name: **group:**

Exercise Sheet 7 – Particle Physics – SS 2016

hand in: Tue 7$^{\text{th}}$ June (after the lecture or at INF 226, 3.104 by 4 pm)

7.1 Parton Distribution functions (8 points)

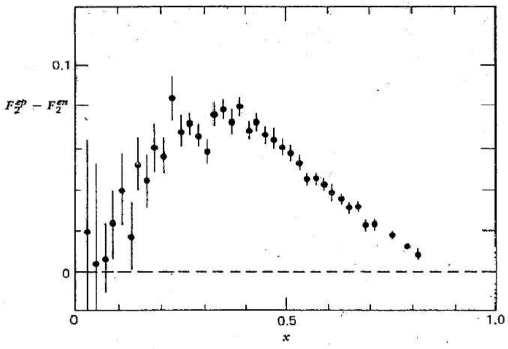

Fig.1.a: The difference $F_2^{ep} - F_2^{en}$ as a function of x, as measured in deep inelastic scattering.

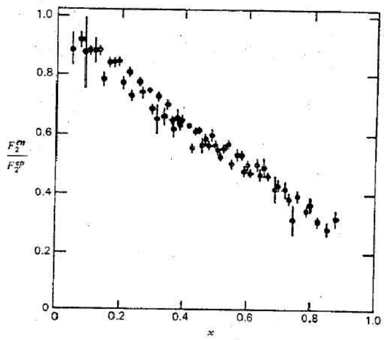

Fig.1.b: The ratio F_2^{en}/F_2^{ep} as a function of x, measured in deep inelastic scattering.

The momentum distribution of quarks and gluons inside the proton can be measured in deep-inelastic scattering. The proton structure function F_2^{ep} is given by:

$$F_2^{ep}/x = \left(\frac{2}{3}\right)^2 [u^p(x) + \bar{u}^p(x)] + \left(\frac{1}{3}\right)^2 [d^p(x) + \bar{d}^p(x)] + \left(\frac{1}{3}\right)^2 [s^p(x) + \bar{s}^p(x)] \tag{1}$$

in which x is the proton momentum fraction carried by the (anti-)quark and $q^p(x)$ ($\bar{q}^p(x)$) denotes the (anti-)quark momentum density of type q ($q = u, d, s$).

a) Express F_2^{ep} in terms of the valence and sea quark momentum densities, $q_v^p(x)$ and $q_s^p(x)$. Assume that the sea quark densities are identical for all quark and antiquark flavours.

b) Electrons are also used to measure the structure of neutrons and the neutron structure function F_2^{en} is defined analogously to F_2^{ep}. How are $u^p(x)$ and $d^p(x)$ connected to $u^n(x)$ and $d^n(x)$?

c) Express F_2^{en} in terms of $q_v^p(x)$ and $q_s^p(x)$.

d) Figure 1a shows the result of a measurement of the difference $F_2^{ep}(x) - F_2^{en}(x)$. What can be learned from this about the valence and sea quark densities in the nucleons?

e) Figure 1b shows the ratio F_2^{en}/F_2^{ep}. What does this mean for the valence and sea quark densities in the limits of $x \to 0$ and $x \to 1$. Calculate the ratio u^p/d^p at $x = 0.8$, assuming that only valence quarks contribute.

f) Some integrals of the quark densities have a simple interpretation. For example, the nucleon momentum fraction carried on average by up quarks is given by $\int_0^1 dx\, x\, u(x)$. Calculate the following integrals and interpret them:

$$\int_0^1 [u(x) - \bar{u}(x)]\, dx \quad \int_0^1 [d(x) - \bar{d}(x)]\, dx \quad \int_0^1 [s(x) - \bar{s}(x)]\, dx \tag{2}$$

7.2 Deep-Inelastic Scattering at HERA (8 points)

Figure 1(left) shows a deep-inelastic scattering event $e^+ p \to e^+ X$ recorded by the H1 experiment at the HERA collider. The positron beam, of energy $E_{e^+} = 27.5$ GeV, enters from the left and the proton beam, of energy $E_P = 820$ GeV, enters from the right. The energy of the outgoing positron is measured to be $E'_{e^+} = 31$ GeV.

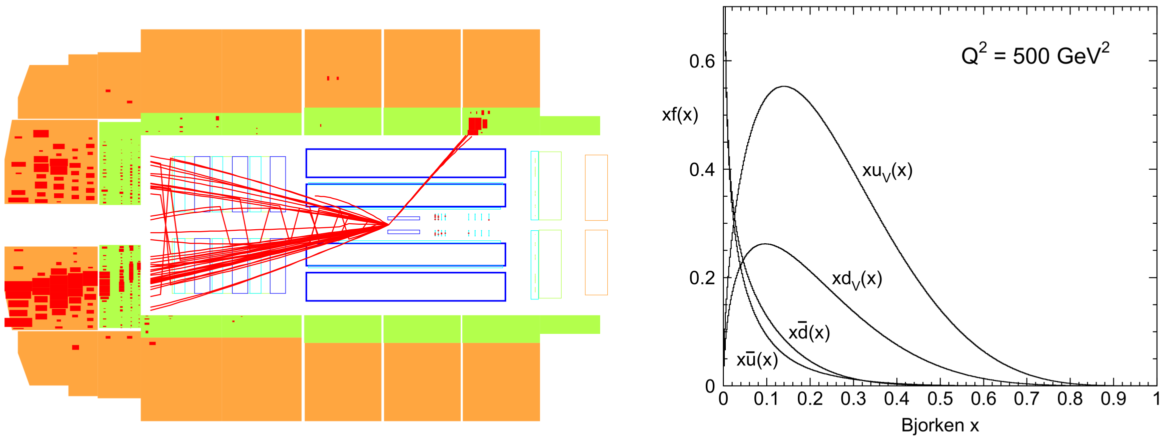

Figure 1: Left: H1 event display in x-z plane, right: Parton distribution functions

a) The Bjorken scaling variable is given by $x = \dfrac{-q^2}{2p_P.q}$ where p_P is the proton four-momentum before the collision and q is the four-momentum of the virtual photon. Show that x can be expressed as

$$x = \frac{E'_{e^+}}{E_P}\left[\frac{1-\cos\theta}{2-(E'_{e^+}/E_{e^+})(1+\cos\theta)}\right] \tag{3}$$

where θ is the angle through which the positron has scattered.

b) Estimate θ from Figure 1 (left) assuming the momentum of the scattered electron lies in the x-z plane. Calculate the values of Q^2, x and y for this event.
Reminder: The energies are not given in the proton's cms!

c) Estimate the invariant mass W_X of the final state hadronic system.

d) Draw quark level diagrams to illustrate the possible origins of this event. Using the parton distribution functions $xu_v(x)$, $xd_v(x)$, $x\bar{u}(x)$, $x\bar{d}(x)$ given in Figure 1 (right), estimate the relative probabilities of the various possible quark-level processes for the event. Neglect contributions from the 2nd and 3rd generation quarks. Assume that the value of Q^2 calculated in b) is close enough to 500 GeV^2 and hence the pdfs in Figure 1 (right) can be used.

Further suggested reading

M. Breidenbach et al. (1969), "Observed behavior of highly inelastic electron-proton scattering" http://journals.aps.org/prl/pdf/10.1103/PhysRevLett.23.935

This is a historical paper that discusses the results of electron-proton inelastic scattering. Please note that some of the conventions are different compare to the lecture.