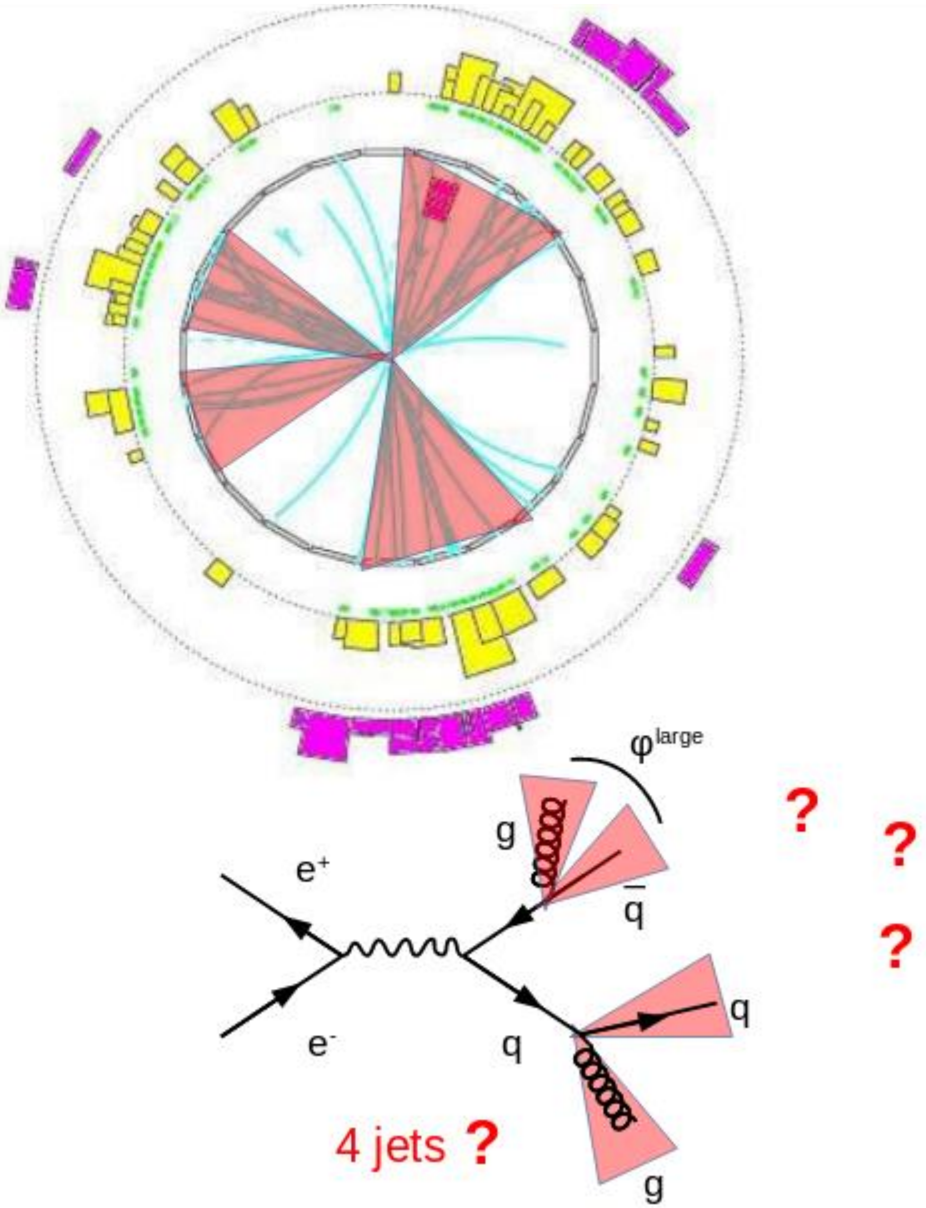


3 jets ?

How many Jets?

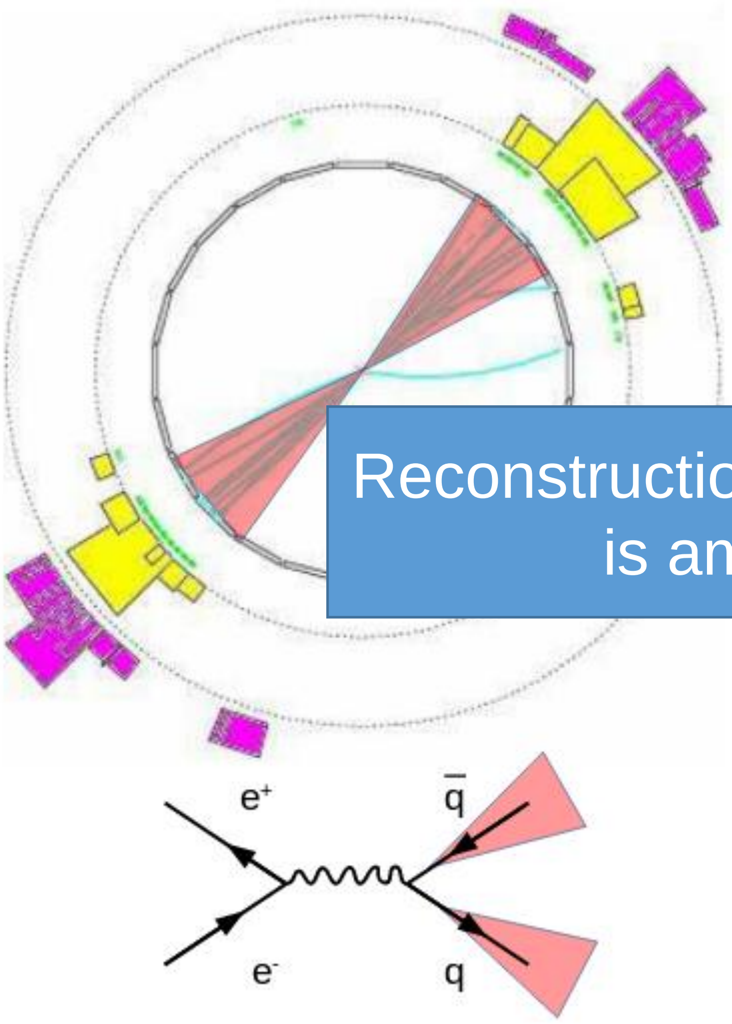


2 clear jets



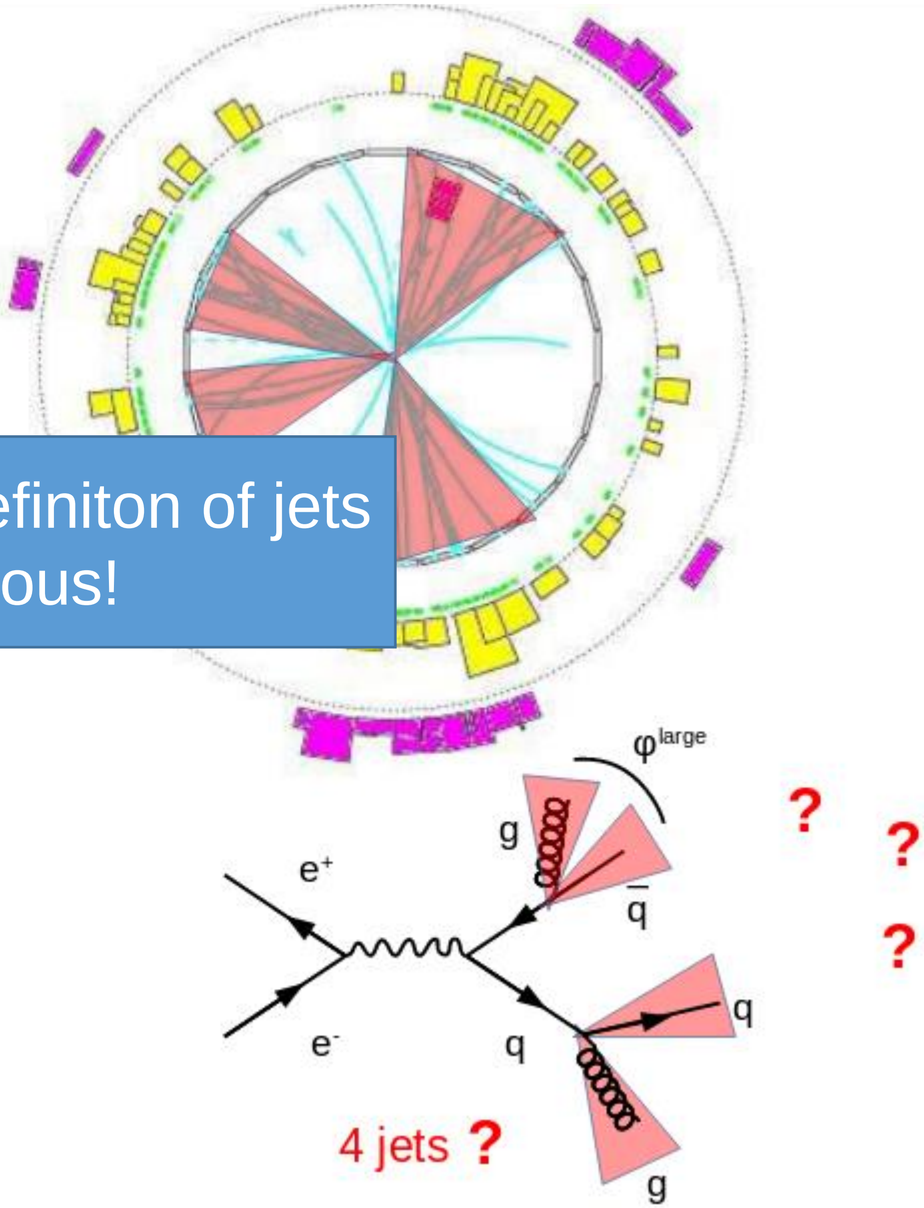
4 jets ?

How many Jets?



2 clear jets

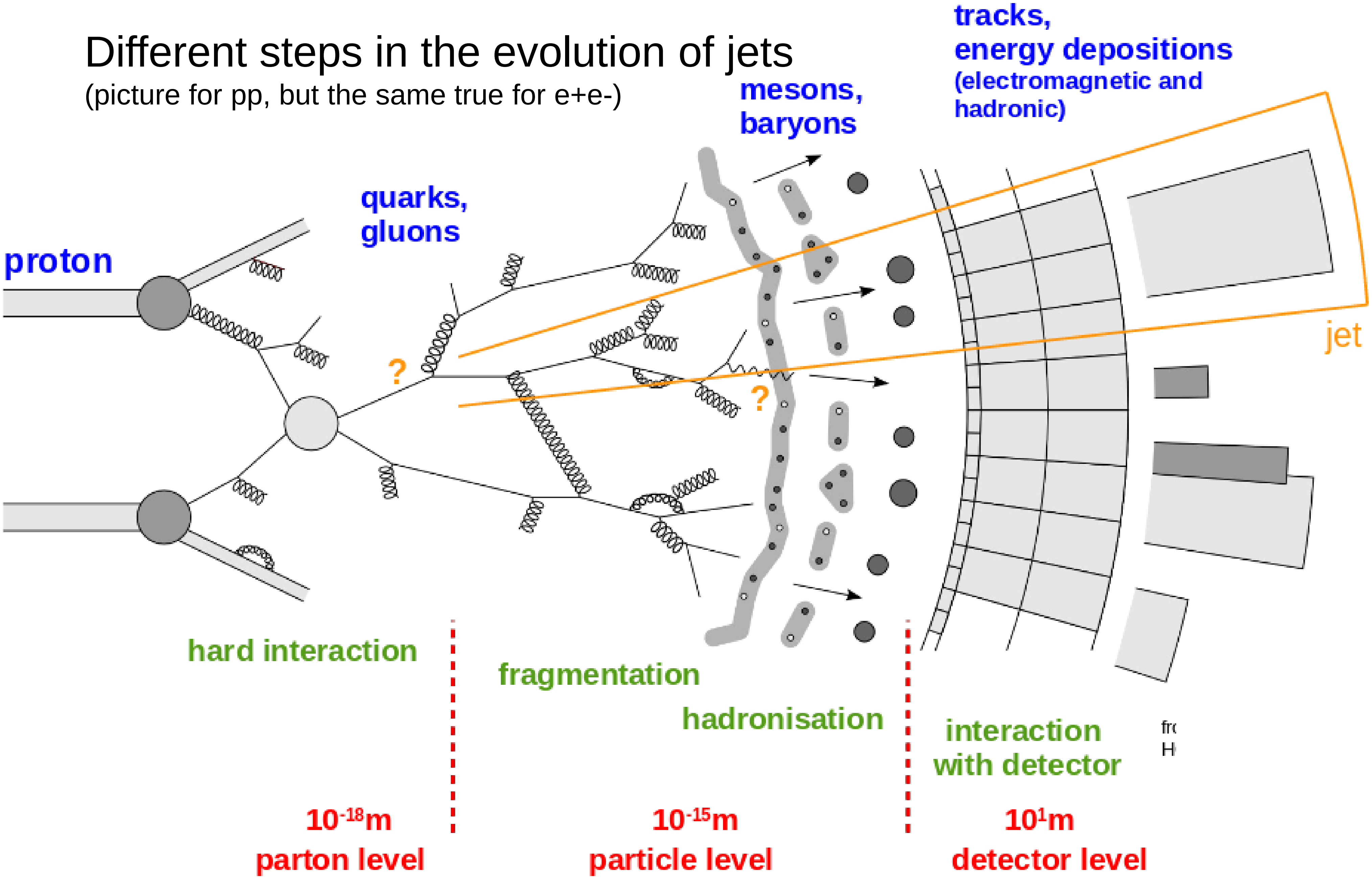
Reconstruction/definiton of jets is ambigious!



4 jets ?

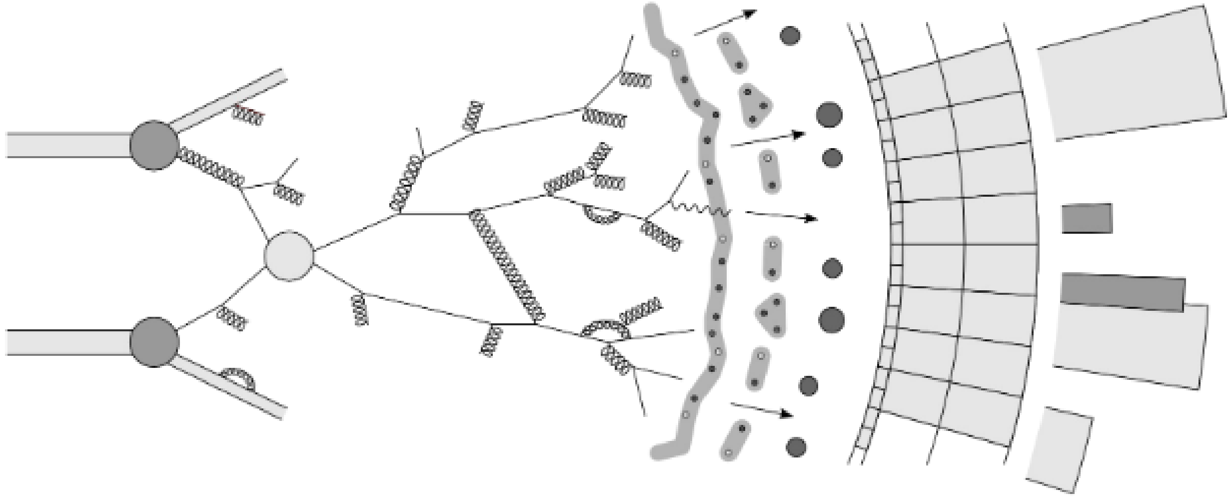
Different steps in the evolution of jets

(picture for pp, but the same true for e+e-)



Requirements on Jet Algorithm

Goal:
connection between detector measurement, final state particle and hard partons



- Standardization of Jet Definition: Snowmass accord (FERMILAB-Conf-90/249-E 1990):

See next slide

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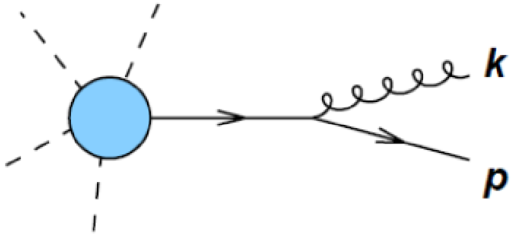
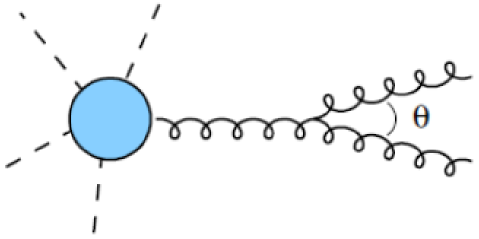
1. Simple to implement in an experimental analysis
2. Simple to implement in the theoretical calculation
3. Defined at any order of perturbation theory
4. Yields finite cross sections at any order of perturbation
   theory
5. Yields a cross section that is relatively insensitive to
   hadronization
```

Reminder: Gluon emmission

divergent

Gluon emission from quark: $\frac{2\alpha_s C_F}{\pi} \overbrace{\frac{dE}{E} \frac{d\theta}{\theta}}$ from gluon: $\frac{2\alpha_s C_A}{\pi} \overbrace{\frac{dE}{E} \frac{d\theta}{\theta}}$



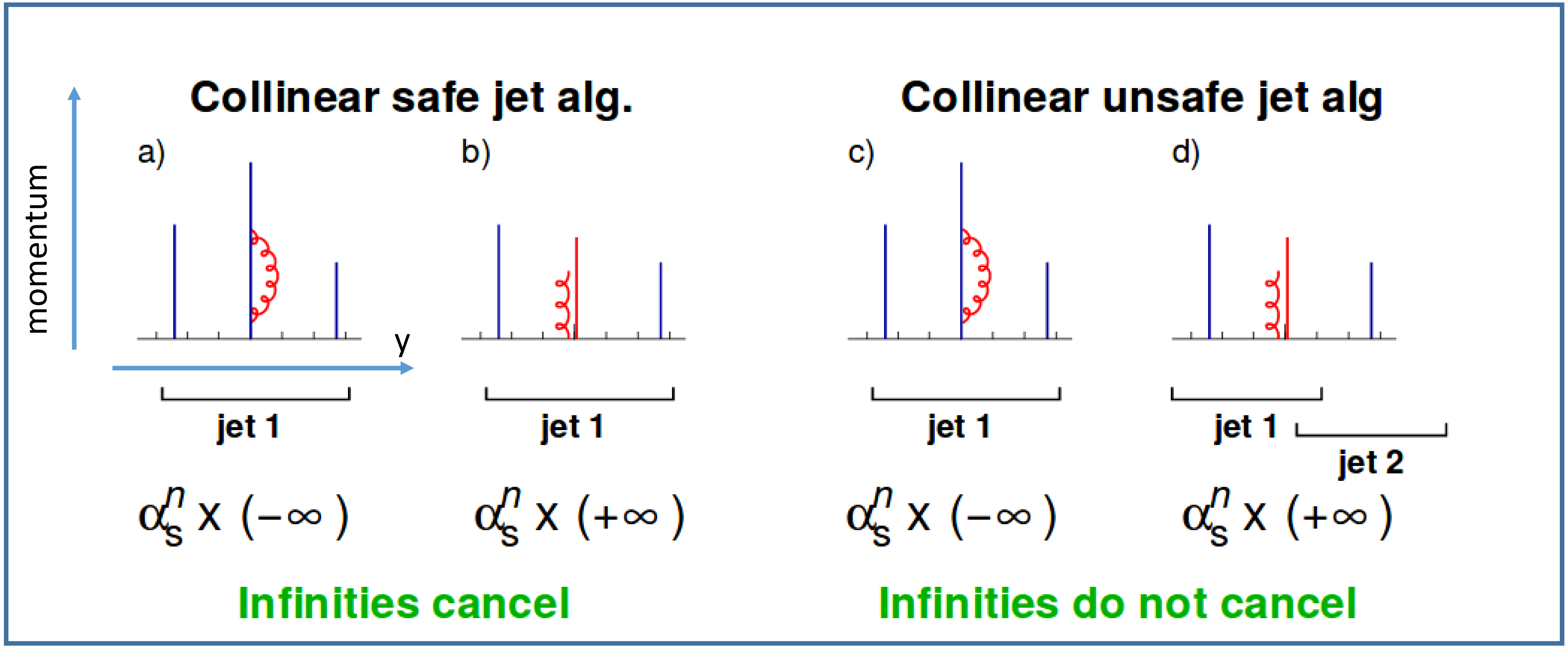


Both expressions valid only for $\theta \ll 1$.

$C_F = 4/3$ v. $C_A = 3$	Gluons have two color charges → probability to emit gluon is higher. Associated color factor C_A is larger

Infrared (1/E) and collinear (1/θ) divergent.

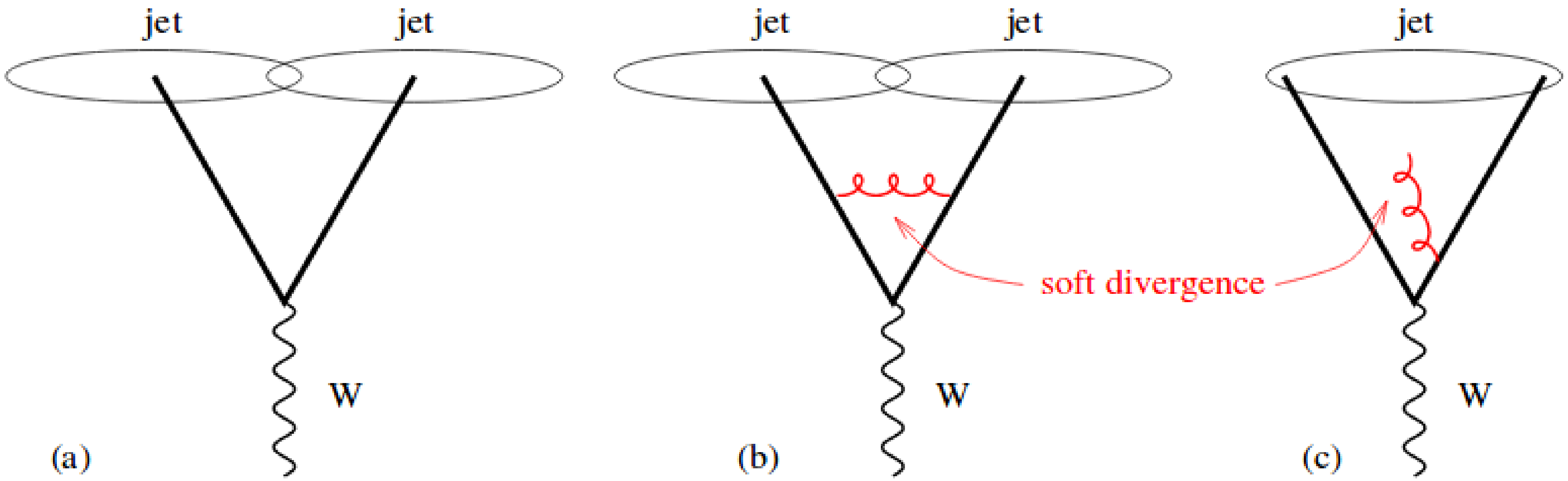
However higher order loop corrections cancel the tree level divergencies.



Need to make sure that the divergencies cancel for each class of number of events individually.

Otherwise comparison of experiment and theory not well defined.

Another example of infrared-unsafe algorithm:



Additional gluon could let the two event look like a one jet event.

Sequential recombination algorithm

Define "distance measure" between particles and do a successive recombination of closest pairs until resulting pseudo particle are too "far way". Remaining pseudo particles are jets. Every particle belongs to exactly one jet.

Durham Jet-Algorithm

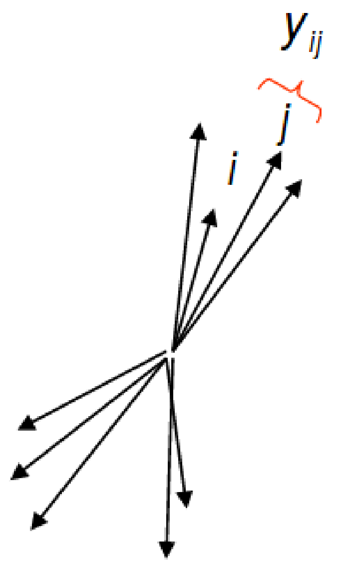
- Most frequent algorithm in e^+e^- exp.
- Particles (group of particles) i and j are not resolved and grouped to a single pseudo particle k if the **resolution parameter y_{ij}**

$$y_{ij} = \frac{2 \cdot \min(E_i^2, E_j^2) \cdot (1 - \cos\theta_{ij})}{s}$$

is smaller than the resolution $y_{cut.}$ The new pseudo particle k is obtained by:

$$E_k = E_i + E_j \quad \vec{p}_k = \vec{p}_i + \vec{p}_j$$

Recursive iteration until all $y_{ij} \geq y_{cut}$.
Remaining pseudo-particle are the final jets.

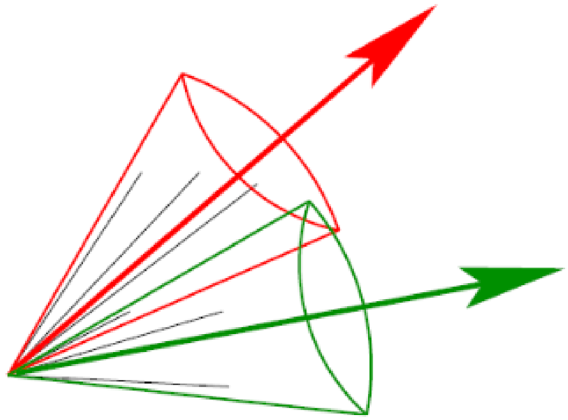


Beside the Durham algorithm there are other algorithms with slightly different definition of y_{ij} and "joining scheme": JADE, Aachen, Cambridge

Cone Algorithm

In cone algorithms jets are defined as the dominant direction of energy flow. Introduce concept of stable cone as a circle of fixed radius R in the plane such that the sum of all the momenta of the particles within the cone points in the direction of the cone-center. Cone algorithms attempt to identify all the stable cones.

Most implementations use a seeded approach to do so: starting from one seed for the centre of the cone, one iterates until the cone is found stable.

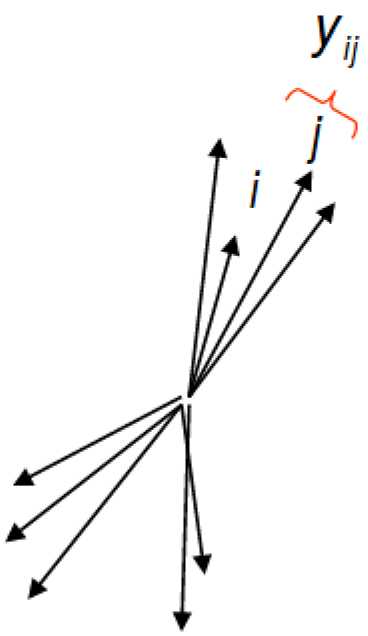


How to find the stable cones?

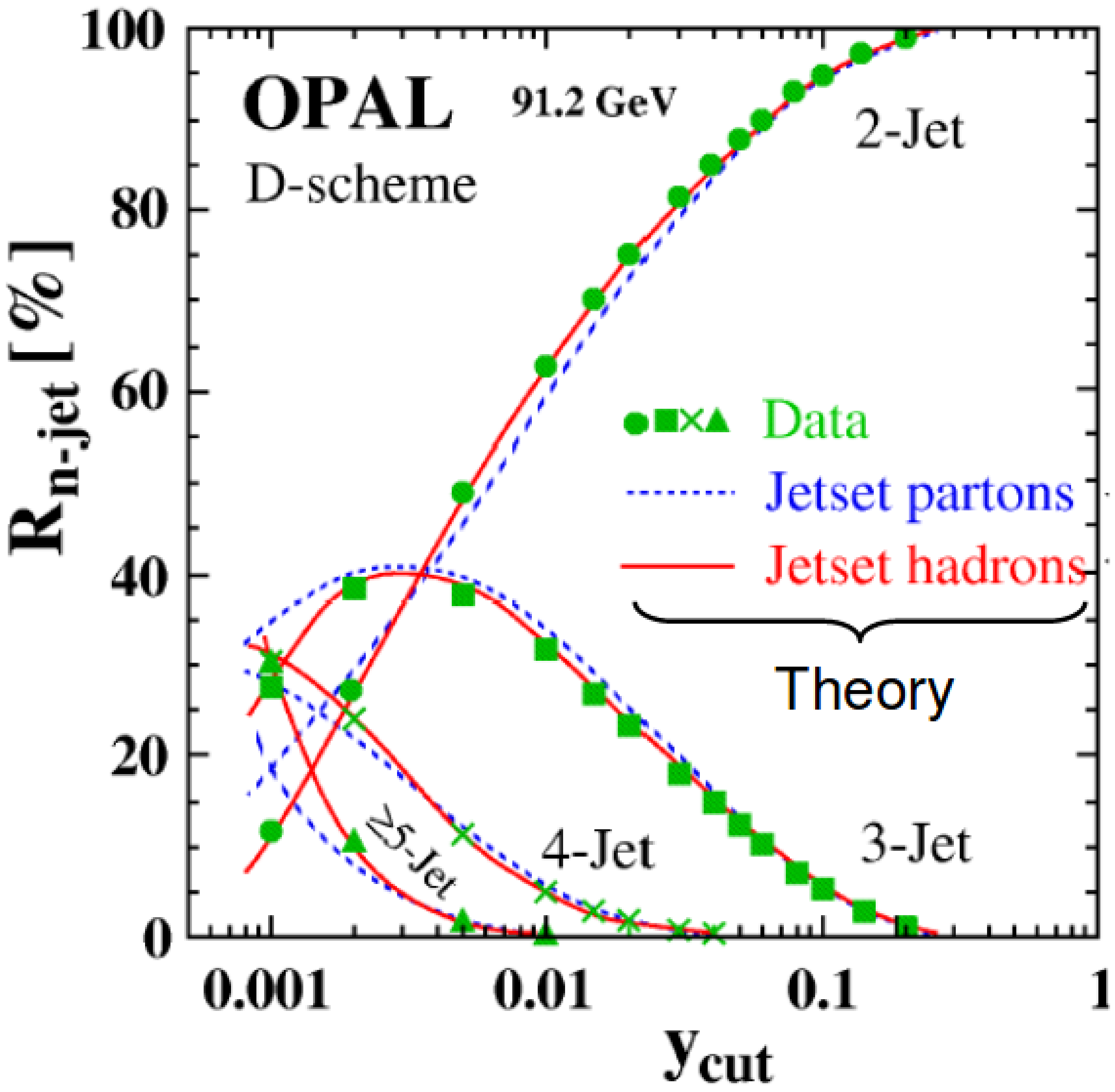
How to avoid that particles belong to two or no cone?

Different implementations of cone algorithms are used at hadron colliders: They differ in the start-point definition, how they find stable cones and how they deal with overlapping jets (split/merge).

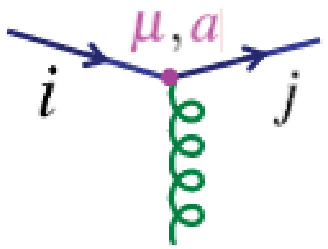
Measurement of Multi-Jet events ≡ Test of SU(3) of QCD



$y_{ij} \geq y_{cut}$



Gauge Group Structure and Color Factor

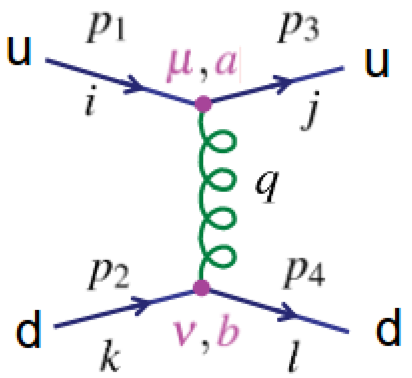


vertex factor ~ $\boxed{-ig_s \frac{1}{2} \lambda^a_{ji} \gamma^\mu}$

λ^a a=1,…,8 Gell-Mann matrices

($T^a = \frac{\lambda^a}{2}$ is one representation of generators of SU (3))

i,j are colors of the quark

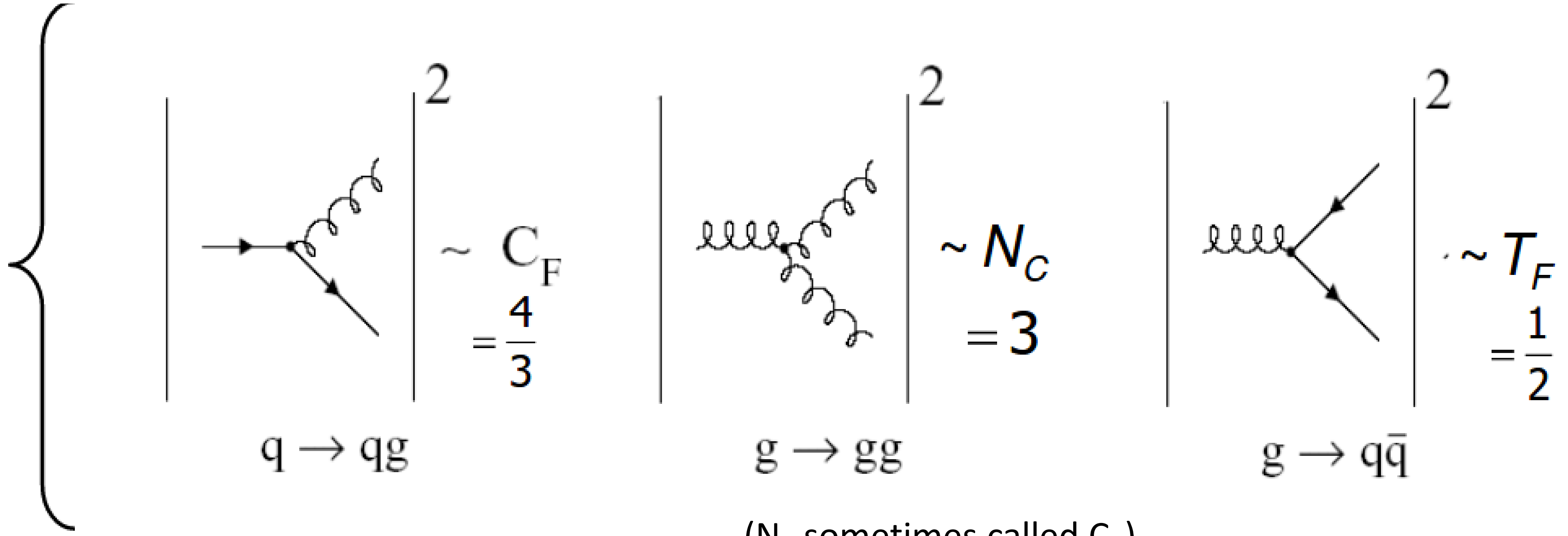


$$\boxed{M = -\frac{g_s^2}{4} \lambda^a_{ji} \lambda^a_{lk} \frac{1}{q^2} g_{\mu\nu} [\overline{u}_u(p_3) \gamma^\mu u_u(p_1)][\overline{u}_d(p_4) \gamma^\nu u_d(p_2)]}$$

Summing over all possible gluons result in color factor: $\boxed{C(ik \to jl) \equiv \frac{1}{4} \sum_{a=1}^{8} \lambda^a_{ji} \lambda^a_{lk}}$

Color Factors relvant for 4-Jet events

Different relative angular distribution



(N_C sometimes called C_A)

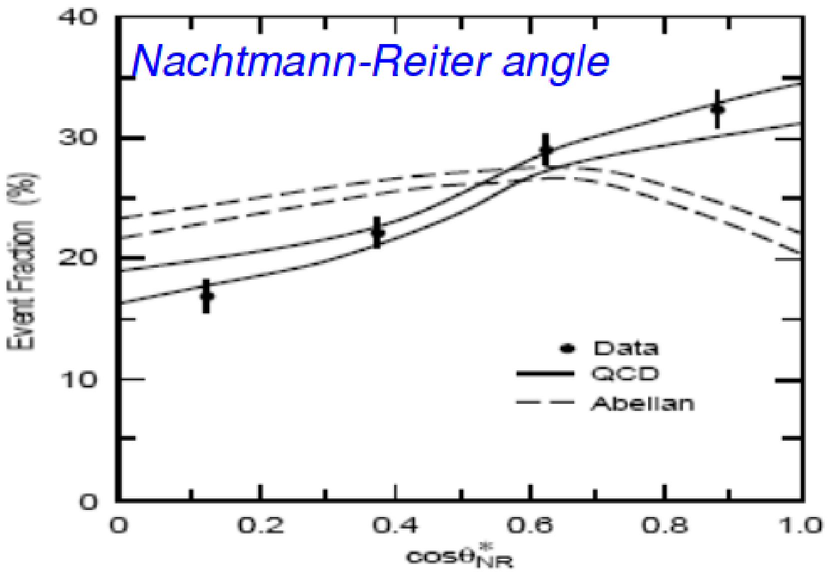
C_F, C_A describe the effective color charge of quarks and gluons.

Angular correlation of jets in 4-jet events

4-jet cross section:

$$\frac{1}{\sigma_0}d\sigma^4 = \left(\frac{\alpha_s C_F}{\pi}\right)^2\left[F_A + \left(1-\frac{1}{2}\frac{N_C}{C_F}\right)F_B + \frac{N_C}{C_F}F_C\right]$$
$$+ \left(\frac{\alpha_s C_F}{\pi}\right)^2\left[\frac{T_F}{C_F}N_f F_D + \left(1-\frac{1}{2}\frac{N_C}{C_F}\right)F_E\right]$$

$F_{A,B,C,D,E}$ are kinematic functions



Exploiting the angular distribution of 4-jets:

- Bengston-Zerwas angle

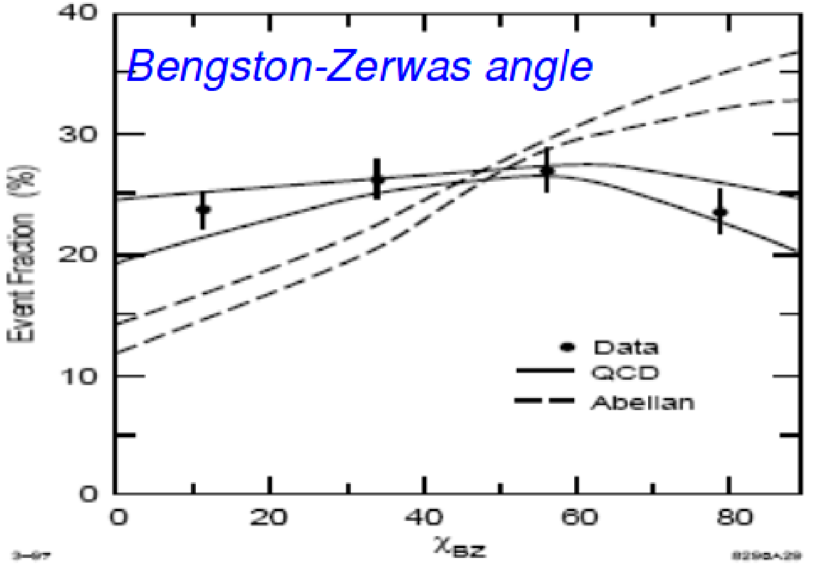
$$\cos\chi_{BZ} \propto (\vec{p}_1\times\vec{p}_2)\cdot(\vec{p}_3\times\vec{p}_4)$$

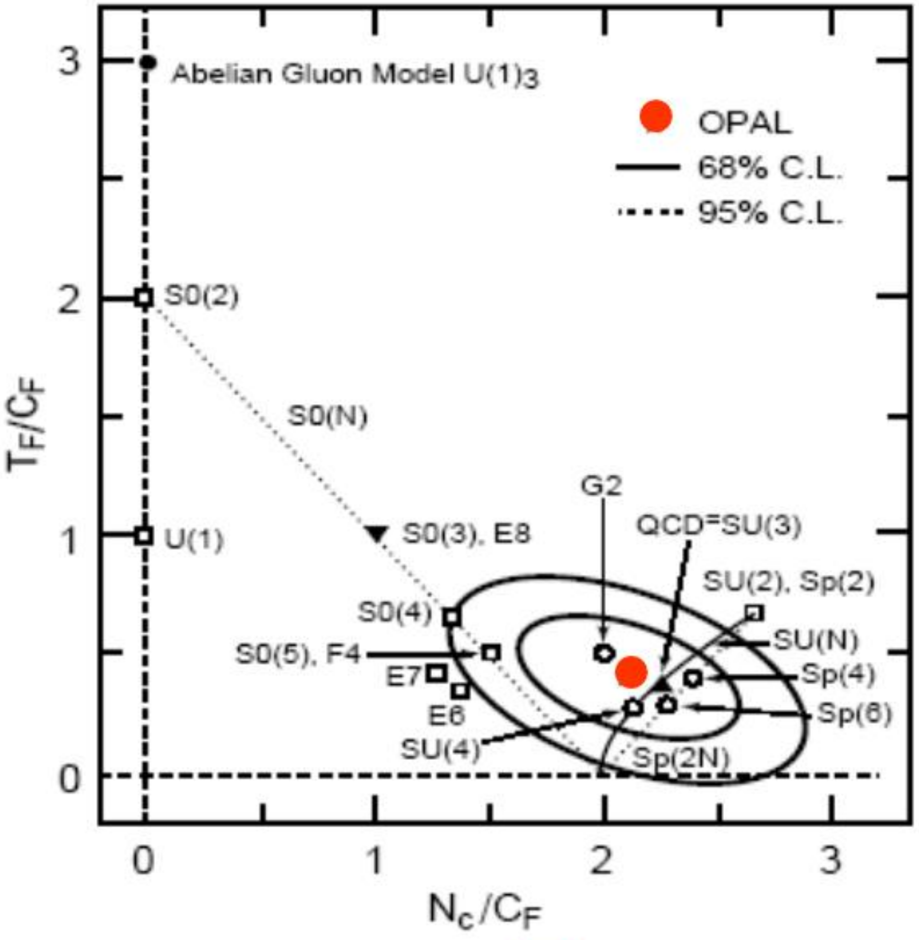
- Nachtmann-Reiter angle

$$\cos\theta_{NR} \propto (\vec{p}_1-\vec{p}_2)\cdot(\vec{p}_3-\vec{p}_4)$$

Allows to measure the ratios T_F/C_F and N_C/C_F

SU(3) predicts: $T_F/C_F = 0.375$ and $N_C/C_F = 2.25$



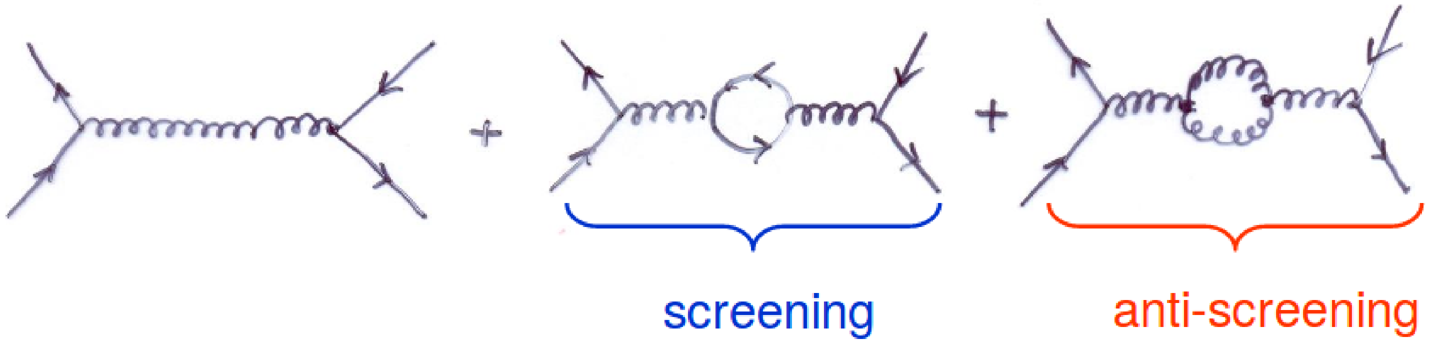


Confirms QCD prediction (SU(3)) and gluon self-coupling:

T_F/C_F = 0.375 and N_C/C_F =2.25

2. Running of the strong coupling constant

Propagator corrections:



Strong coupling $\alpha_s(Q^2)$

$$\boxed{\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\underbrace{\frac{1}{12\pi}(33 - 2n_f)}_{\beta_0 = \frac{1}{12\pi}(33-2n_f)}\log\frac{Q^2}{\mu^2}}}$$

n_f = active quark flavors

μ^2 = renormalization scale

conventionally $\mu^2 = M_Z^2$

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \log(Q^2/\Lambda^2_{QCD})}$$

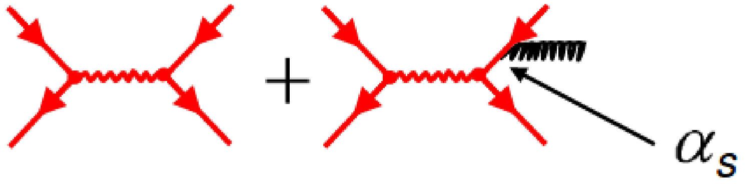
scale at which perturbative description of QCD breaks

Measurement of Q^2 dependence of α_s

➡ α_s measurements are done at given scale Q^2: $\alpha_s(Q^2)$

a) α_s from total hadronic cross section

$e^+e^- \rightarrow q\bar{q}(g)$



Final state gluon radiation.

$$\sigma_{had}(s) = \sigma_{had}^{QED}(s)\left[1 + \frac{\alpha_s(s)}{\pi} + 1.411 \cdot \frac{\alpha_s(s)^2}{\pi^2} + \ldots\right]$$

$$R_{had} = \frac{\sigma(ee \rightarrow hadrons)}{\sigma(ee \rightarrow \mu\mu)} = 3\sum Q_q^2\left\{1 + \frac{\alpha_s}{\pi} + 1.411\frac{\alpha_s^2}{\pi^2} + \ldots\right\}$$

$\boxed{\Rightarrow \alpha_s(s)}$

b) α_s from hadronic event shape variables

3-jet rate: $R_3 \equiv \dfrac{\sigma_{3-jet}}{\sigma_{had}}$ depends on α_s

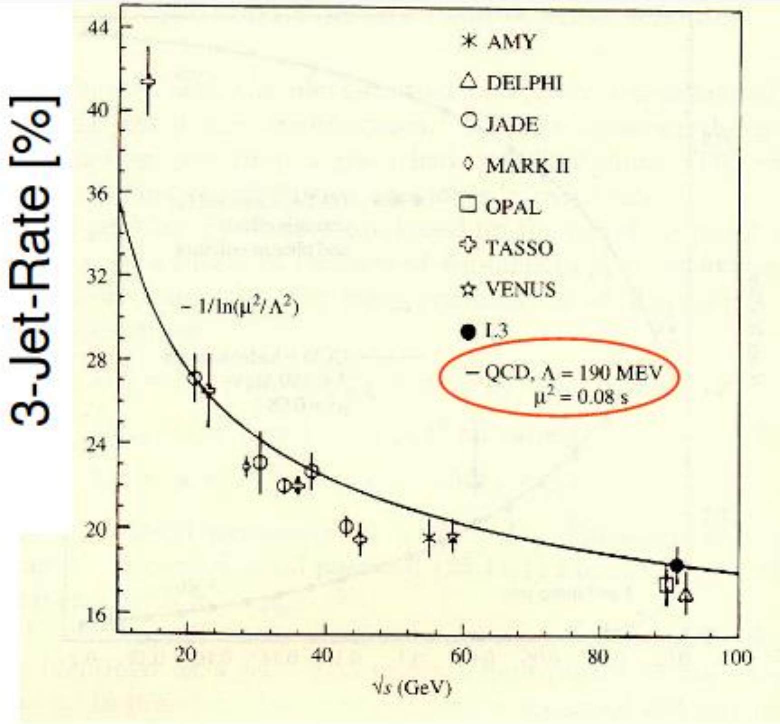
3-jet rate is measured as function of a jet resolution parameter y_{cut}

QCD calculation provides a theoretical prediction for $R_3^{theo}(\alpha_s , y_{cut})$

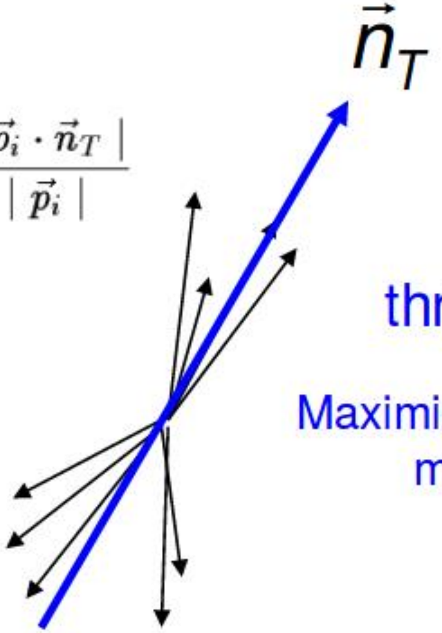
$\rightarrow$ fit $R_3^{theo}(\alpha_s , y_{cut})$ to the data to determine α_s

Similarly other event shape variables (sphericity, thrust,…) can be used to obtain a prediction for α_s

$\Rightarrow \alpha_s(s)$

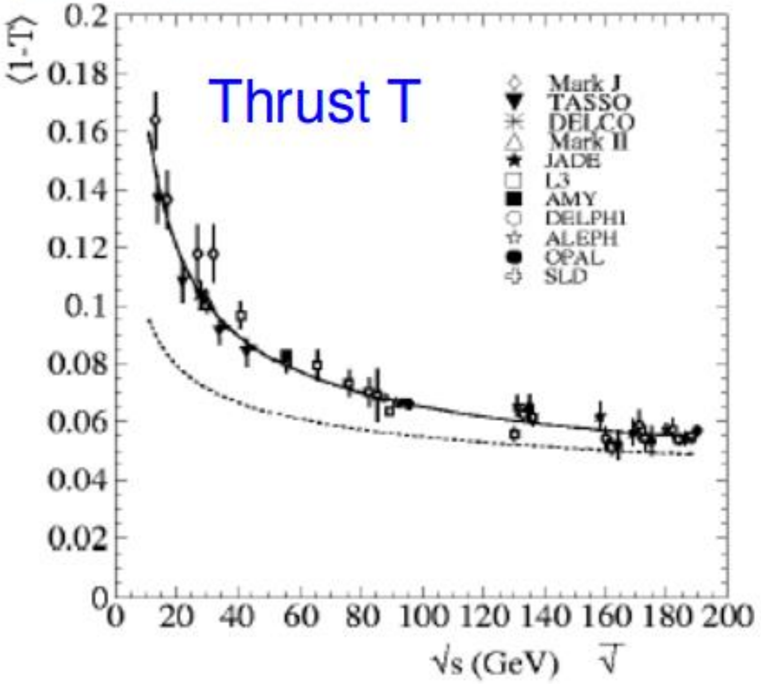


$$T = \max \frac{\sum_i |\vec{p}_i \cdot \vec{n}_T|}{\sum_i |\vec{p}_i|}$$



thrust axis

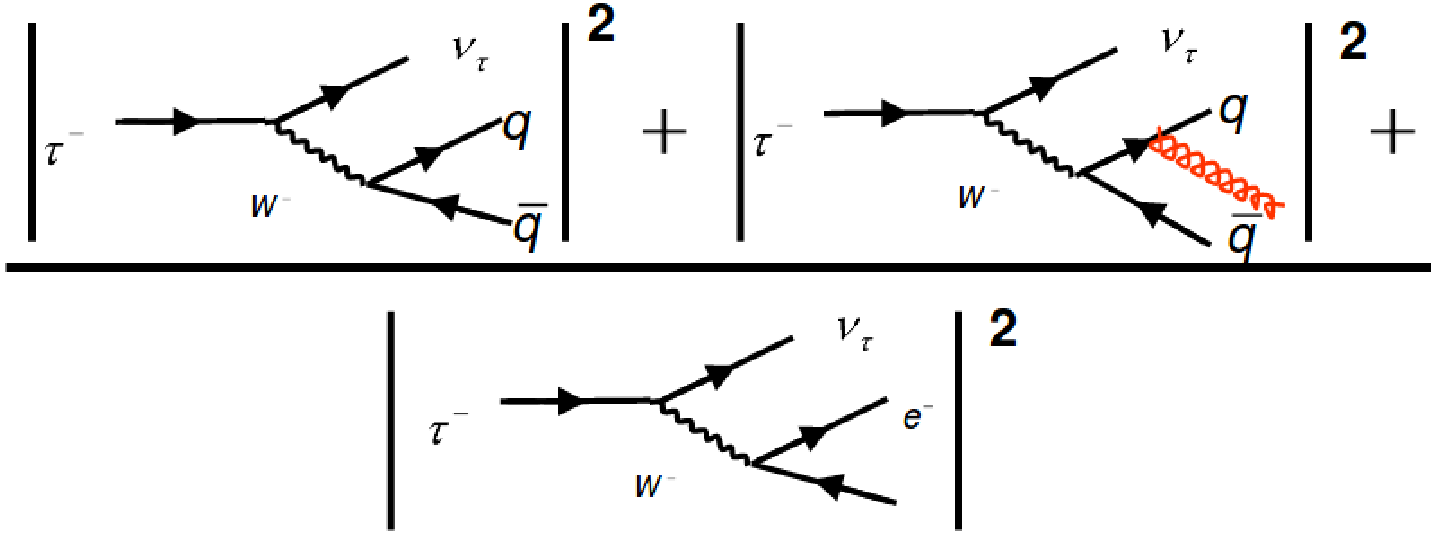
Maximizes longitudinal momentum



c) α_s from hadronic τ decays

$$R^{\tau}_{had} = \frac{\Gamma(\tau \to \nu_\tau + Hadrons)}{\Gamma(\tau \to \nu_\tau + e\bar{\upsilon}_e)} \sim f(\alpha_s)$$

$R^{\tau}_{had} =$



$$R^{\tau}_{had} = R^{\tau,0}_{had}\left(1 + \frac{\alpha_s(m_\tau^2)}{\pi} + \ldots\right)$$

d) α_s from DIS (deep inelastic scattering)

Running of α_s and asymptotic freedom

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \log(Q^2/\Lambda^2_{QCD})}$$