name: **group:**

Exercise Sheet 4 – Particle Physics – SS 2016

hand in: Tue 17^{rd} May (after the lecture or at INF 226, 3.104 by 4 pm)

4.1 Intrinsic parity of fermions (5 points)

The parity operator $\hat{P}$ is given by the first Dirac matrix γ^0. The parity of a particle (or anti-particle) at rest is called *intrinsic parity*. Show that particles and anti-particles represented by the spinors u_1, u_2 and v_1, v_2 introduced in the lecture and given in exercise (3.3) have intrinsic parity 1 and -1, respectively.

4.2 Positron discovery (5 points)

In the Dirac theory negative energy solutions are interpreted as antiparticle states. The positron was the first such state to be observed (see Phys. Rev. 43 (1933) 491) by C. D. Anderson. Read the article and answer the following questions:

a) Why can electrons and protons be excluded as the source of the observed tracks?
b) Which process, according to the author, produced the positron?
c) How do you measure the energy loss of the particle in the lead plate?
d) What is the energy of the positron in Figure 1?
e) What was apparently the common opinion on the constituents of a neutron in 1933?
f) What was the original goal of the experiment?

4.3 Cross section (5 points)

Given that the total reaction cross section of the reacgtion $e^+e^- \to \mu^+\mu^-$ is described to good accuracy by the first order QED cross section,

a) How many muon pairs do you expect to be produced per day at an e^+e^--collider with luminosity $\mathcal{L} = 10^{30} cm^2 s^{-1}$ at a beam energy of 10 GeV?
b) How many muon pairs can be detected with a typical collider detector that features full azimuthal coverage and a polar angle acceptance of $30^\circ < \theta < 150^\circ$. Assume a detection efficiency of $\varepsilon = 90\%$ for a muon with momentum above 3 GeV.

4.4 Spin and the Dirac equation (5 points)

In non-relativistic quantum mechanics the angular momentum operator $\hat{L} = \hat{r} \times \hat{p}$ commutes with the Hamiltonian of the free Schrödinger equation and thus orbital angular momentum is a conserved quantity.

a) Show that $\hat{L}$ is not conserved for a system described by the Dirac equation.
b) Compute the commutation relation for the Hamiltonian of the Dirac equation $\hat{H}_D = \alpha \cdot p + \beta m$ and the operator

$$\hat{S} = \frac{1}{2}\begin{pmatrix} \hat{\sigma} & 0 \\ 0 & \hat{\sigma} \end{pmatrix}$$

where $\hat{\sigma}$ are the Pauli spin-matrices.
c) Using results a) and b) show that the total angular momentum $\hat{J}$ is conserved in relativistic quantum mechanics.
d) What are the eigenvalues of $\hat{S}^2$ applied to a Dirac spinor Ψ? How do you interpret the result?

4.5 Optional: Gamma Matrices (5 extra points)

The fifth γ matrix is defined as:

$$\gamma^5 := i\gamma^0\gamma^1\gamma^2\gamma^3$$

a) Using the anti-commutation rules for γ^μ

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

to show **representation independently** that:

$$\gamma^5\gamma^5 = 1 \quad \text{and} \quad \{\gamma^5, \gamma^\mu\} = 0$$

b) Show that γ^5 does not commute with $H = \vec{\alpha}\vec{p} + \beta m$.

c) Show that the operator

$$P_\pm := \frac{1}{2}(1 \pm \vec{\Sigma} \cdot \vec{p})$$

acts as a projector (i.e. $P_\pm^2 \Psi = P_\pm \Psi$) on Dirac spinors ($\vec{\Sigma} = 2\hat{S}$).

d) Show that at high energies ($E \gg m$):

$$\gamma^5 \begin{pmatrix} u_A \\ u_B \end{pmatrix} \approx \vec{\Sigma} \cdot \vec{p} \begin{pmatrix} u_A \\ u_B \end{pmatrix}$$

where u_A, u_B correspond to the two component vectors introduced in the lecture