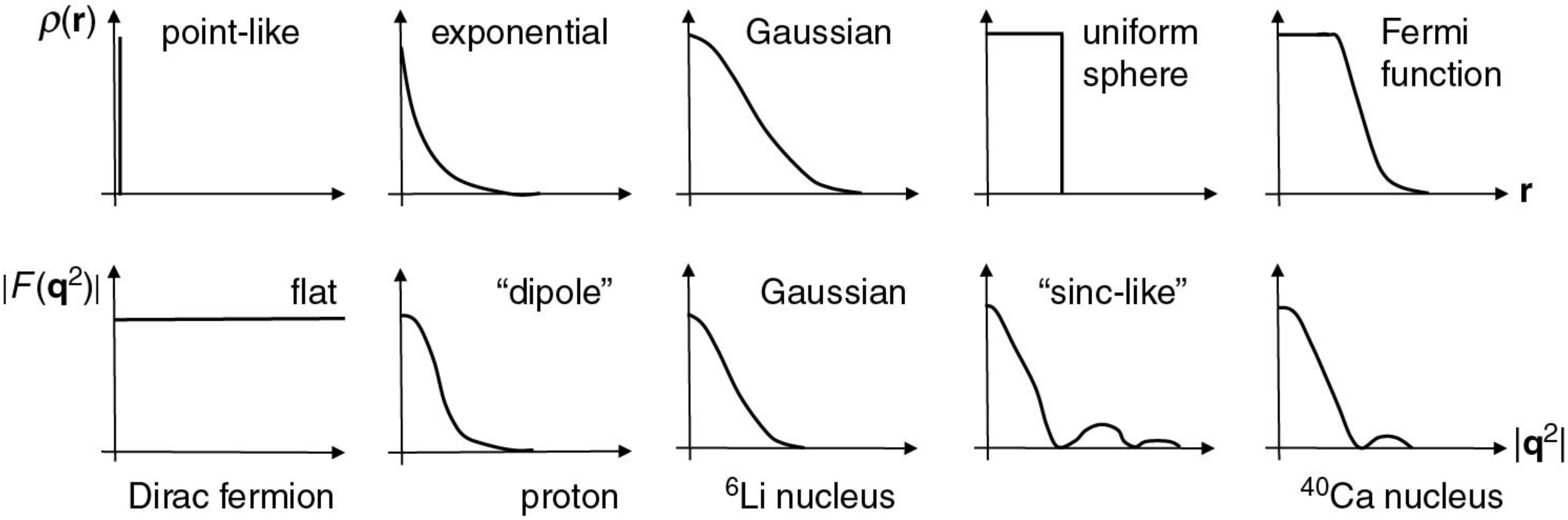
$$u_\uparrow = N \begin{pmatrix} c \\ s e^{i\phi} \\ \frac{p}{E+m} c \\ \frac{p}{E+m} s e^{i\phi} \end{pmatrix} \quad u_\downarrow = N \begin{pmatrix} -s \\ c e^{i\phi} \\ \frac{p}{E+m} s \\ -\frac{p}{E+m} c e^{i\phi} \end{pmatrix} \quad v_\uparrow = N \begin{pmatrix} \frac{p}{E+m} s \\ -\frac{p}{E+m} c e^{i\phi} \\ -s \\ c e^{i\phi} \end{pmatrix} \quad v_\downarrow = N \begin{pmatrix} \frac{p}{E+m} c \\ \frac{p}{E+m} s e^{i\phi} \\ c \\ s e^{i\phi} \end{pmatrix}$$

$$N = \sqrt{E+m}$$

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Form-Factors



Cambridge University Press 2013

Dirac Scattering

$$\left(\frac{d\sigma}{d\Omega}\right)_{Dirac} = \left(\frac{d\sigma}{d\Omega}\right)_{Ruth} \left(\cos^2\theta/2 + \frac{Q^2}{2m_p^2c^2}\sin^2\theta/2\right)$$

$Q^2 = -q^2$ (4-momentum transfer)

el. WW: Mott scattering for für high relativistic particles

mag. WW: depend on Q^2 introduced by considering recoil (m_p is not much larger than Q^2)

$$\left(\frac{d\sigma}{d\Omega}\right)_{Dirac} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} (1 + 2\tau\tan^2\theta/2)$$

$\tau = \frac{Q^2}{4m_p^2}$

Rosenbluth Formular

$$\left(\frac{d\sigma}{d\Omega}\right) =$$

$$\frac{\alpha^2}{4E_1^2\sin^4(\theta/2)}\frac{E_3}{E_1}\left(\frac{G_E^2(Q^2)+\tau G_M^2(Q^2)}{1+\tau}\cos^2\theta/2 + 2\tau G_M^2(Q^2)\sin^2\theta/2\right)$$

$$= \left(\frac{d\sigma}{d\Omega}\right)_0 \left(\frac{G_E^2(Q^2)+\tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\tan^2\theta/2\right)$$

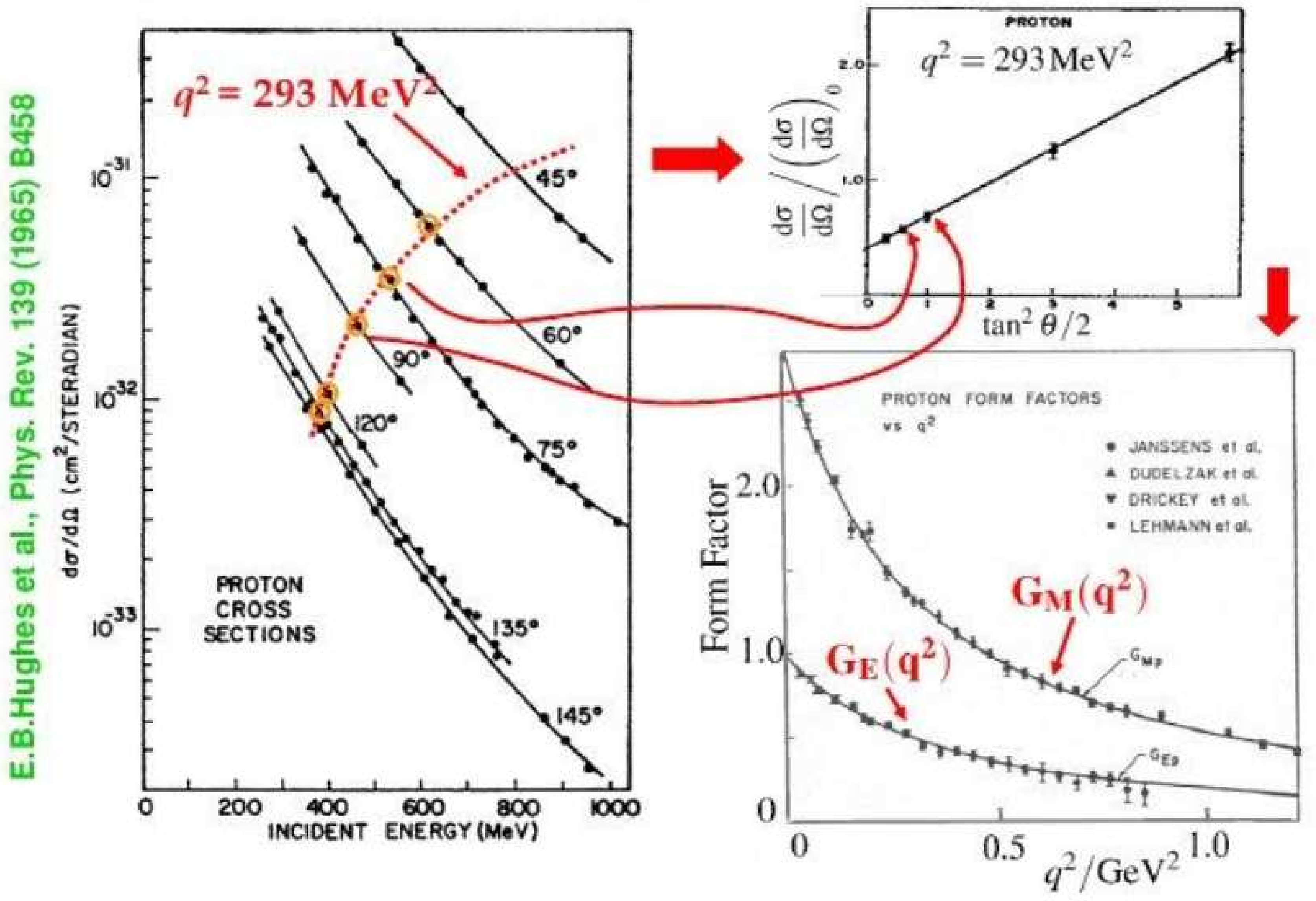
$$\tau = \frac{Q^2}{4m_p^2}$$

$$\left(\frac{d\sigma}{d\Omega}\right) / \left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{G_E^2(Q^2)+\tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\tan^2\theta/2$$

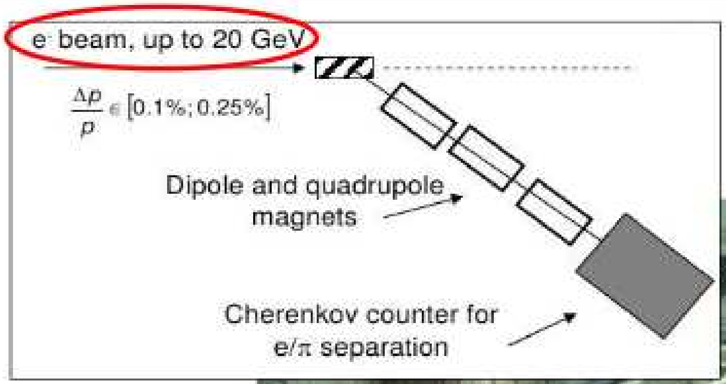
$\rightarrow$ Measure $\left(\frac{d\sigma}{d\Omega}\right) / \left(\frac{d\sigma}{d\Omega}\right)_0$ for fixed Q^2 at different $\tan^2\theta/2$ values!

Slope and absizze determine G_M and G_E at this Q^2 value.

Measurement of el. and mag. Formfactors

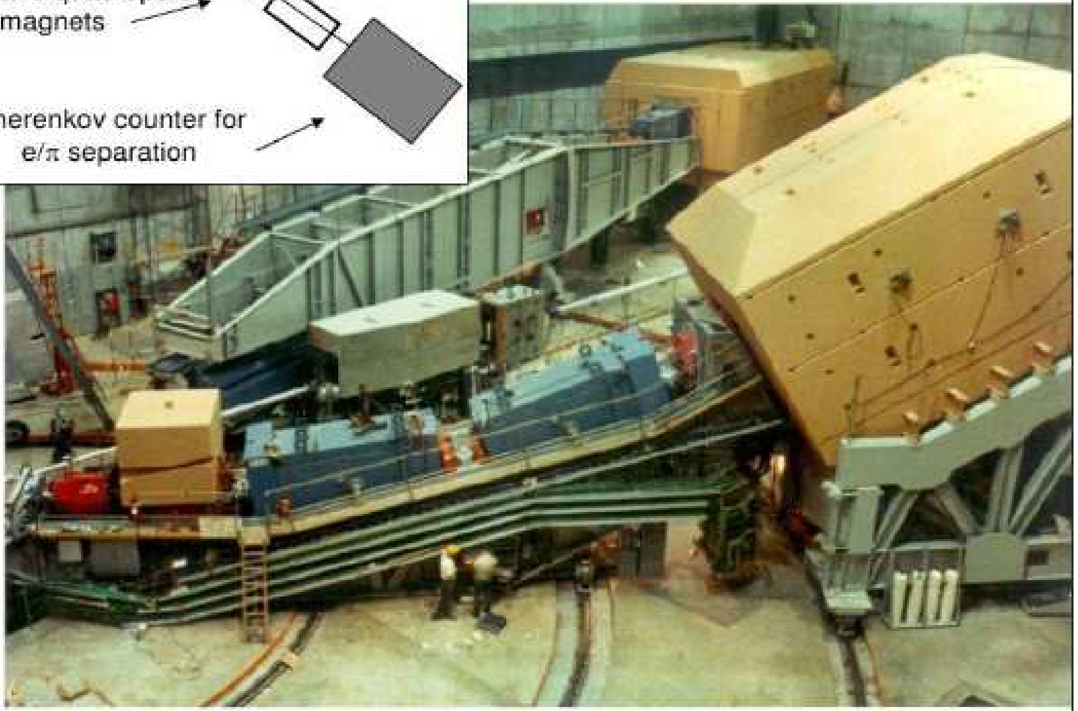


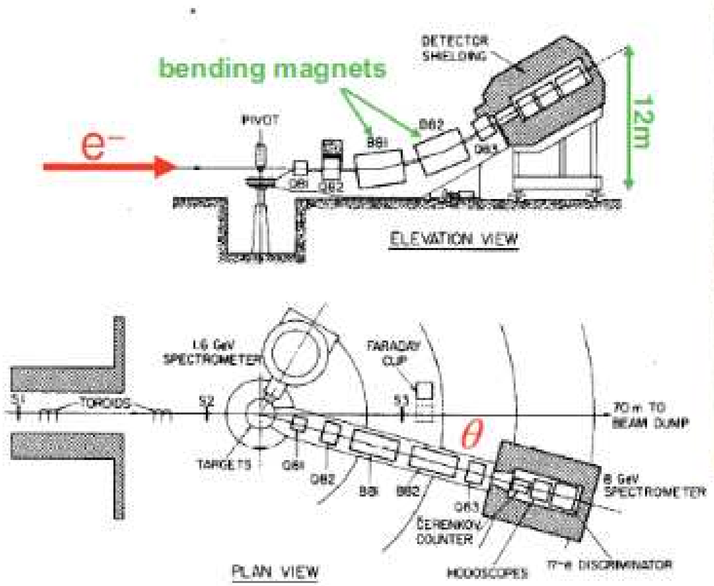
SLAC Experiment



Spectrometer at fixed θ

$\frac{\Delta p}{p} \sim 0.1\%$ $\Delta\theta \sim 0.7$mrad

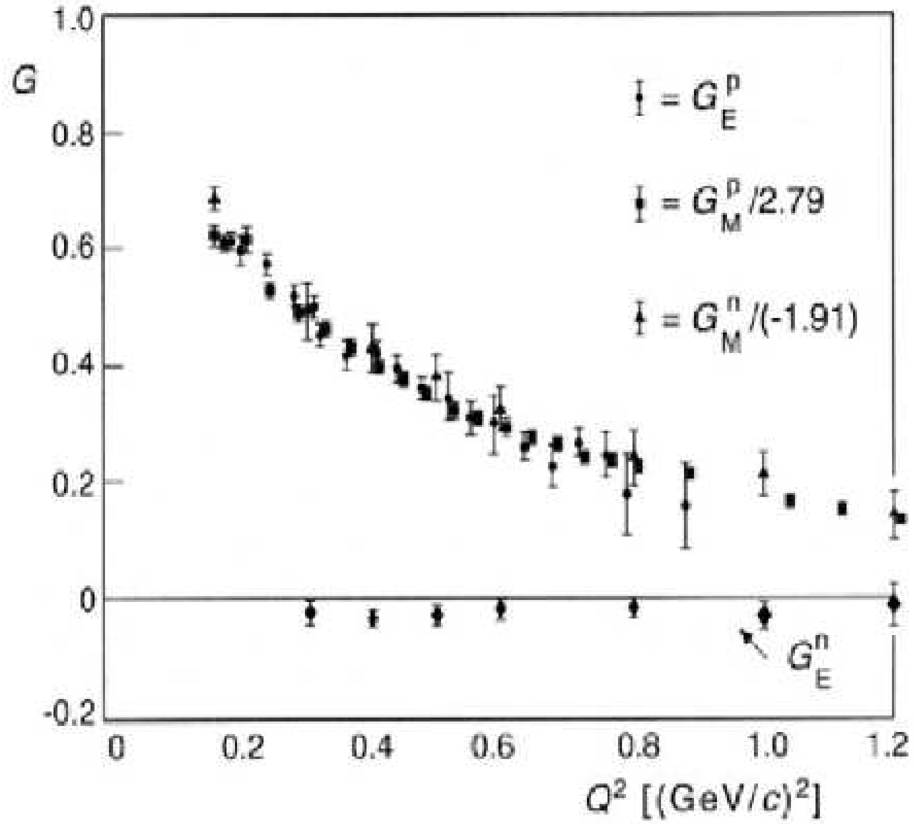




P.N.Kirk et al., Phys Rev D8 (1973) 63

Measurement of el. and mag. Formfactors

$$G_E^p(Q^2) = \frac{G_M^p}{2.79} = \frac{G_M^n}{-1.91} = G^{Dipole}(Q^2)$$



Charge and magnetic moment have give the same size for the proton. Both quantities are equally distributed in the proton.

$$G(Q^2) = \frac{1}{(1+Q^2/0.71\ GeV^2)^2}$$

$$\langle r^2 \rangle = -6 \frac{dF(Q^2)}{dQ^2}\Big|_{Q^2=0}$$

$$\sqrt{\langle r^2 \rangle\ Dipole} = 0.81\ fm$$

Charge/magnetic moment distribution

$\rho(r) = \rho(0)\, e^{-ar}$ $\quad a = 4.27\ fm^{-1}$