name: **group:**

Exercise Sheet 3 – Particle Physics – SS 2016

hand in: Tue 10$^{\text{rd}}$ May (after the lecture or at INF 226, 3.104 by 4 pm)

3.1 Phase Space Integration (5 points)

Prove that

$$\sigma = \frac{1}{(2\pi)^2}\frac{1}{4p_i^*\sqrt{s}}\int |M_{fi}|^2\delta(\sqrt{s}-E_1-E_2)\delta^3(\vec{p}_1+\vec{p}_2)\frac{d^3\vec{p}_1}{2E_1}\frac{d^3\vec{p}_2}{2E_2}$$

can be simplified to

$$\sigma = \frac{1}{64\pi^2 s}\cdot\frac{p_f^*}{p_i^*}\int |M_{fi}|^2 d\Omega^*$$

by solving the integration in the center-of-mass frame and exploiting features of the δ-function. **Hint:** Some guidance can be obtained from the M. Thomson's textbook.

3.2 Fermi's Golden Rule (5 points)

Fermi's Golden rule:

$$\Gamma_{fi} = 2\pi|T_{fi}|^2\rho(E_f) \tag{1}$$

relates a transition rate from an initial state $|i>$ to a final state $|f>$ to the matrix element and the available phase space. The cross section can then be calculated as:

$$\sigma = \frac{\Gamma_{fi}}{F}. \tag{2}$$

Now, consider a *collinear* scattering:

$$A+B\rightarrow C+D \tag{3}$$

and show that the incoming flux defined as:

$$F := 2E_A 2E_B|\vec{v}_A-\vec{v}_B| \tag{4}$$

is Lorentz invariant. **Hint:** Bring the expression to a manifestly Lorentz invariant form. Some guidance can be obtained from the M. Thomson's textbook.

3.3 Free particle spinors (5 points)

The two ($i=1,2$) free particle solutions to the Dirac equation are given by $\psi_i = ae^{+i(\boldsymbol{p}\cdot\boldsymbol{x}-Et)}u_i(p)$ and the two free antiparticle solutions are given by $\psi_{i+2} = ae^{-i(\boldsymbol{p}\cdot\boldsymbol{x}-Et)}v_i(p)$ where a is a real normalisation factor, $N=\sqrt{E+m}$ and the spinors u_i and v_i are given by:

$$u_1 = N\begin{pmatrix}1\\0\\\frac{p_z}{E+m}\\\frac{p_x+ip_y}{E+m}\end{pmatrix},\quad u_2 = N\begin{pmatrix}0\\1\\\frac{p_x-ip_y}{E+m}\\\frac{-p_z}{E+m}\end{pmatrix}\quad\text{and}\quad v_1 = N\begin{pmatrix}\frac{p_x-ip_y}{E+m}\\\frac{-p_z}{E+m}\\0\\1\end{pmatrix},\quad v_2 = N\begin{pmatrix}\frac{p_z}{E+m}\\\frac{p_x+ip_y}{E+m}\\1\\0\end{pmatrix}.$$

Let the subscripts A and B denote the upper and lower two-component column vectors of the spinors:

$$u=\begin{pmatrix}u_A\\u_B\end{pmatrix}\quad\text{and}\quad v=\begin{pmatrix}v_A\\v_B\end{pmatrix}.$$

a) It was shown in the lecture that solving the Dirac equation using above spinor representation leads to coupled equations for e.g. u_A in terms of u_B:

$$u_A = \frac{\boldsymbol{\sigma} \cdot \boldsymbol{p}}{E - m}\, u_B \quad \text{and} \quad u_B = \frac{\boldsymbol{\sigma} \cdot \boldsymbol{p}}{E + m}\, u_A$$

Using above equations, show that E and $\boldsymbol{p}$ must obey the usual relativistic energy-momentum relation.

b) Show for u_1 that in the non-relativistic limit, where $\beta \equiv v/c \ll 1$, $|u_B|$ (the lower components) are smaller than $|u_A|$ (the upper components) by a factor $\approx v/c$.

3.4 Particles, antiparticles and charge conjugation (5 points)

In the Dirac-Pauli representation, the charge conjugation operator $\hat{C}$, which transforms a particle wave function ψ into the corresponding charge-conjugate wave function ψ_c, is given by

$$\hat{C}\psi = \psi_c = i\gamma^2\psi^* \qquad \text{with} \qquad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} .$$

a) Find the charge-conjugates of particle spinors ψ_1 and ψ_2 and compare them with antiparticle spinors ψ_3 and ψ_4. Interpret the result.

b) The Dirac equation for an electron with charge $q = -e$ in the presence of an electromagnetic field $A^\mu = (\phi, \boldsymbol{A})$ is given by

$$\gamma^\mu(\partial_\mu - ieA_\mu)\psi + im\psi = 0 \quad .$$

Calculate the corresponding equation after charge-conjugation and interpret the result.