name: **group:**

Exercise Sheet 8 – Particle Physics – SS 2016

hand in: Tue 14th June (after the lecture or at INF 226, 3.104 by 4 pm)

8.1 Isospin multiplets (6 points)

Mesons are bound states of a quark and an antiquark ($q\bar{q}'$). Consider combinations of u and d quarks with $\bar{u}$ and $\bar{d}$ antiquarks. These are the only quarks that carry (non-vanishing) isospin.

The isospin combinations (upper entry: $I_3 = 1/2$, lower entry: $I_3 = -1/2$)

$$\begin{pmatrix} u \\ d \end{pmatrix} \otimes \begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} \tag{1}$$

yield composed isospin states $|I, I_3\rangle$, an isospin triplet (the pions) and an isospin singlet (the η meson).

Isospin ladder operators act on these composed states as follows:

$$I_{\pm} |I, I_3\rangle = \sqrt{I(I+1) - I_3(I_3 \pm 1)}\, |I, I_3 \pm 1\rangle \,. \tag{2}$$

a) Write the pions and the η meson in terms of quark-antiquark states.
b) What is the isospin I and its third component I_3 for π^+, π^0, π^-, and η?
c) Use the isospin ladder operators, starting from the π^+, to show that the pions form a triplet.
d) Use the isospin ladder operators on the η to show that it is a singlet state.

8.2 Magnetic moment of the proton and the neutron (6 points)

Using the proton wave funtion from the lecture,

a) Perform an ”isospin-reflection” to obtain the corresponding wave function for the neutron.

b) Determine the ratio between the neutron and proton magnetic moments and compare your result with the experimental value. *Hint:* The magnetic moments of baryons can be obtained by combining the Dirac-moments of the involved quarks (keep in mind the quark charges). Thus all that is needed is adding the quark moments with the correct sign in the absolute-squared baryon wave function.

HINT: The total magnetic moment of a baryon is the vector sum of the magnetic moments of the three constituent quarks $\vec{\mu} = \vec{\mu}^1 + \vec{\mu}^2 + \vec{\mu}^3$ where $\vec{\mu}^i$ is the magnetic moment operator which acts on the ith quark. The magnetic moment of the proton/neutron can thus be evaluated computing the expectation value of μ.

The magnetic moment of an u quark with spin up is e.g. given by $\mu_u =< u\uparrow |\mu_z| u\uparrow >= (+2/3)\frac{e\hbar}{m_u}(+1/2)$ Assume isospin symmetry ($m_u = m_d$).

c) Use the measured value of the proton magnetic moment to obtain an estimate of the quark mass (again assuming isospin symmetry).

8.3 Observation of a Hyperon with strangeness minus three (8 points)

The discovery of the Ω^- baryon reported in 1964 (see Physical Review Letters 12 (8) 204 (1964) or http://teachers.web.cern.ch/teachers/archiv/HST2001/bubblechambers/omegaminus.pdf) was a great triumph in the study of symmetries in the quark model, as its existence has been predicted by American physicist Murray Gell-Mann in 1962 and independently by Israeli physicist Yuval Ne'eman. The letter reports on the observation of only one event, which the authors believe to be an example of the production and decay of an Ω^-. Let's verify their arguments!

A photograph of the event and the corresponding line diagram is shown in the figure below. The table contains measured quantities of all tracks.

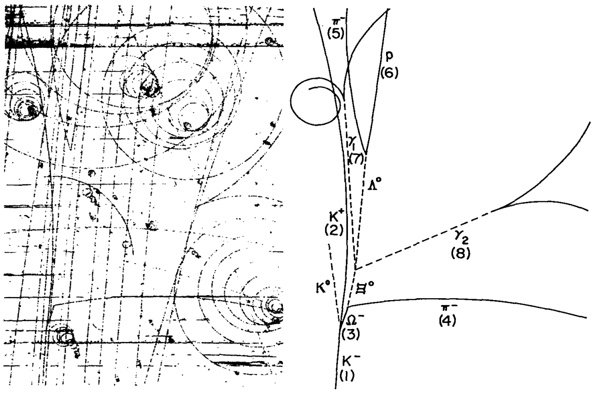

FIG. 2. Photograph and line diagram of event showing decay of Ω^-.

Table I. Measured quantities.

Track	Azimuth (deg)	Dip (deg)	Momentum (MeV/c)
1	4.2 ±0.1	1.1 ±0.1	4890 ±100
2	6.9 ±0.1	3.3 ±0.1	501 ±5.5
3	14.5 ±0.5	-1.5 ±0.6	...
4	79.5 ±0.1	-2.7 ±0.1	281 ±6
5	344.5 ±0.1	-12.0 ±0.2	256 ±3
6	9.6 ±0.1	-2.5 ±0.1	1500 ±15
7	357.0 ±0.3	3.9 ±0.4	82 ±2
8	63.3 ±0.3	-2.4 ±0.2	177 ±2

a) Download the publication and answer the following questions:

   i) What arguments led to the prediction of the observed particle?

   ii) How are p^+, γ, and $K^\pm$ identified in the experiment?

   iii) Why is it known that there are no non-observed particles involved in the measured Ω^- decay?

   iv) How is the strangeness ($S = -3$) of the Ω^- determined?

b) Use the measured track quantities as given in table I to derive the mass and momentum of the observed Ω^- and its neutral decay products. Compare your result with the numbers given in the publication and the numbers published in the PDG (only masses) and explain differences. Use the calculated momentum vector of the Ω^- to derive the azimuthal angle ϕ and the dip, and compare these values with the corresponding numbers of track 3.
   Hint: The dip of the tracks, as given in table I is defined w.r.t the xy-plane.

8.4 Optional: Isospin and ratios of nucleon decay rates (5 extra points)

The collision of a pion with a nucleon can lead to the formation of a nucleon resonance.

a) The resonances with isospin $I = 1/2$ are N^0 ($I_3 = -1/2$) and N^+ ($I_3 = 1/2$). Calculate the ratio $\dfrac{\Gamma(N^+ \to n\pi^+)}{\Gamma(N^+ \to p\pi^0)}$ from isospin invariance. Neglect phase space differences due to the different final state masses. You can find relevant Clebsch-Gordan coefficients at http://pdg.lbl.gov/2002/clebrpp.pdf.

b) A resonance X decays via the strong interaction to $n\pi^0$ (branching ratio BR = 18%) and to $p\pi^-$ (BR = 36%). What is the isospin of X?