name: **group:**

Exercise Sheet 11 – Particle Physics – SS 2016

hand in: Tue 5th July (after the lecture or at INF 226, 3.104 by 4 pm)

11.1 Goldhaber experiment (5 points)

Read the paper *Helicity of Neutrinos. Phys. Rev.,109, 1015 (1958)* and answer the following questions.

a) When, where and by whom was the experiment performed?

b) What is the lifetime of Eu^{152m}?

c) The following reactions have been studied.

$$e^- + \mathrm{Eu}^{152m} \to \mathrm{Sm}^{152*}(1-) + \nu, \tag{1}$$

$$\mathrm{Sm}^{152*}(1-) \to \mathrm{Sm}^{152}(0+) + \gamma. \tag{2}$$

Consider the two-body decay Eq. (2) in the center of mass system of $\mathrm{Sm}^{152*}(1-)$. What is the energy difference ΔE between $\mathrm{Sm}^{152*}(1-)$ and $\mathrm{Sm}^{152}(0+)$?. How large is the photon energy compared to ΔE? Explain your answer.

d) How large is the photon energy compared to ΔE in the lab frame in which Eu^{152m} can be considered at rest? Distinguish both cases of photons emitted in the direction of flight of the $\mathrm{Sm}^{152*}(1-)$ (forward photons) and in the opposite direction (backward photons).

e) The photon is recorded in the resonant scattering on $\mathrm{Sm_2O_3}$

$$\mathrm{Sm}^{152}(0+) + \gamma \to \mathrm{Sm}^{152*}(1-). \tag{3}$$

What is the threshold of the photon energy for this reaction to happen? Which of the photons discussed in (d) are detected? How are the flight directions of these photons correlated to the flight directions of the neutrinos in Eq. (1)?

f) What is the correlation between the helicities of the photons and the neutrinos?

g) Between the source and the resonant scattering there is a magnet. What is the spin alignment of the electrons in the magnet when the field inside the magnet is pointing up (following the convention in the paper, *up* is the direction from the resonant scatterer to the source)? What happens to photons traversing the magnet? Discuss the two cases of parallel and anti-parallel spins of the electrons and the photons. Assuming that the density of polarized electrons in the magnet is n_e, what is the mean free path length of the photons in the magnet?

h) What is the measured asymmetry $\delta = (N_- - N_+)/\left[\frac{1}{2}(N_- + N_+)\right]$ between numbers of photons recorded with different magnetic orientations? What is the reported δ for 100% negatively polarized photons? What is the reported photon polarization and how was it derived? According to the paper, give two reasons why the measured photon polarization is not 100%.

11.2 Intrinsic parity (4 points)

a) Calculate the intrinsic parity of the pion in the quark model. (Hint: It is a ground state meson.)

b) Deuterons (spin $J = 1$) can capture negative pions in an atomic S-state and a commonly observed subsequent decay is

$$d + \pi^- \rightarrow n + n \tag{4}$$

What is the parity of the deuteron?

11.3 Parity violation in pion decays (5 points)

Shortly after the result of the Wu-experiment was published, Leo Lederman and colleagues performed another experiment - suggested by Li and Yang - to confirm parity violation as well. Protons where shot on a fixed target. Pions produced in this reaction were filtered out. The pions decay into muons which are then stopped in a carbon target. The subsequent decays of the muons into electrons were studied:

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \tag{5}$$

$$\mu^+ \rightarrow e^+ + \overline{\nu}_\mu + \nu_e \tag{6}$$

The rate of emitted electrons under different angles relative to the incident flight direction of the muons was measured. Justify your answers to the following questions:

a) Is the original pion beam polarized?

b) Are the emitted muons polarized?

c) Do you expect the electrons to be emitted isotropically?

d) What is the signature of parity violation in this experiment? Which of the both reactions above are violating parity?

11.4 Pion decay rate (6 points)

In the decay of a pion to a lepton and neutrino, i.e. $\pi^- \rightarrow \ell^- + \overline{\nu}_\ell$, it is seen that the pion decays to muons much more often than electrons. Let's explore why this happens.

a) The Lorentz invariant phase space for this 2-body decay is given by

$$\rho = \tfrac{1}{4\pi^2} \tfrac{p_\ell}{m_\pi} d\Omega,$$

Compute the relative phase space ratio for the decay to electron and to muon respectively.

b) Write down the most general form of the matrix element, treating the weak interaction as a contact interaction (i. e. use $q^2 \ll m_W^2$). Assume that the hadronic current can be written as

$$j_\pi = \frac{g_w}{2\sqrt{2}} f_\pi p_\pi^\mu$$

with the pion decay constant f_π. Can you think of a reason for why this current can be parametrized in this why?

c) Show that in the limit of vanishing neutrino masses the matrix element can be written as

$$\mathcal{M} = \left(\frac{g_W}{2m_W}\right)^2 f_\pi m_\ell \sqrt{(m_\pi^2 - m_\ell^2)}$$

Hint: work in the pion rest-frame

d) Compute the relative branching ratio:

$$\tfrac{BR(\pi^- \rightarrow e^- \overline{\nu}_e)}{BR(\pi^- \rightarrow \mu^- \overline{\nu}_\mu)}.$$