name: **group:**

Exercise Sheet 6 – Particle Physics – SS 2016

hand in: Tue 31st May (after the lecture or at INF 226, 3.104 by 4 pm)

6.1 Elastic scattering (5 points)

In fixed-target electron-proton elastic scattering, where E_1 and E_3 are, respectively, initial and final energy of the electron:

$$Q^2 = 2m_p(E_1 - E_3) = 2m_pE_1y \text{ and } Q^2 = 4E_1E_3\sin^2(\theta/2)$$

a) Use these relations to show that

$$\sin^2\left(\frac{\theta}{2}\right) = \frac{E_1}{E_3}\frac{m_p^2}{Q^2}y^2$$

and hence

$$\frac{E_3}{E_1}\cos^2\left(\frac{\theta}{2}\right) = 1 - y - \frac{m_p^2y^2}{Q^2}$$

b) Assuming azimuthal symmetry and using

$$E_3 = \frac{E_1m_p}{m_p + E_1(1-\cos\theta)} \text{ and } Q^2 = \frac{2m_pE_1^2(1-\cos\theta)}{m_p + E_1(1-\cos\theta)},$$

show that

$$\frac{d\sigma}{dQ^2} = \left|\frac{d\Omega}{dQ^2}\right|\frac{d\sigma}{d\Omega} = \frac{\pi}{E_3^2}\frac{d\sigma}{d\Omega}$$

c) Using the results of (a) and (b) show that the Rosenbluth equation,

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2\sin^4(\theta/2)}\frac{E_3}{E_1}\left(\frac{G_E^2 + \tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2} + 2\tau G_M^2\sin^2\frac{\theta}{2}\right),$$

can be written in the Lorentz-invariant form

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\frac{G_E^2 + \tau G_M^2}{1+\tau}\left(1 - y - \frac{m_p^2y^2}{Q^2}\right) + \frac{1}{2}y^2G_M^2\right]$$

6.2 Fixed-target inelastic scattering (5 points)

A fixed-target e^-p scattering experiment consists of an electron beam of energy 20 GeV and a fixed angle spectrometer that can detect scattered electrons with energies greater than 5 GeV.

a) What is the maximum energy an electron scattered at an angle of $\theta = 6^\circ$ with respect to the directon of the incoming electron beam can have?

b) Find the range of x that can be measured with the setup described in part a).

c) Find the energy resolution needed to measure x with a precision of 1 % or better over the full range from part b). Assume that the energy of the incoming electron beam as well as the angular position of the spectrometer are known precisely.

6.3 Form factors (5 points)

For a spherically symmetric charge distribution $\rho(r)$, where

$$\int \rho(r) d^3\mathbf{r} = 1,$$

show that the form factor can be expressed as

$$\begin{aligned} F(q^2) = &\ \frac{4\pi}{q}\int_0^\infty r\sin(qr)\rho(r)dr, \\ \simeq &\ 1 - \frac{1}{6}q^2 < R^2 > + \ldots, \end{aligned}$$

where $< R^2 >$ is the mean square charge radius. Hence show that

$$< R^2 > = -6\left[\frac{dF(q^2)}{dq^2}\right]_{q^2=0}$$

6.4 Form Factors (5 points)

Elastic $e^- p \to e^- p$ scattering via the exchange of a single photon can be described by the Rosenbluth formula

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4(\frac{\theta}{2})}\frac{E_3}{E_1}\left(\frac{G_E^2 + \tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2} + 2\tau G_M^2 \sin^2\frac{\theta}{2}\right).$$

Here E_1 and E_3 are the energy of the incoming and outgoing electron, respectively; $\tau = Q^2/4m_p^2$ and $G_E(Q^2)$ ($G_M(Q^2)$) is the electric (magnetic) form factor of the proton.

a) Re-express the Rosenbluth formula in terms of $\left(\frac{d\sigma}{d\Omega}\right)_0$, which is the Mott scattering cross section of

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{\alpha^2}{4E_1^2 \sin^4(\frac{\theta}{2})}\frac{E_3}{E_1}\cos^2\frac{\theta}{2}.$$

b) The ratio of

$$\left(\frac{d\sigma}{d\Omega}\right) / \left(\frac{d\sigma}{d\Omega}\right)_0$$

is linear in $\tan^2(\theta/2)$. Express the gradient and the intersect as functions of $G_E(Q^2)$ and $G_M(Q^2)$.

c) Obtain values for $G_E(0.292\,\mathrm{GeV}^2)$ and $G_M(0.292\,\mathrm{GeV}^2)$ from Figure 1.

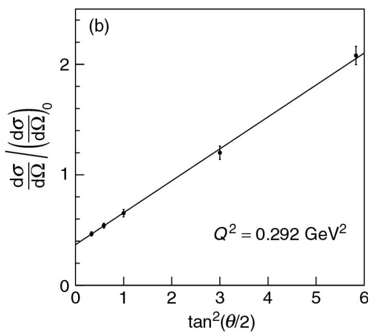

Figure 1: Low energy $e^- p \to e^- p$ elastic scattering data.