name: **group:**

Exercise Sheet 12 – Particle Physics – SS 2016

hand in: Tue 12th July (after the lecture or at INF 226, 3.104 by 4 pm)

12.1 B Meson Decays (5 points)

The quark mixing is described in the Standard Model by the Cabibbo Kobayashi Maskawa (CKM) matrix:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{1}$$

a) Which CKM matrix elements are involved in Feynman Diagram 1 ($B^+ \to \bar{D}^0 K^+$)?

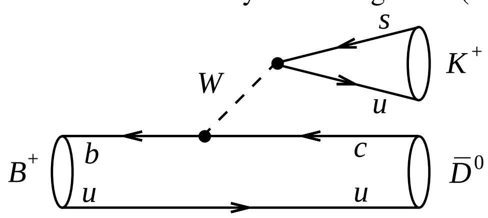

Diagram 1

b) Diagram 2 also contributes to $B^+ \to \bar{D}^0 K^+$:

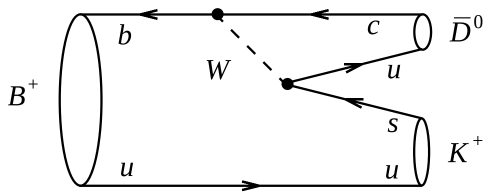

Diagram 2

Why is the partial decay width of Diagram 2 expected to be smaller by a factor of 1/3 with respect to the partial decay width of Diagram 1?

c) Draw the corresponding Feynman diagram for $B^+ \to D^0 K^+$. Which CKM matrix elements are involved? Calculate the ratio $\Gamma(B^+ \to D^0 K^+)/\Gamma(B^+ \to \bar{D}^0 K^+)$, considering only Diagram 1 for $B^+ \to \bar{D}^0 K^+$.

12.2 Charm quark decay (5 points)

a) Estimate the lifetime of the charm quark ($m_c = 1.25$ GeV) by employing the muon decay formula (partial width $\Gamma(\mu^- \to e^- \nu_\mu \bar{\nu}_e) \approx \frac{G_F^2 m_\mu^5}{192\pi^3}$) and taking into account CKM mixing. *Hint: the referred formula is an approximation for any flavour-changing weak decay with mas of the initial particle $m_X \ll m_W$.*

b) Compare the obtained result with the observed lifetime of the D^+ meson ($c\bar{d}$). Why is the description not perfect?

12.3 Kaon oscillations (7 points)

It is known that the properties of some weakly interacting neutral particles vary with time. For instance, a kaon, originally produced as a $\bar{K}^0$ oscillates into a K^0 and back. This mixing appears because K^0 and $\bar{K}^0$ are no eigenstates of the weak interaction; instead, they are combinations of the two weak eigenstates K_1^0 and K_2^0 (the small CP violation in the kaon system is ignored):

$$|K^0(t=0)\rangle = 1/\sqrt{2}\left(|K_1^0(t=0)\rangle + |K_2^0(t=0)\rangle\right), \; |\bar{K}^0(t=0)\rangle = 1/\sqrt{2}\left(|K_1^0(t=0)\rangle - |K_2^0(t=0)\rangle\right), \tag{2}$$

The time dependence of K_1^0 and K_2^0 is given by:

$$|K_1^0(t)\rangle = |K_1^0(t=0)\rangle\, e^{-im_1t-\frac{1}{2}\Gamma_1 t}, \; |K_2^0(t)\rangle = |K_2^0(t=0)\rangle\, e^{-im_2t-\frac{1}{2}\Gamma_2 t} \tag{3}$$

where the m_i are the masses and the Γ_i the decay widths.

a) Draw one of the Feynman diagrams for the transition of a K^0 to a $\bar{K}^0$.
b) Write down the time dependence of the wave function of an initial $\bar{K}^0$ beam.
c) What are the most abundant K^0 and $\bar{K}^0$ decays? Why is it not possible to tell K^0 and $\bar{K}^0$ apart in these decays?
d) As K^0 and $\bar{K}^0$ cannot be identified by their decays, one uses the fact that their strong interaction properties differ. $K^0 p \to \Lambda^0 \pi^+$ is allowed, whereas $\bar{K^0} p \to \Lambda^0 \pi^+$ is forbidden. Draw the Feynman diagrams to see why.
e) Using the reaction in d), it is possible to measure the probability $|\langle K_0(t=0)|\bar{K}_0(t)\rangle|^2$ that an original $\bar{K}^0$ converted into a K^0. Compute this probability and show that it oscillates with a frequency proportional to $m_1 - m_2$.
f) The mean lifetimes are $0.895 \cdot 10^{-10}$s for K_1^0 and $5.1 \cdot 10^{-8}$s for K_2^0. The time period of the oscillation, divided by 2π, is $1.9 \cdot 10^{-10}$s. Compute the ratios of $\Gamma_1 - \Gamma_2$ and $m_1 - m_2$ to the mean mass $(m_1 + m_2)/2 = 498$ MeV.