Quark Flavour Physics

The known fundamental matter comes in three generations, carrying the same charges under the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$

Flavour is the feature that distinguish the generations.

Open question in flavour physics:

- mass hierarchie $m(\nu) < eV$, $m(u) \sim MeV$, $m(t) \sim 171\,Ge$
- structure of the interaction

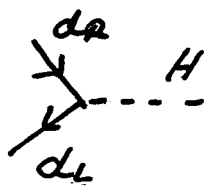
Both effects parametrized in the SM, but origin not understood.

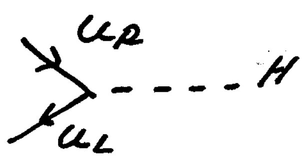
Additional interessting effect: CP violation only observed in flavor changing hadronic processes

1) Flavour within the SM

After spontanous breaking of EW symmetry the quark's Yukawa terms give rise to masses and mixing:

$$\mathcal{L}_Y^{Quark} = -\frac{v}{\sqrt{2}}\left(\bar{d}_L Y_d d_R + \bar{u}_L Y_u u_R\right) + h.c.$$





Y_d, Y_u: 3×3 matrices

$$d_L = \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \qquad u_L = \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L$$

The CKM matrix:

Strong and EM Interaction does not distinguish between (u, c, t) and (d, s, b) quarks.

Coupling strengths are the same for all up-type and all down-type quarks.

⇒ Strong and EM eigenstates are degenerated

⇒ can define mass eigenstates which are eigenstates of strong EM interaction as well

However flavour eigenstates ≠ mass eigenstates

Rewrite $\mathcal{L}_Y^{\text{quark}}$ in form of mass eigenstates:

$$M_u = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} = \frac{v}{\sqrt{2}} V_{L,u} Y_u V_{R,u}^\dagger$$

$$M_d = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} = \frac{v}{\sqrt{2}} V_{L,d} Y_d V_{R,d}^\dagger$$

$V_{A,q} V_{A,q}^\dagger = 1$ unitary matrices

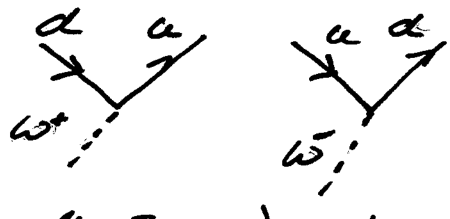
$$\mathcal{L}_Y^{\text{quarks}} = -\bar{\tilde{d}}_L M_d \tilde{d}_R - \bar{\tilde{u}}_L M_u \tilde{u}_R + h.c.$$

↑ mass eigenstates

$$\Rightarrow \quad \bar{\tilde{d}}_L = \bar{d}_L V_{L,d}^\dagger \qquad \tilde{d}_R = V_{R,d}\, d_R$$

$$\bar{\tilde{u}}_L = \bar{u}_L V_{L,u}^\dagger \qquad \tilde{u}_R = V_{R,u}\, u_R$$

charged current interaction



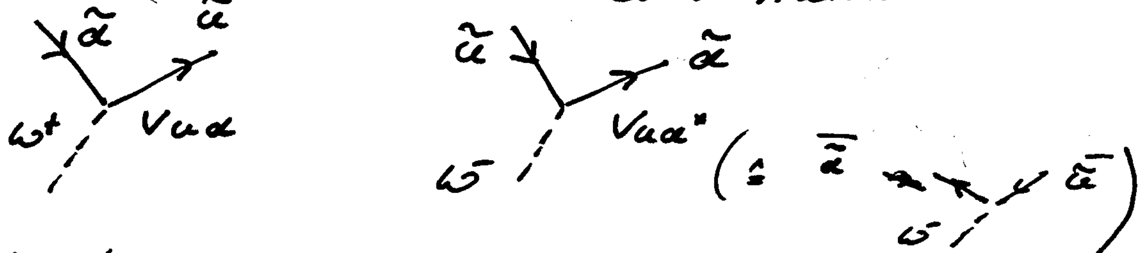
$$\mathcal{L}_{CC} = \frac{-g}{\sqrt{2}} \left(\bar{u}_L \gamma^\mu W_\mu^+ d_L + \bar{d}_L \gamma^\mu W_\mu^- u_L \right) + h.c.$$

$$\mathcal{L}_{CC} = \frac{-g}{\sqrt{2}} \left(\bar{\tilde{u}}_L V_{L,u} \gamma^\mu W^+_\mu V_{L,d}^\dagger \tilde{d}_L + \bar{\tilde{d}}_L V_{L,d} \gamma^\mu W^- V_{L,u}^\dagger \tilde{u}_L \right.$$

$$= \frac{-g}{\sqrt{2}} \left(\bar{\tilde{u}}_L \gamma^\mu W^+ \underbrace{V_{L,u} V_{L,d}^\dagger}_{V_{CKM}} \tilde{d}_L + \bar{\tilde{d}}_L \gamma^\mu W^- \underbrace{V_{L,d} V_{L,u}^\dagger}_{V_{CKM}^\dagger} \tilde{u}_L \right)$$

$V_{A,q}$ are unitary matrices $\Rightarrow$ V_{CKM} is a unitary matrix
If up-type and down-type quark matrices cannot be diagonalised simultaneously, there is a net effect of the basis change of the charge current interaction:

If $V_{L,u} \neq V_{L,d}^\dagger$ $\Rightarrow$ non trivial mixing matrix.
Flavor structure is encoded in CKM matrix



[Why don't we see quark mixing for neutral currents?]

Apply CP transformation. $W^- \to W^+$
incoming particle
$\Rightarrow$ outgoing particle

$$\mathcal{L}_{CC}^{CP} = \frac{-g}{\sqrt{2}} \left(\bar{\tilde{d}}_L \gamma^\mu W^-_\mu V_{CKM}^T \tilde{u}_L + \bar{\tilde{u}}_L \gamma^\mu W^+_\mu V_{CKM}^* \tilde{d}_L \right)$$

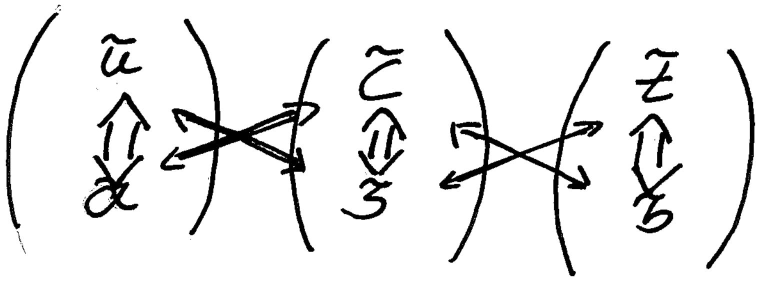
If $V_{CKM} = V_{CKM}^*$ $\Rightarrow$ $\mathcal{L}_{CC}^{CP} = \mathcal{L}_{CC}$
$\Rightarrow$ CP conservation

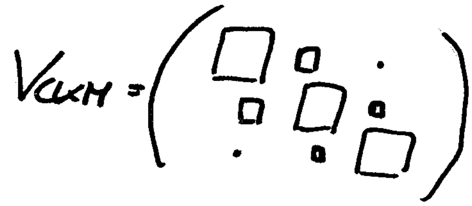
Complex phase of CKM matrix is prerequisit for CP-violation!

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

CKM $\hat{=}$ Cabibo - $\underbrace{\text{Kobayashi - Maskawa}}$

Nobel prize in 2008
(model for CPV + prediction of third quark family)





size of "□" represents strength of coupling

Transition within the same family is CKM favoured!

Parameters of the CKM matrix

9 complex numbers => 18 parameters
- 9 unitarity conditions
- 5 relative quark phases

4 independent parameters

3 rotation angles + 1 phase
$\Uparrow$
CP-Violation!

Phases of left-handed quark fields are unobservable
=> possible to redefine

$u_L \to e^{i\varphi_u} u_L \qquad c_L \to e^{i\varphi_c} c_L \qquad t_L \to e^{i\varphi_t} t_L$

$d_L \to e^{i\varphi_d} d_L \qquad s_L \to e^{i\varphi_s} s_L \qquad b_L \to e^{i\varphi_b} b_L$

RH quark fields are rotated simultaneously to keep mass term real.

$$V_{CKM} \to \begin{pmatrix} e^{-i\varphi_u} & & \\ & e^{-i\varphi_c} & \\ & & e^{-i\varphi_t} \end{pmatrix} V_{CKM} \begin{pmatrix} e^{i\varphi_d} & & \\ & e^{i\varphi_s} & \\ & & e^{i\varphi_b} \end{pmatrix}$$

$$V_{\alpha j} \to \exp[i(\varphi_j - \varphi_\alpha)] V_{\alpha j}$$

6 phases => 5 relativ phase differences

One possible parametrisation of the CKM matrix is

$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$= \ldots$ see slides

$c_{ij} = \cos\theta_{ij} \qquad s_{ij} = \sin\theta_{ij}$

This parametrisation is exact and completely general.
Phenomenologically, the CKM matrix is hierarcical

$\theta_{13} \ll \theta_{23} \ll \theta_{12} \ll 1$ and has an order $O(1)$ phase.

A useful parametrisation making the hierarchy transparent is due to Wolfenstein:

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & +\lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & +A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

$\lambda \approx 0.225$ $\quad A \sim 0.81$ $\quad \bar{\rho} = 0.14$ $\quad \bar{\eta} = 0.34$

In total 10 parameters in the quark flavour sector of the SM: 4 parameters of the CKM matrix + 6 quark masses.

Unitarity triangles

The unitarity of the CKM matrix $V_{CKM} V_{CKM}^{+} = 1$ leads to experimental testable constraints.
Three of the nine relations are of the type:

$$|V_{xd}|^2 + |V_{xs}|^2 + |V_{xb}|^2 = 1$$

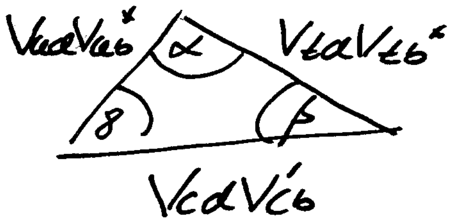
this expresses the fact that the sum of the propabilities that up-type and down-type quark d, s, b is 1.

Taking the first and the third column of V_{CKM} one finds:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

„db" triangle

This can be depicted as triangle in the complex plane.



$$\alpha = \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right]$$

$$\beta = \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right]$$

$$\gamma = \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]$$

There exist 6 different triangles, all with same area

The „db" triangle is called THE unitarity triangle. It was extensively tested at the B-Factories BELLE and BaBar.

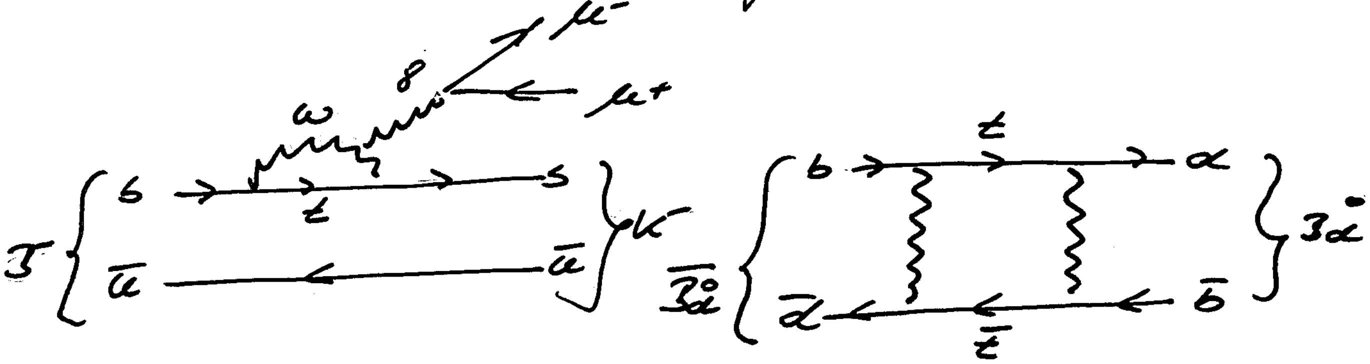
It is customary to rescale the unitarity triangle by normalisation of the triangle basis to one. For the rescaled triangle the position of the apex is given by the complex number $\bar{\rho} + i\bar{\eta}$

For current status of the CKM matrix see slides

2 Mixing of neutral mesons

The quark mixing results into several interesting "loop" effects.

Flavor changing neutral currents are allowed at loop level, however forbidden at tree-level



Neutral meson mixing

$|P^0\rangle$: $K^0 = |d\bar{s}\rangle$ $D^0 = |\bar{u}c\rangle$ $B_d^0 = |\bar{b}d\rangle$ $B_s^0 = |\bar{b}s\rangle$

$|\bar{P}^0\rangle$: $\bar{K}^0 = |\bar{d}s\rangle$ $\bar{D}^0 = |u\bar{c}\rangle$ $\bar{B}_d^0 = |b\bar{d}\rangle$ $\bar{B}_s^0 = |b\bar{s}\rangle$

discovery	1960	2007 [very slow]	1987	2006 [very fast]

Mixing Phenomenology (applies to all neutral mesons)

$$i\frac{d}{dt}\begin{pmatrix} B^0(t) \\ \bar{B}^0(t) \end{pmatrix} = \left(M - \frac{i}{2}\Gamma \right) \begin{pmatrix} B^0(t) \\ \bar{B}^0(t) \end{pmatrix}$$

Flavour states are no mass eigenstates

Mass eigenstates $\quad |B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle$

with m_L, Γ_L „light"

$|B_H\rangle = p|B^0\rangle - q|\bar{B}^0\rangle$

with m_H, Γ_H

$$|B_{H,L}(t)\rangle = |B_{H,L}\rangle\, e^{-i m_{H,L} t}\, e^{-\frac{1}{2}\Gamma_{H,L} t}$$

Flavour eigenstates:

$$|B^0\rangle = \frac{1}{2p}\left(|B_L\rangle + |B_H\rangle\right) \qquad \bar{B}^0 = \frac{1}{2q}\left(|B_L\rangle - |B_H\rangle\right)$$

complex coefficients p, q : $|p|^2 + |q|^2 = 1$

$$|B^0(t)\rangle = \frac{1}{2p}\left(p|B^0\rangle + q|\bar{B}^0\rangle\right) e^{-i m_L t}\, e^{-\frac{1}{2}\Gamma_L t} + \frac{1}{2p}\left(p|B^0\rangle - q|\bar{B}^0\rangle\right) e^{-i m_H t}\, e^{-\frac{1}{2}\Gamma_H t}$$

$$A\left(B_0(t=0) \to B_0(t)\right) = \langle B^0 | B^0(t)\rangle$$

$$= \frac{1}{2}\, e^{-i m_L t}\, e^{-\frac{1}{2}\Gamma_L t} + \frac{1}{2}\, e^{-i m_H t}\, e^{-\frac{1}{2}\Gamma_H t}$$

$$P(B^0 \to B^0) = |A|^2 = \frac{1}{4}\left[e^{-\Gamma_L t} + e^{-\Gamma_H t} + 2e^{-(\Gamma_L+\Gamma_H)t/2} \cos\Delta m t\right.$$

$\Delta m = m_H - m_L$

$$P(B^0 \to \bar{B}^0) = |A(B^0 \to \bar{B}^0)|^2 = \frac{1}{4}\left|\frac{q}{p}\right|^2 \left[e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_L+\Gamma_H)} \cos\Delta m t\right.$$

$$P(\bar{B}^0 \to B^0) = |A(\bar{B}^0 \to B^0)|^2 = \frac{1}{4}\left|\frac{p}{q}\right|^2 \left[e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_L+\Gamma_H)t/2} \cos\Delta m t\right.$$

CP-Violation in mixing:

$$P(B^0 \to \overline{B^0}) \neq P(\overline{B^0} \to B^0)$$

$$\Rightarrow \left|\frac{q}{p}\right| \neq \underline{1}$$