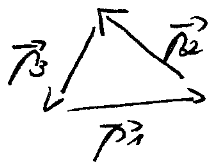
Dalitz plot

– how to visualize matrix elements

3-body decay $\quad A \to 1 + 2 + 3$

in CMS: $\quad \vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$



3 momenta define one plane

masses of initial and final state particles are given

=> 3 momenta define final state

=> $3 \times 3 = 9$ parameters

energy and momentum conservation $\quad p_A = p_1 + p_2 + p_3$

=> 4 constraints

=> 5 independent variables describe the decay.

Choice: 2 angles to define normal vector of decay plane, 1 angle defines orientation of momentum triangle in the plane

$$\Gamma_{fi} = \frac{(2\pi)^4}{\underset{\substack{\uparrow \\ m_A}}{2E_A}} \int |M_{fi}|^2 \, \delta(p_A - p_1 - p_2 - p_3) \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} \frac{d^3p_3}{2E_3(2\pi)^3}$$

If A scalar or spin averaged matrix element => $|M_{fi}|^2$ does not depend on the 3 angles!

$$\Rightarrow \Gamma_{fi}(E_1, E_3) = \frac{1}{64\pi^3} \frac{1}{m_A} \int |M_{fi}|^2 \, dE_1 \, dE_3$$

(E_1, E_2: choice of remaining two independent variables)

$$m_{23}^2 = (E_2+E_3)^2 - (\vec{p}_2+\vec{p}_3)^2 = (m_A - E_1)^2 - \vec{p}_1^{\,2}$$

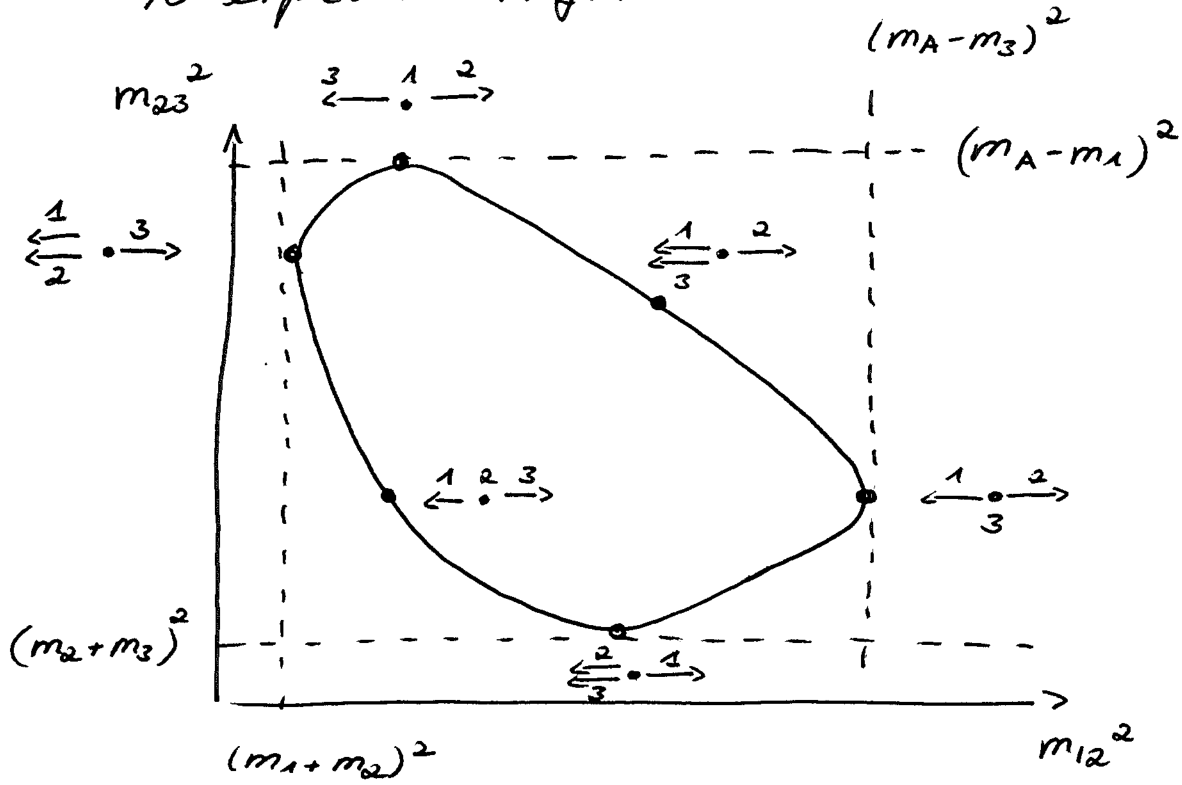
$$= m_A^2 + m_1^2 - 2 m_A E_1$$

$$\frac{d m_{23}}{d E_1} = -2 m_A$$

$$\Rightarrow \quad dE_1\, dE_3 = \frac{1}{4 m_A^2}\, d m_{23}^2\, d m_{12}^2$$

$$d\Gamma(m_{23}^2, m_{12}^2) = \frac{1}{256\, m_A^3} |M_{fi}|^2\, d m_{12}\, d m_{23}$$

$\Rightarrow$ Dalitz plot : experimental method to explore $|M_{fi}|^2$



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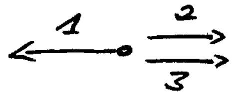
max of $m_{23}^2 \Rightarrow E_1$ minimal $\Rightarrow \vec{p}_1 = 0$

$\max(m_{23}^2) = (m_A - m_1)^2$

min of $m_{23}^2 \Rightarrow E_1$ maximal $\Rightarrow |\vec{p}_1|$ maximal

$\min(m_{23}^2) = (m_2 + m_3)^2$



$V_2 = V_3$

How do you expect the events to be distributed in the kinematically allowed area of the Dalitz plot?

If $|M_{fi}|$ does not depend on E_1, E_2, Dalitz plot is evenly distributed, if not, sign of intermediate resonances

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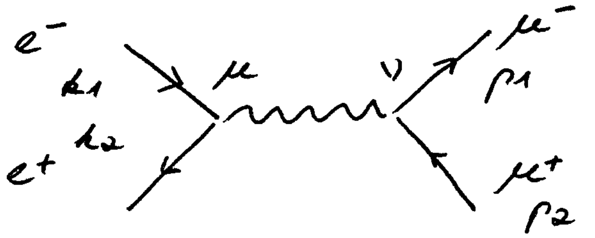
e^+e^- scattering experiments

Remember:



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$$-i\mathcal{M}_{fi} = \underbrace{[\bar{v}_2 (ieQ_e\gamma^\mu) u_1] \frac{-g_{\mu\nu}}{q^2} [\bar{u}_3 (ieQ_\mu \gamma^\nu) v_4]}_{\text{Spinors describe a specific spin state of the fermions}}$$

For non-polarized incoming particles and for non-observation of final state spin one observes unpolarized cross-sections.

$\Rightarrow$ need to average over possible initial spin states and sum over all final spin states

$$|\overline{\mathcal{M}_{fi}}|^2 = \frac{1}{4} \sum_{s_e, s_e'} \sum_{s_\mu, s_\mu'} |\mathcal{M}_{fi}|^2$$

"—" sign for spin averaged

$$= 2e^4 \frac{t^2+u^2}{s^2}$$

use Mandelstam variables

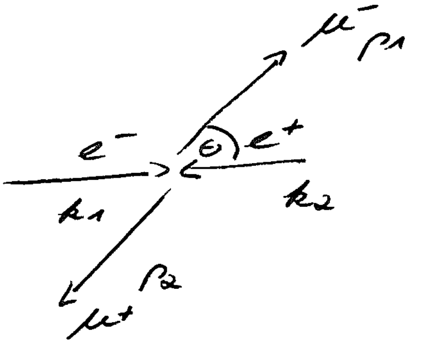
[Note we had in last lecture
$|\mathcal{M}_{fi}|^2 = \sum\sum_{s_e s_e' s_\mu s_\mu'} |\mathcal{M}_{fi}|^2 = 8e^4 \frac{t^2+u^2}{s^2}$]

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\frac{1}{s}|M_{fi}|^2$$

$$= \frac{e^4}{32\pi^2}\frac{1}{s}\frac{t^2+u^2}{s^2}$$

Compute s, t, u in the limit of very high relativistic particles

CMS



$$|\vec{p}_i| = |\vec{p}_f| = |\vec{p}_1| = |\vec{p}_2| = |\vec{k}_1| = |\vec{k}_2|$$

$$m_e, m_\mu \sim 0 \Rightarrow p_1^2 = p_2^2 = p_3^2 = p_4^2 = 0$$

$$s = (k_1+k_2)^2 = 4E_i$$

$$t = (k_1-p_1)^2 = -2k_1p_1 = -2(E_1E_3 - \vec{k}_i\vec{p}_i)$$

$$= -2E_i^2(1-\cos\theta)$$

$$u = (k_1-p_2)^2 = -2E_i^2(1+\cos\theta)$$

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{32\pi^2}\frac{1}{s}\left[(1-\cos\theta)^2 + (1+\cos\theta)^2\right]$$

$$= \frac{e^4}{32\pi^2}\frac{1}{s}(1+\cos^2\theta) \qquad e^2 = 4\pi\alpha$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{cms} = \underbrace{\frac{\alpha^2}{4s}}_{\text{flux factor}}\underbrace{(1+\cos^2\theta)}_{\text{matrix element}}$$

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$$\sigma_{tot} = \frac{4\pi\alpha^2}{3s} \simeq \frac{87\text{ nb GeV}^2}{s}$$

What is the origin of the angular distribution?

$$\bar{u}\gamma^\mu u = (\bar{u}_L + \bar{u}_R)\gamma^\mu(u_L + u_R)$$

$$= \bar{u}_L\gamma^\mu u_L + \underbrace{\cancel{\bar{u}_L\gamma^\mu u_R}}_{=0} + \underbrace{\cancel{\bar{u}_R\gamma^\mu u_L}}_{=0} + \bar{u}_R\gamma^\mu u_R$$

Proof:

$$u_L = \tfrac{1}{2}(1-\gamma^5)u \qquad u_R = \tfrac{1}{2}(1+\gamma^5)u$$

$$\bar{u}_L = u_L^\dagger\gamma^0 = u^\dagger\tfrac{1}{2}(1-\gamma^5)^\dagger\gamma^0$$

$$= \bar{u}\,\tfrac{1}{2}(1+\gamma^5)$$

$$[\gamma^5 = \gamma^{5\dagger} \quad \gamma^5\gamma^0 = -\gamma^0\gamma^5]$$

$$\bar{u}_R = \bar{u}\,\tfrac{1}{2}(1-\gamma^5)$$

$$\bar{u}_L\gamma^\mu u_R = \tfrac{1}{4}\bar{u}(1+\gamma^5)\gamma^\mu(1+\gamma^5)u$$

$$= \tfrac{1}{4}\bar{u}\gamma^\mu(1-\gamma^5)(1+\gamma^5)u$$

$$= 0$$

Illustration: ⬅ spin $\vec{s}$

$\longrightarrow$ momentum $\vec{p}$

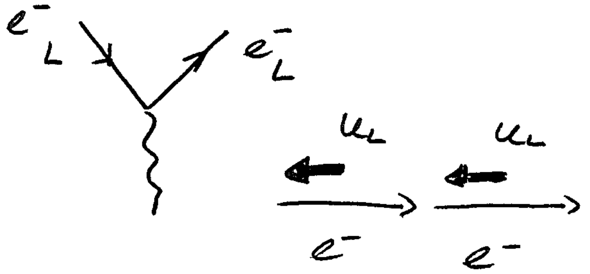
Helicity $h = \dfrac{\vec{s}\,\vec{p}}{|\vec{s}||\vec{p}|}$

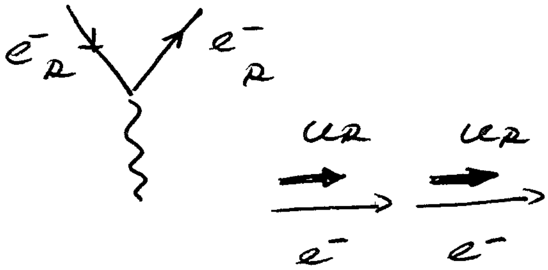
$h = +1$ — massless particles $\Rightarrow$ right handed chirality

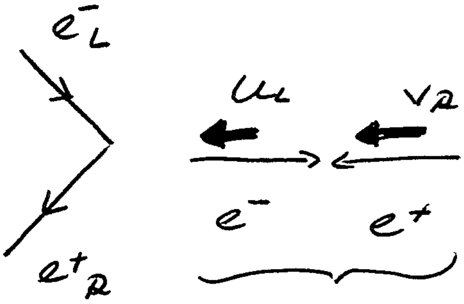
$h = -1$ — left handed chirality

Four allowed combinations contribute to process $e^+e^- \to \mu^+\mu^-$

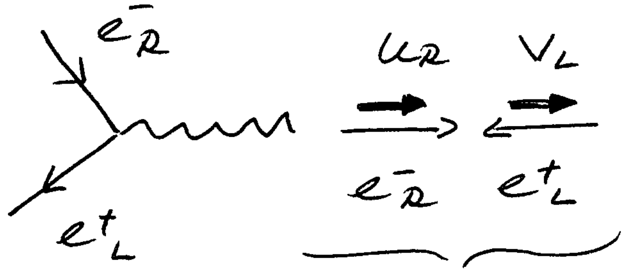
$LR \to LR$
$RL \to LR$
$LR \to RL$
$RL \to RL$







photon
spin = 1 !



photon
spin = 1

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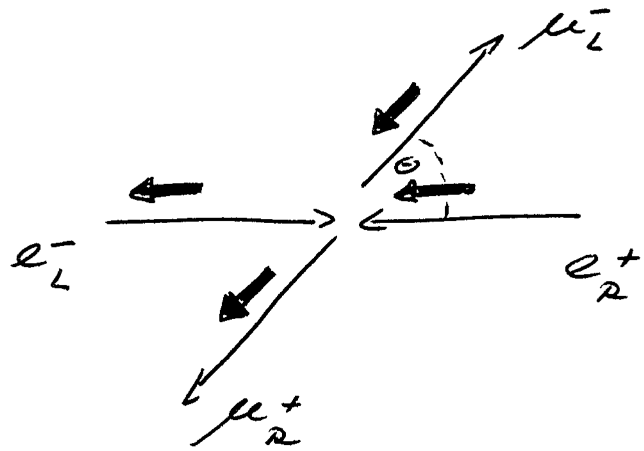
Angular distribution:

Scattering can be treated as change of quantization axis.

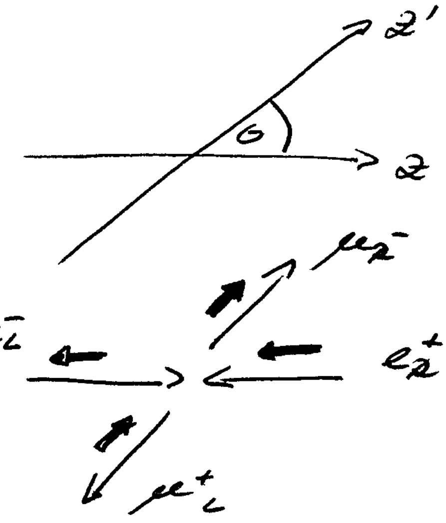
CMS



rotation

Axis z $\longrightarrow$ Axis z'

$$\left.\begin{matrix}J=1\\ m_z=-1\end{matrix}\right\} d^J_{-1,-1} \left\{\begin{matrix}J=1\\ m_{z'}=-1\end{matrix}\right.$$



$$\left.\begin{matrix}J=1\\ m_z=-1\end{matrix}\right\} d^J_{1,-1} \left\{\begin{matrix}J=1\\ m_{z'}=+1\end{matrix}\right.$$

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$d^1_{-1,-1} = d^1_{1,1} = \frac{1}{2}(1+\cos\theta)$ [LR -> LR, RL -> RL]

$d^1_{-1,1} = d^1_{1,-1} = \frac{1}{2}(1-\cos\theta)$ [LR -> RL, RL -> LR]

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}(1+\cos\theta)^2 + \frac{1}{4}(1-\cos\theta)^2 \sim 1+\cos^2\theta$$

Angular distribution is an effect of vector coupling i.e. γ^μ!

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