PDG parametrization: 3 Euler angles $\theta_{23}, \theta_{13}, \theta_{12}$ and 1 Phase δ

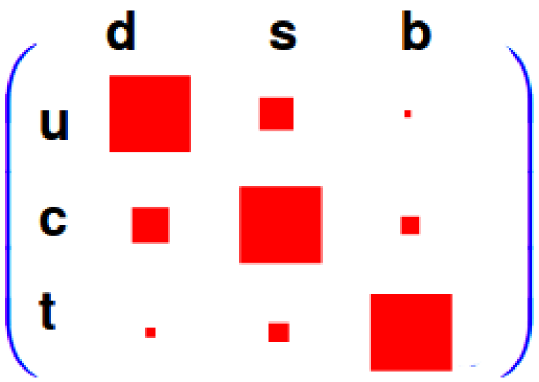
$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$

Wolfenstein Parametrisation:

$V_{CKM} =$



λ, A, ρ, η with $\lambda = 0.22$

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

$|V_{ub}| \times e^{-i\gamma}$

$|V_{td}| \times e^{-i\beta}$

Reflects the “hierarchical structure” of the CKM matrix.

Unitarity triangles:

Unitarity condition of the CKM matrix can be described by an triangle relations in the complex plane:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \quad \text{(db)}$$

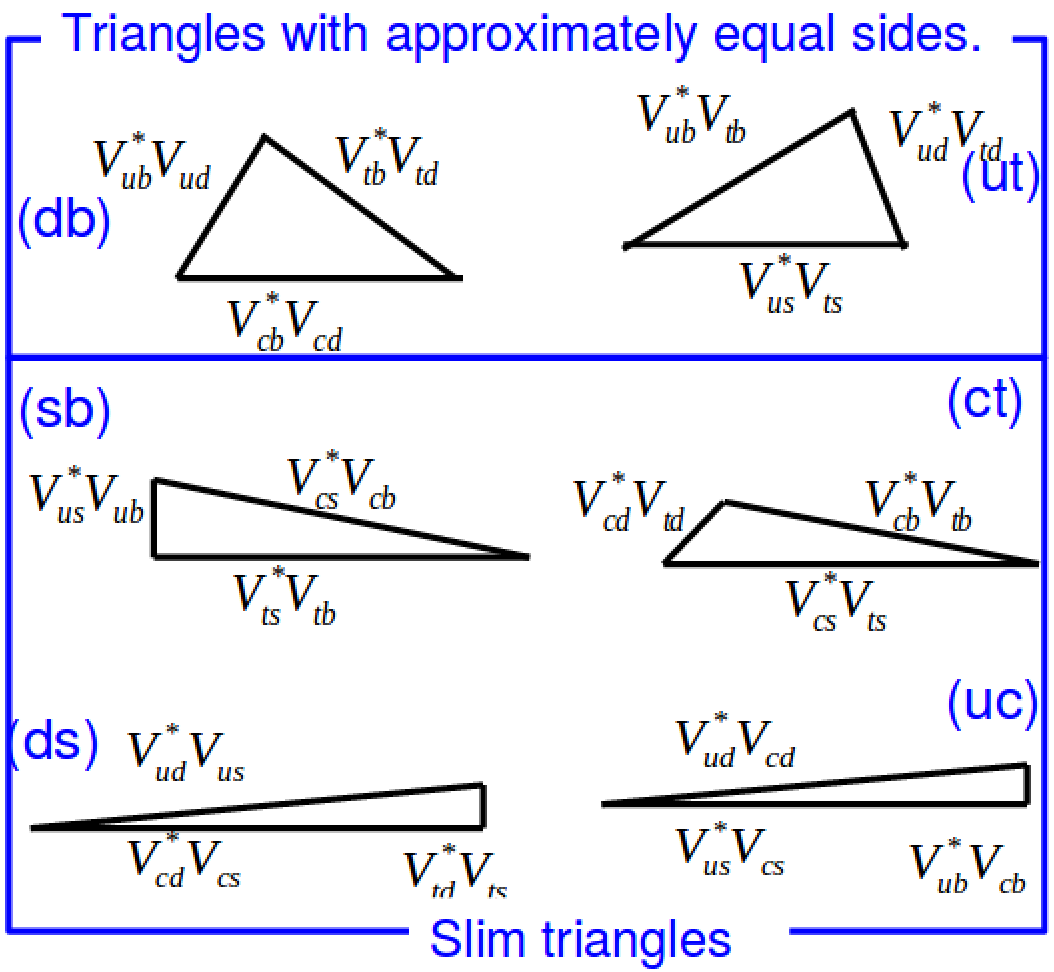
$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0 \quad \text{(sb)}$$

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0 \quad \text{(ds)}$$

$$V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* = 0 \quad \text{(ut)}$$

$$V_{cd}V_{td}^* + V_{cs}V_{ts}^* + V_{cb}V_{tb}^* = 0 \quad \text{(ct)}$$

$$V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* = 0 \quad \text{(uc)}$$

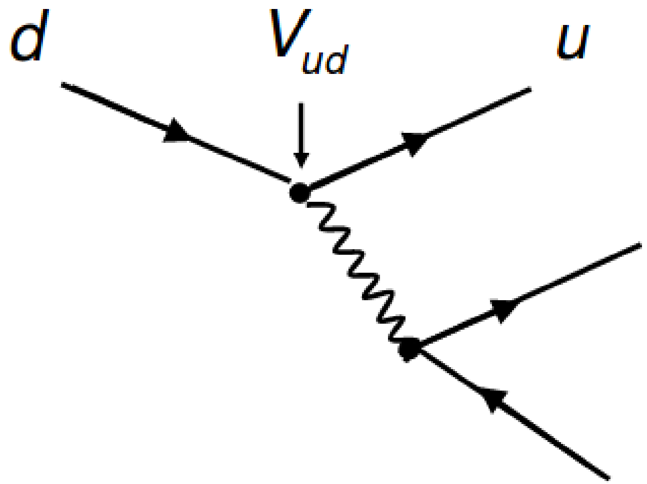


All 6 triangles have the same area → A measure of CP violation.

Determination of the CKM matrix elements

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

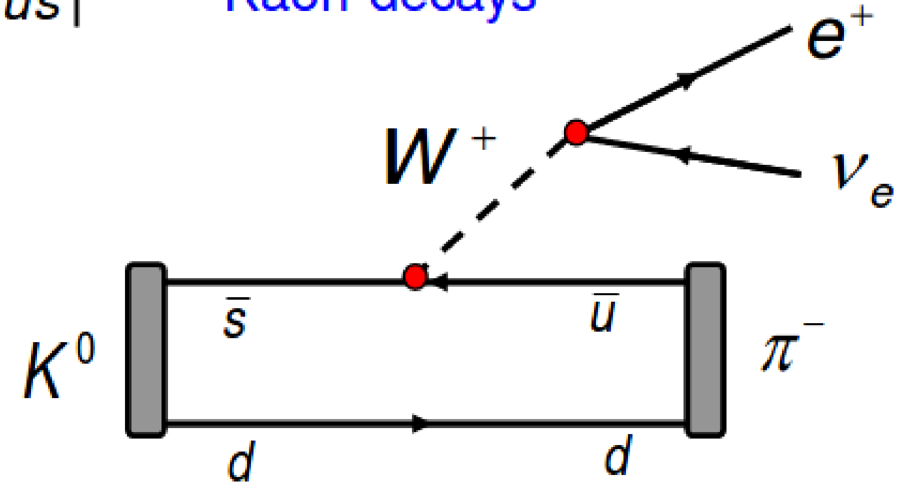
$|V_{ud}|$ Nuclear beta-decays ($0^+ \to 0^+$ beta decays, neutron decay)

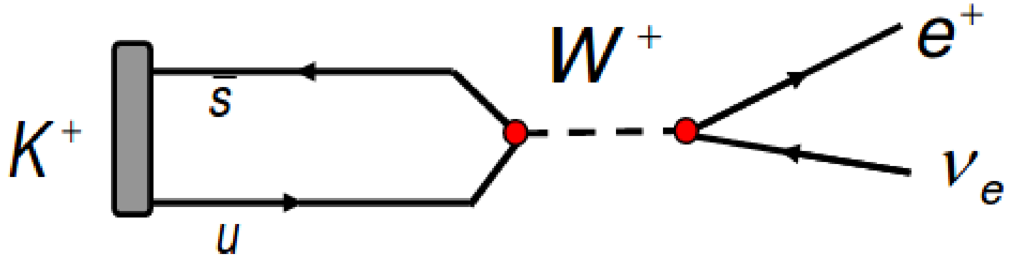


$$|V_{ud}| = 0.97425 \pm 0.00022$$

$|V_{us}|$ Kaon-decays

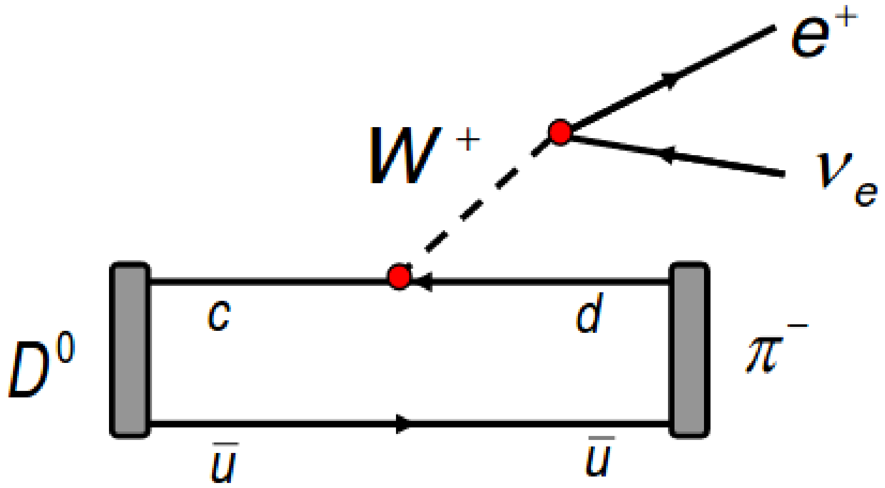
$$|V_{us}| = 0.2252 \pm 0.0009$$





Problem: Kaon and pion form factors

$|V_{cd}|$ D-meson decays



Form faktor!

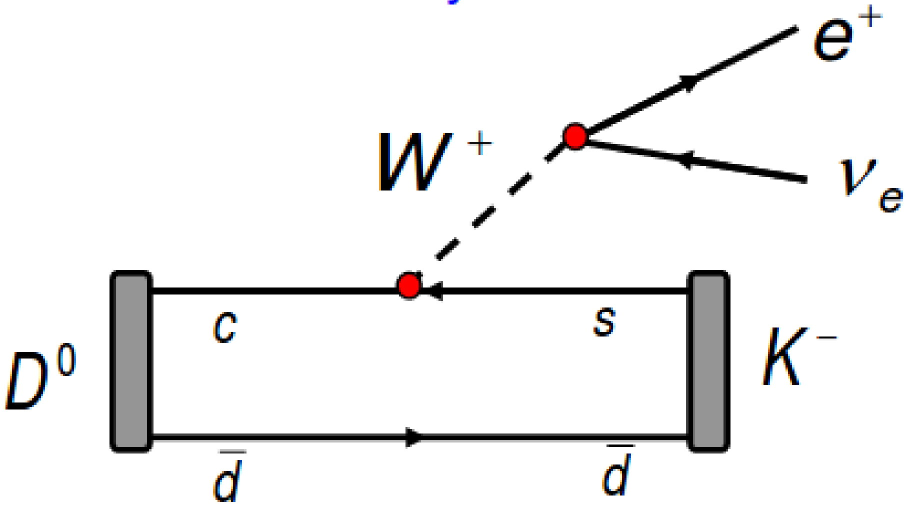
$|V_{cd}|$ More precise: Double muon production in neutrino scattering

$$\sigma(\nu_\mu + d \to \mu + D + X \to \mu + \mu + Y) \sim |V_{cd}|^2$$

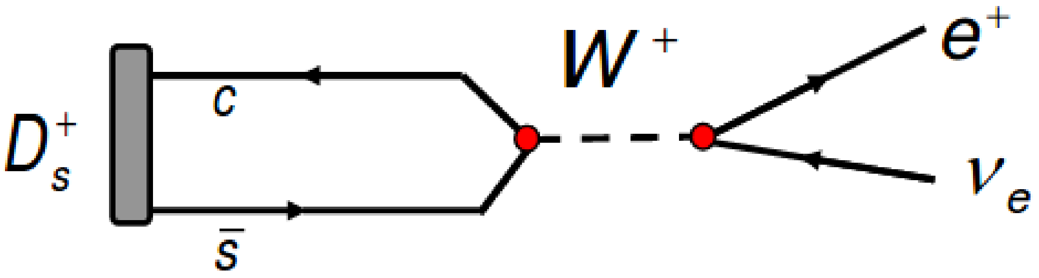
$$|V_{cd}| = 0.230 \pm 0.011$$

$|V_{cs}|$ Tagged on-shell W→cs decays at LEP II: $|V_{cs}| = 0.94^{+0.32}_{-0.26} \pm 0.13$

D-decays



Form factors!

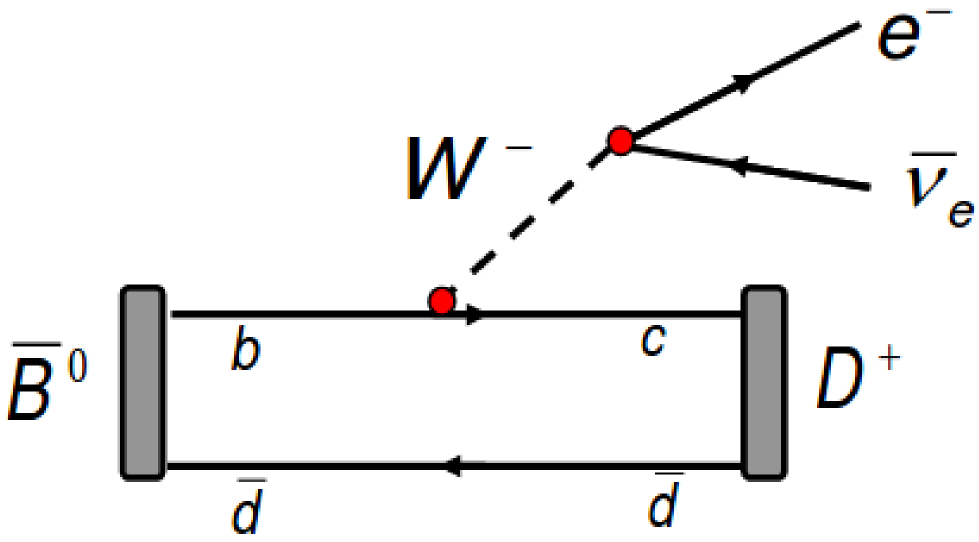


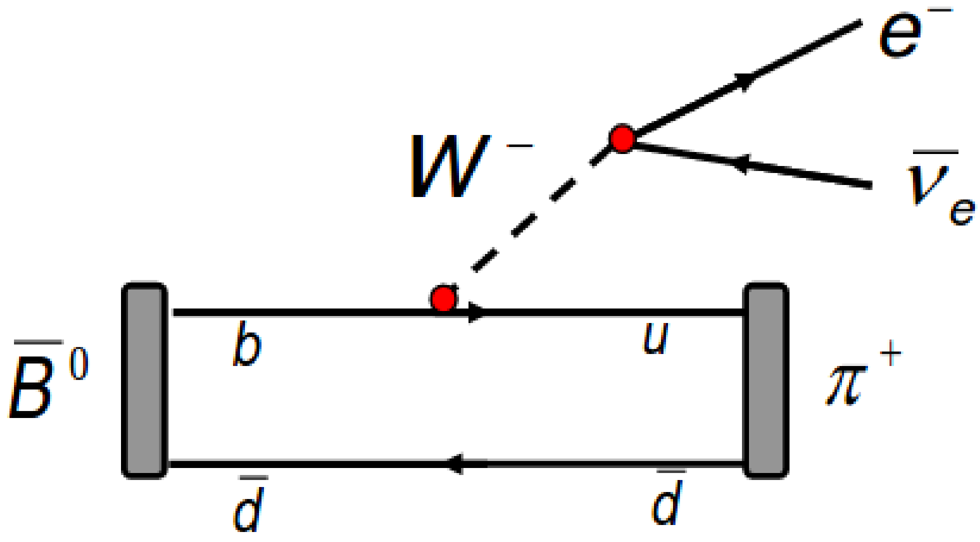
$|V_{cs}| = 1.023 \pm 0.036$

$|V_{cb}|$

$|V_{ub}|$

Semi-leptonic B decays





$|V_{cb}| = (40.6 \pm 1.3) \times 10^{-3}$

$|V_{ub}| = (3.89 \pm 0.44) \times 10^{-3}$

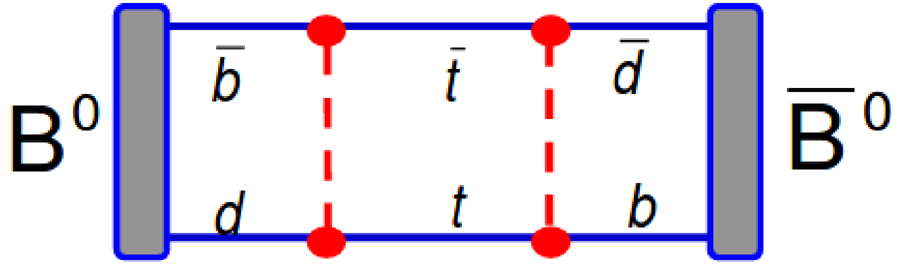
$|V_{tb}|$

Single top-quark production at hadron colliders: W→ tb → Wb + b

$|V_{td}|$

$|V_{ts}|$

Can be measured only via virtual effects: top quark decays nearly entirely to b-quarks.



B_d and B_s oscillation: Next section.

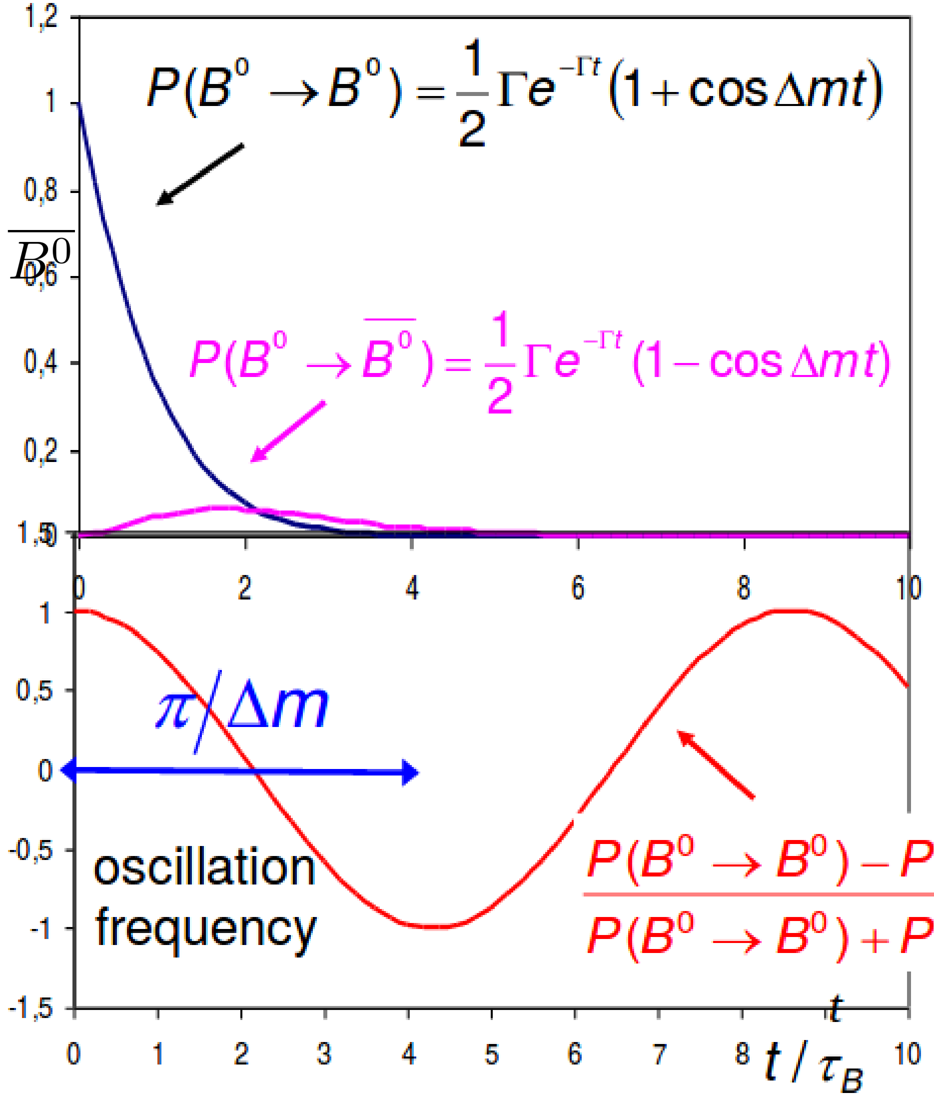
Determination of CKM Phases:

In the Wolfenstein parametrization at order $O(\lambda^4)$ (λ^6) only 2 (3) of the CKM matrix elements have non-trivial phases: V_{td}, V_{ub} (V_{ts}).

The CKM phases are measured via CP violation in B decays.

General remark:
B decays provide access to the modulus of 4 CKM elements and of two CKM phases. That's the reason B decays are studied very intensively.

$B^0 - \overline{B^0}$ oscillation



$$\Delta m = m_H - m_L$$

Simplification for

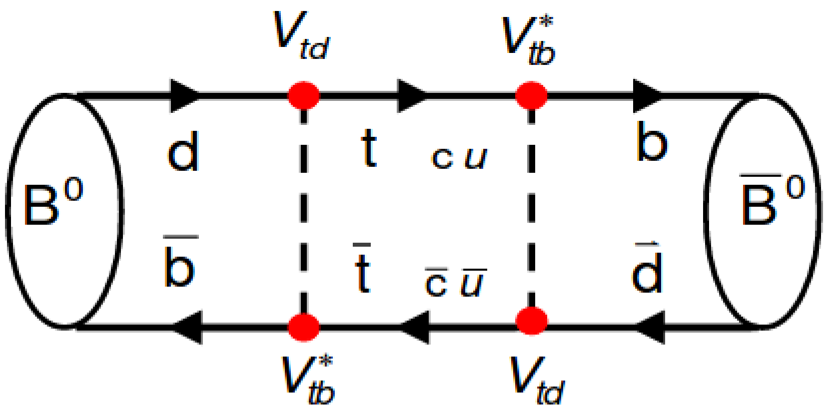
$$\Gamma_H \approx \Gamma_L \approx \Gamma$$

Mixing asymmetry

$$\frac{P(B^0 \to B^0) - P(B^0 \to \overline{B^0})}{P(B^0 \to B^0) + P(B^0 \to \overline{B^0})}$$

Standard Model preditions

$B_d^0 - \overline{B}_d^0$



$\Delta m_d \sim m_t^2 \cdot O(\lambda^6)$

Dominant contribution from top-loop:

$\eta_B = 0.55 \pm 0.01$

NLO QCD

$$\Delta m_d = \frac{G_F^2}{6\pi^2} m_B f_B^2 B_B (V_{td} V_{tb}^*)^2 m_W^2 \eta_B F\left(\frac{m_t^2}{m_W^2}\right)$$

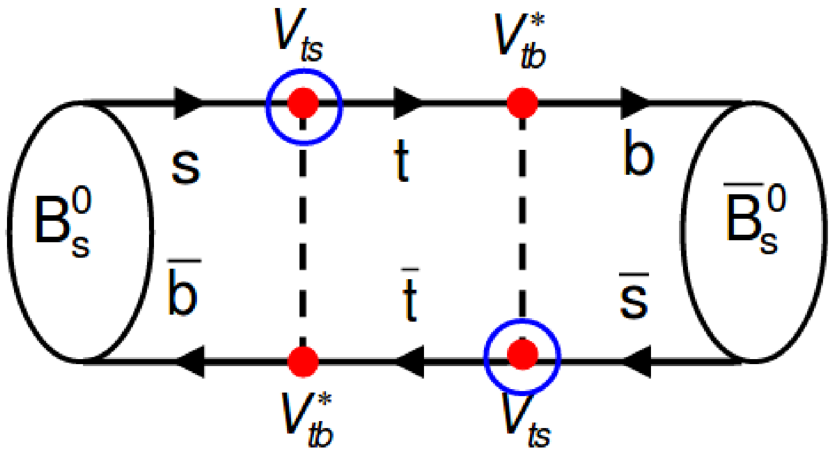
e.w. correction

$f_B^2 B_B = (235 \pm 33 \pm 12)^2 MeV^2$ from lattice QCD

Describes the binding of the quarks to a meson

Prediction for $B_s^0 - \overline{B_s^0}$ oscillation

$B_s^0 - \overline{B}_s^0$



$\Delta m_s \sim m_t^2 \cdot O(\lambda^4)$

$$\Delta m_s \sim (V_{ts} V_{tb}^*)^2$$

Oscillation is about 35 times stronger than in the case of B_d
(V_{ts} much larger than V_{td})

B oscillation:
Deactivation of GIM(*) suppression because of large top mass:

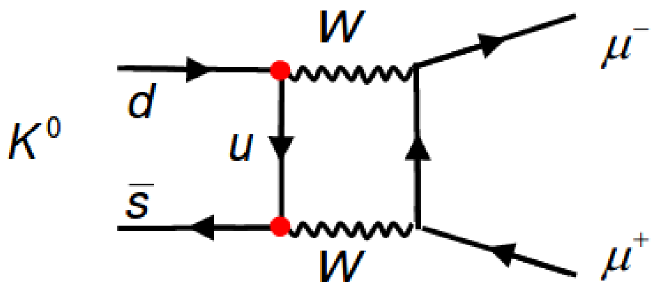
What would be the mixing if all quarks had the same masses?

(*) Glashow, Iliopoulos, Maiani, 1970, see next page.

Missing FCNC and GIM mechanism

Historical retrospect

FCNC in the 3 quark model: $K^0 \rightarrow \mu^+\mu^-$



$$M \sim \sin\theta_c \cos\theta_c$$

Theoretically one predicts large BR, in contradiction with experimental limits for this decay:

$$\frac{BR(K_L \rightarrow \mu^+\mu^-)}{BR(K_L \rightarrow all)} = (7.2 \pm 0.5) \cdot 10^{-9}$$

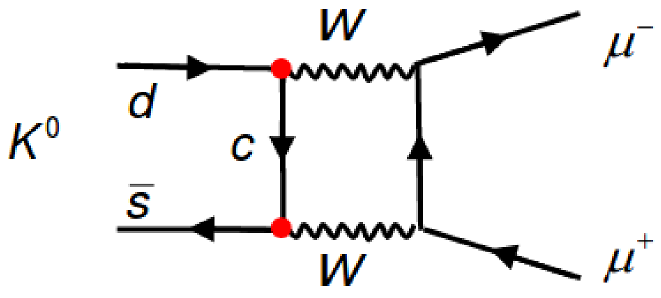
Proposal by Glashow, Iliopoulos, Maiani, 1970:

GIM

There exists a fourth quark which builds together with the s quark a second doublet:

$$\begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ -\sin\theta_c \cdot d + \cos\theta_c \cdot s \end{pmatrix}$$

→ Additional Feynman-Graph for $K^0 \rightarrow \mu\mu$ which compensates the first one:

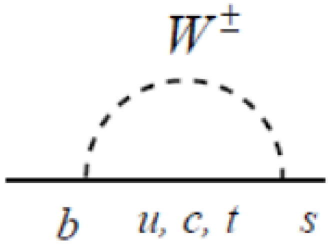


$$M \sim -\sin\theta_c \cos\theta_c$$

Prediction of a fourth quark:
Mass prediction BR=f(m_c,…)

GIM Suppression

Example: FCNC process b→s (“penguin process” as in B→K*γ)



$$\mathcal{A}(b \to s)_{\rm SM} = V_{ub}V_{us}^* A_u + V_{cb}V_{cs}^* A_c + V_{tb}V_{ts}^* A_t$$

where A_q denote the sub-amplitudes for the 3 possible internal quark. A_q depend on the quark masses only:

$$A_q = A(m_q^2/M_W^2)$$

Using the unitarity of the CKM matrix, especially: $\sum_i V_{ib}V_{is}^* = 0$

the total amplitude can be rewritten:

$$\mathcal{A}(b \to s)_{\rm SM} = V_{tb}V_{ts}^*(A_t - A_c) + V_{ub}V_{us}^*(A_u - A_c)$$

In case of approx. equal quark masses, total amplitude vanishes: GIM suppression.

For large top quark mass: $A(b \to s)_{SM} = V_{tb}V_{ts}^* \cdot \frac{m_t^2}{m_W^2}$ GIM suppression inactive

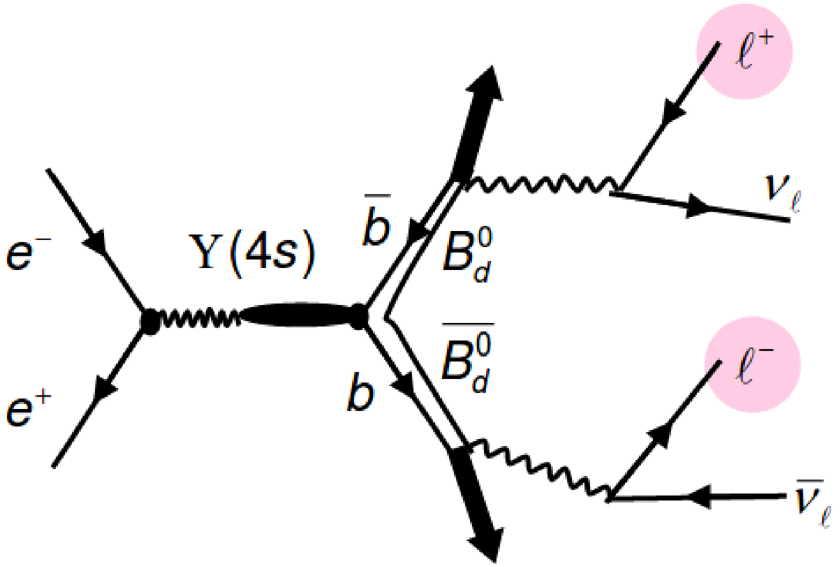
Discovery of B^0 mixing

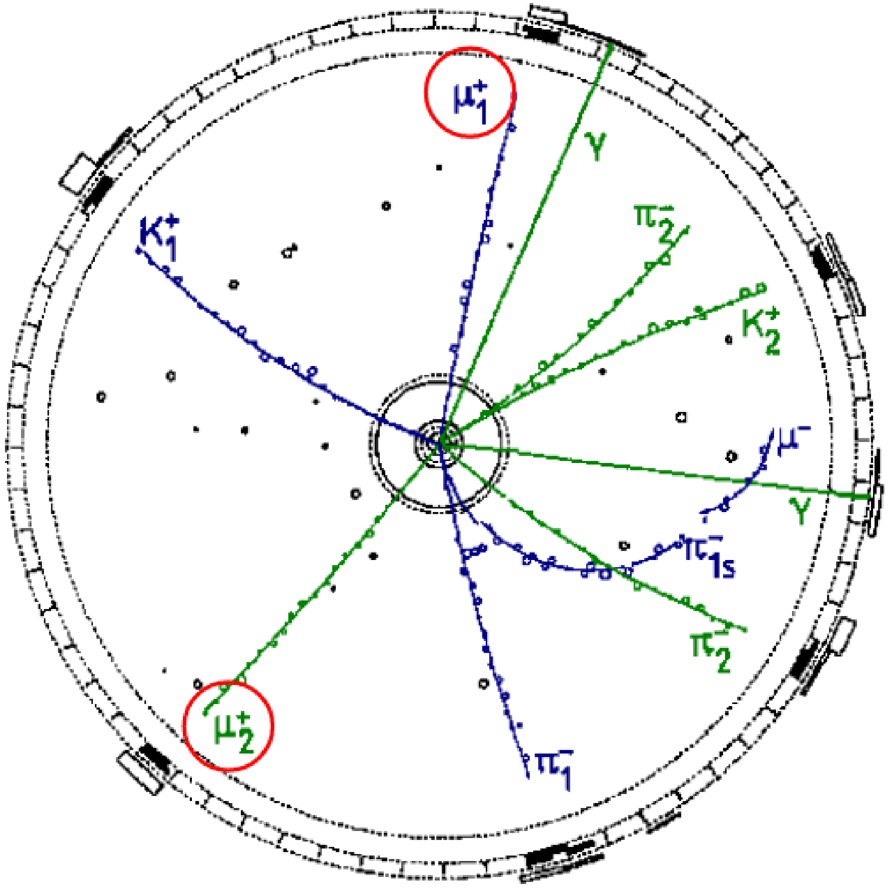
ARGUS 1987

First e^+e^- B factory at DESY:

at $\sqrt{s} = 10.58$ GeV :
$e^+e^- \to \Upsilon(4S) \to B^0\bar{B}^0$

$\sigma(B\bar{B}) \approx 1\text{nb}$





$B^0 \to D^{*-}\mu^+\nu_\mu$, $D^{*-} \to \bar{D}^0\pi_s^-$, $\bar{D}^0 \to K^+\pi^-$

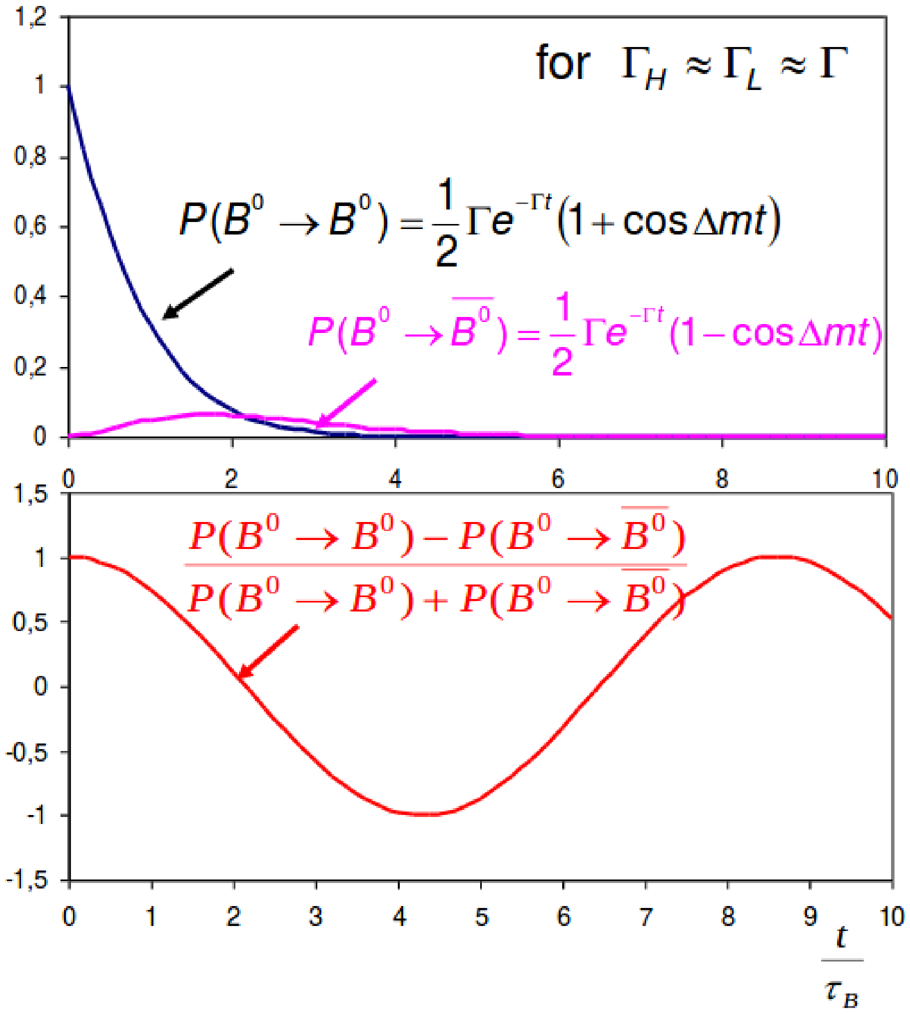
$B^0 \to D^{*-}\mu^+\nu_\mu$, $D^{*-} \to D^-\pi^0$, $\pi^0 \to \gamma\gamma$, $D^- \to K^+\pi^-\pi$

resulted in first lower bound on the top mass

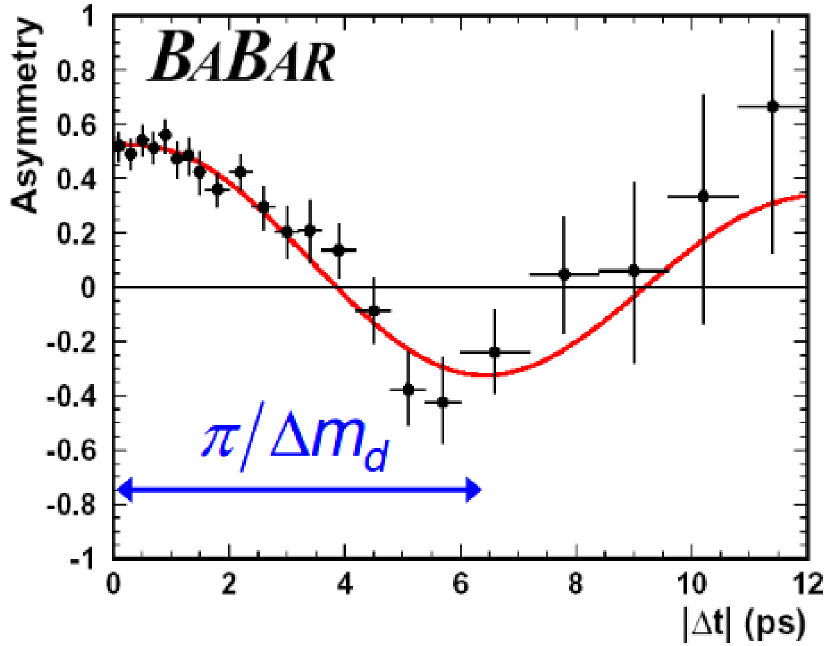
Unmixed: $B^0\bar{B}^0 \to \ell^+\ell^-$

Mixed:
$B^0B^0 \to \ell^+\ell^+$
$\bar{B}^0\bar{B}^0 \to \ell^-\ell^-$
Same charge

Experimental Status of B_d meson mixing



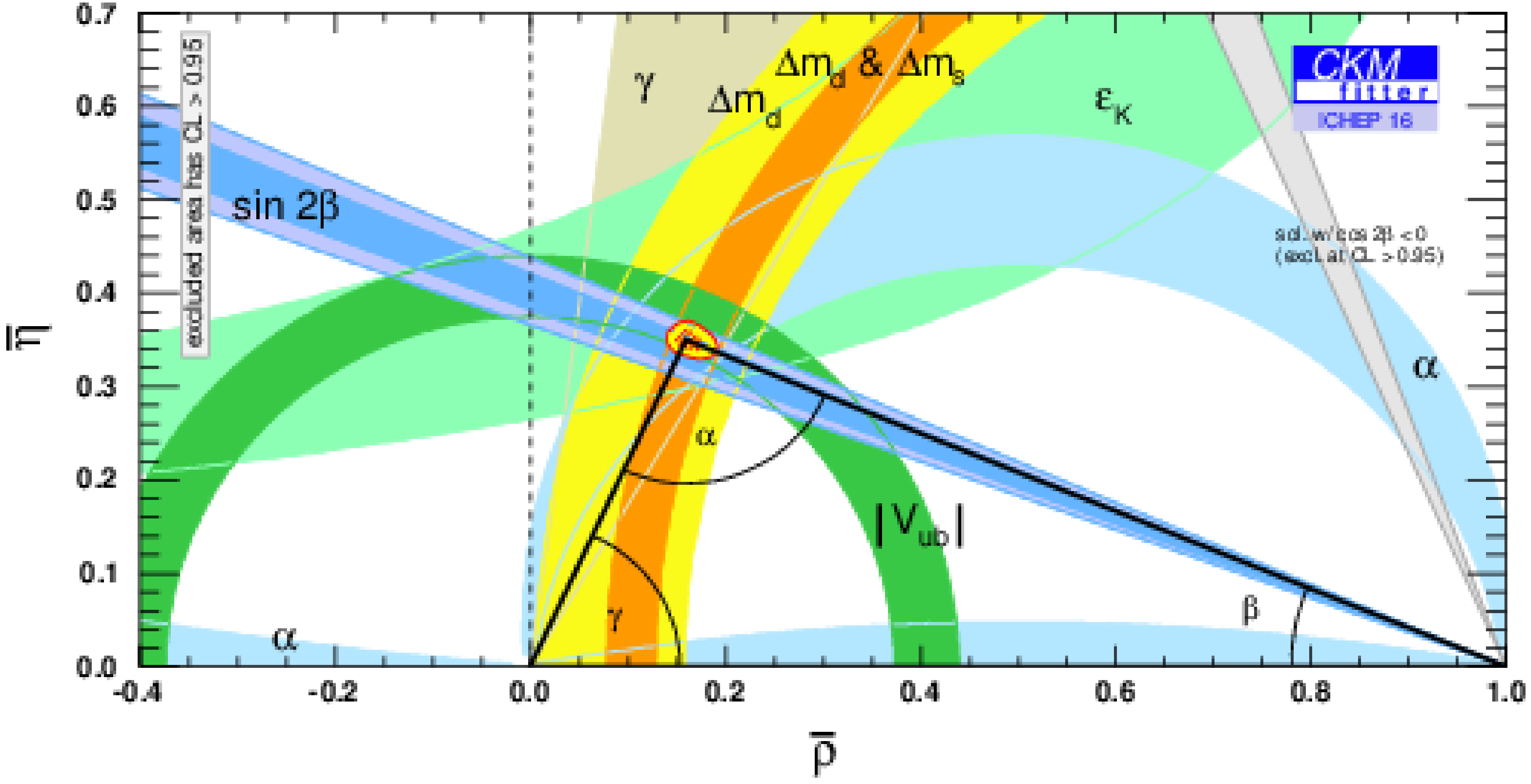
$$A = \frac{unmixed - mixed}{unmixed + mixed}$$



$$\Delta m_d = 0.506 \pm 0.006 \pm 0.004\,\mathrm{ps}^{-1}$$

$$\approx \frac{0.774}{\tau_B}$$

Status of the unitarity triangle



Sofar discussed only length of sides (absolute values of CKM matrix elements), angles come from measurements of CP violation