name: **group:**

Exercise Sheet 9 – Particle Physics – SS 2016

hand in: Tue 14$^{\text{th}}$ June (after the lecture or at INF 226, 3.104 by 4 pm)

9.1 Running of α_s (6 points)

a) Prove that the value of μ in

$$\alpha_s(|q^2|) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi}(11n - 2f)\ln\frac{|q^2|}{\mu^2}} \tag{1}$$

is arbitrary. The quantity n is the number of colours and f is the number of active flavours.
Hint: Suppose physicist A uses μ_a and physicist B uses μ_b. Assume that A's version of the equation is correct, and prove that B's version is also correct.

b) Show that with $\ln\Lambda^2 = \ln\mu^2 - \dfrac{12\pi}{(11n-2f)\alpha_s(\mu^2)}$, in which Λ and μ are in units of GeV, (1) can be written

$$\alpha_s(|q^2|) = \frac{12\pi}{(11n - 2f)\ln\frac{|q^2|}{\Lambda^2}} \tag{2}$$

c) The parameter Λ depends on the number of active flavours f, i.e., the number of quark flavours that contribute in the higher order loops that lead to the running of α_s. Assume that only those quarks contribute which have a mass $m_q \leq \sqrt{|q^2|}/2$. Contributions from heavier virtual quarks are strongly suppressed.
Assume the b-quark mass to be 5 GeV and use $\Lambda(f=5) = 0.2$ GeV. Calculate $\Lambda(f=4)$ from the requirement that α_s has to be continuous.

9.2 Measurements of α_s (8 points)

The strong coupling α_s has been measured in the reaction $e^+e^- \to$ hadrons. The ratio to muon pair production is called R_{had}:

$$R_{\text{had}} = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

Including first order QCD corrections to the QED cross section, the ratio is given by

$$R_{\text{had}}^{(1)} = 3\sum_i \left(\frac{Q_i}{e}\right)^2 (1 + \frac{\alpha_s}{\pi}) \tag{3}$$

in which the sum extends over active quark flavours ($\sqrt{s} \geq 2m_q$).

a) Which QCD correction Feynman diagrams contribute in equation 3?

b) The CLEO Collaboration has measured[1] $R_{\text{had}}(\sqrt{s} = 10.52\,GeV) = 3.56 \pm 0.01$. What is (according to (3)) the value and uncertainty of α_s? Hint: only the four lightest quark flavours contribute.

c) Find and describe briefly two other methods for measuring α_s (search the literature or the internet).

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[1] Ammar et al., Phys. Rev. D57 (1998) 1350, `http://arxiv.org/abs/hepex/9707018`

9.3 Colour factor and quark–anti-quark potential (6 points)

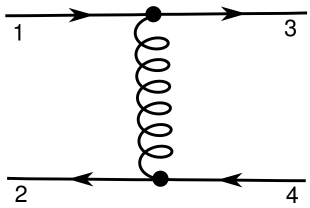
The potential energy U between a quark and an antiquark at short range (1 gluon exchange) is given by

$$U(r) = -\frac{\alpha_s}{r} f$$

with r the distance between the quarks and the colour factor

$$f = \frac{1}{4} \sum_{\alpha=1}^{8} \left(c_3^{+} \lambda^{\alpha} c_1\right) \left(c_2^{+} \lambda^{\alpha} c_4\right)$$

in which c_i is the colour vector of particle i ($i = 1, \ldots, 4$; see diagram):



$$c(R) = c(\overline{R}) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \qquad c(G) = c(\overline{G}) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \qquad c(B) = c(\overline{B}) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

and the λ^{α} are the Gell-Mann matrices.

Consider the colour configuration $q(R) + \bar{q}(\overline{G}) \to q(R) + \bar{q}(\overline{G})$ in the above diagram.

a) Which gluon(s) contribute(s) to this exchange?
b) Calculate f.
c) Is the force between the quarks repulsive or attractive?
d) Bonus point: Sketch the quark-antiquark potential beyond the short distance regime in the limit of heavy quarks. Describe the difference to the e^+e^- potential and comment on the physical consequences.