

Rutherford Scattering

► $E_{kin,e} \ll m_e \ll m_p$

→ recoil ignored (p_p before and after scattering = 0)

→ implies magnetic IA (spin-flip) is ignored

→ $s = E_{CMS}^2 = m_p^2$

→ non-relativistic limit (no helicity conservation)

► $\lambda \gg r_{target}$

λ : de Broglie wave length of incoming electron

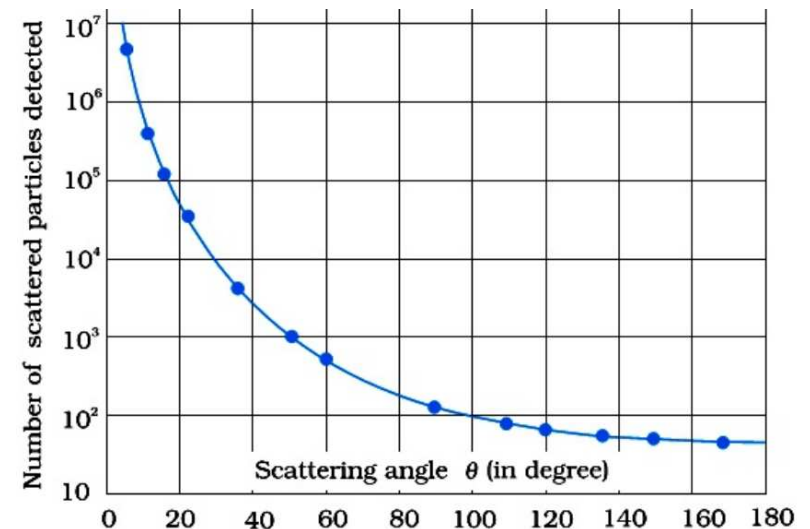
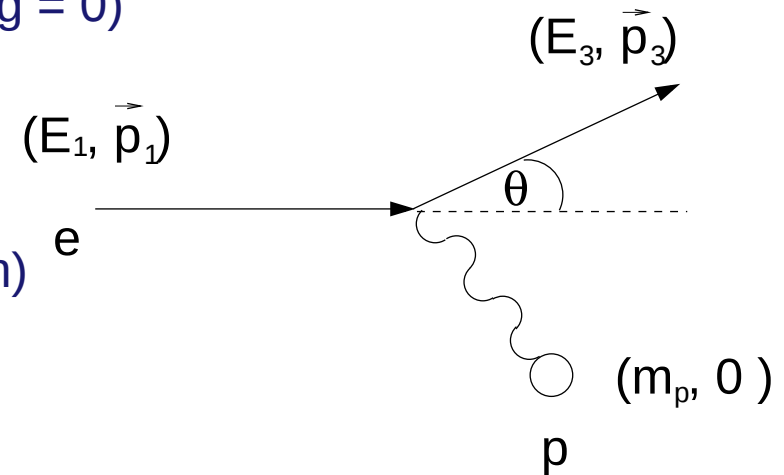
r_{target} : size of target particle (proton)

► point-like scattering particles, spin=1/2

→ Dirac spinors describe the process

$$\left(\frac{d\sigma}{d\Omega} \right)_{Ruth} = \frac{\alpha^2}{16E_{kin}^2} \frac{1}{\sin^4 \theta/2}$$

$$|q| = 2|\vec{p}| \sin \theta/2$$



Mott Scattering

► $m_e \ll E_e \ll m_p$

→ recoil still neglected (no spin flip of proton)

relativistic limit for description of electron

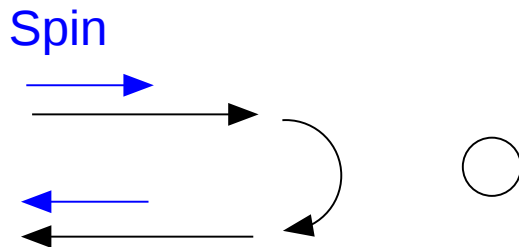
→ helicity $\equiv$ chirality

→ helicity conservation

$$\left(\frac{d\sigma}{d\Omega} \right)_{Mott} = \left(\frac{d\sigma}{d\Omega} \right)_{Ruth} \cos^2 \theta / 2$$

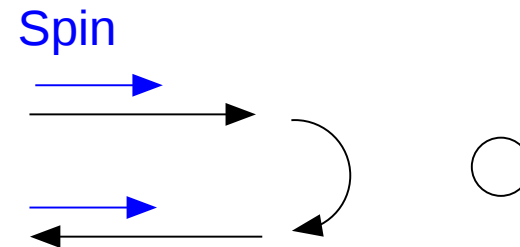
backward scattering is suppressed

(simultaneous conservation of angular momentum and helicity not possible)



Helizitaet erhalten

Drehimpuls nicht erhalten



Helizitaet nicht erhalten

Drehimpuls erhalten

Extended Charge Distribution

$$\left(\frac{d\sigma}{d\Omega}\right)_{extended} = \left(\frac{d\sigma}{d\Omega}\right)_{point} |F(\vec{q}^2)|^2$$

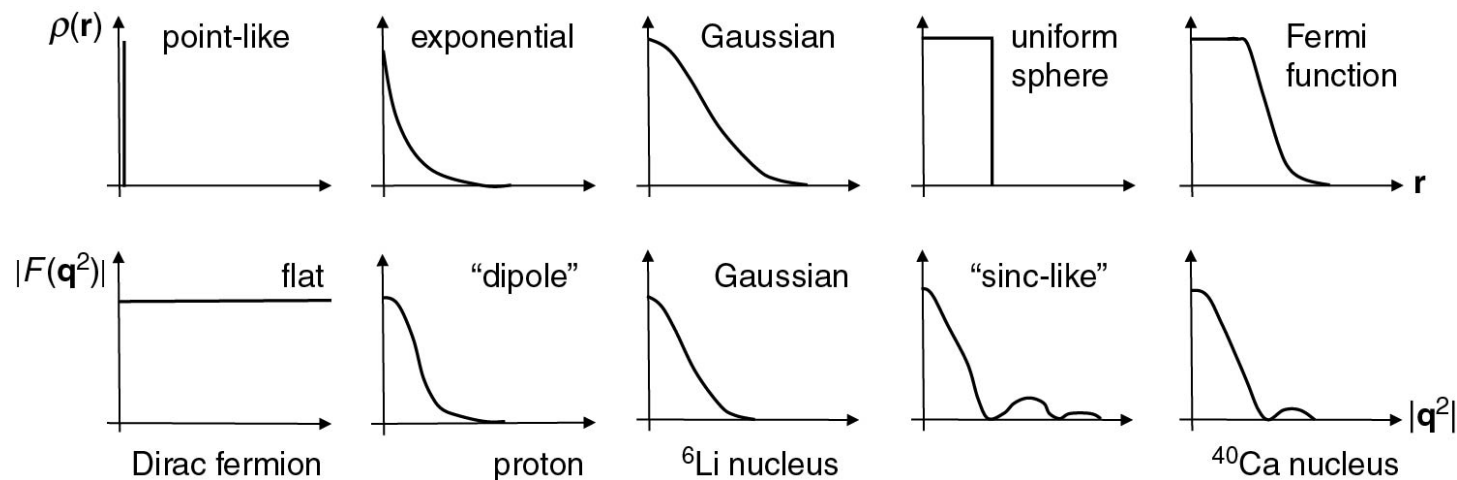
$\vec{q} = (\vec{p} - \vec{p}') \quad 3\text{-momentum transfer}$

Form-factor is fourier transformed charge distribution of proton

This interpretation was derived und the assumption of no recoil of proton

$\rightarrow m_p \gg m_e, p_p = 0$ before and after scattering

$\rightarrow E_1 = E_3 \rightarrow q^2 = -\vec{q}^2$



Dirac Scattering

$$\left(\frac{d\sigma}{d\Omega}\right)_{Dirac} = \left(\frac{d\sigma}{d\Omega}\right)_{Ruth} \frac{E_3}{E_1} \left(\cos^2 \theta / 2 + \frac{Q^2}{2m_p^2 c^2} \sin^2 \theta / 2 \right)$$

$Q^2 = -q^2$ (4-momentum transfer)

el. IA: Mott scattering for high relativistic particles

mag. IA: depend on Q^2 introduced by considering recoil

(m_p is not much larger than Q^2)

$$\left(\frac{d\sigma}{d\Omega}\right)_{Dirac} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \frac{E_3}{E_1} (1 + 2\tau \tan^2 \theta / 2)$$

$$\tau = \frac{Q^2}{4m_p^2}$$

Rosenbluth Formular

$$\begin{aligned}\left(\frac{d\sigma}{d\Omega}\right) &= \frac{\alpha^2}{4E_1^2 \sin^4(\theta/2)} \frac{E_3}{E_1} \left(\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1 + \tau} \cos^2 \theta/2 + 2\tau G_M^2(Q^2) \sin^2 \theta/2 \right) \\ &= \left(\frac{d\sigma}{d\Omega}\right)_0 \left(\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1 + \tau} + 2\tau G_M^2(Q^2) \tan^2 \theta/2 \right) \\ \tau &= \frac{Q^2}{4m_p^2}\end{aligned}$$

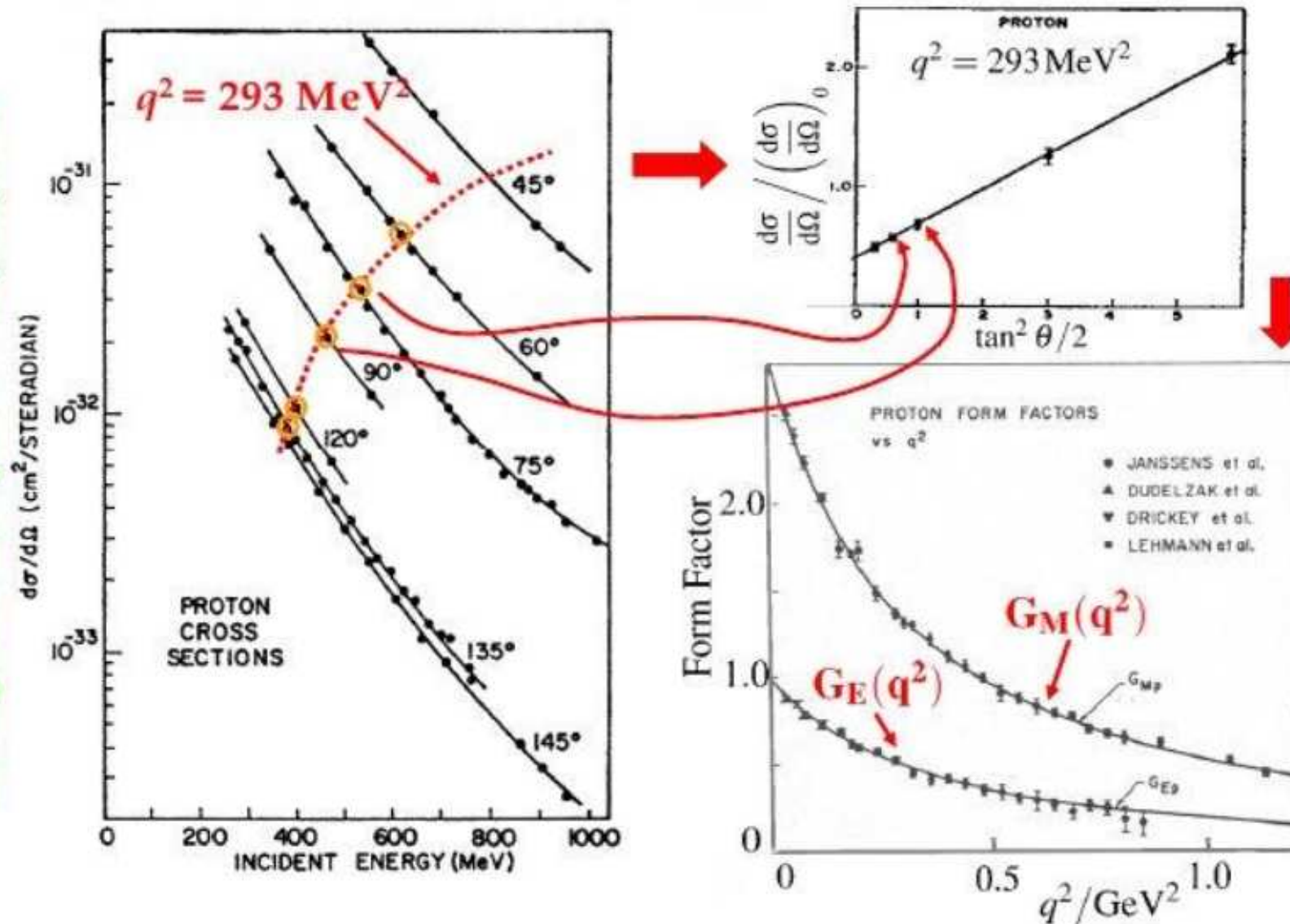
$$\left(\frac{d\sigma}{d\Omega}\right) / \left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1 + \tau} + 2\tau G_M^2(Q^2) \tan^2 \theta/2$$

→ Measure $\left(\frac{d\sigma}{d\Omega}\right) / \left(\frac{d\sigma}{d\Omega}\right)_0$ for fixed Q^2 at different $\tan^2 \theta/2$ values!

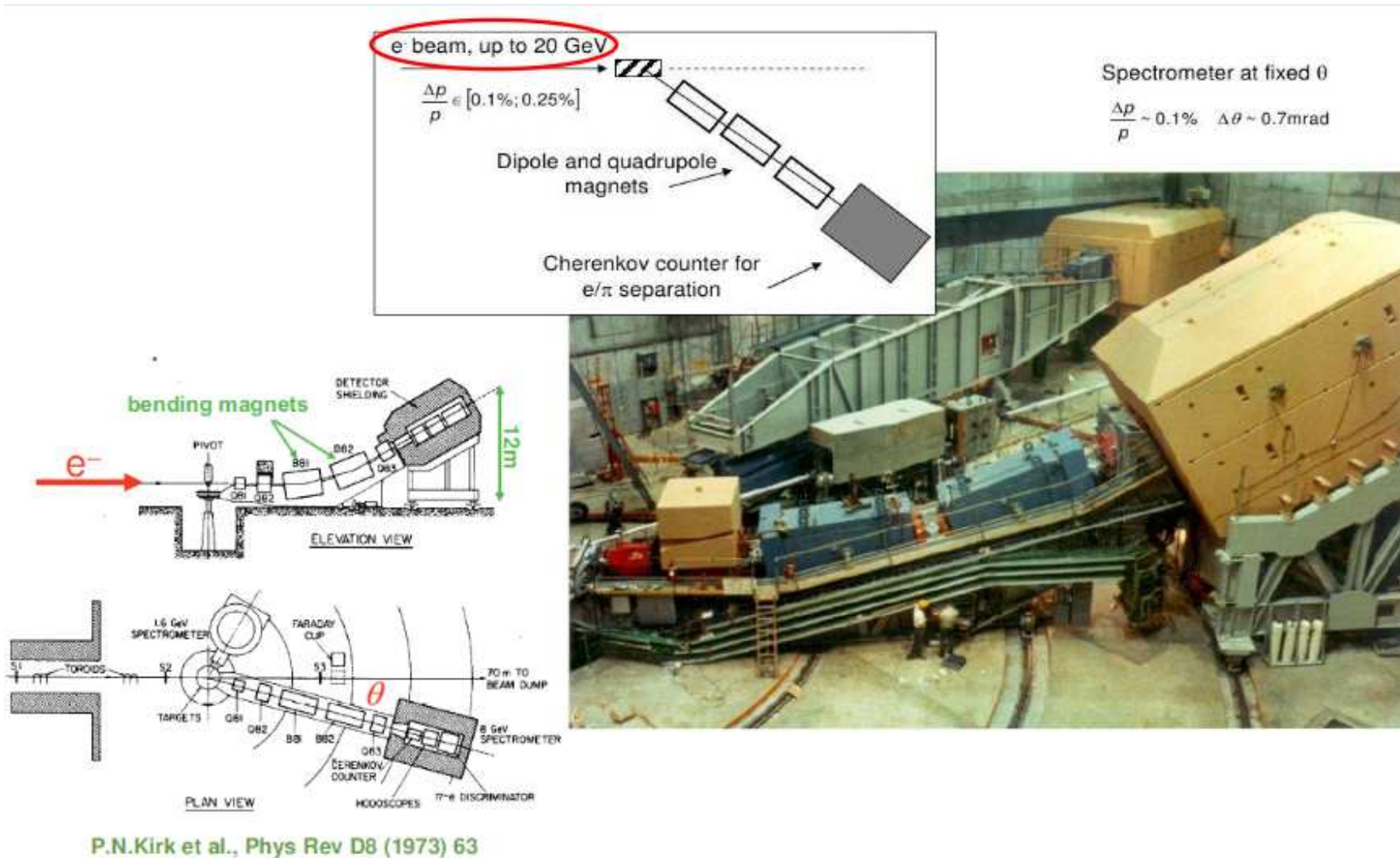
Slope and absizze determine G_M and G_E at this Q^2 value.

Measurement of el. and mag. Formfactors

E.B. Hughes et al., Phys. Rev. 139 (1965) B458

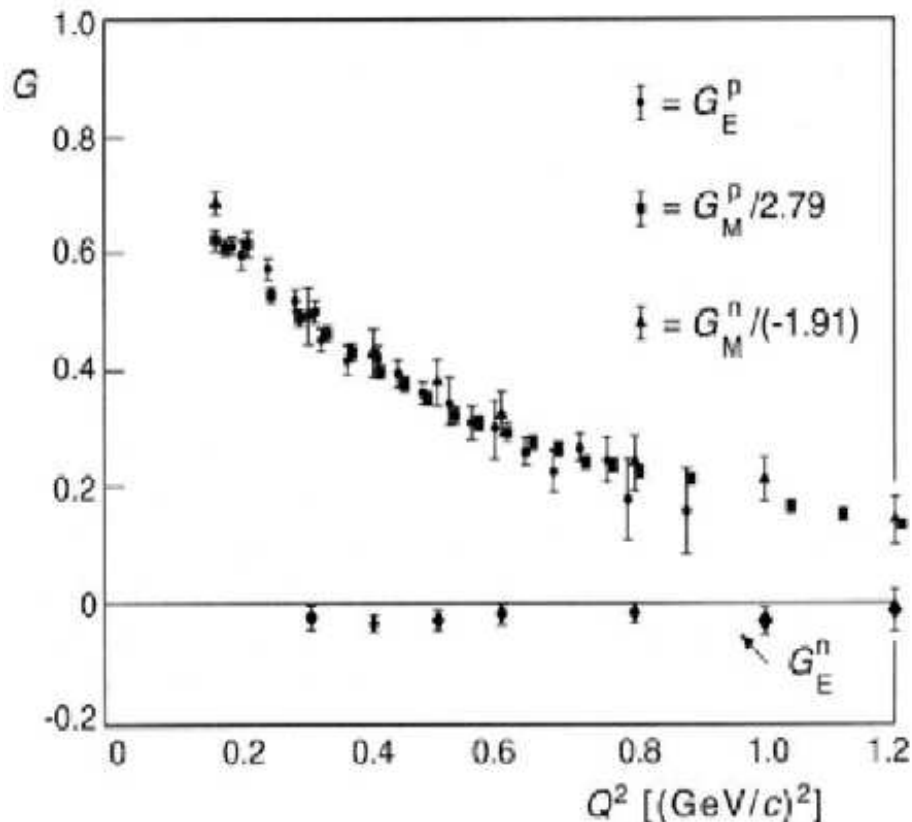


SLAC Experiment



Measurement of el. and mag. Formfactors

$$G_E^p(Q^2) = \frac{G_M^p}{2.79} = \frac{G_M^n}{-1.91} = G^{\text{Dipole}}(Q^2)$$



Charge and magnetic moment have give the same size for the proton. Both quantities are equally distributed in the proton.

$$G(Q^2) = \frac{1}{(1+Q^2/0.71 \text{ GeV}^2)^2}$$

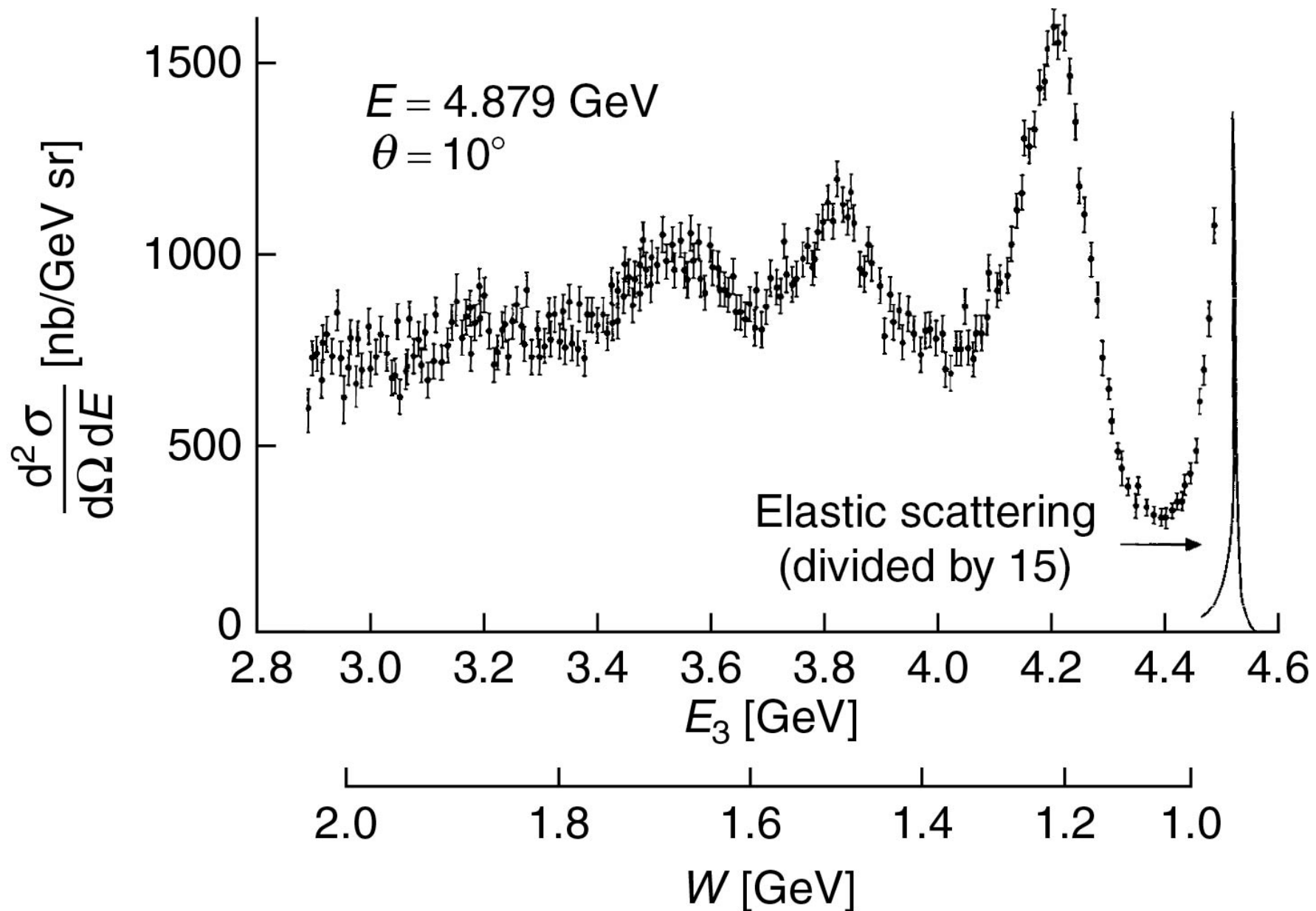
$$\langle r^2 \rangle = -6 \frac{dF(Q^2)}{dQ^2} \Big|_{Q^2=0}$$

$$\sqrt{\langle r^2 \rangle \text{ Dipole}} = 0.81 \text{ fm}$$

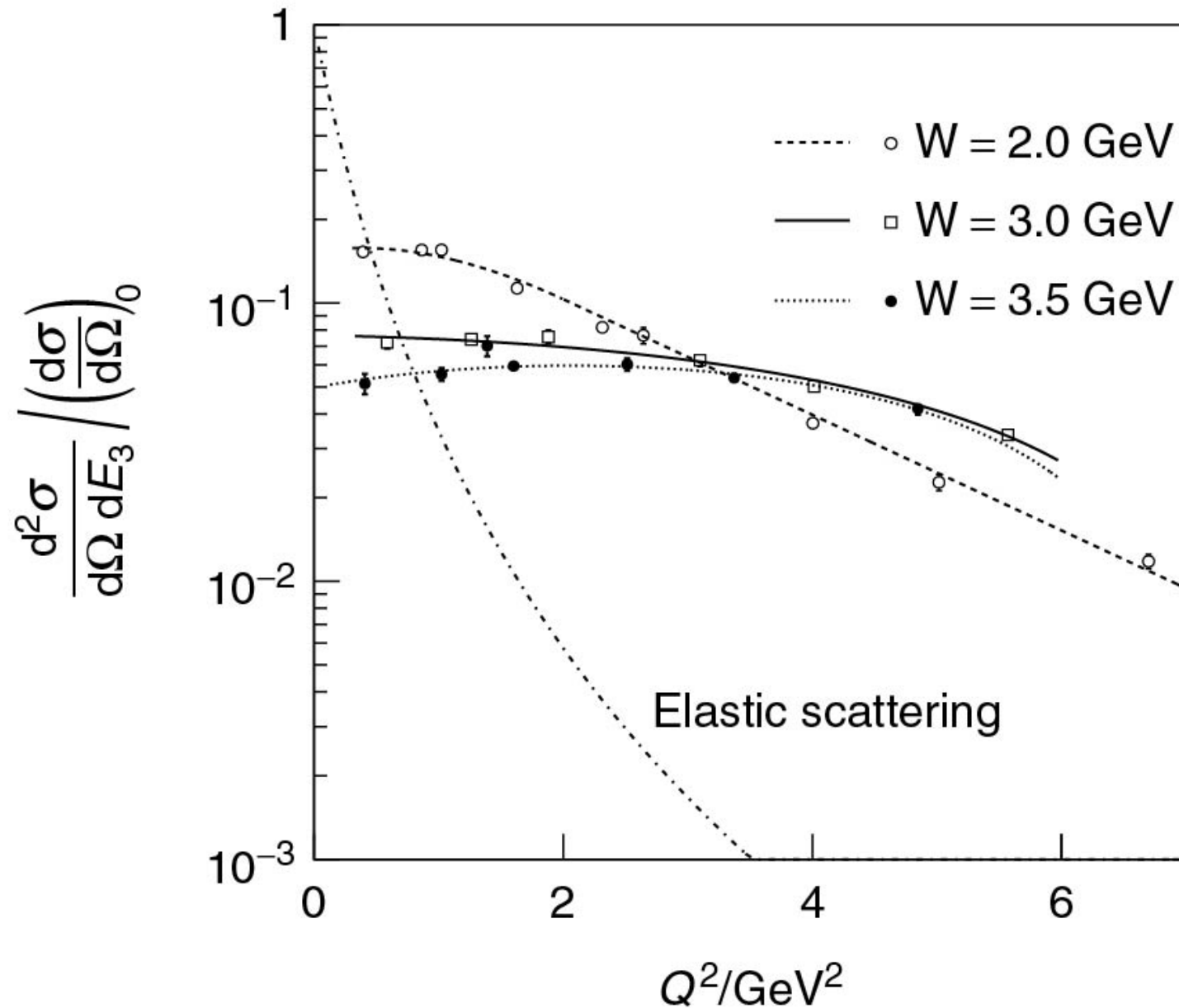
Charge/magnetic moment distribution

$$\rho(r) = \rho(0) e^{-ar} \quad a = 4.27 \text{ fm}^{-1}$$

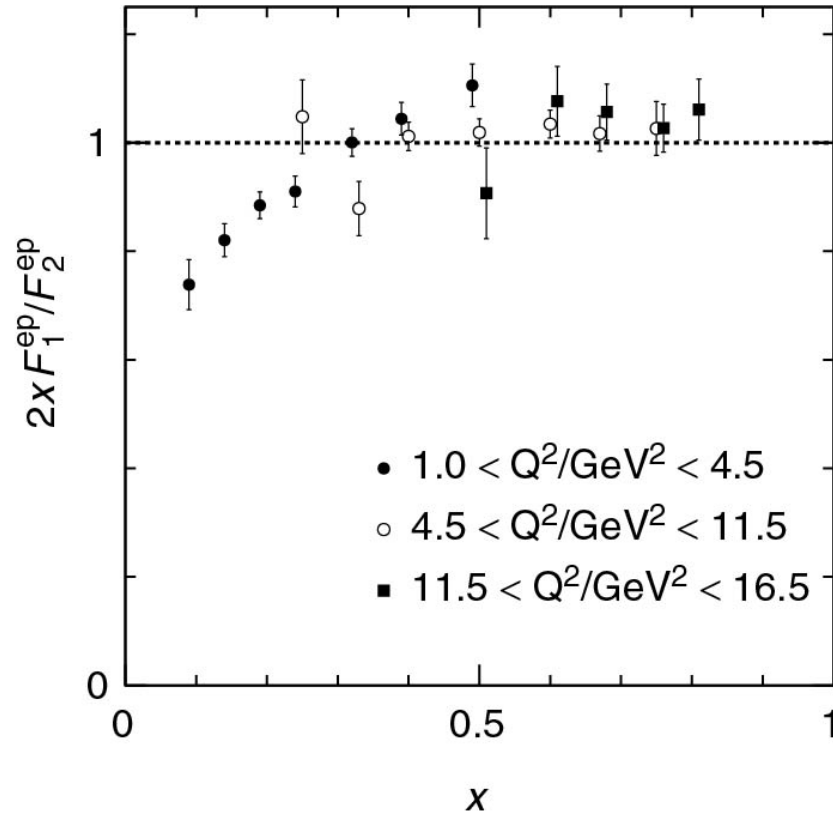
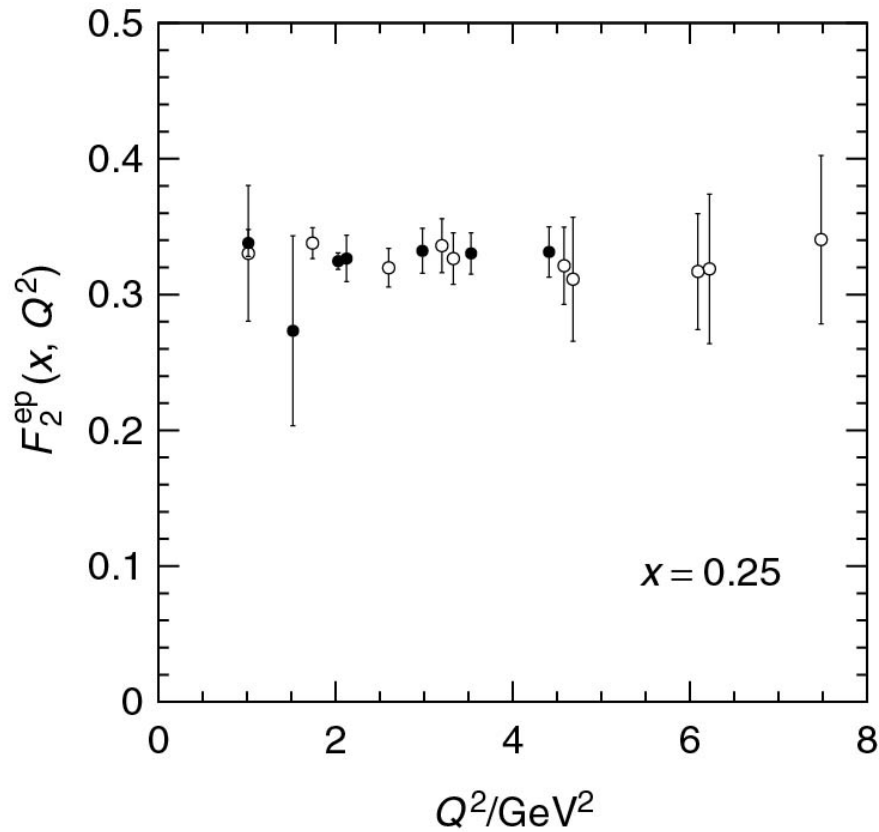
Inelastic scattering



Inelastic scattering

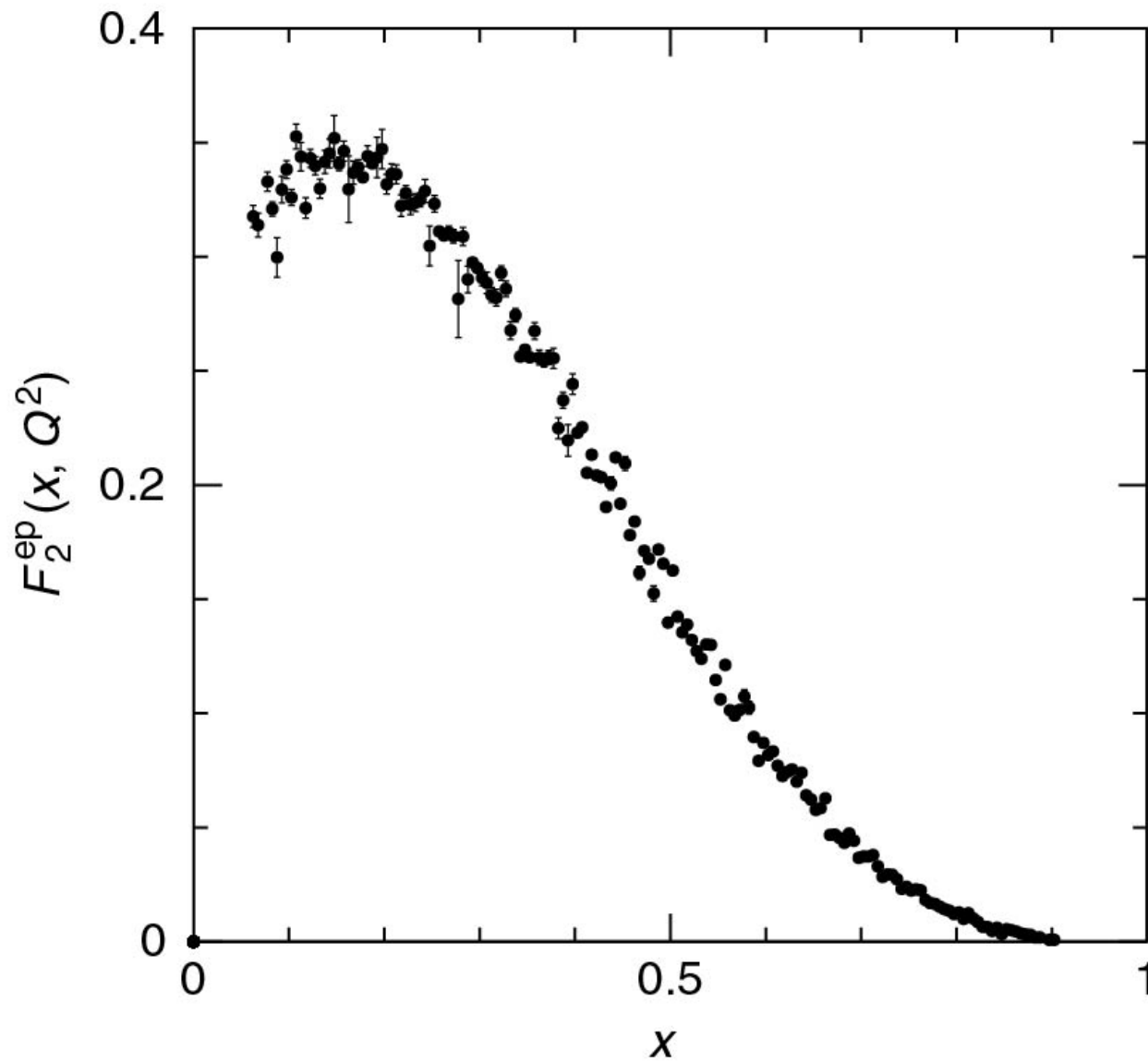


Inelastic scattering

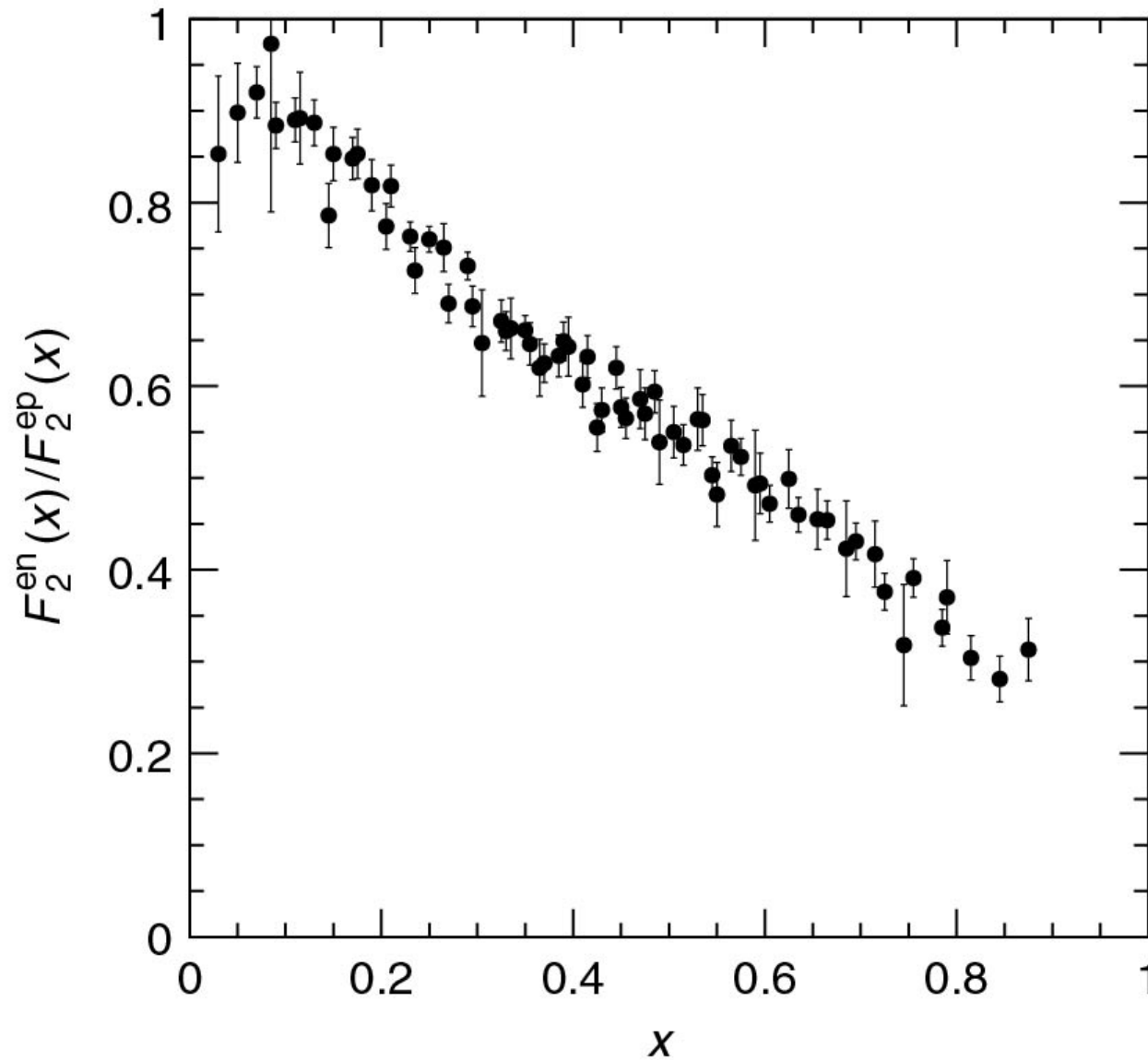


The proton consists of Dirac (point-like Spin 1/2) particles!

Measurement of structure function



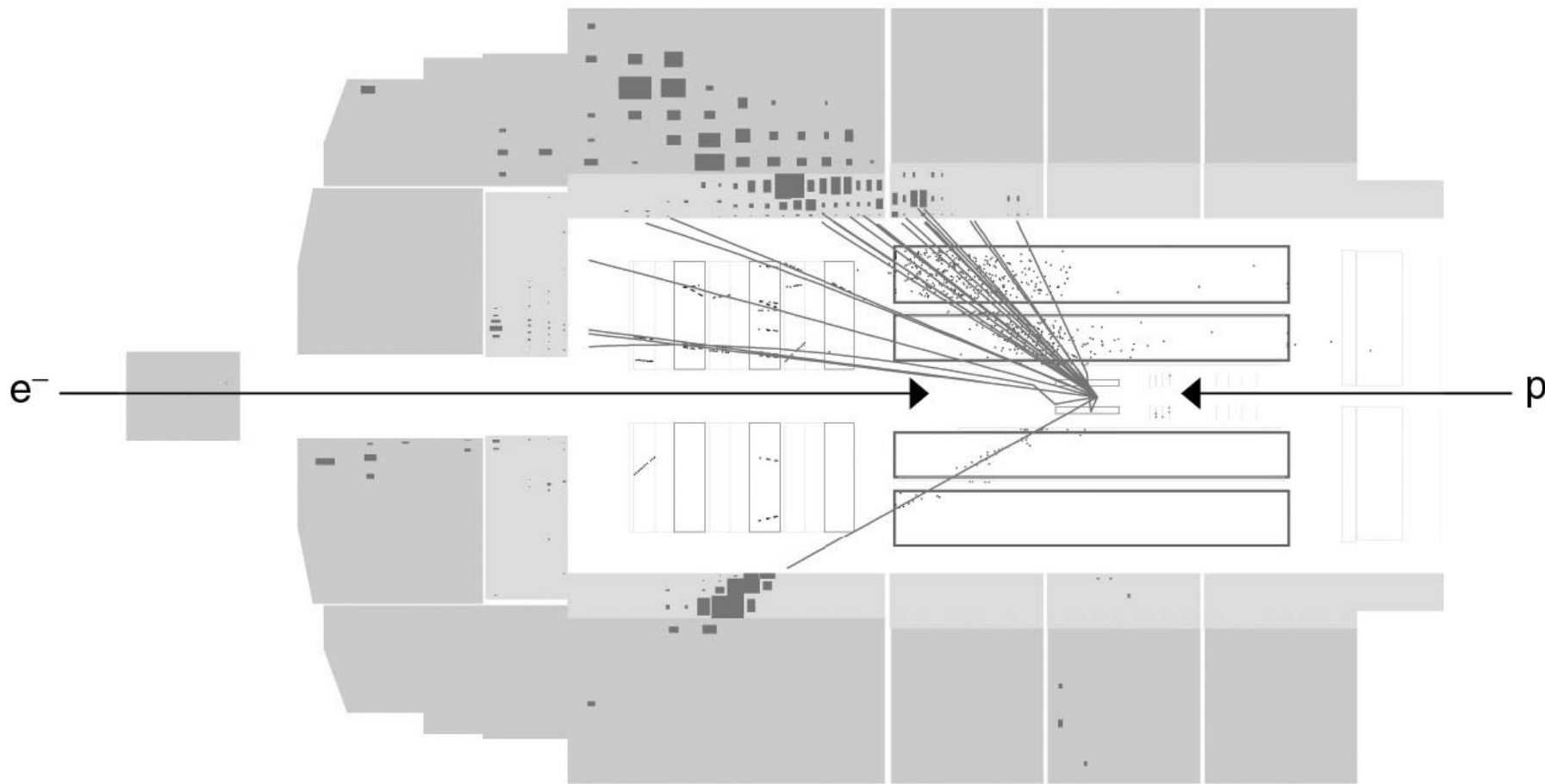
Ratio of Form Factors



H1 at HERA

Two detectors measuring structure functions located at HERA/DESY

(1991-2007): ZEUS & H1



$$E_e = 27.5 \text{ GeV}, E_p = 820 \text{ or } 920 \text{ GeV}$$

Structure functions

