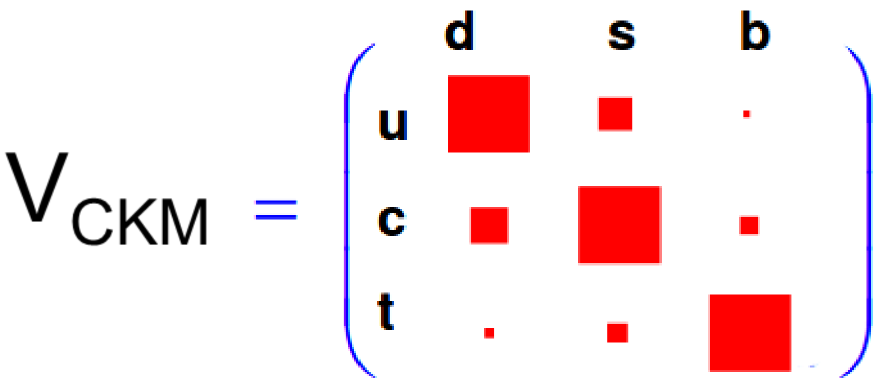
Wolfenstein Parametrisation:



λ, A, ρ, η with $\lambda = 0.22$

$|V_{ub}| \times e^{-i\gamma}$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

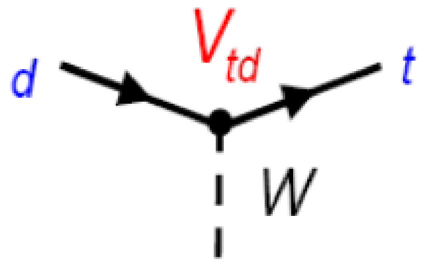
$|V_{td}| \times e^{-i\beta}$

Reflects the “hierarchical structure” of the CKM matrix.

CKM under CP Transformation

Quarks

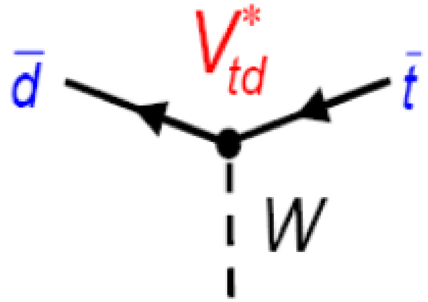
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



CP

Anti-quarks:

$$\begin{pmatrix} \bar{d}' \\ \bar{s}' \\ \bar{b}' \end{pmatrix} = \begin{pmatrix} V^*_{ud} & V^*_{us} & V^*_{ub} \\ V^*_{cd} & V^*_{cs} & V^*_{cb} \\ V^*_{td} & V^*_{ts} & V^*_{tb} \end{pmatrix} \begin{pmatrix} \bar{d} \\ \bar{s} \\ \bar{b} \end{pmatrix}$$

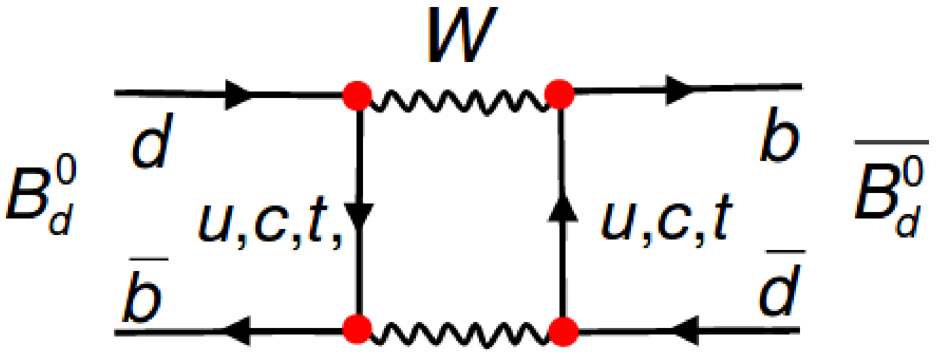


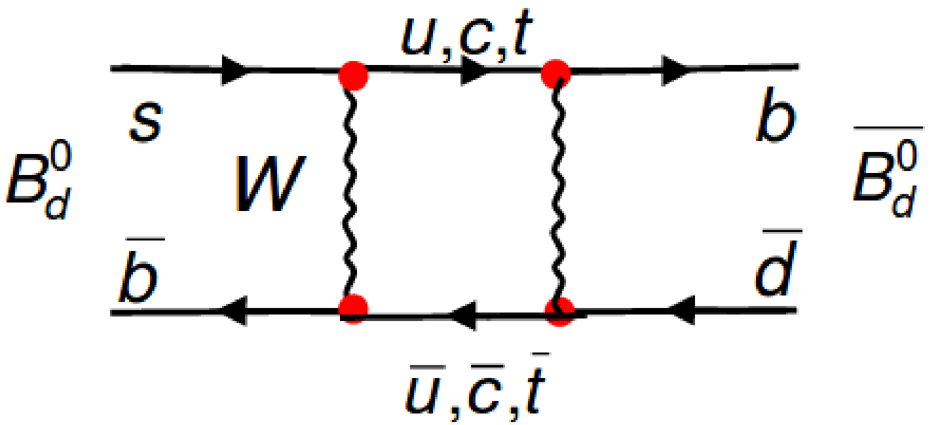
Weak (CKM) phases change sign under CP transformation!

The quark mixing results into several interesting “loop” effects:
Standard Model predicts at loop-level: Flavor Changing Neutral Currents

(forbidden at tree-level)

Mixing of neutral mesons, e.g.: $B_d^0 \quad \Leftrightarrow \quad \overline{B_d^0}$





Neutral mesons:

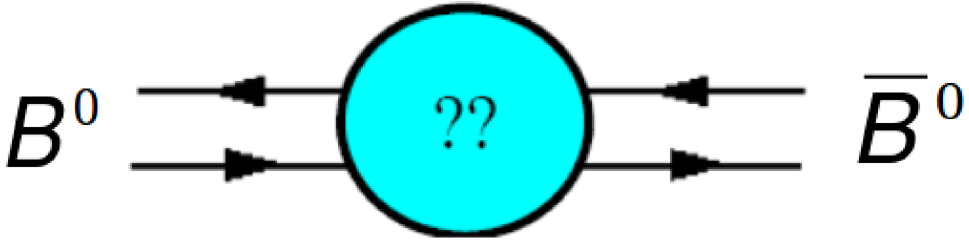
$$\left|P^0\right\rangle: \quad K^0 = \left|d\bar{s}\right\rangle \quad D^0 = \left|\bar{u}c\right\rangle \quad B_d^0 = \left|d\bar{b}\right\rangle \quad B_s^0 = \left|s\bar{b}\right\rangle$$

$$\left|\overline{P^0}\right\rangle: \quad \overline{K^0} = \left|\bar{d}s\right\rangle \quad \overline{D^0} = \left|\bar{u}c\right\rangle \quad \overline{B_d^0} = \left|d\bar{b}\right\rangle \quad \overline{B_s^0} = \left|s\bar{b}\right\rangle$$

discovery of mixing 1960 2007 1987 2006

Phenomenology of Mixing

Applies to all neutral mesons!



$$i\frac{d}{dt}\begin{pmatrix} B^0(t) \\ \overline{B}^0(t) \end{pmatrix} = \underbrace{\left(\mathbf{M} - \frac{i}{2}\boldsymbol{\Gamma}\right)}_{\mathbf{H}}\begin{pmatrix} B^0(t) \\ \overline{B}^0(t) \end{pmatrix}$$

Flavor states
= No mass
eigenstates

Diagonalizing H:

Mass eigenstates: $|B_L\rangle = p|B^0\rangle + q|\overline{B^0}\rangle$ with m_L, Γ_L light

$|B_H\rangle = p|B^0\rangle - q|\overline{B^0}\rangle$ with m_H, Γ_H heavy

$$\boxed{\text{complex coefficients} \quad |p|^2 + |q|^2 = 1}$$

$$|B_{H,L}(t)\rangle = |B_{H,L}(0)\rangle \cdot e^{-im_{H,L}t} \cdot e^{-\frac{1}{2}\Gamma_{H,L}t}$$

Flavor eigenstates: $|B^0\rangle = \frac{1}{2p}\left(|B_L\rangle + |B_H\rangle\right)$ $\quad |\overline{B}^0\rangle = \frac{1}{2q}\left(|B_L\rangle - |B_H\rangle\right)$

Phenomenology of Mixing

$$\underbrace{P(B^0 \to B^0) = P(\overline{B^0} \to \overline{B^0})}_{\text{CPT}} = \frac{1}{4}\left[e^{-\Gamma_L t} + e^{-\Gamma_H t} + 2e^{-(\Gamma_L + \Gamma_H)t/2} \cos \Delta m t\right]$$

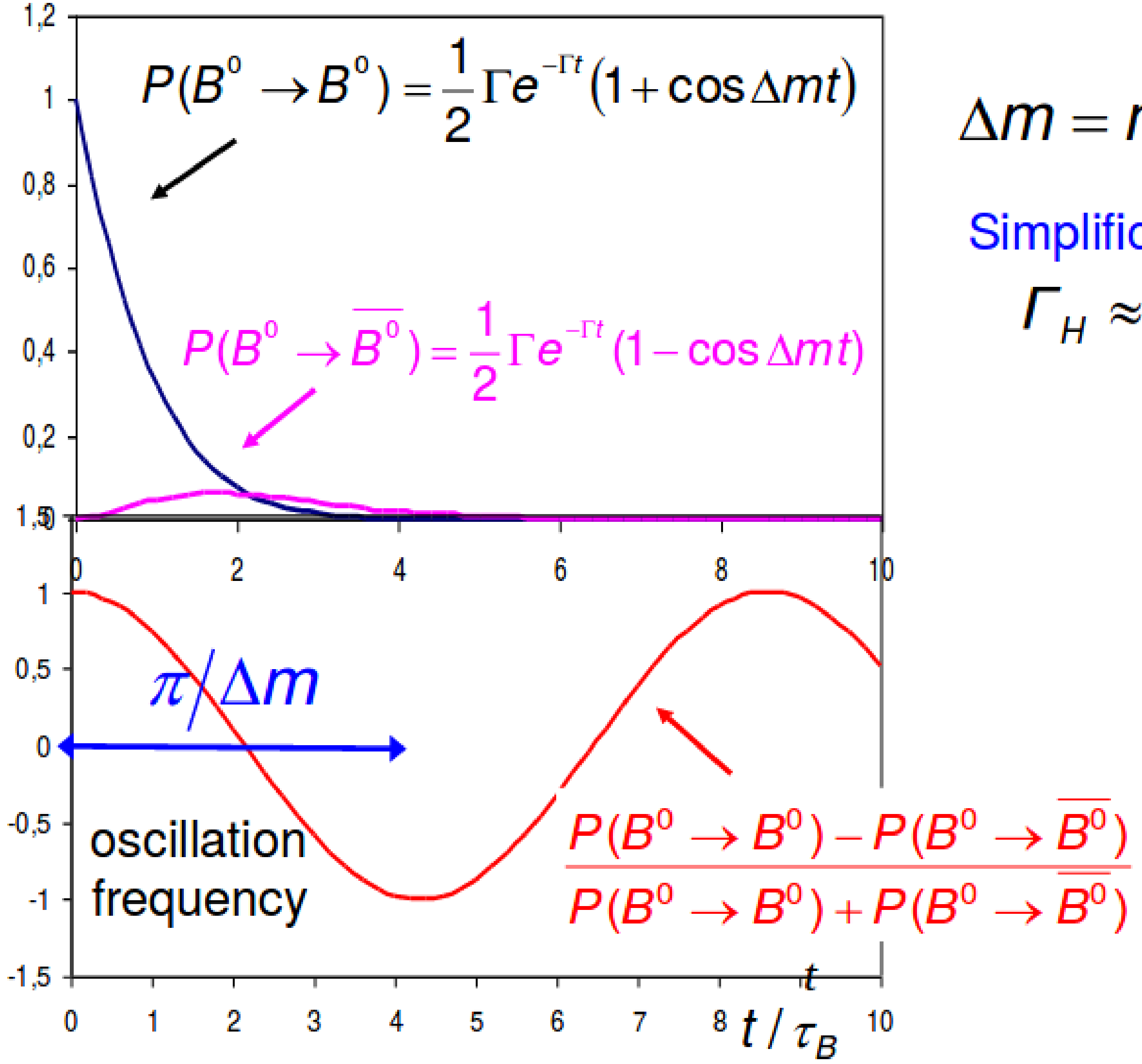
$$P(B^0 \to \overline{B^0}) = \frac{1}{4}\left|\frac{q}{p}\right|^2 \left[e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_L + \Gamma_H)t/2} \cos \Delta m t\right] \qquad \Delta m = m_H - m_L$$

$$P(\overline{B^0} \to B^0) = \frac{1}{4}\left|\frac{p}{q}\right|^2 \left[e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_L + \Gamma_H)t/2} \cos \Delta m t\right]$$

CP - violation in mixing:

$$P(B^0 \to \overline{B^0}) \neq P(\overline{B^0} \to B^0) \Rightarrow \left|\frac{q}{p}\right| \neq 1$$

$B^0 - \overline{B^0}$ oscillation



$$P(B^0 \to B^0) = \frac{1}{2}\Gamma e^{-\Gamma t}(1 + \cos \Delta m t)$$

$$P(B^0 \to \overline{B^0}) = \frac{1}{2}\Gamma e^{-\Gamma t}(1 - \cos \Delta m t)$$

$\pi / \Delta m$

oscillation frequency

$$\frac{P(B^0 \to B^0) - P(B^0 \to \overline{B^0})}{P(B^0 \to B^0) + P(B^0 \to \overline{B^0})}$$

Mixing asymmetry

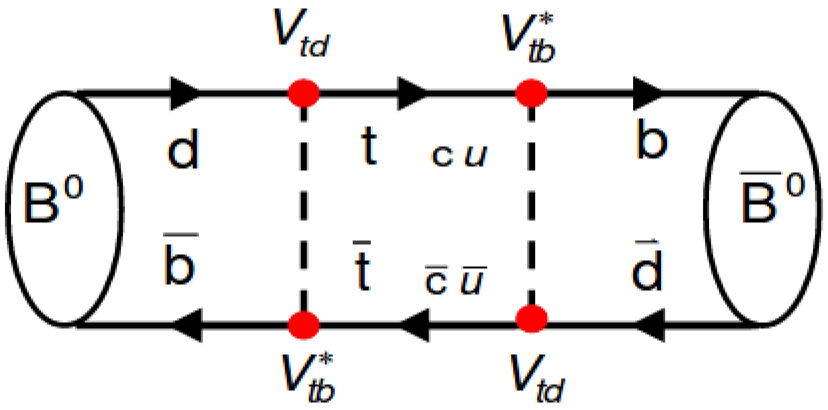
$$\Delta m = m_H - m_L$$

Simplification for

$$\Gamma_H \approx \Gamma_L \approx \Gamma$$

Standard Model predictions

$B_d^0 - \overline{B}_d^0$



$\Delta m_d \sim m_t^2 \cdot O(\lambda^6)$

Dominant contribution from top-loop:

$\eta_B = 0.55 \pm 0.01$

NLO QCD

$$\Delta m_d = \frac{G_F^2}{6\pi^2} m_B f_B^2 B_B (V_{td} V_{tb}^*)^2 m_W^2 \eta_B F\left(\frac{m_t^2}{m_W^2}\right)$$

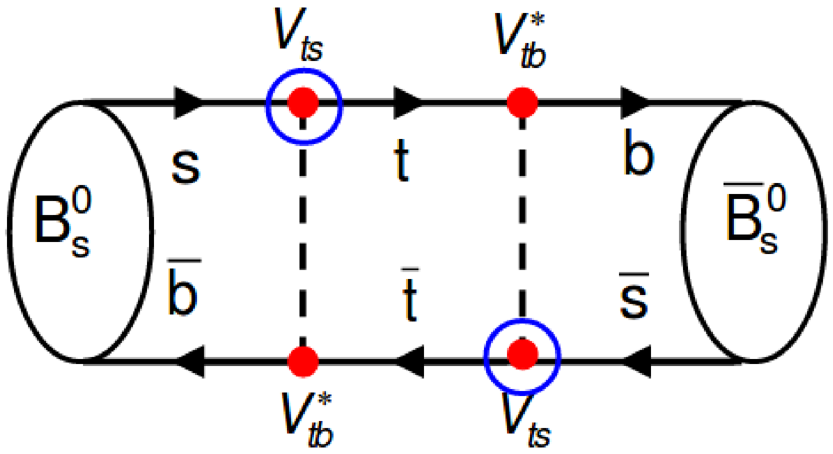
e.w. correction

$f_B^2 B_B = (235 \pm 33 \pm 12)^2 MeV^2$ from lattice QCD

Describes the binding of the quarks to a meson

Prediction for $B_s^0 - \overline{B_s^0}$ oscillation

$B_s^0 - \overline{B}_s^0$



$\Delta m_s \sim m_t^2 \cdot O(\lambda^4)$

$$\boxed{\Delta m_s \sim (V_{ts} V_{tb}^*)^2}$$

Oscillation is about 35 times stronger than in the case of B_d
(V_{ts} much larger than V_{td})

B oscillation:
Deactivation of GIM(*) suppression because of large top mass:

What would be the mixing if all quarks had the same masses?

(*) Glashow, Iliopoulos, Maiani, 1970, see next page.

Historical interlute

1970: Rare Kaon Decays

Observed branching ratio $K_L \to \mu^+\mu^-$

$$\frac{BR(K_L \to \mu^+\mu^-)}{BR(K_L \to all)} = (7.2 \pm 0.5) \times 10^{-9}$$

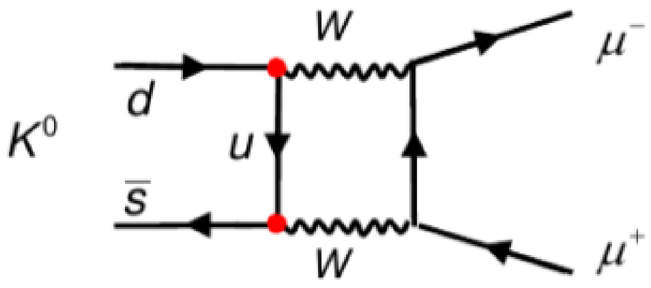
In contradiction with theoretical expectations in the 3 quark model $(d' = d\cos\theta_c + s\sin\theta_c)$

➡ Glashow, Iliopolus, Maiani (1970):

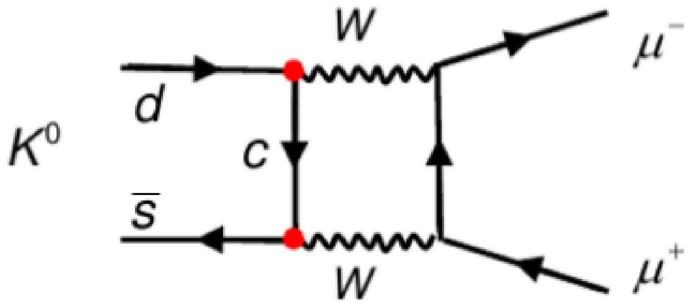
Prediction of a 2nd up type quark, additional Feynman graph cancels the “u box graph”

GIM mechanism

The study of this rare decay resulted in accidentally correct prediction of $m_c \sim 1.5 GeV$



$M \sim \sin\theta_c \cos\theta_c$

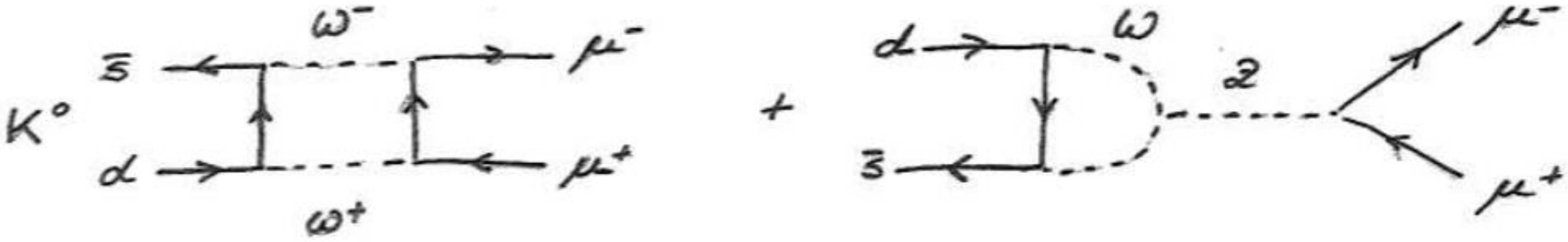


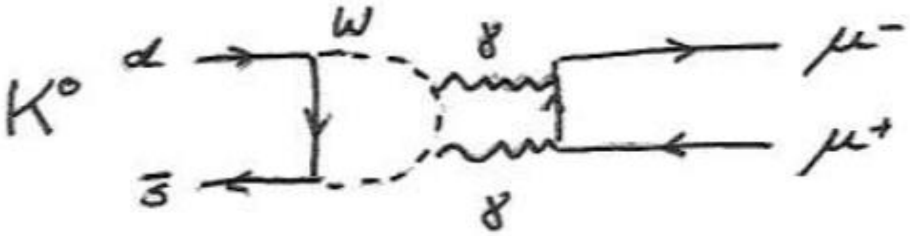
$M \sim -\sin\theta_c \cos\theta_c$

Historical interlute

Additional Diagrams

short range contribution



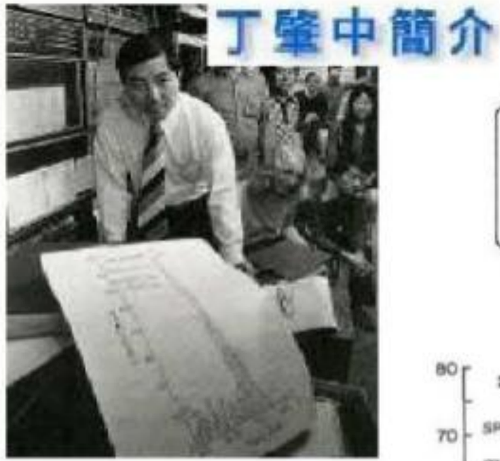


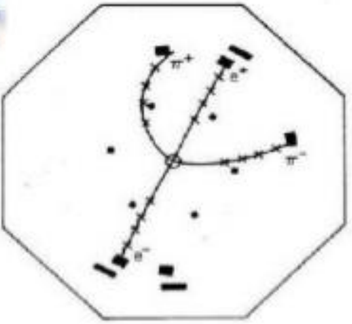
+ long range contributions

Prediction of Charm Quark Mass was per chance correct, however triggered a lot of activities.

1974: Discovery of J/Ψ

Historical interlute





BNL experiment
(S. Ting et. al)
(Berkley national laboratory)

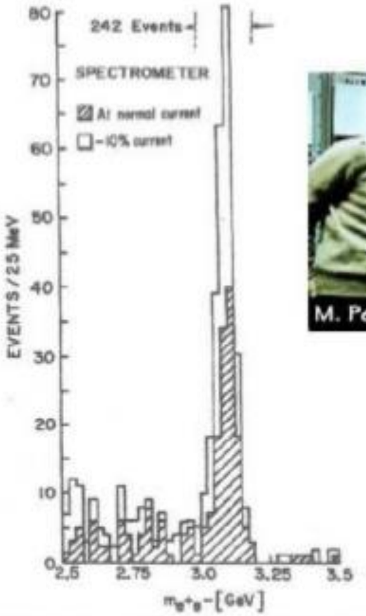


FIG. 2. Mass spectrum showing the existence of *J*. Results from two spectrometer settings are plotted showing that the peak is independent of spectrometer currents. The run at reduced current was taken two months later than the normal run.

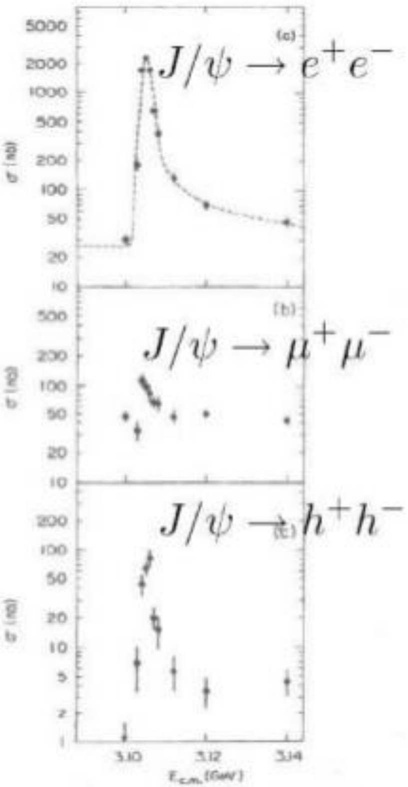


FIG. 1. Cross section versus energy for (a) multihadron final states, (b) e^+e^- final states, and (c) $\mu^+\mu^-$, $\pi^+\pi^-$, and K^+K^- final states. The curve in (a) is the expected shape of a δ-function resonance folded with the Gaussian energy spread of the beams and including radiative processes. The cross sections shown in (b) and (c) are integrated over the detector acceptance. The total hadron cross section, (a), has been corrected for detection efficiency.

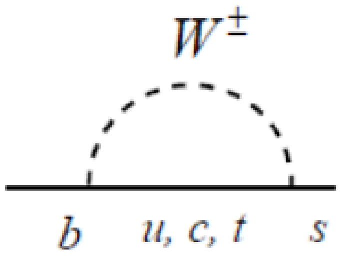
SLAC: Mark I
(Standford linear accelerator complex)

$$p + Be \rightarrow ? + X \rightarrow e^+ + e^- X$$

$$e^+ + e^- \rightarrow e^+ + e^- / \mu^+ + \mu^- / h^+ + h^-$$

GIM supression:

Example: FCNC process b→s ("penguin process" as in B→K*γ)



$$\mathcal{A}(b \to s)_{\text{SM}} = V_{ub}V_{us}^* A_u + V_{cb}V_{cs}^* A_c + V_{tb}V_{ts}^* A_t$$

where A_q denote the sub-amplitudes for the 3 possible internal quark. A_q depend on the quark masses only:

$$A_q = A(m_q^2/M_W^2)$$

Using the unitarity of the CKM matrix, especially: $\sum_i V_{ib}V_{is}^* = 0$

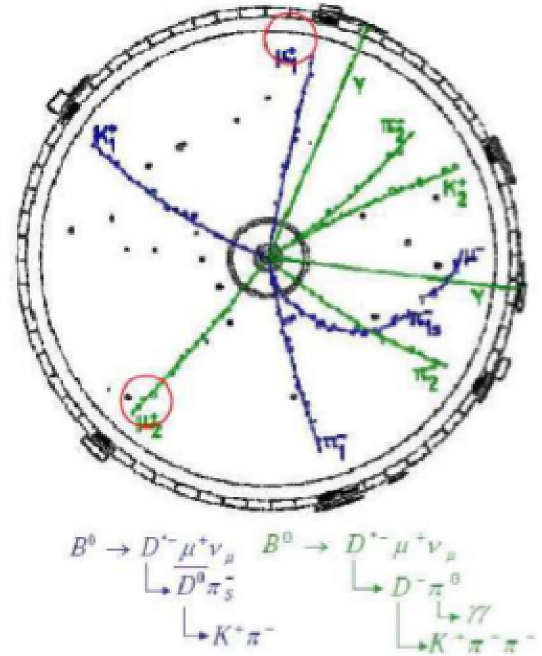
the total amplitude can be rewritten:

$$\mathcal{A}(b \to s)_{\text{SM}} = V_{tb}V_{ts}^*(A_t - A_c) + V_{ub}V_{us}^*(A_u - A_c)$$

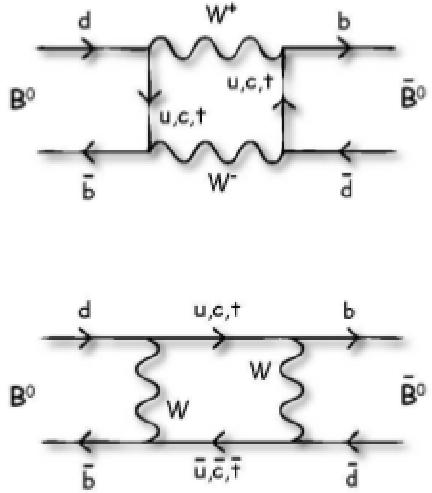
In case of approx. equal quark masses, total amplitude vanishes: GIM suppression.

For large top quark mass: $A(b \to s)_{SM} = V_{tb}V_{ts}^* \cdot \frac{m_t^2}{m_W^2}$ GIM suppression inactive

1986: B^0 Oscillation at ARGUS



$$e^+e^- \to Y(4S) \to B^0\overline{B^0}$$



Time integrated mixing rate: $\chi_d = \int P_{mixed}(t) \cdot e^{-t/\tau} dt = 0.17 \pm 0.05$

25 mixed events:

$B^0\overline{B^0} \to \ell^-\ell^-$

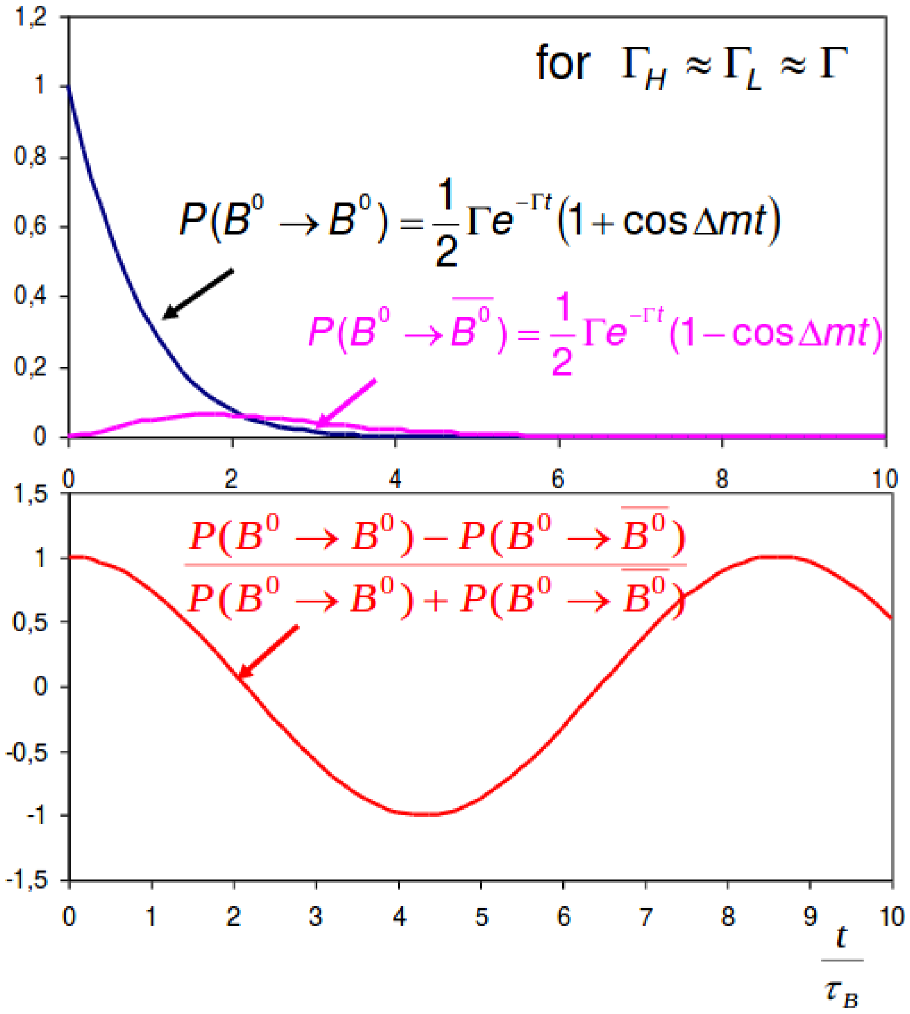
$B^0\overline{B^0} \to \ell^+\ell^+$

250 unmixed events:

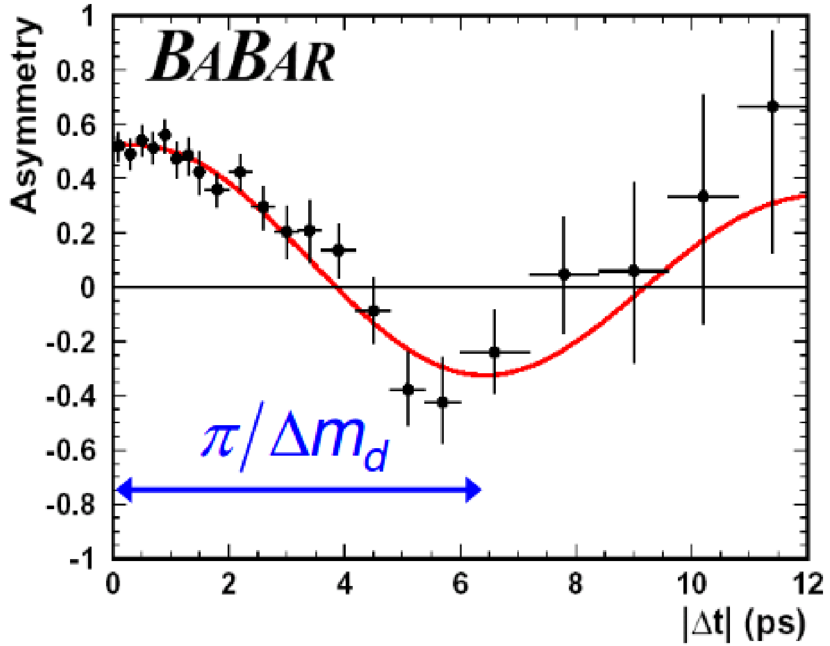
$B^0\overline{B^0} \to \ell^+\ell^-$

First indication for a heavy top quark $m_t > 40$ GeV!

Experimental Status of B_d meson mixing



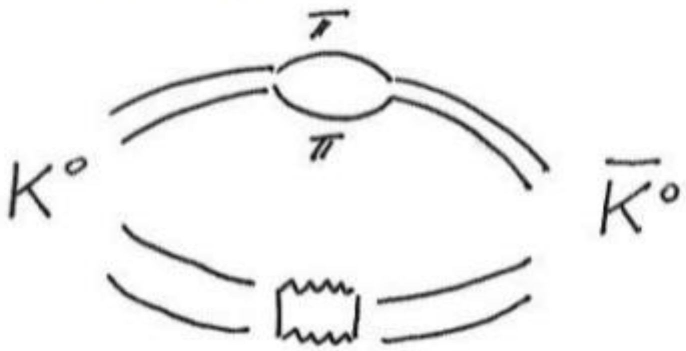
$$A = \frac{unmixed - mixed}{unmixed + mixed}$$



$$\Delta m_d = 0.506 \pm 0.006 \pm 0.004\,\mathrm{ps}^{-1}$$

$$\approx \frac{0.774}{\tau_B}$$

long range contribution $\Delta\Gamma$



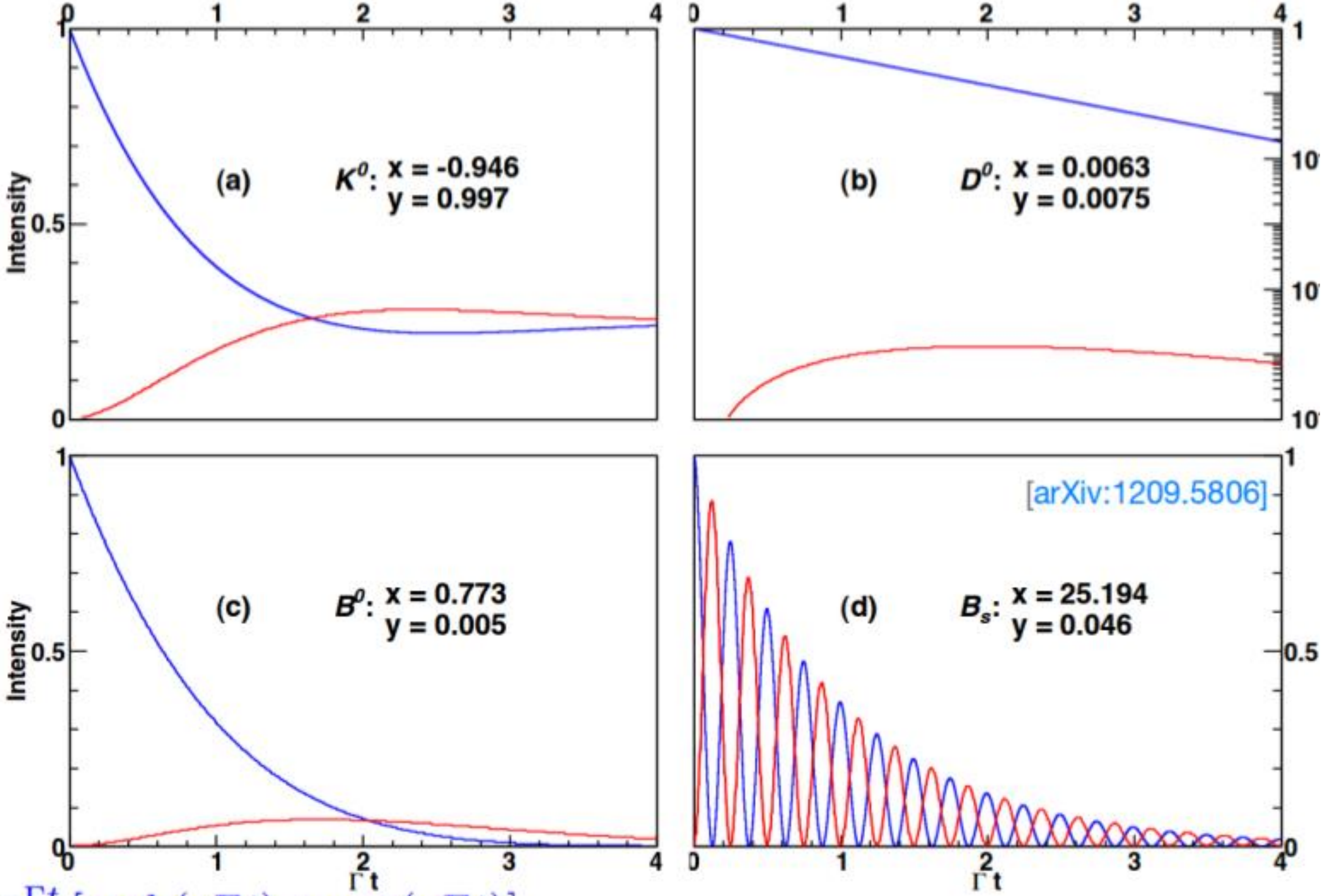
short range contribution Δm

Same concept for all neutral meson systems, however Different choice of parameters.

	$K^0/\overline{K^0}$	$D^0/\overline{D^0}$	$B^0/\overline{B^0}$	$B^0_s/\overline{B^0_s}$
τ [ps]	89.3	0.415	1.564	1.47
	51700			
Γ [ps^{-1}]	$5.61\cdot10^{-3}$	2.4	0.643	0.62
y=$\frac{\Delta\Gamma}{2\Gamma}$	0.9966	0.008	0.0075	0.059
Δm [ps^{-1}]	$5.301\cdot10^{-3}$	0.16	0.506	17.8
x=$\frac{\Delta m}{\Gamma}$	0.945	0.010	0.768	26.1

Blue line:
given a P^0, at t=0,
the probability of
finding a P^0 at t

Red Line:
given a P^0, at t=0,
the probability of
finding a $\bar{P}^0$ at t

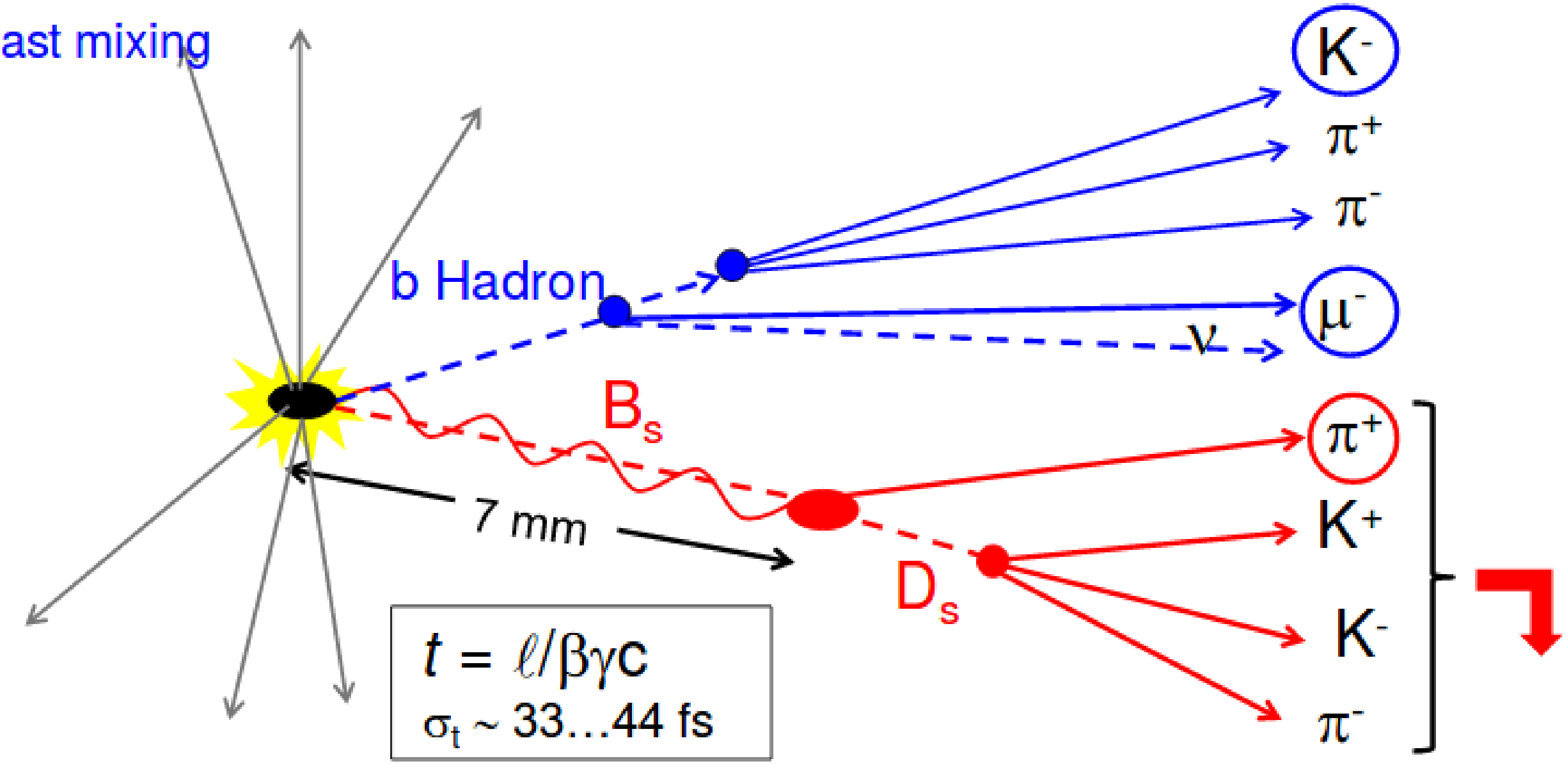


$$|\langle P^0(0)|P^0(t)\rangle|^2 \propto e^{-\Gamma t}\left[\cosh(y\Gamma t) + \cos(x\Gamma t)\right]$$

$$|\langle P^0(0)|\overline{P}^0(t)\rangle|^2 \propto e^{-\Gamma t}\left[\cosh(y\Gamma t) - \cos(x\Gamma t)\right]$$

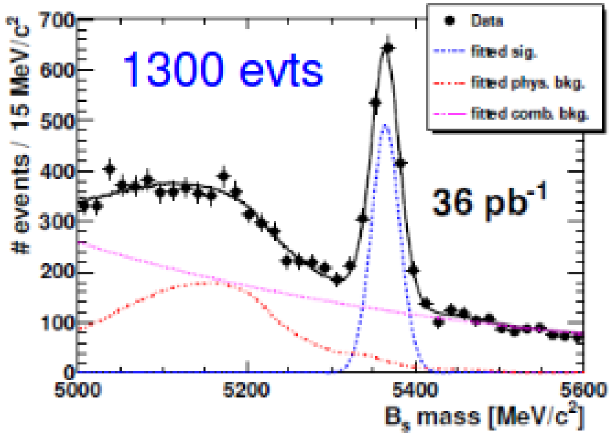
B_s – Mixing measurement at LHC

very fast mixing



$t = \ell/\beta\gamma c$

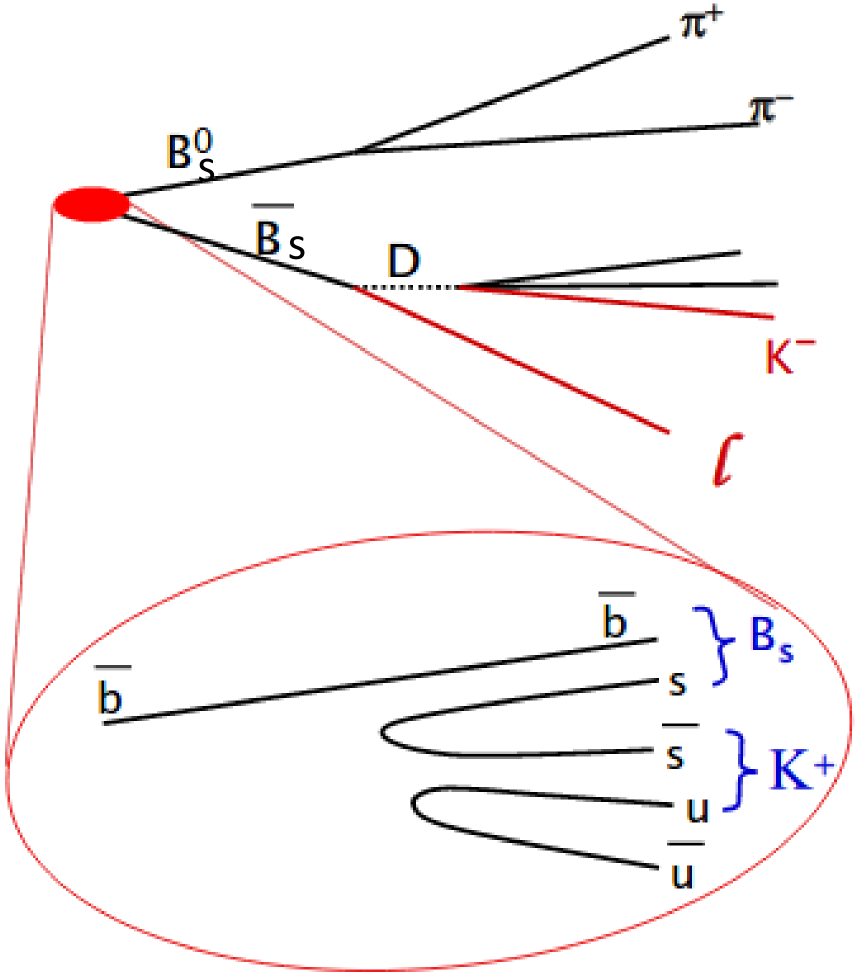
$\sigma_t \sim 33 \ldots 44$ fs



Analysis steps:

- B_s reconstruction: $B_s \rightarrow D_s \pi$ (self-tagging)
- Measurement of proper decay time
- Tagging of production flavor

Flavor Tagging & B_s Mixing



- Lepton
- Kaon
- Vertex charge

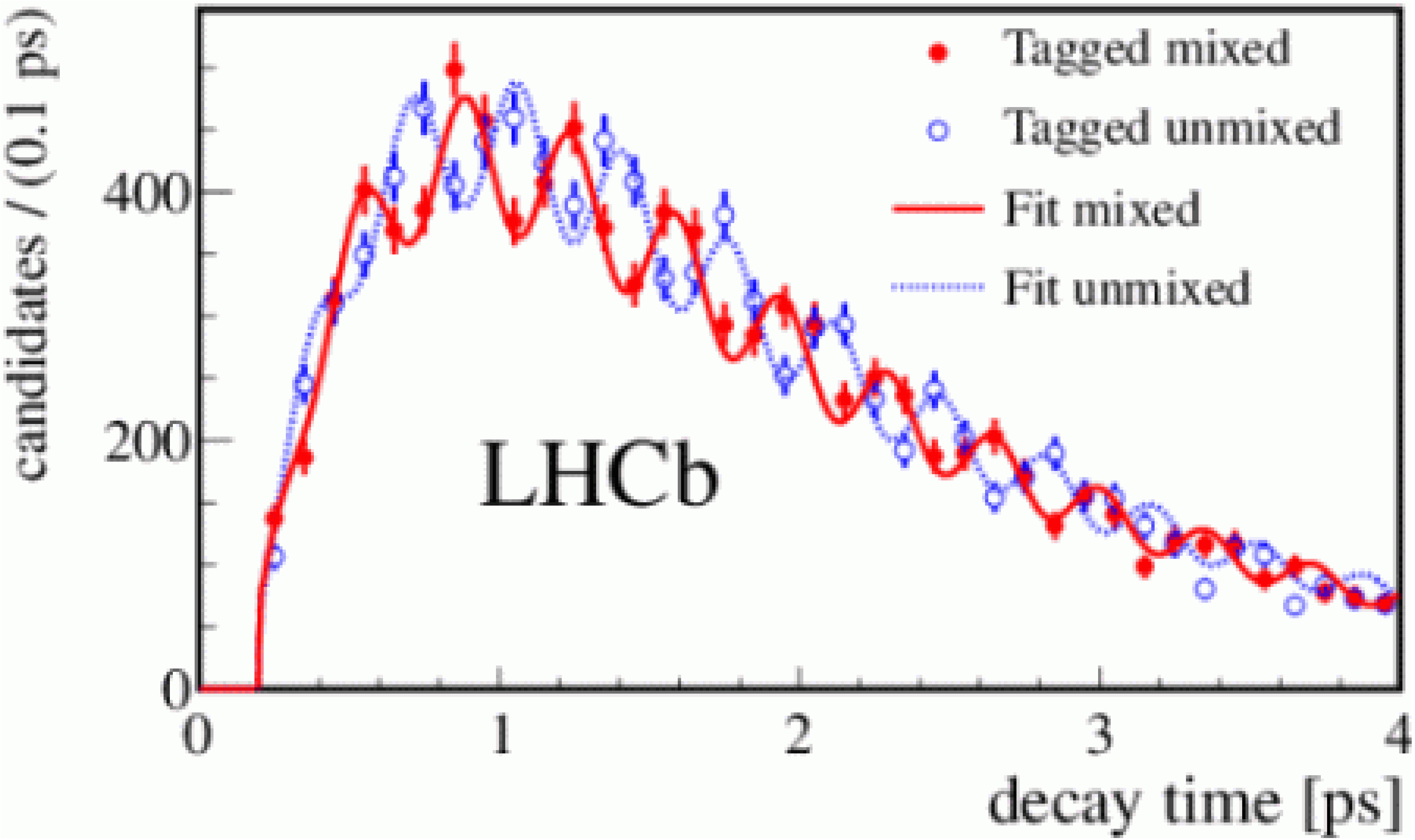
Other B: "opposite"

- Fragmentation hadron "same side"

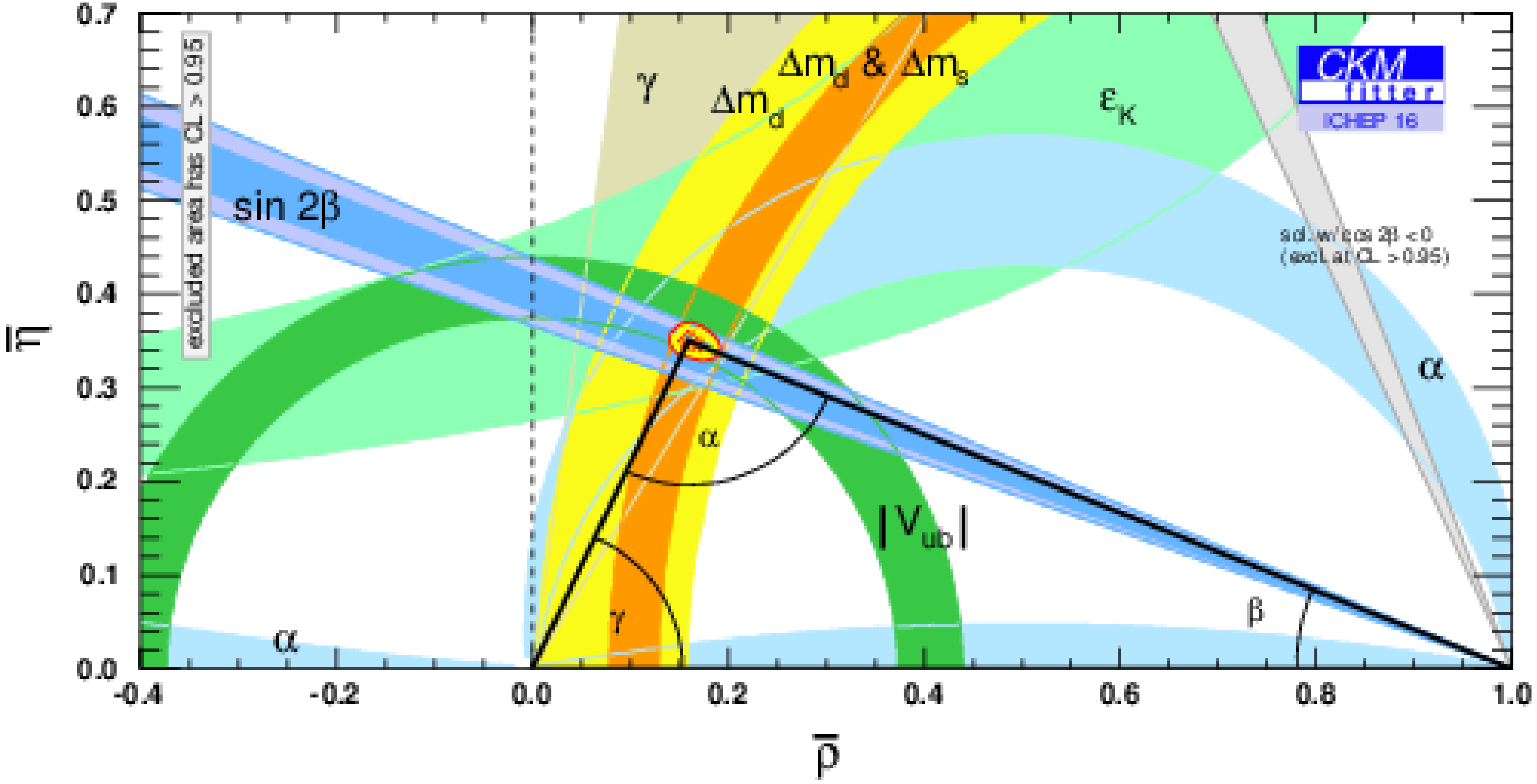
Figure of merit: $\varepsilon D^2 \sim 4.3\%$

- Tagging efficiency ε ~34%
- Dilution $D = (1 - 2\omega) \sim 32\%$

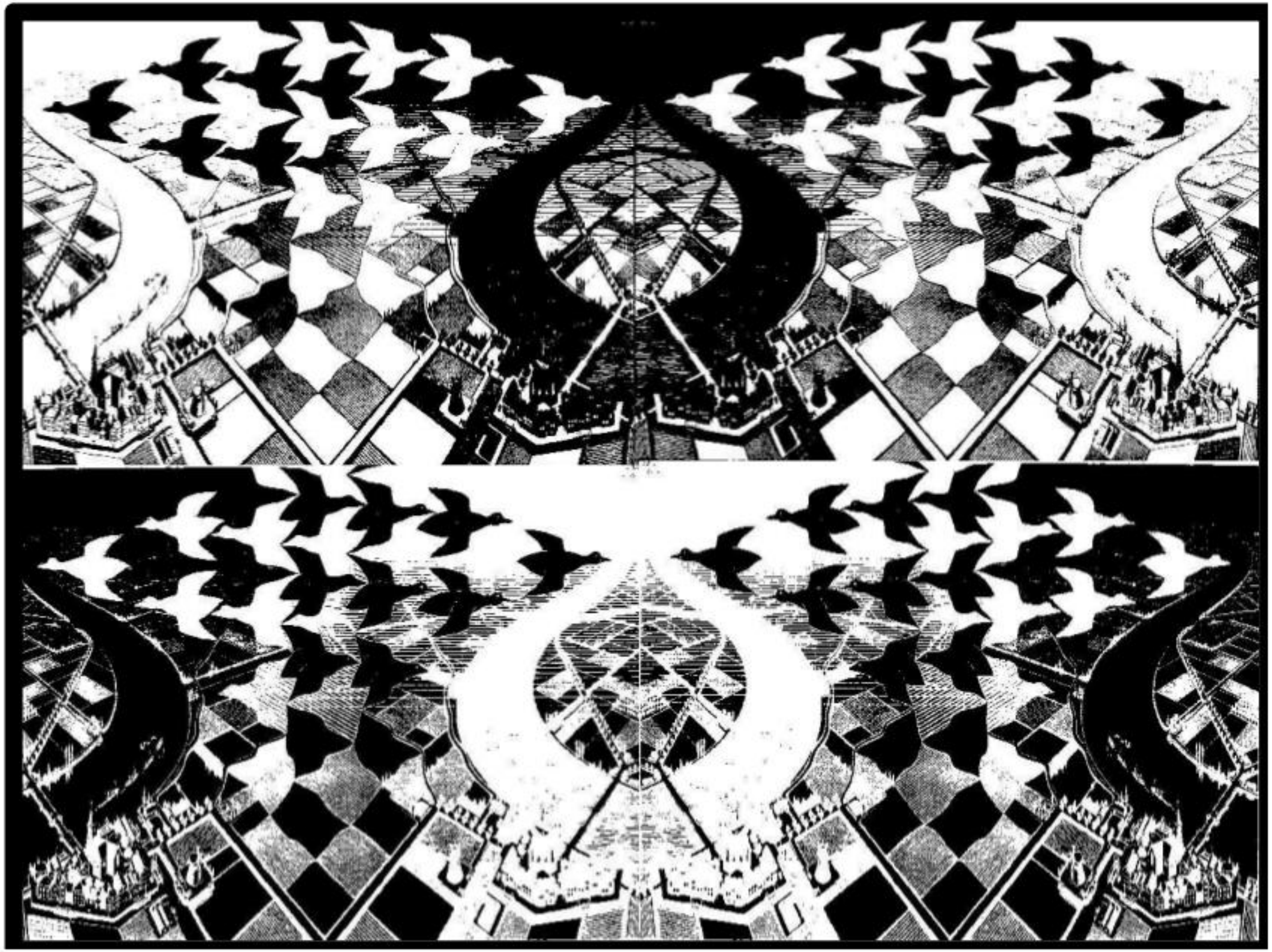
ω = mistag probability


candidates / (0.1 ps)
400
200
0
0
1
2
3
4
decay time [ps]
LHCb
● Tagged mixed
○ Tagged unmixed
Fit mixed
Fit unmixed

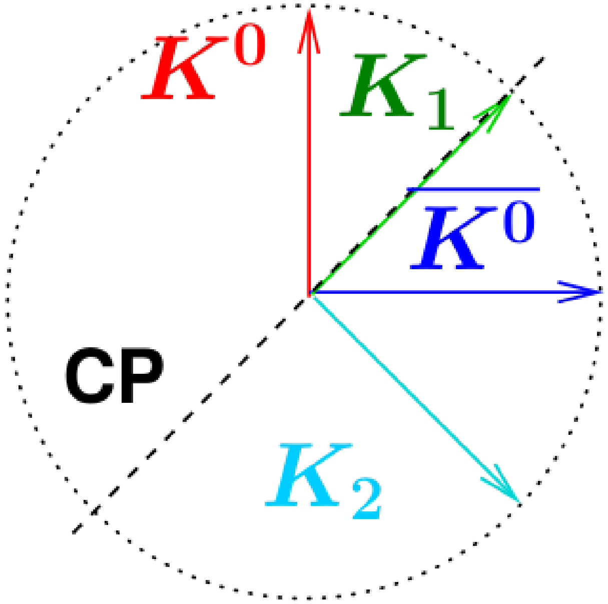
Status of the unitarity triangle



Sofar discussed only length of sides (absolute values of CKM matrix elements), angles come from measurements of CP violation



Neutral Meson Mixing



$$CP(K^0) = \overline{K^0}$$
$$CP(\overline{K^0}) = K^0$$

$$K_1 = \frac{1}{\sqrt{2}}(K^0 + \overline{K^0})$$
$$CP(K_1) = +K_1$$

$$K_2 = \frac{1}{\sqrt{2}}(K^0 - \overline{K^0})$$
$$CP(K_2) = -K_2$$

K^0, $\overline{K^0}$: flavour eigenstates; clear defined quark content ($K^0 = |d\bar{s}>$, $\overline{K^0} = |\bar{d}s>$)

K_1, K_2: CP eigenstates

K_S, K_L: mass eigenstates

(with clear defined mass and lifetime, $\psi_{S/L}(t) = e^{-im_{S/L}t}e^{-\Gamma_{S/L}t/2}$)

in absence of CPV: $K_S = K_1, K_L = K_2$

Kaon Mixing

$$|\mathbf{K_S}> = p|\mathbf{K^0}> + q|\overline{\mathbf{K^0}}>, \quad |\mathbf{K_S(t)}> = |\mathbf{K_S}> e^{-\frac{\Gamma_S}{2}t} e^{-im_S t}$$

$$|\mathbf{K_L}> = p|\mathbf{K^0}> - q|\overline{\mathbf{K^0}}>, \quad |\mathbf{K_L(t)}> = |\mathbf{K_L}> e^{-\frac{\Gamma_L}{2}t} e^{-im_L t}$$

$|p|^2 + |q|^2 = 1$ complex coefficients; $q = p = \frac{1}{\sqrt{2}} \Leftrightarrow \mathbf{K_S} = \mathbf{K_1}, \mathbf{K_L} = \mathbf{K_2}$

Flavour eigenstates:

$$|\mathbf{K^0}> = \frac{1}{2p}(|\mathbf{K_S}> + |\mathbf{K_L}>)$$

$$|\overline{\mathbf{K^0}}> = \frac{1}{2q}(|\mathbf{K_L}> - |\mathbf{K_S}>)$$

time development of originally (at t=0) pure $\mathbf{K^0}$ and $\overline{\mathbf{K^0}}$ states:

$$|\mathbf{K^0(t)}> = \frac{1}{2p}(|\mathbf{K_S(t)}> + |\mathbf{K_L(t)}>)$$

$$|\overline{\mathbf{K^0}}\mathbf{(t)}> = \frac{1}{2q}(|\mathbf{K_L(t)}> - |\mathbf{K_S(t)}>)$$

Kaon Mixing

$$P(\mathbf{K^0} \to \overline{\mathbf{K^0}}) =< \mathbf{K^0(t)} | \overline{\mathbf{K^0}} >=$$
$$\frac{1}{4}|\frac{q}{p}|^2 \left(e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_L+\Gamma_H)t/2}\cos\Delta m t\right)$$

$$P(\overline{\mathbf{K^0}} \to \mathbf{K^0}) =< \overline{\mathbf{K^0}}\mathbf{(t)} | \mathbf{K^0} >=$$
$$\frac{1}{4}|\frac{p}{q}|^2 \left(e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_L+\Gamma_H)t/2}\cos\Delta m t\right)$$

CP conserved: $P(\mathbf{K^0} \to \overline{\mathbf{K^0}}) = P(\overline{\mathbf{K^0}} \to \mathbf{K^0})$

$$\Leftrightarrow$$

$$|\frac{q}{p}| = 1$$

(+ normalisation $q^2 + p^2 = 1$)

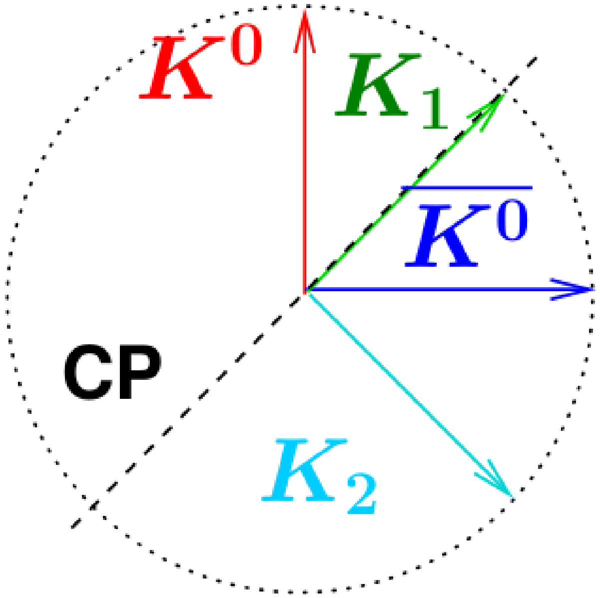
$$\Leftrightarrow$$

$$q = p = \frac{1}{\sqrt{2}}$$

$$\Leftrightarrow$$

$$\boldsymbol{K_S = K_1, K_L = K_2}$$

Neutral Meson Mixing



$$CP(K^0) = \overline{K^0}$$
$$CP(\overline{K^0}) = K^0$$

$$K_1 = \frac{1}{\sqrt{2}}(K^0 + \overline{K^0})$$
$$CP(K_1) = +K_1$$

$$K_2 = \frac{1}{\sqrt{2}}(K^0 - \overline{K^0})$$
$$CP(K_2) = -K_2$$

$$P(\Psi(\pi)) = P(\Psi(q)) \cdot P(\Psi(\overline{q})) \cdot (-1)^{L=0} = 1 \cdot -1 \cdot 1 \cdot \Psi(\pi) = -\Psi(\pi)$$
$$C(\Psi(\pi)) = C(\Psi(q\overline{q})) = (-1)^{L+S} \cdot \Psi(q\overline{q}) = +\Psi(\pi)$$

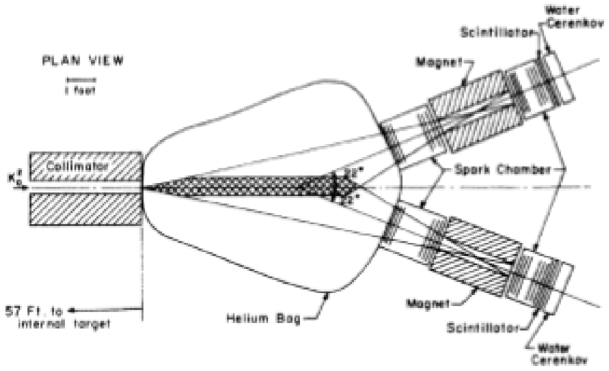
$$CP(\Psi(\pi^+\pi^-)) = CP(\Psi(\pi^+)) \cdot CP(\Psi(\pi^-)) \cdot (-1)^{L=0} = +\Psi(\pi^+\pi^-)$$
$L = 0$ in $K^0 \to \pi^+\pi^-$

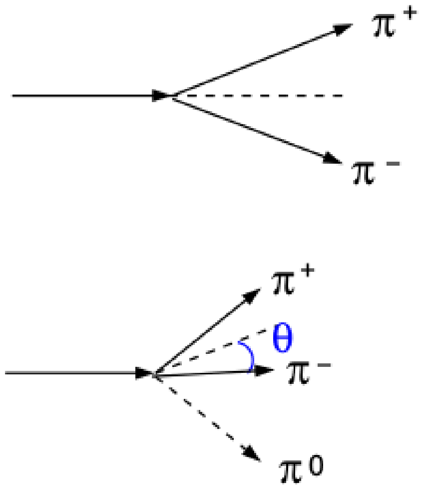
$$CP(\Psi(\pi^+\pi^-\pi^0)) = CP(\Psi(\pi^-))^3 \cdot (-1)^L = -\Psi(\pi^+\pi^-\pi^0)$$
$L = 0$ in $K^0 \to \pi^+\pi^-\pi^0$

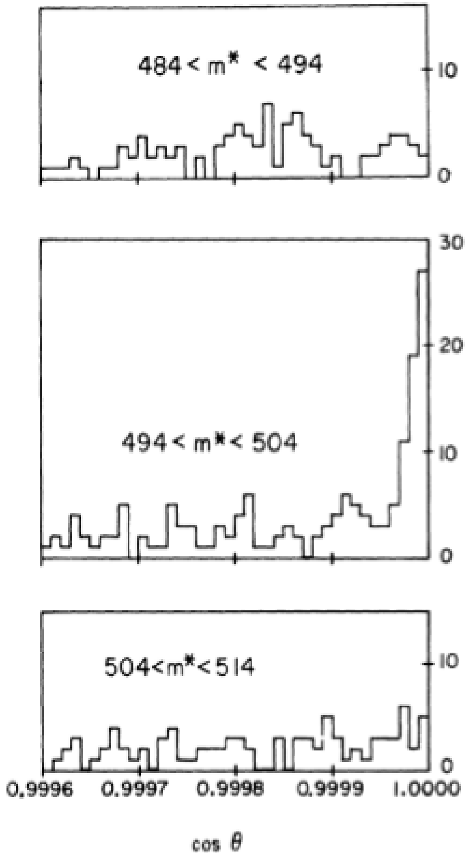
If there is no CPV in decay, then: $\mathrm{K_1} \to \pi^+\pi^-$; $\mathrm{K_2} \to \pi^+\pi^-\pi^0$

1964: Discovery of CPV

- produce K^0, wait long enough for K_S component to decay away $\rightarrow$ pure K_L beam
- search for CP violation: $K_L \rightarrow \pi^+\pi^-$
  $\rightarrow$ excess of 56 events: BR($K_L \rightarrow \pi^+\pi^-$) $\sim 2 \times 10^{-3}$





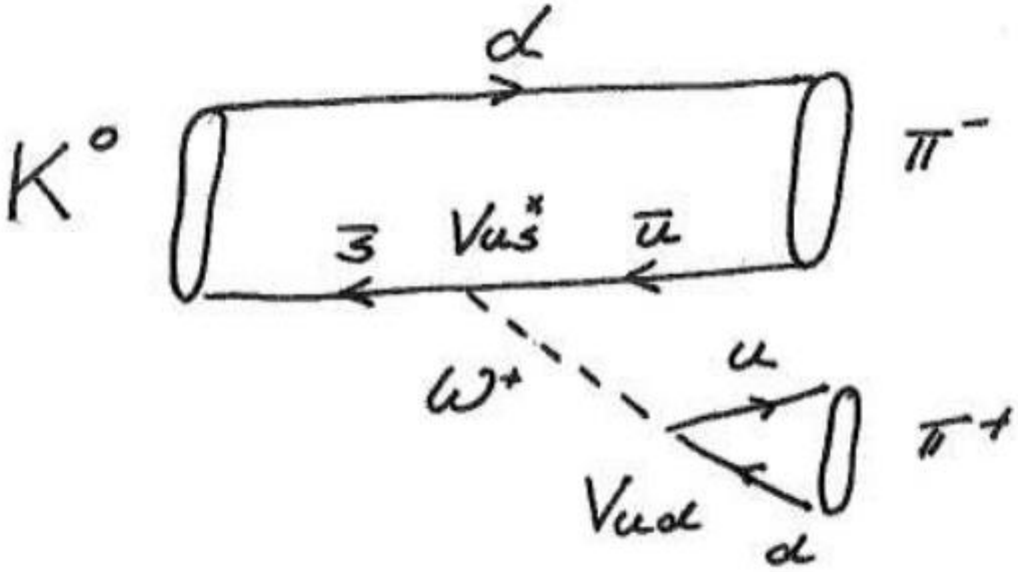


mass eigenstates ≠ CP eigenstates: $|\mathbf{K_L}>= \frac{1}{\sqrt{1+|\epsilon^2|}}(|\mathbf{K_2}> +\epsilon|\mathbf{K_1}>)$

CP=-1 CP=+1

Nobel prize for Cronin and Fitch in 1980

Weak and Strong Phases



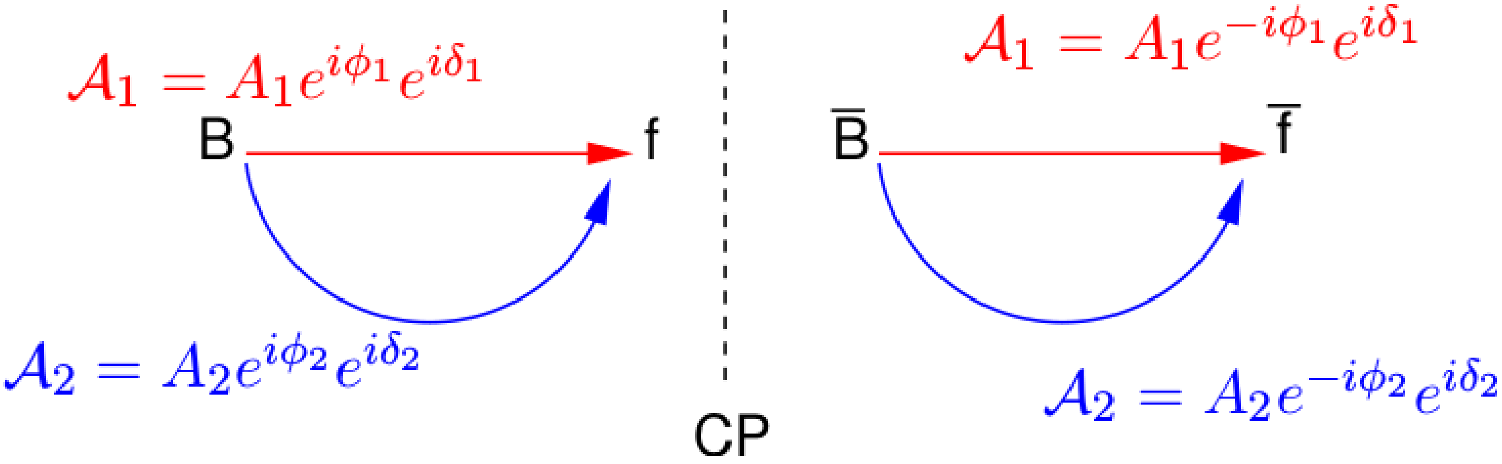
Weak phases are related to involved CKM elements: $\phi_{weak} = arg(V_{us}^{*} V_{ud})$

Strong phases δ comes often (but not always) from the hadronisation.

Definition of strong phase:

phase which doesn't change sign under CP transformation.

CP Violation



$\mathcal{A}_1 = A_1 e^{i\phi_1} e^{i\delta_1}$

B → f

$\mathcal{A}_2 = A_2 e^{i\phi_2} e^{i\delta_2}$

CP

$\mathcal{A}_1 = A_1 e^{-i\phi_1} e^{i\delta_1}$

$\overline{\text{B}}$ → $\overline{\text{f}}$

$\mathcal{A}_2 = A_2 e^{-i\phi_2} e^{i\delta_2}$

$$|\mathcal{A}|^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\Delta\phi + \Delta\delta)$$

$$|\mathcal{A}|^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(-\Delta\phi + \Delta\delta)$$

$\mathcal{A}_1$ and $\mathcal{A}_2$ need to have different weak phases ϕ and different strong phases δ.

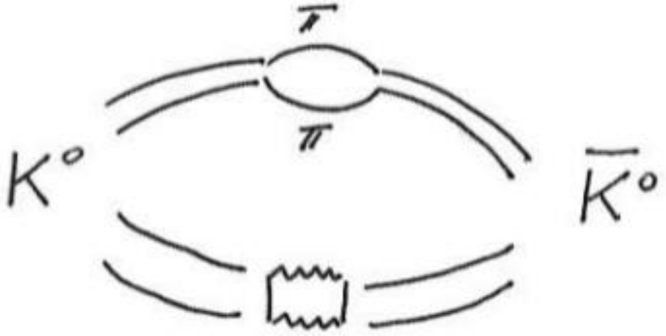
For sizable (measurable) effects both amplitudes should have about same size, and both phase differences have to be sizable.

To conclude on weak phases, strong phases need to be known/measured.

CPV in Kaon System

Interfering amplitudes which cause CPV in mixing:

long range contribution $\Delta\Gamma$



short range contribution Δm

Interfering amplitudes which cause CPV in decay: