name: **group:**

Exercise Sheet 5 – Particle Physics – SS 2016

hand in: Tue 24th May (after the lecture or at INF 226, 3.104 by 4 pm)

5.1 Chirality and Helicity (6 points)

Chirality and helicity are two different concepts which are frequently confused with each other. Helicity is the projection of a particle's spin onto its direction of motion. Chiral states are the eigenstates of the γ^5-matrix.

a) A spinor can be decomposed into left- and right-handed chiral states via the following projection operations

$$u = \left(\frac{1-\gamma^5}{2}\right)u + \left(\frac{1+\gamma^5}{2}\right)u = u_L + u_R.$$

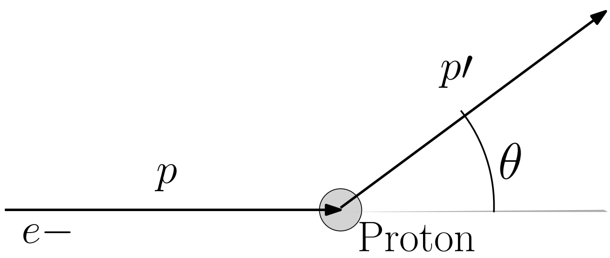
Calculate the chiral states of the right-handed helicity spinor. Do not assume that $E \gg m$.

b) Calculate the value of $\frac{p}{E+m}$ for an electron with 1 MeV of energy and for an electron with 1 GeV of energy.

c) For each of these electron energies, how do the helicity and chiral eigenstates differ from each other?

5.2 $e^- + p$ scattering (14 points)

Consider the electromagnetic scattering of electrons with four momentum $(E, \vec{p})$ off protons at rest. Due to its large mass the proton can be considered at rest also after the scattering.



a) Write down the four momenta of the electron and proton before and after the scattering process. Do NOT apply any assumptions about the electron energy.

b) Write down the Dirac spinors for helicity +1 and -1 eigenstates of the incoming and outgoing electron.

c) Show that the electron currents for all helicity combinations are as follows:

$$\bar{u}_{h=+1}(p\prime)\gamma^\mu u_{h=+1}(p) = (E+m)((1+\alpha^2)c, 2\alpha s, 2i\alpha s, 2\alpha c) \quad (1)$$

$$\bar{u}_{h=+1}(p\prime)\gamma^\mu u_{h=-1}(p) = (E+m)((1-\alpha^2)s, 0, 0, 0) \quad (2)$$

$$\bar{u}_{h=-1}(p\prime)\gamma^\mu u_{h=+1}(p) = (E+m)((\alpha^2-1)s, 0, 0, 0) \quad (3)$$

$$\bar{u}_{h=-1}(p\prime)\gamma^\mu u_{h=-1}(p) = (E+m)((1+\alpha^2)c, 2\alpha s, -2i\alpha s, 2\alpha c) \quad (4)$$

where $c = cos\frac{\theta}{2}$, $s = sin\frac{\theta}{2}$ and $\alpha = \frac{|\vec{p}|}{E+m}$. Which combinations vanish in the relativistic limit ($E \gg m_e$) and which ones in the non-relativistic limit ($|\vec{p}| \to 0$)? Hint: See Slide 19: Equation 1 in Lecture 6.

d) Write down the Dirac spinors for helicity +1 and -1 eigenstates of the ”incoming” and ”outgoing” proton. Show that the proton currents are as follows:

$$
\begin{aligned}
\bar{u}_{h=+1}(p\prime)\gamma^{\mu}u_{h=+1}(p) =& 2m_p((1,0,0,0) && (5)\\
\bar{u}_{h=+1}(p\prime)\gamma^{\mu}u_{h=-1}(p) =& 0 && (6)\\
\bar{u}_{h=-1}(p\prime)\gamma^{\mu}u_{h=+1}(p) =& 0 && (7)\\
\bar{u}_{h=-1}(p\prime)\gamma^{\mu}u_{h=-1}(p) =& 2m_p((1,0,0,0) && (8)\\
& && (9)
\end{aligned}
$$

e) Compute the spin averaged matrix element square for the scatter process in the non-relativistic limit. Express the four-momentum transfer q in terms of the momentum of the incoming electron and the scattering angle θ.

f) Use the two-body phase space computation from the lecture to write down the differential cross-section of the scatter process. You have derived the Rutherford scattering formula! Hint($E \ll m_p$ implies that the lab frame = CM frame.)

g) For extra points (+1): Which other well known formula do you get if you use the relativistic limit instead of the non-relativistic one.