

History of EM unification

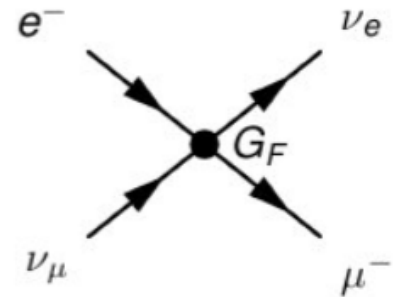
- **1934** Fermi-Theory: **point-like interaction (no q^2 dependence)**

$$M_{fi} = \frac{G_F}{\sqrt{2}} g_{\mu\nu} (\bar{\Psi} \gamma^\mu (1 - \gamma^5) \Psi) (\bar{\Psi} \gamma^\nu (1 - \gamma^5) \Psi)$$

approach still valid for $q^2 < m(W)$

However divergence of electron-neutrino cross section at high CME ($\sqrt{s}$)

$$\sigma \propto G_F^2 s$$

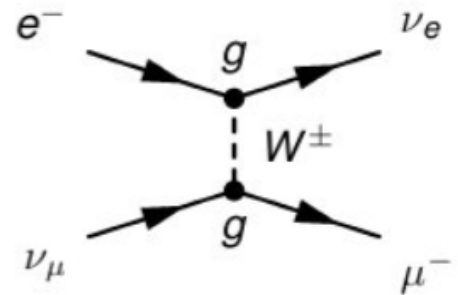


- Fix: exchange of massive boson $W^\pm$: V-A theory

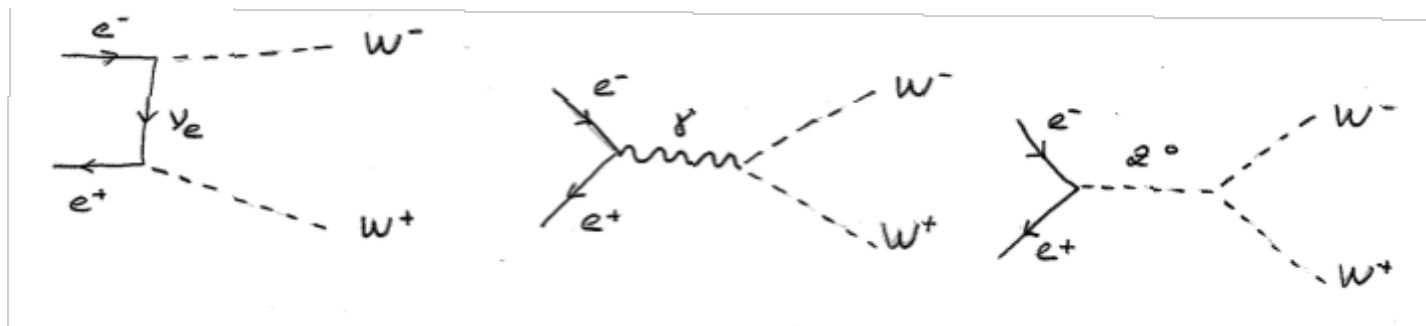
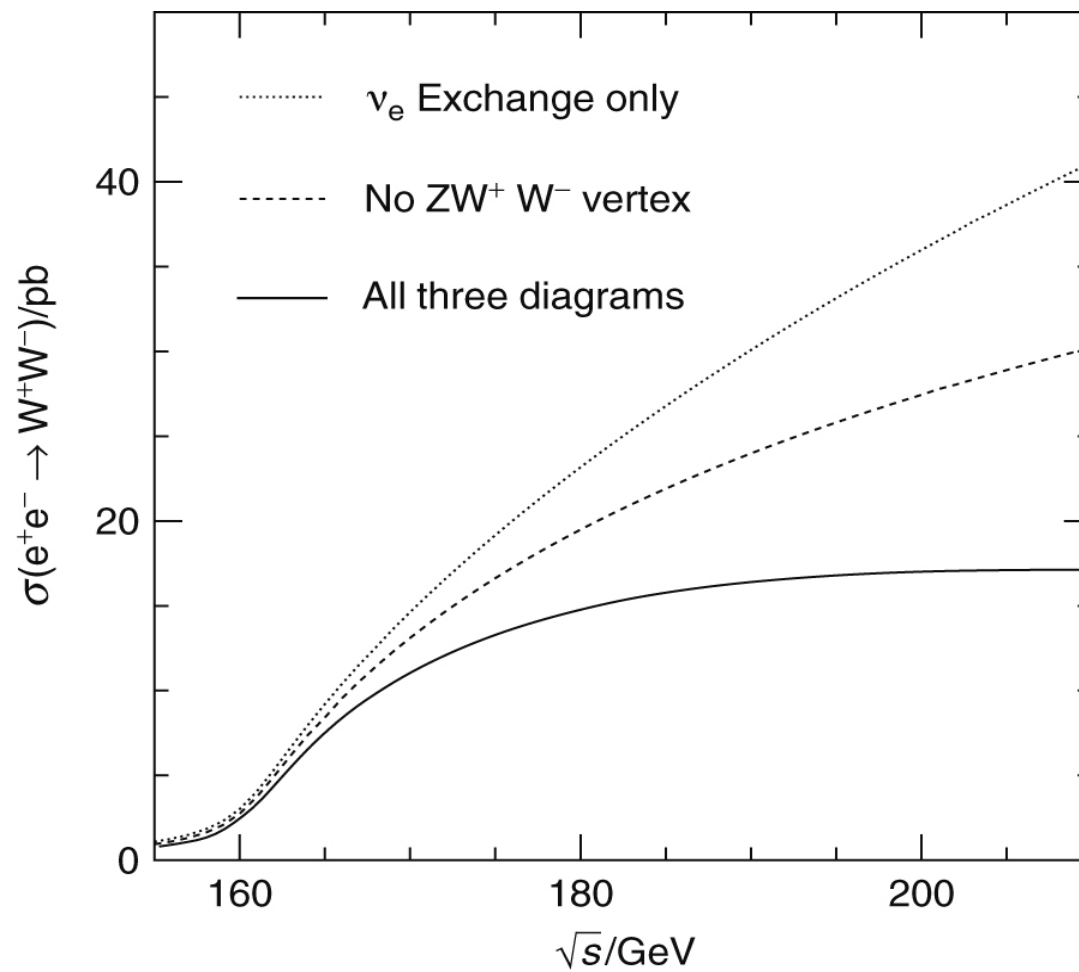
$$M_{fi} = \left(\frac{g_W}{\sqrt{2}} \bar{\Psi} \gamma^\mu \frac{1-\gamma^5}{2} \Psi \right) \frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2} \left(\frac{g_W}{\sqrt{2}} \bar{\Psi} \gamma^\nu \frac{1-\gamma^5}{2} \Psi \right)$$

$$G_F = \frac{\sqrt{2} g_W^2}{8 M_W^2}$$

$$\sigma \propto \frac{G_F^2 M_W^2}{s + M_W^2} s$$



Still remaining problem, divergence of W pair production cross-section



Standard model of particle physics $\rightarrow$ electroweak unification

Glashow, Salam and Weinberg (1967): Electroweak Unification

- in analogy to strong IA, introduce weak isospin

left handed particles form isospin doublets, $T = \frac{1}{2}, T_3 = \pm \frac{1}{2}$

right handed particles form isospin singlets $T = 0$

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}, u_R, d_R, c_R, s_R, t_R, b_R$$

$$\begin{pmatrix} \nu_{e,L} \\ e_L^- \end{pmatrix}, \begin{pmatrix} \nu_{\mu,L} \\ \mu_L^- \end{pmatrix}, \begin{pmatrix} \nu_{\tau,L} \\ \tau_L^- \end{pmatrix}, e_R^-, \mu_R^-, \tau_R^-$$

Electro-weak quantum numbers

Leptons	T	T_3	Q	Y
ν_e	$\frac{1}{2}$	$+\frac{1}{2}$	0	-1
e_L^-	$\frac{1}{2}$	$-\frac{1}{2}$	-1	-1
e_R^-	0	0	-1	-2

Quarks	T	T_3	Q	Y
u_L	$\frac{1}{2}$	$+\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{3}$
d_L	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$
u_R	0	0	$\frac{2}{3}$	$\frac{4}{3}$
d_R	0	0	$-\frac{1}{3}$	$-\frac{2}{3}$

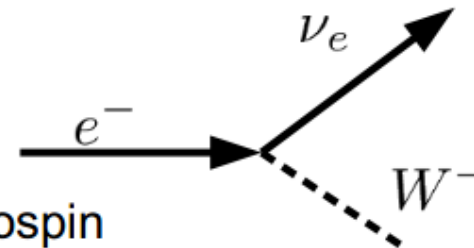
W_1, W_2, W_3 generators of $SU(2)_{iso}$

$W^\pm = W_1 \mp iW_2$ ladder operators

W_3 eigenvalue: third component of weak isospin

$$Y = 2[Q - T_3]$$

Weak IA described by $SU_{iso}(2) \times U_Y(1)$



4 mass less gauge fields W_1, W_2, W_3 and B

couple to left handed particles only

couples to left and right handed particles

Glashow, Salam and Weinberg (1967): Electroweak Unification

$SU_{iso}(2) \times U_Y(1)$ contains subgroup $U_{em}(1)$

$B \neq A$ and $W_3 \neq A$ elm. IA does not couple to left handed neutrinos

Electro-weak quantum numbers

Leptons	T	T_3	Q	Y
ν_e	$\frac{1}{2}$	$+\frac{1}{2}$	0	-1
e_L	$\frac{1}{2}$	$-\frac{1}{2}$	-1	-1
e_R	0	0	-1	-2

Quarks	T	T_3	Q	Y
u_L	$\frac{1}{2}$	$+\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{3}$
d_L	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$
u_R	0	0	$\frac{2}{3}$	$\frac{4}{3}$
d_R	0	0	$-\frac{1}{3}$	$-\frac{2}{3}$

Solution: A is linear combination of W_3 and B

$$\begin{aligned}
 A_\mu &= B_\mu \cos \theta_W + W_\mu^3 \sin \theta_W && \leftarrow \text{massless photon} \\
 Z_\mu &= -B_\mu \sin \theta_W + W_\mu^3 \cos \theta_W && \leftarrow \text{massive Z boson} \\
 B_\mu &= A_\mu \cos \theta_W - Z_\mu \sin \theta_W \\
 W_\mu^3 &= A_\mu \sin \theta_W + Z_\mu \cos \theta_W
 \end{aligned}$$

Weinberg angle θ_W defined by couplings of A and Z.

from experiment: $\sin^2 \theta_W = 0.23143 \pm 0.00015$

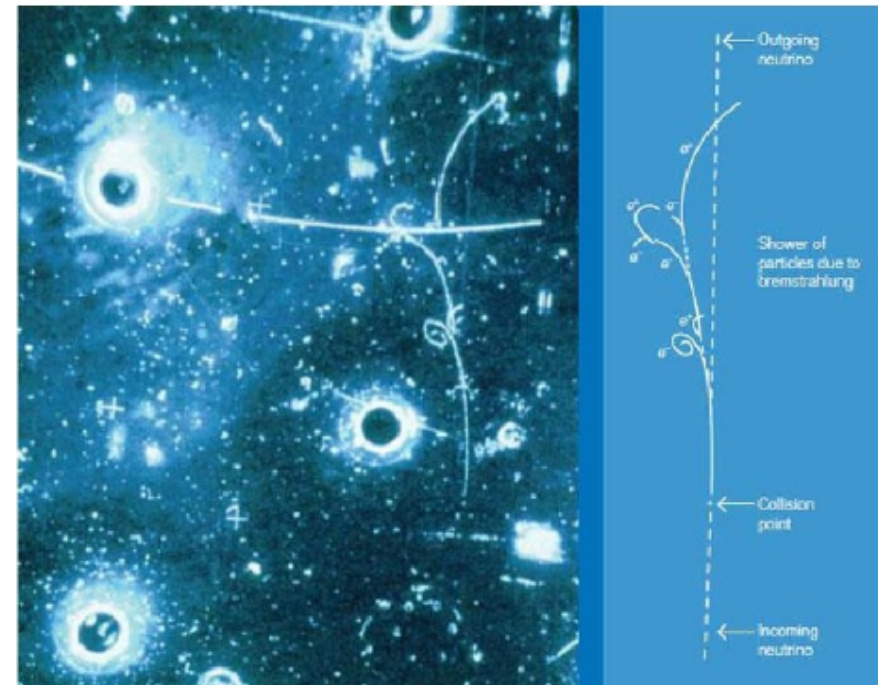
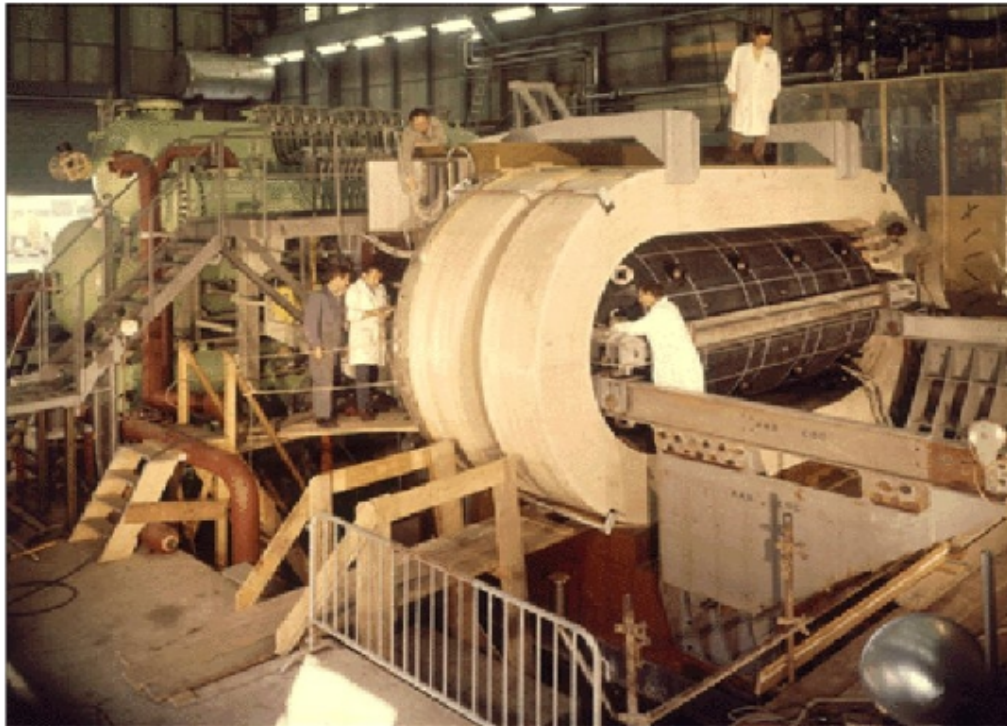
$$\cos \theta_W = \frac{M_W}{M_Z}$$

Problem, no mass term in Lagrangian (otherwise spoils gauge invariance, thus is not renormalizable)

→ require dedicated mechanism to create masses → HIGGS mechanism (introduced 1964)

Discovery of neutral current

NC was first theoretical introduced in GSW theory in 1969 and then discovered by the Gargamell experiment in 1973



$$\bar{\nu}_{\mu} + e^{-} \rightarrow \bar{\nu}_{\mu} + e^{-}$$

Despite this experimental “proof” GSW model not widely accepted before 1977 t' Hooft and Veltmann demonstrated renormalization of this theory (all divergences cancel)

1979 Nobel prize for Glashow, Salam and Weinberg



Sheldon Glashow, Abdus Salam, and Steven Weinberg sharing the Nobel Prize, 1979

1999 Nobel prize for 't Hooft and Veltman



Gerardus 't Hooft



Martinus J.G. Veltman

The Nobel Prize in Physics 1999 was awarded jointly to Gerardus 't Hooft and Martinus J.G. Veltman *"for elucidating the quantum structure of electroweak interactions in physics"*

1984: Nobel Prize for the discovery of the Z and W boson



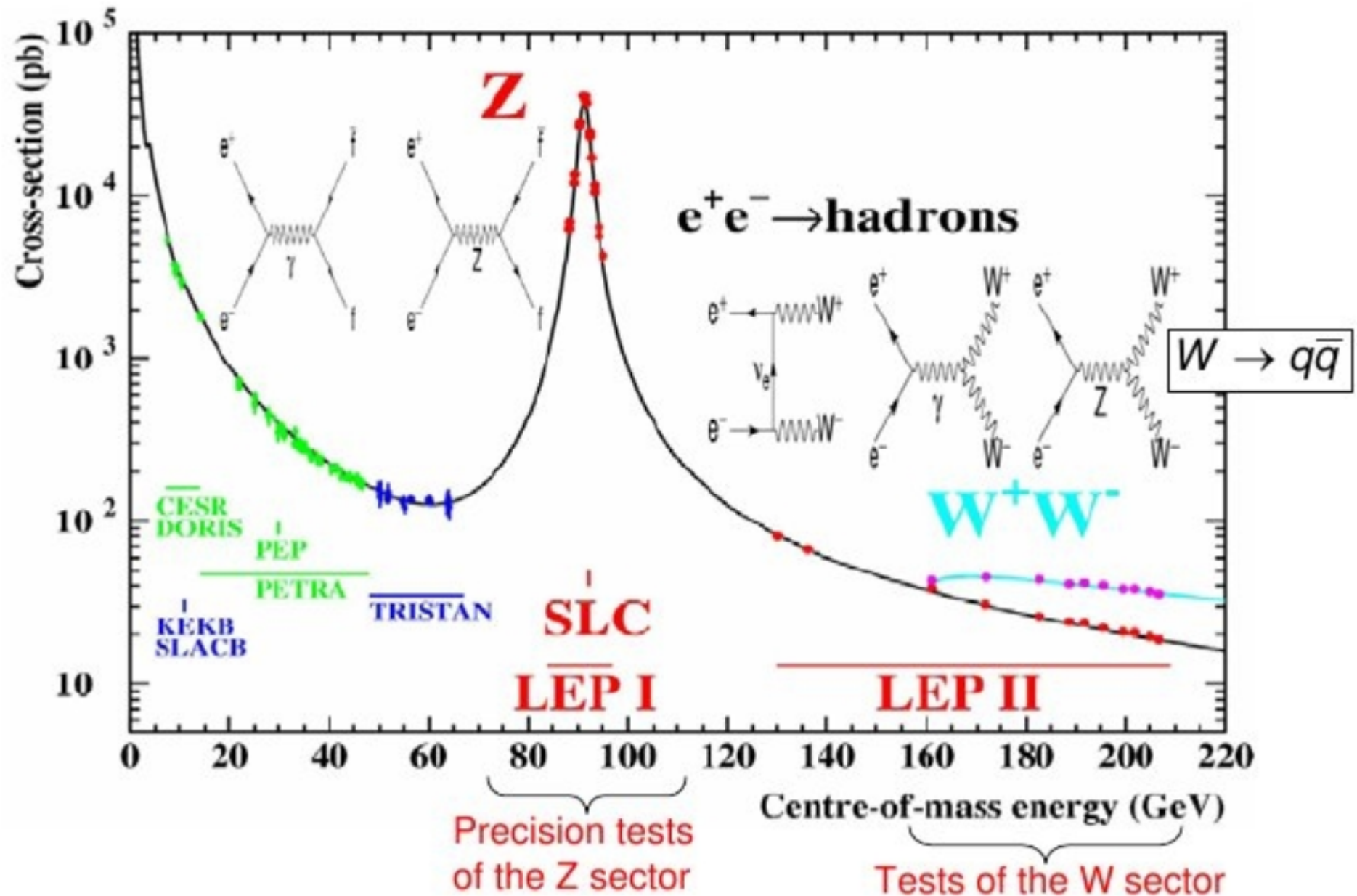
Carlo Rubbia



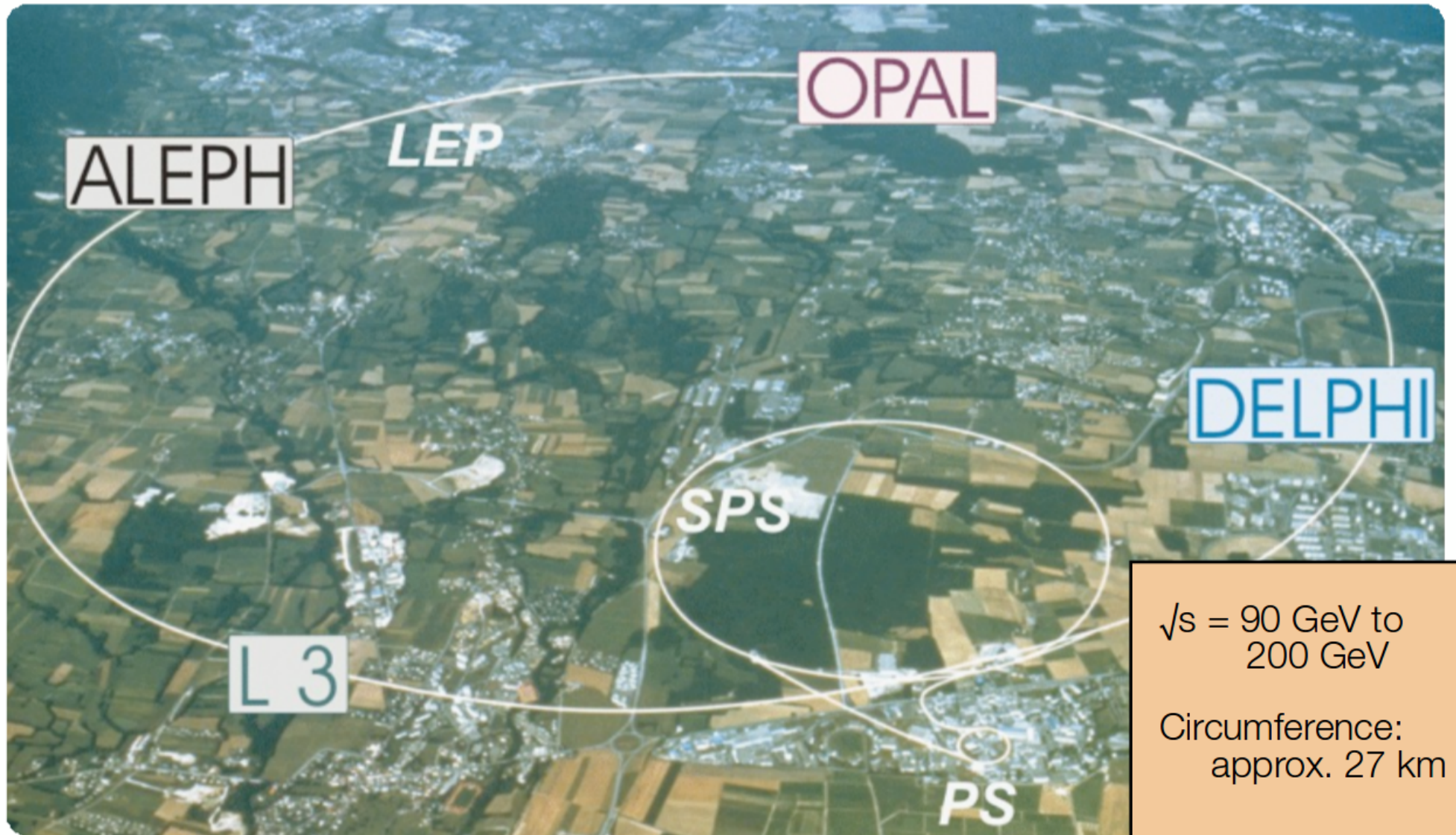
Simon van der Meer

The Nobel Prize in Physics 1984 was awarded jointly to Carlo Rubbia and Simon van der Meer *"for their decisive contributions to the large project, which led to the discovery of the field particles W and Z, communicators of weak interaction"*

Precision Test of the Standard Model at LEP (e^+e^-)



The LEP Collider



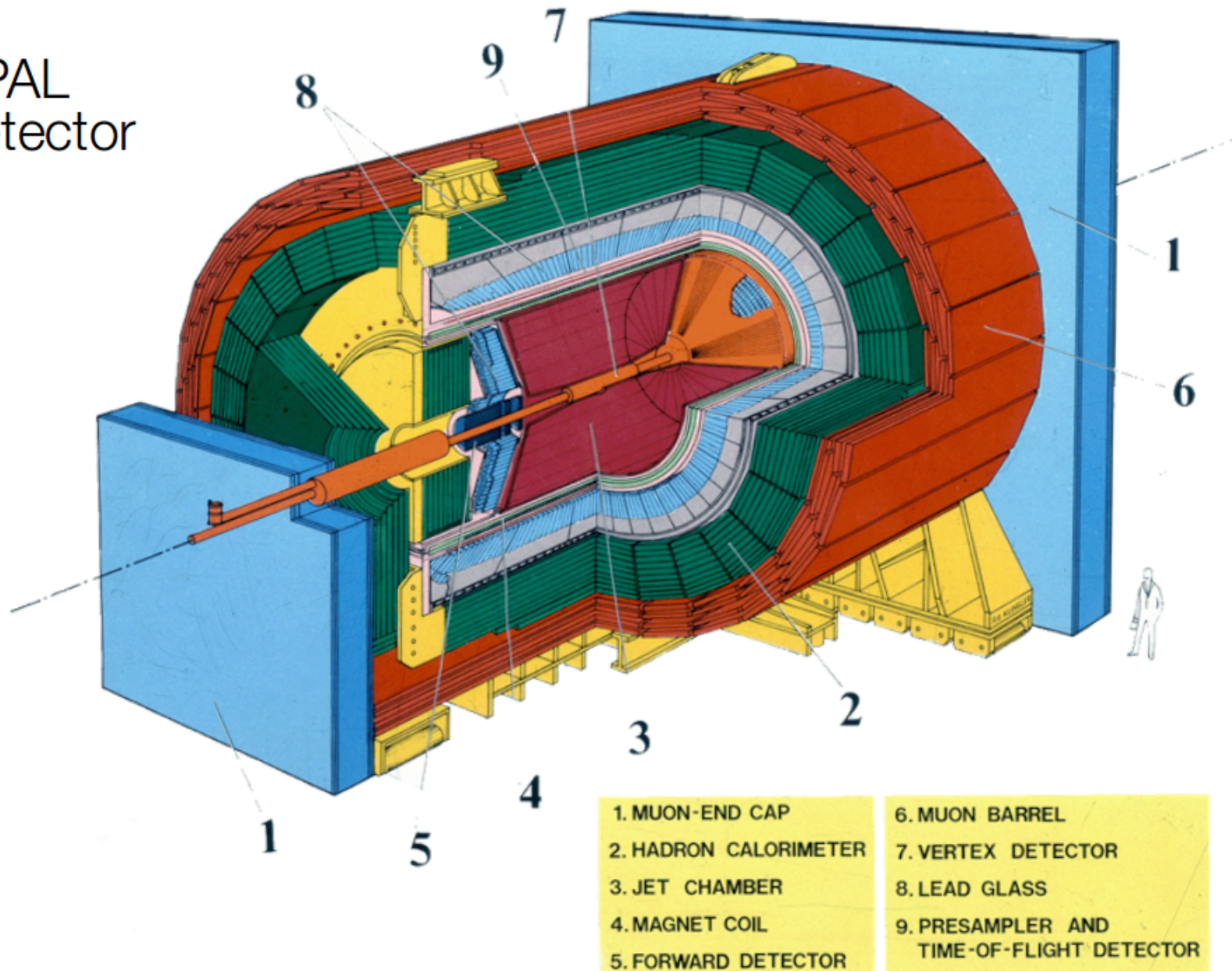
$\sqrt{s} = 90 \text{ GeV to } 200 \text{ GeV}$

Circumference:
approx. 27 km

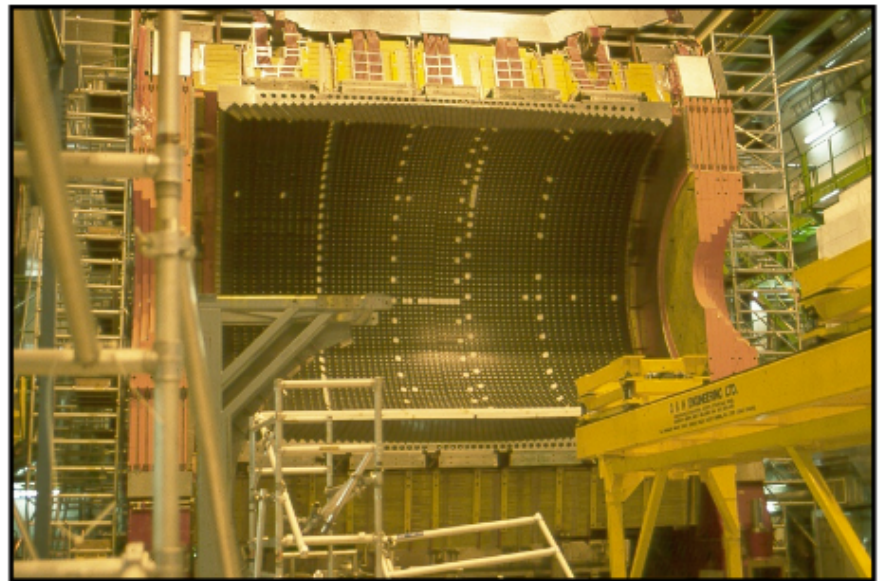
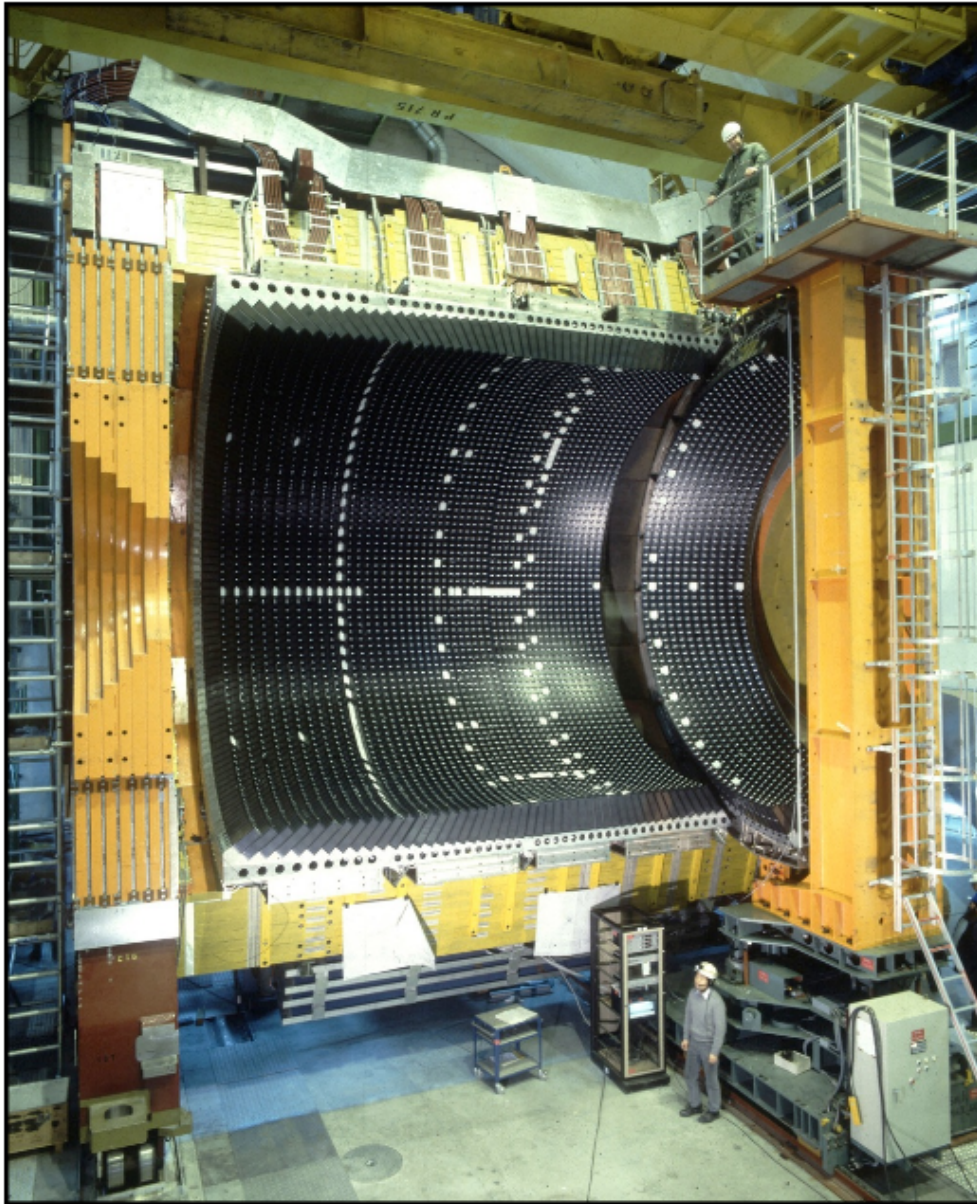
4 Experiments

The LEP Experiments - OPAL

OPAL
Detector



The LEP Experiments - OPAL



Opal Calorimeter

The LEP Experiments - OPAL

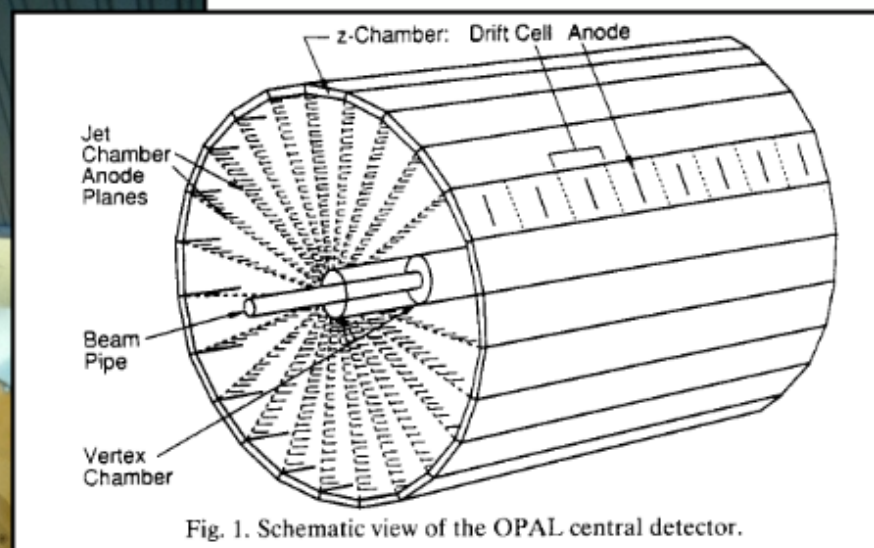
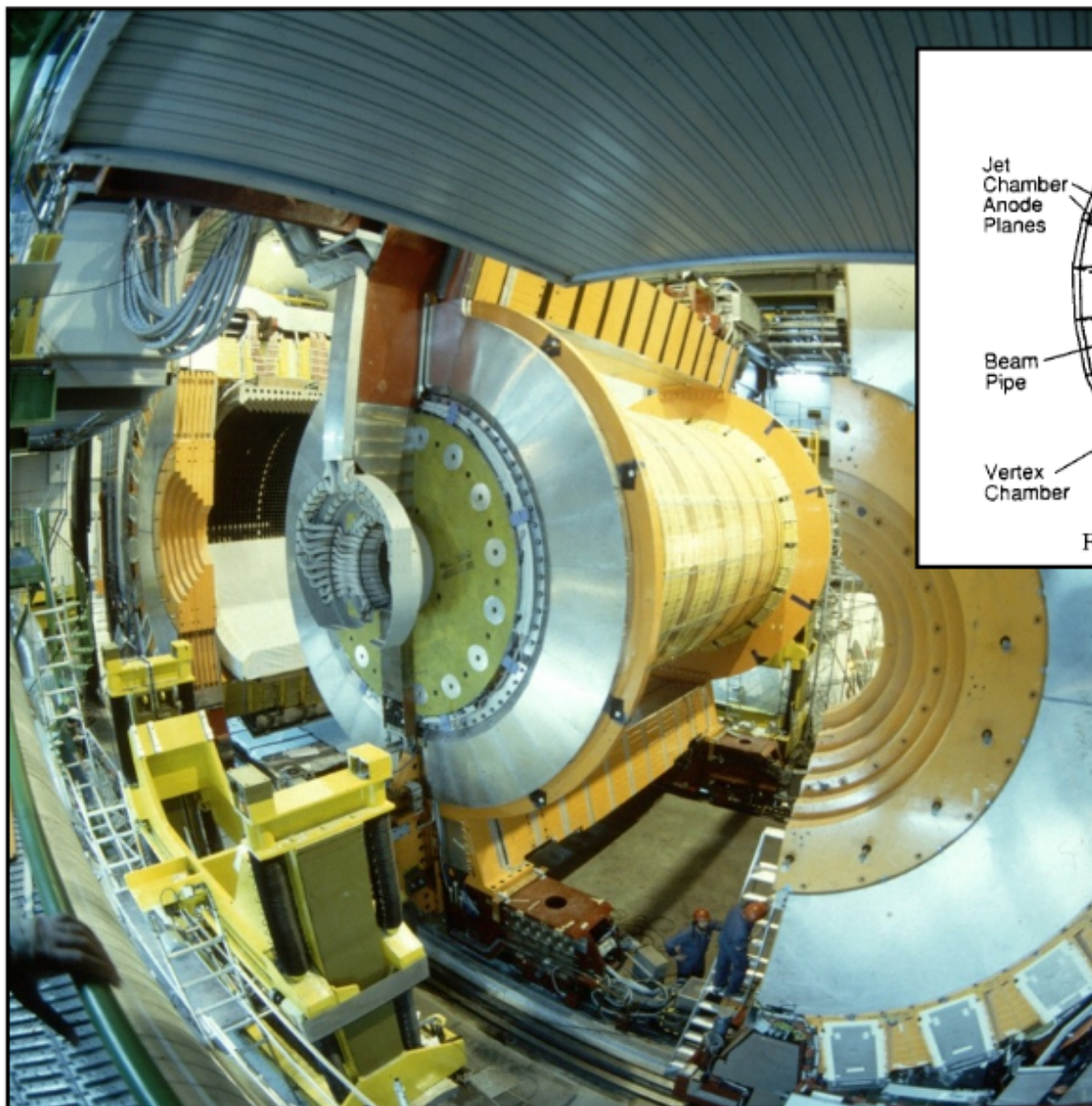
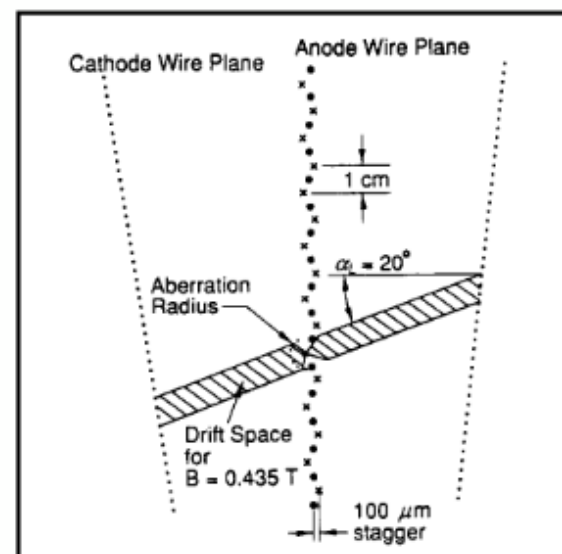


Fig. 1. Schematic view of the OPAL central detector.



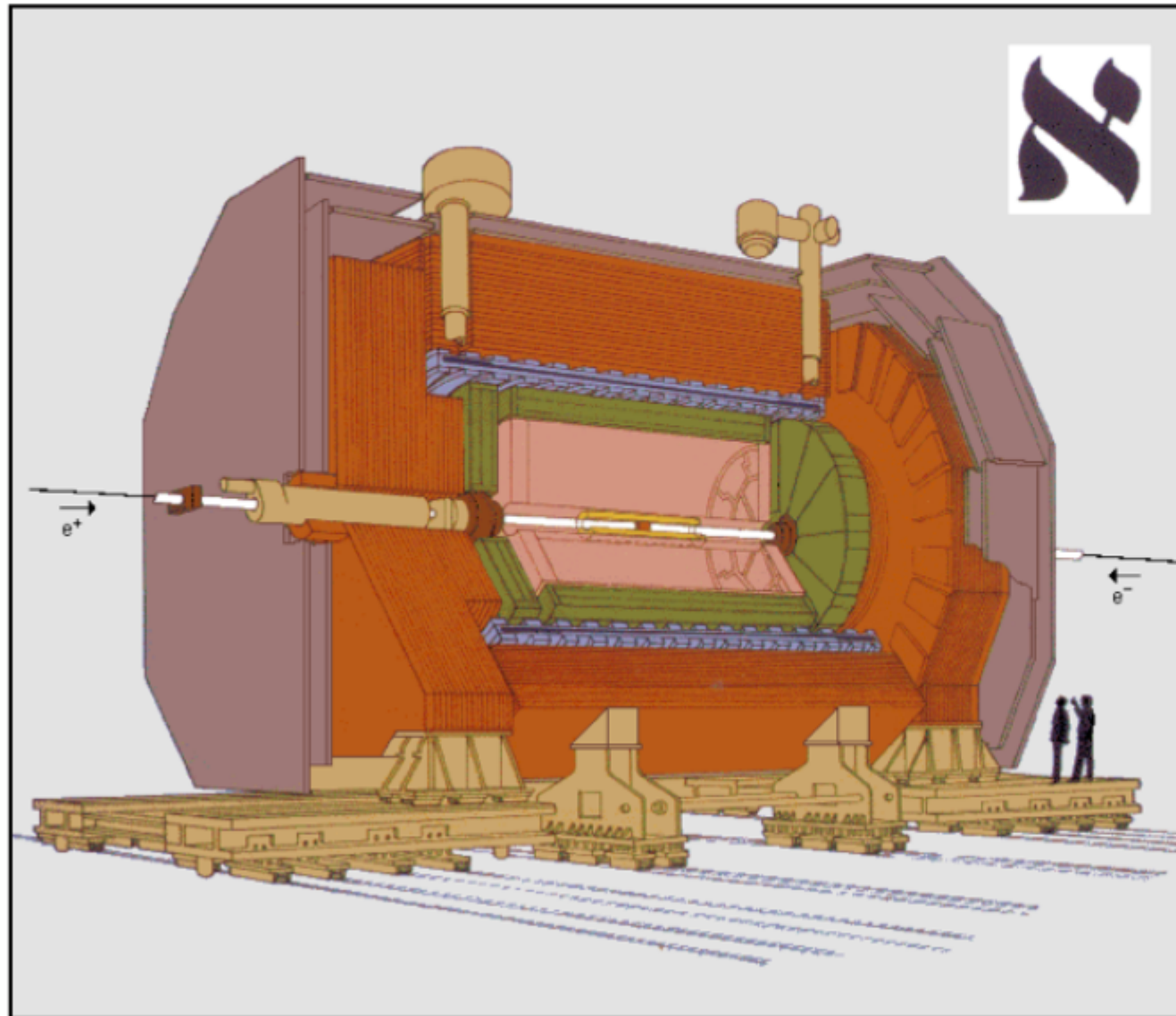
Opal Jet Chamber

The LEP Experiments - OPAL



OPAL Jet Chamber installation

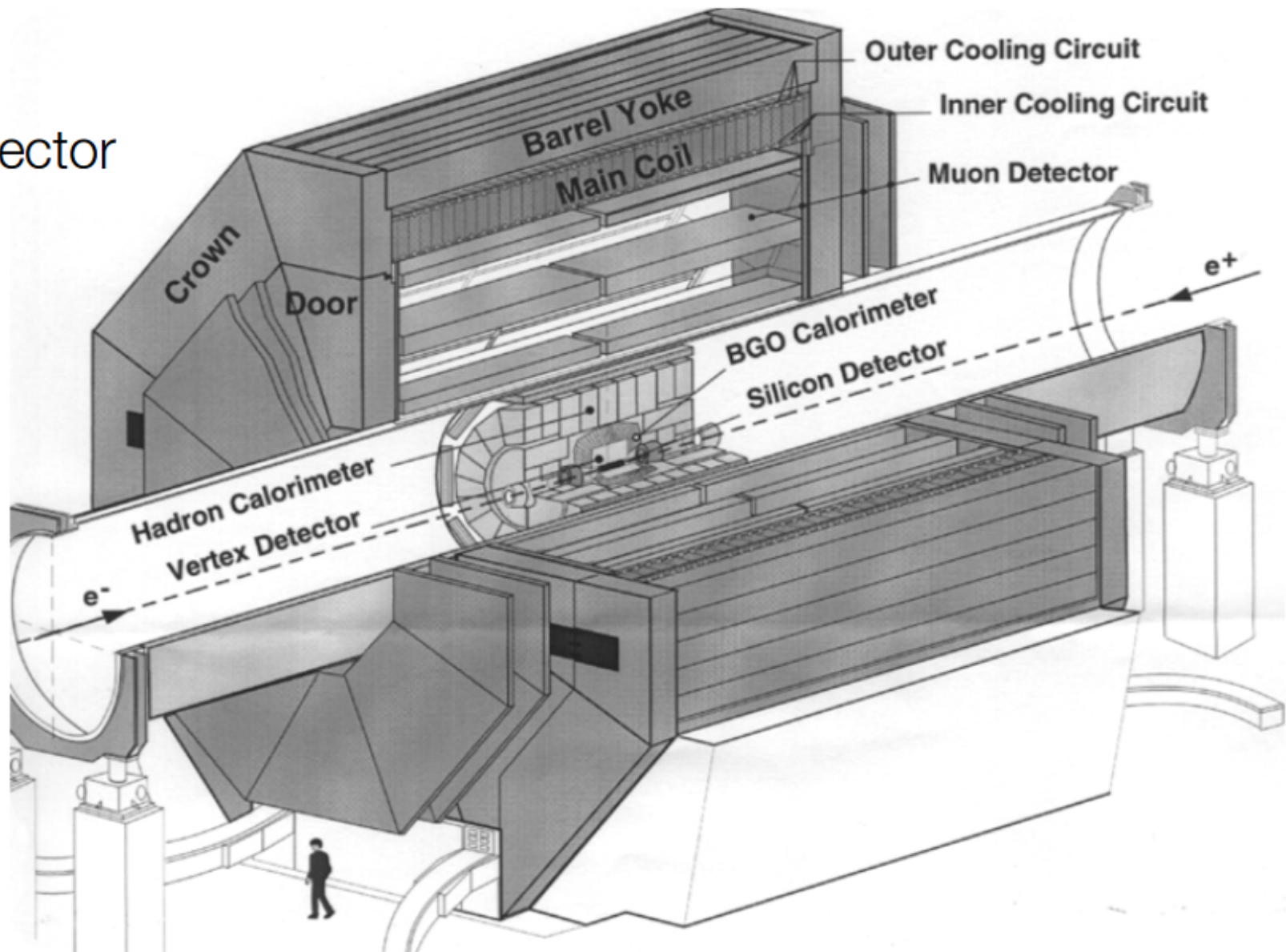
The LEP Experiments - ALEPH



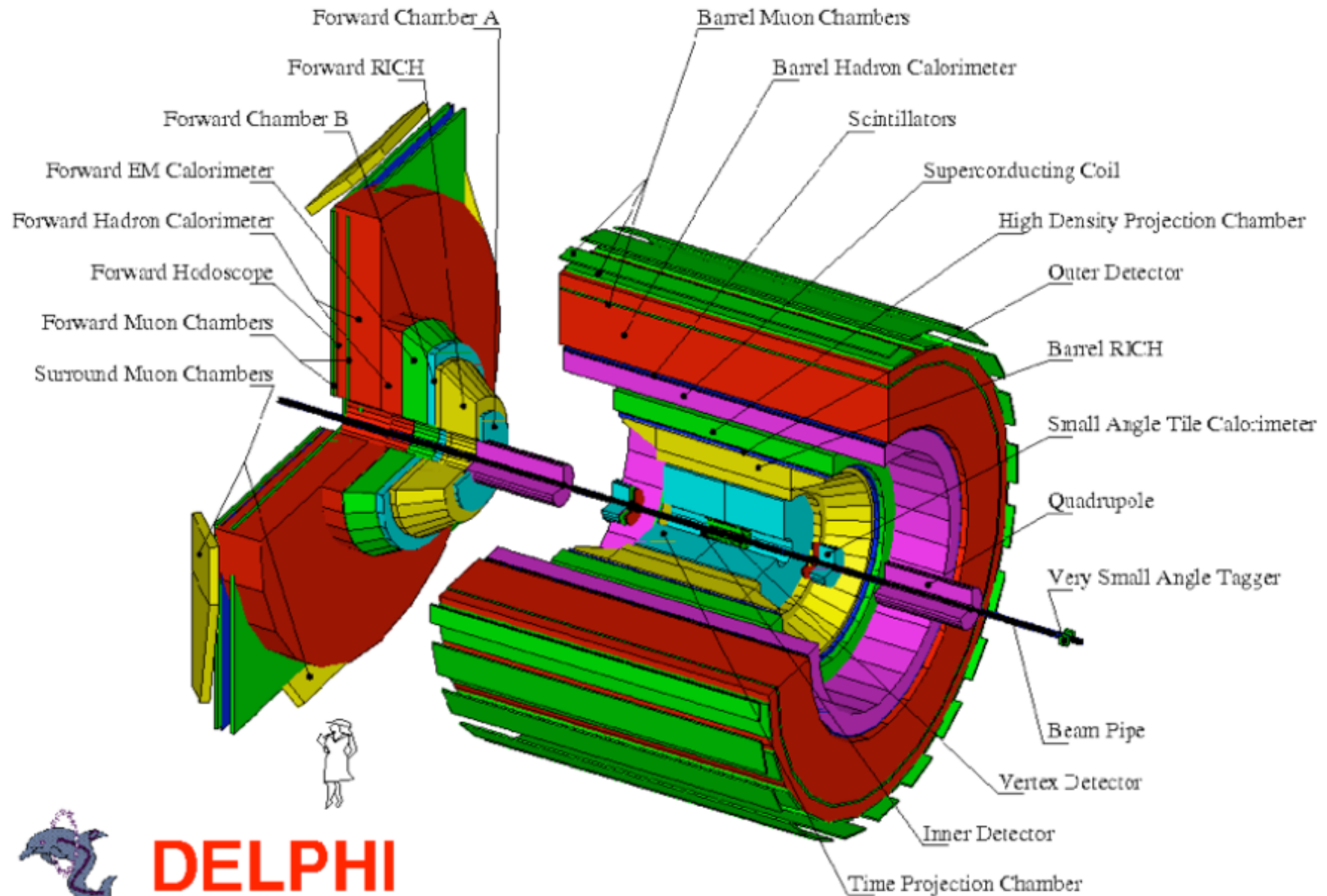
- Vertex Detector
- Inner Tracking Chamber
- Time Projection Chamber
- Electromagnetic Calorimeter
- Superconducting Magnet Coil
- Hadron Calorimeter
- Muon Chambers
- Luminosity Monitors

The LEP Experiments - L3

L3 Detector



The LEP Experiments - DELPHI



Precision Test of the Z sector at LEP

$$|M|^2 = \left| \text{diagram with } \gamma \text{ exchange} + \text{diagram with } Z \text{ exchange} \right|^2$$

for $e^+ e^- \rightarrow \mu^+ \mu^-$

s-channel only!

$$M_\gamma = -e^2 (\bar{\mu} \gamma_\mu \mu) \frac{1}{q^2} (\bar{e} \gamma^\mu e)$$

$$M_Z = -\frac{g^2}{\cos^2 \theta_W} \left[\bar{\mu} \gamma^\nu \frac{1}{2} (g_V^\mu - g_A^\mu \gamma^5) \mu \right] \underbrace{\frac{g_{\nu\rho} - q_\nu q_\rho / M_Z^2}{(q^2 - M_Z^2) + iM_Z \Gamma_Z}}_{\text{Z propagator considering a finite Z width}} \left[\bar{e} \gamma^\rho \frac{1}{2} (g_V^e - g_A^e \gamma^5) e \right]$$

Z propagator considering
a finite Z width

One finds for the differential cross section:

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s} \left[\underbrace{F_\gamma(\cos\theta)}_{\gamma} + \underbrace{F_{\gamma Z}(\cos\theta)}_{\gamma/Z \text{ interference}} \frac{s(s-M_Z^2)}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2} + \underbrace{F_Z(\cos\theta)}_Z \frac{s^2}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2} \right]$$

Vanishes at $\sqrt{s} \approx M_Z$

$$F_\gamma(\cos\theta) = Q_e^2 Q_\mu^2 (1 + \cos^2\theta) = (1 + \cos^2\theta)$$

$$F_{\gamma Z}(\cos\theta) = \frac{Q_e Q_\mu}{4\sin^2\theta_W \cos^2\theta_W} [2g_V^e g_V^\mu (1 + \cos^2\theta) + 4g_A^e g_A^\mu \cos\theta]$$

$$F_Z(\cos\theta) = \frac{1}{16\sin^4\theta_W \cos^4\theta_W} [(g_V^e)^2 + (g_A^e)^2] [(g_V^\mu)^2 + (g_A^\mu)^2] (1 + \cos^2\theta) + 8g_V^e g_A^e g_V^\mu g_A^\mu \cos\theta]$$

Forward-backward asymmetry

$$\frac{d\sigma}{d\cos\theta} \sim (1 + \cos^2\theta) + \frac{8}{3} A_{FB} \cos\theta \quad \text{with} \quad A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

at Z pole ($\sqrt{s} = M_Z$):

$$A_{FB} = 3 \frac{g_V^e g_A^e g_V^\mu g_A^\mu}{((g_V^e)^2 + (g_A^e)^2)((g_V^\mu)^2 + (g_A^\mu)^2)}$$

$$\sigma_{tot} = \sigma_Z = \frac{4\pi}{3s} \frac{\alpha^2}{16\sin^4\theta_W \cos^4\theta_W} ((g_V^e)^2 + (g_A^e)^2) ((g_V^\mu)^2 + (g_A^\mu)^2) \frac{s^2}{(s-M_Z^2)^2 + (M_Z\Gamma_Z)^2}$$

$\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ at Z pole:

$$\sigma_{tot} = \sigma_Z = \frac{4\pi}{3s} \frac{\alpha^2}{16 \sin^4 \theta_W \cos^4 \theta_W} ((g_V^e)^2 + (g_A^e)^2) ((g_V^\mu)^2 + (g_A^\mu)^2) \frac{s^2}{(s - M_Z^2)^2 + (M_Z \Gamma_Z)^2}$$

$$\sigma_z(\sqrt{s} = M_Z) = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_\mu}{\Gamma_Z^2}$$

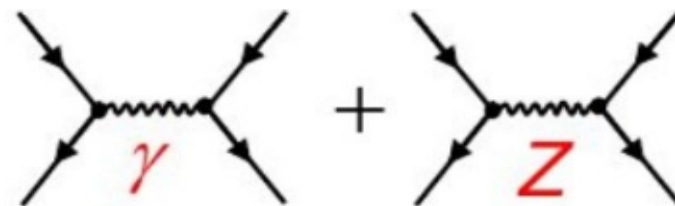
$$\Gamma_f = \frac{\alpha M_Z}{12 \sin^2 \theta_W \cos^2 \theta_W} ((g_V^f)^2 + (g_A^f)^2)$$

$$\Gamma_Z = \sum_f \Gamma_f$$

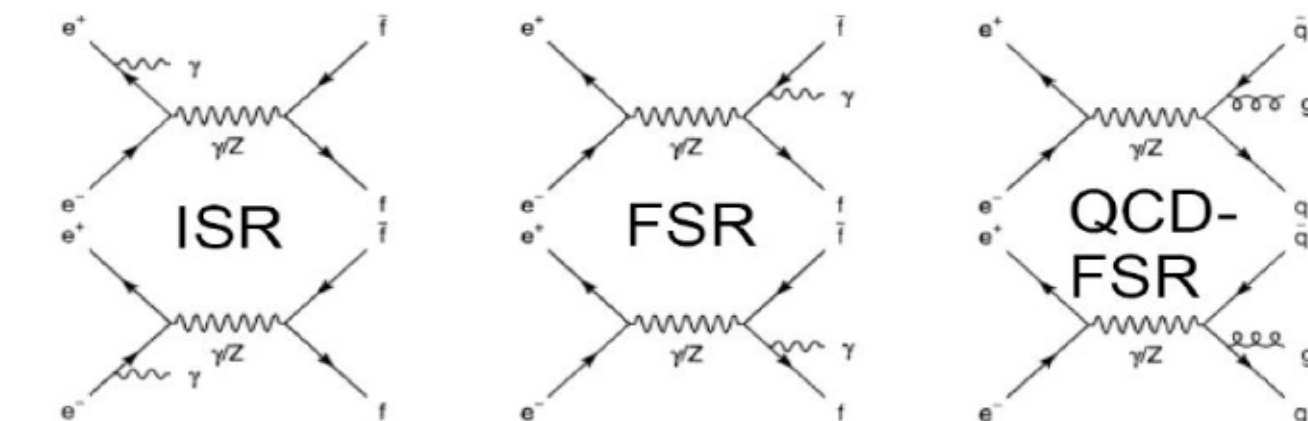
Cross sections and widths
can be calculated within the
Standard Model if all
parameters are known



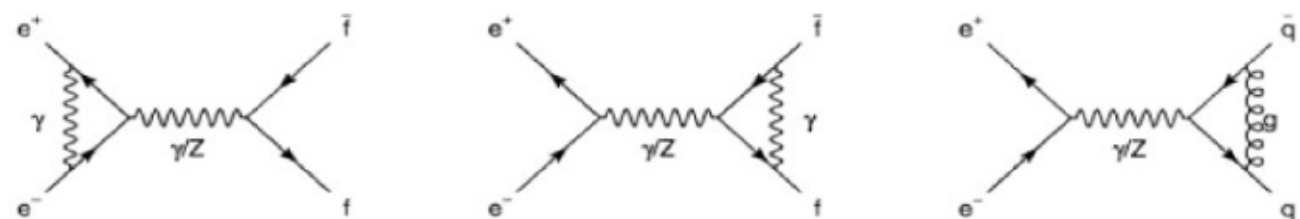
We like to measure:



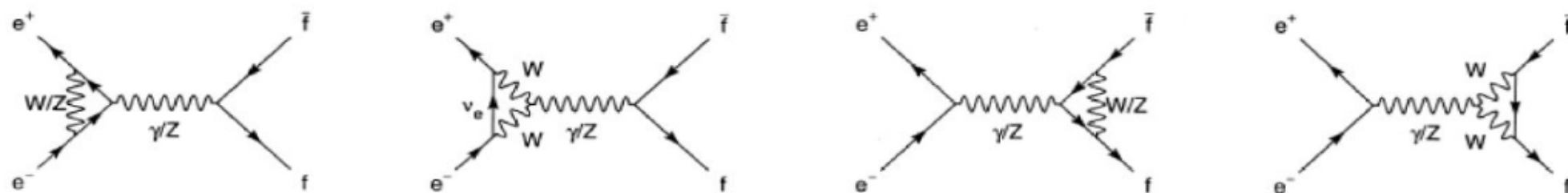
but there are many sizeable (but computable) higher order corrections:



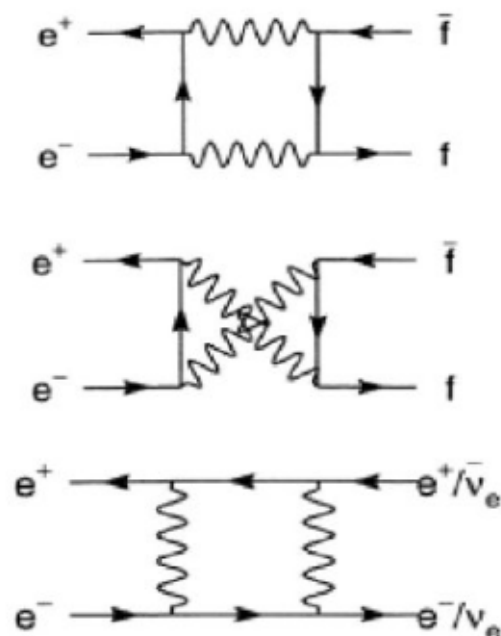
QED- und QCD-Vertex-Korrekturen



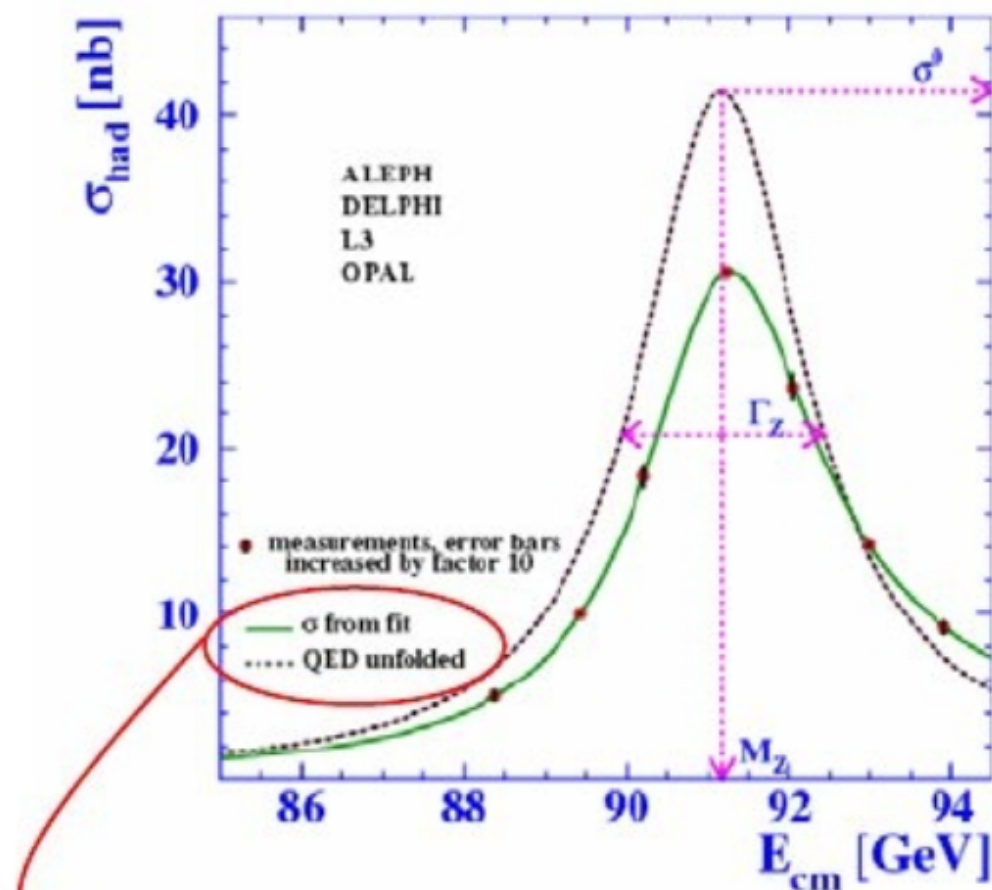
Elektroschwache Vertex-Korrekturen



Box-Korrekturen



Measurement of Z line shape



Resonance curve:

$$\sigma(s) = 12\pi \frac{\Gamma_e \Gamma_\mu}{M_Z^2} \cdot \frac{s}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

Peak: $\sigma_0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_\mu}{\Gamma_Z^2}$

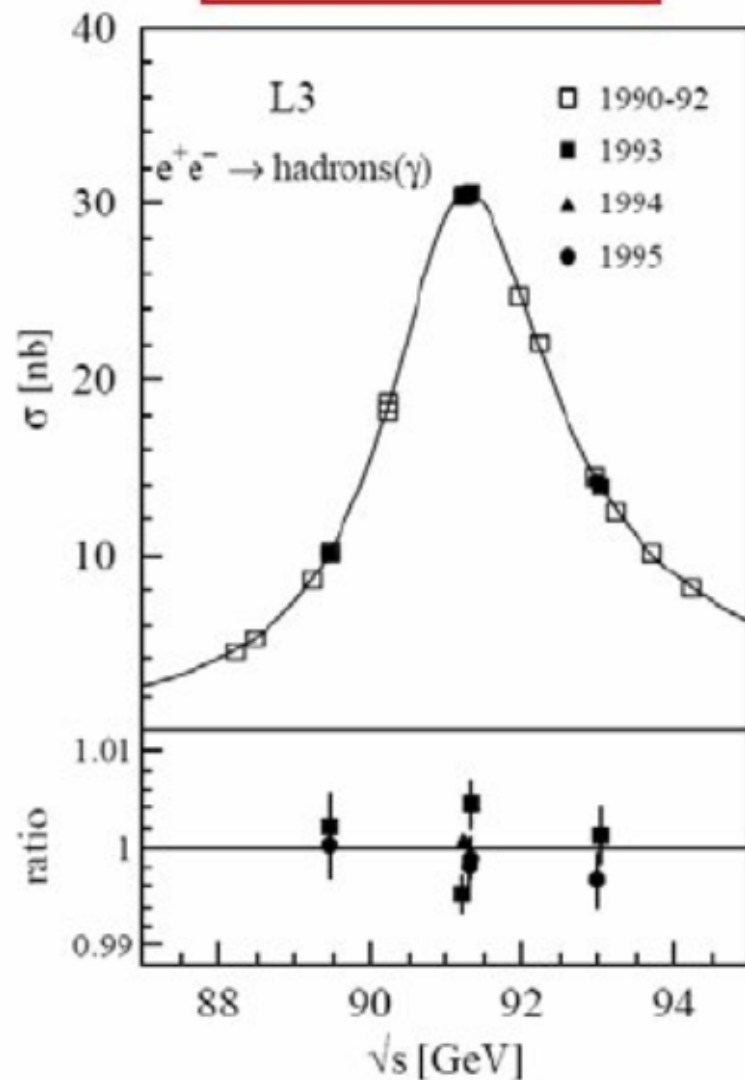
- Resonance position $\rightarrow M_Z$
- Height $\rightarrow \Gamma_e \Gamma_\mu$
- Width $\rightarrow \Gamma_Z$

Initial state Bremsstrahlung corrections

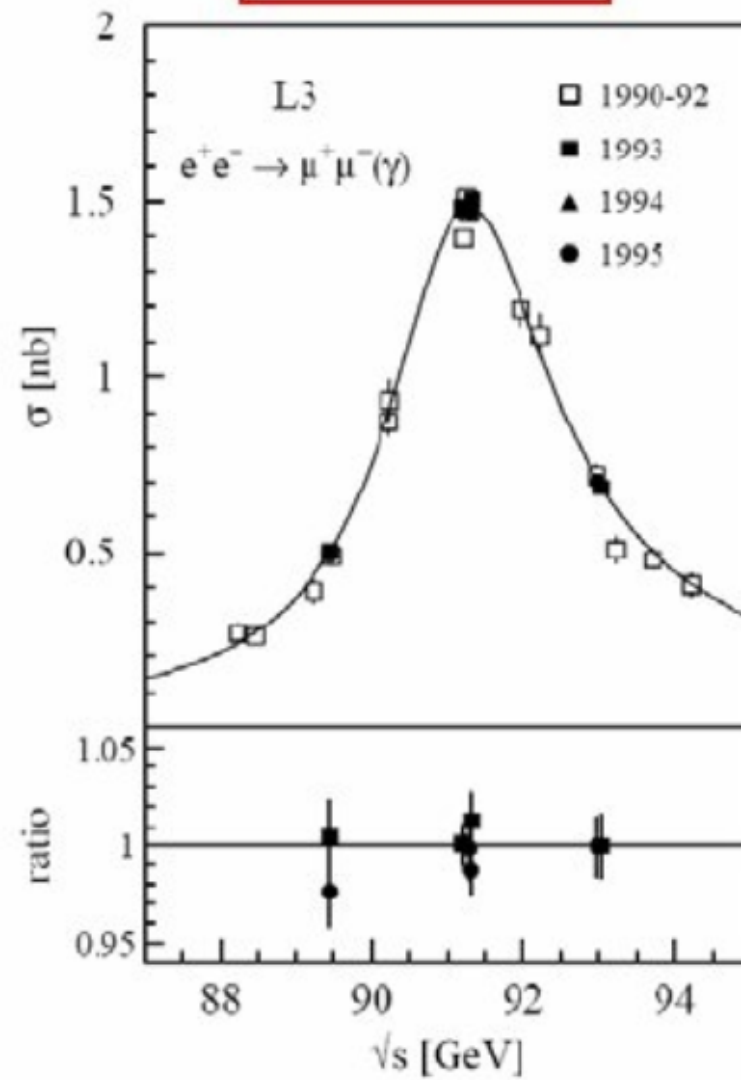
$$\sigma_{ff(\gamma)} = \int_{4m_f^2/s}^1 G(z) \sigma_{ff}^0(zs) dz \quad z = 1 - \frac{2E_\gamma}{\sqrt{s}}$$

Energy scan is more precise than reconstructed invariant mass, advantage of e^+e^- !

$$e^+ e^- \rightarrow \text{hadrons}$$



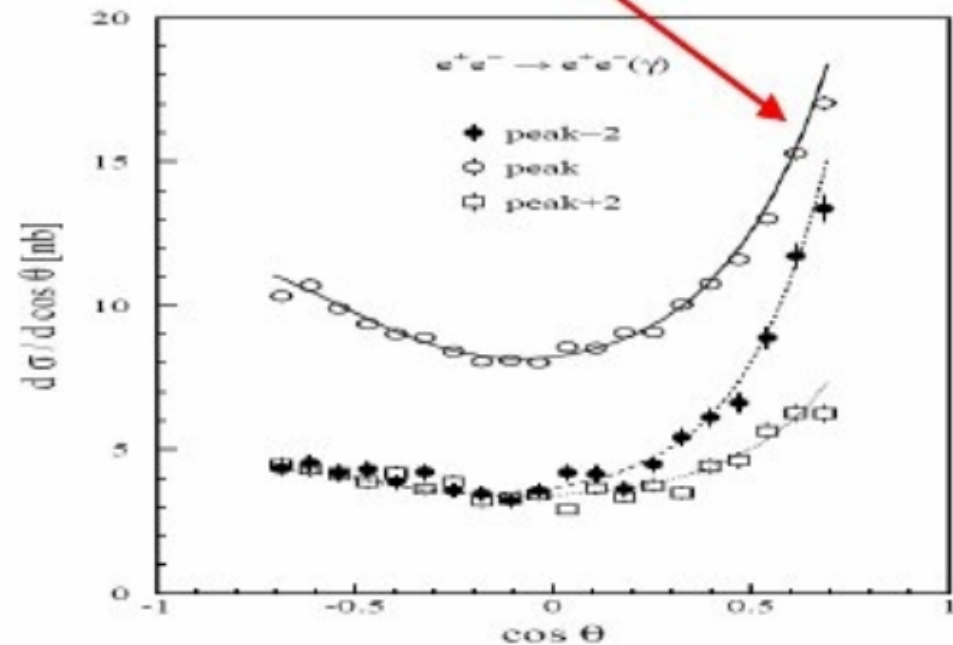
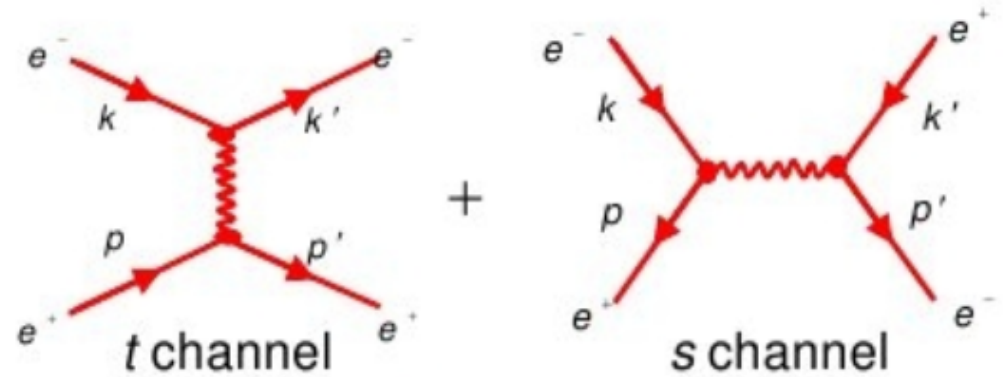
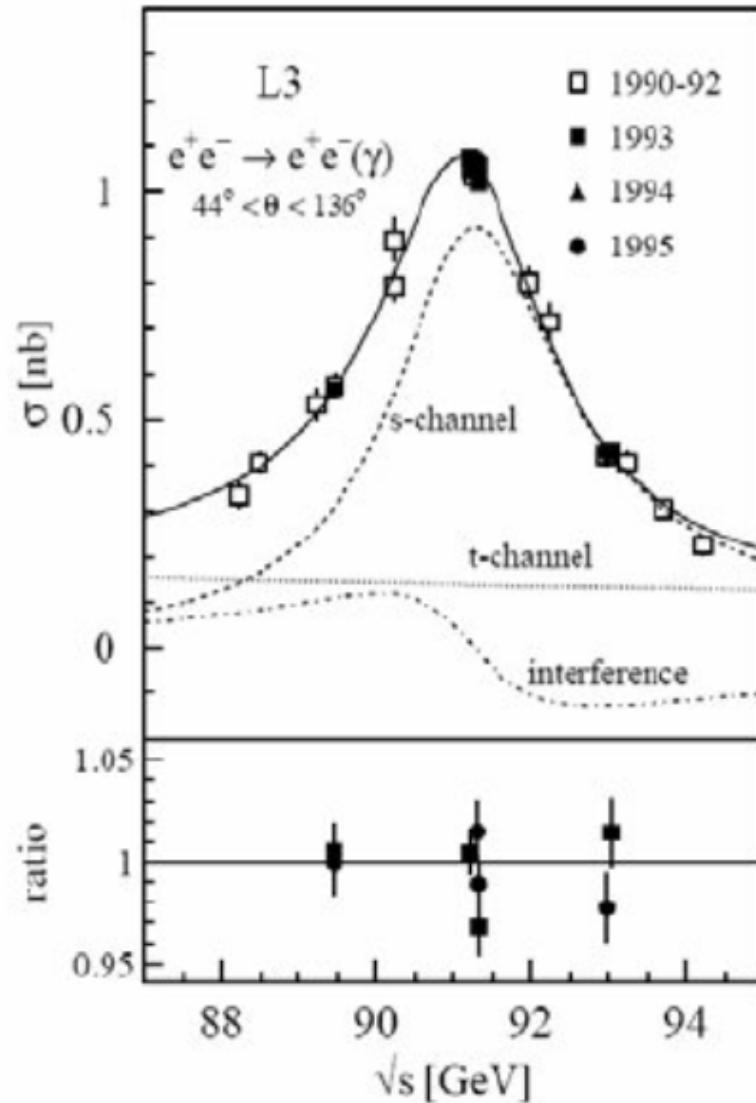
$$e^+ e^- \rightarrow \mu^+ \mu^-$$



Resonances look the same, independent of the final state: propagator is the same!

$$e^+ e^- \rightarrow e^+ e^-$$

t channel contribution $\rightarrow$ forward peak



Phase shift around the pole of Breit-Wigner of s-channel process, explains the shape of the interference term.

Z line shape parameters (LEP average)

M_Z	=	91.1876 ± 0.0021 GeV	± 23 ppm (*)
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Γ_Z	=	2.4952 ± 0.0023 GeV
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Γ_{had}	=	1.7458 ± 0.0027 GeV
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Γ_e	=	0.08392 ± 0.00012 GeV
------------	---	---------------------------

Γ_μ	=	0.08399 ± 0.00018 GeV
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Γ_τ	=	0.08408 ± 0.00022 GeV
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± 0.09 %

3 leptons are treated independently



test of lepton universality

Γ_Z	=	2.4952 ± 0.0023 GeV
------------	---	-------------------------

Γ_{had}	=	1.7444 ± 0.0022 GeV
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Γ_e	=	0.083985 ± 0.000086 GeV
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Assuming lepton universality: $\Gamma_e = \Gamma_\mu = \Gamma_\tau$

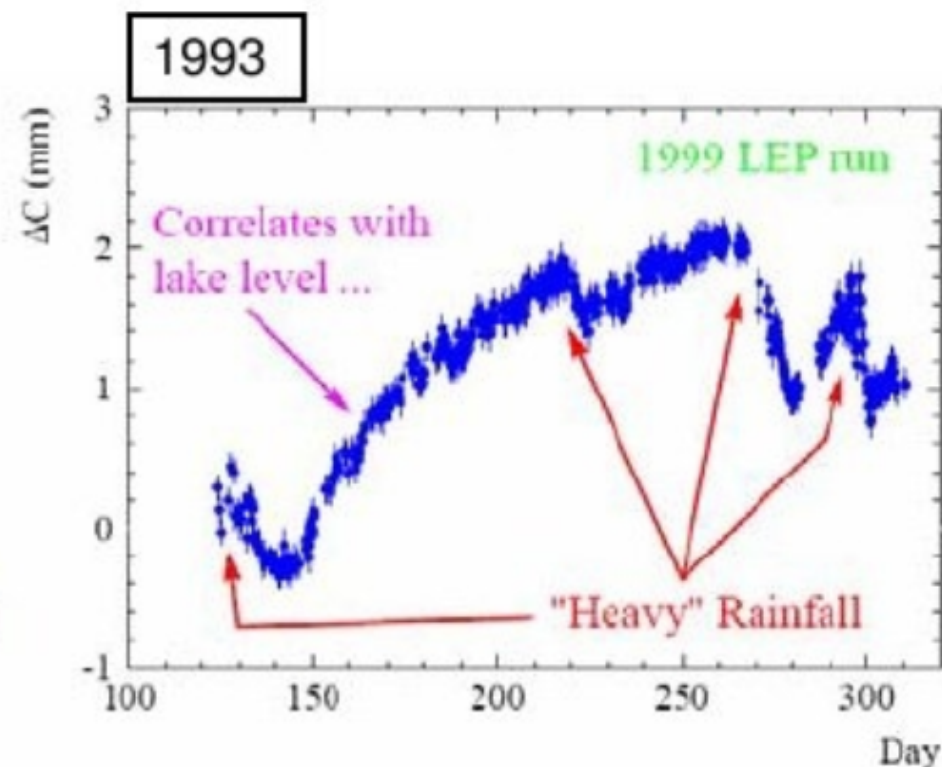
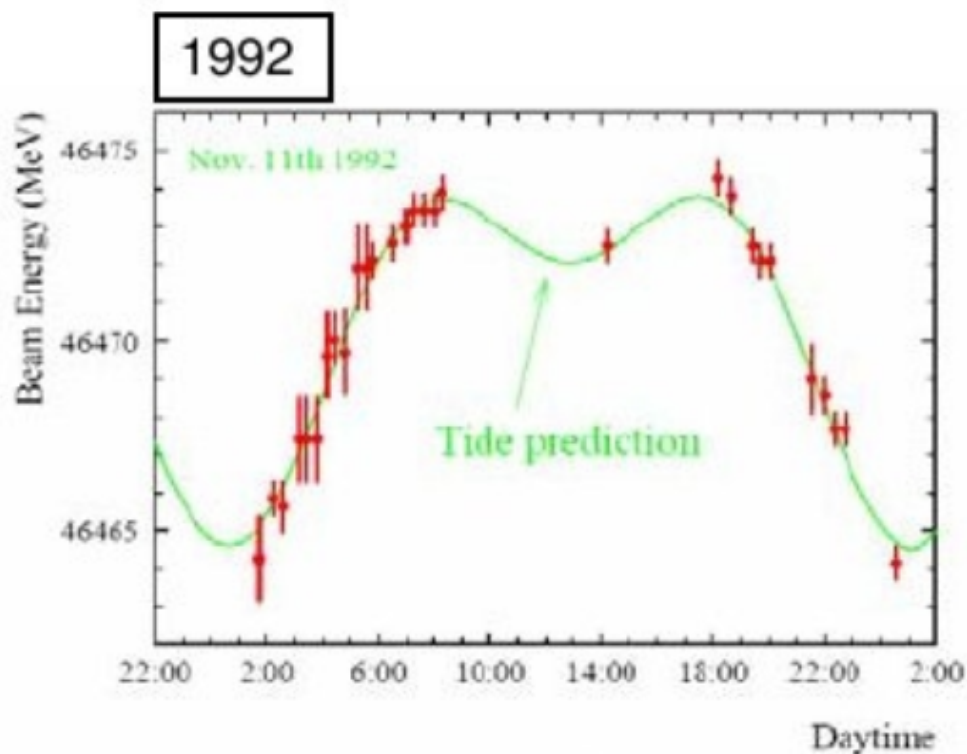
*) error of the LEP energy determination: ± 1.7 MeV (19 ppm)

LEP energy calibration: Hunting for ppm effects

Changes of the circumference of the LEP ring changes the energy of the electrons:

- tide effects
- water level in lake Geneva

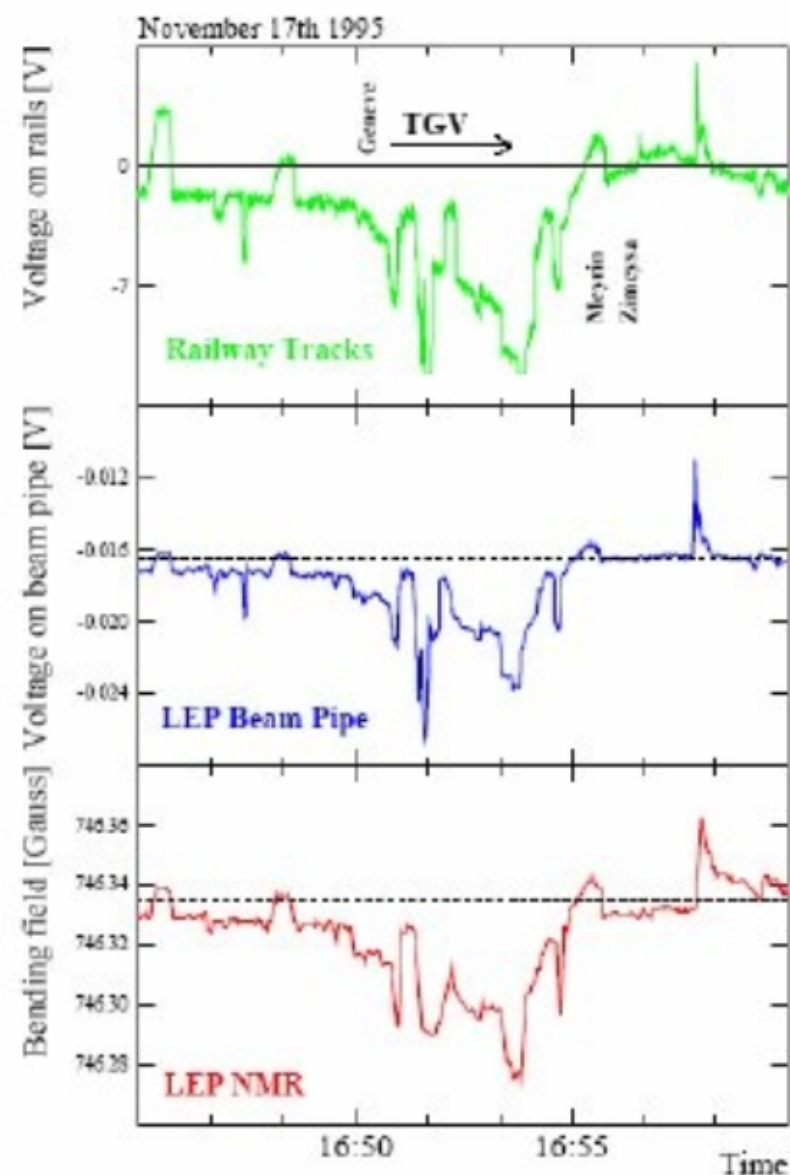
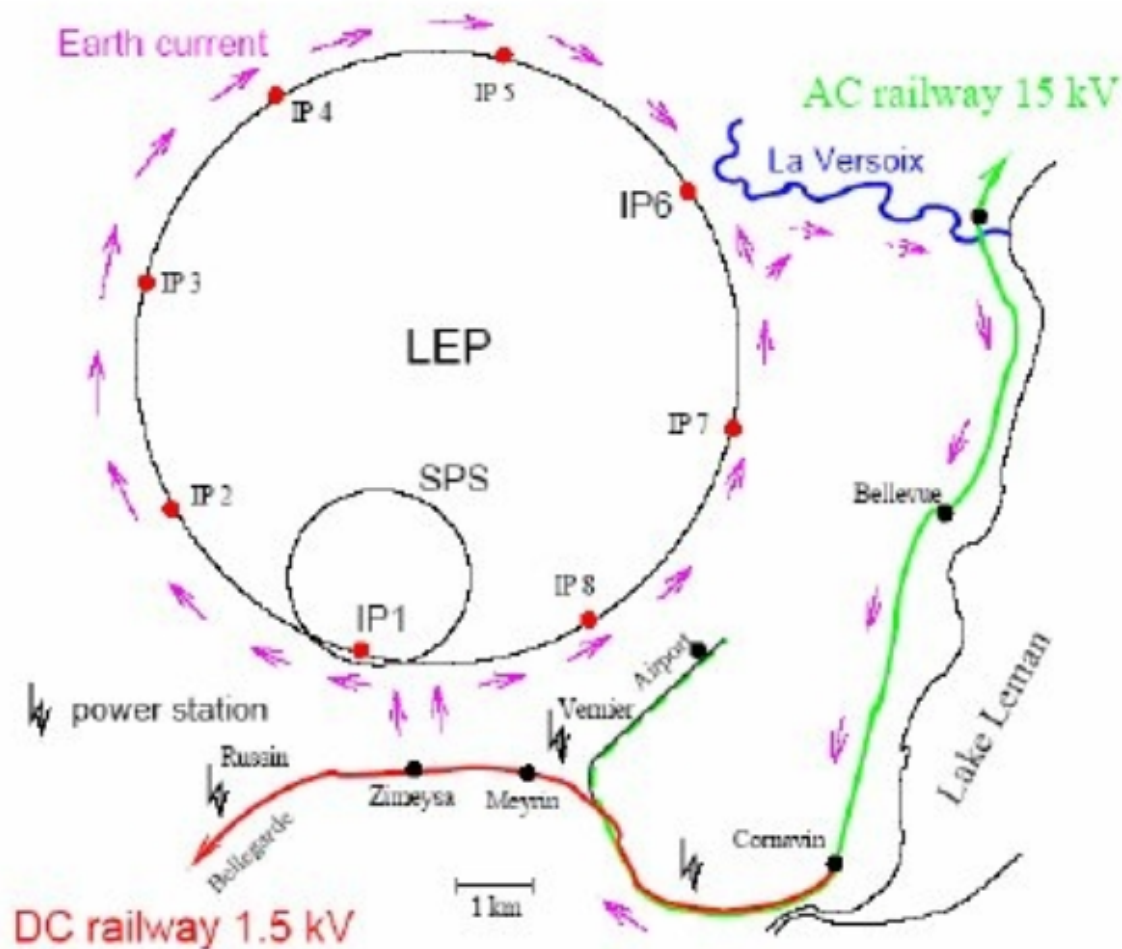
Changes of LEP circumference
 $\Delta C = 1 \dots 2 \text{ mm} / 27 \text{ km}$ ($4 \dots 8 \times 10^{-8}$)



Effect of the French "Train a Grande Vitesse" (TGV)



Vagabonding currents ($\sim 1\text{A}$) from trains



3 “light” neutrino species!

In the Standard Model:

$$\Gamma_Z = \Gamma_{\text{had}} + 3 \cdot \Gamma_\ell + \underbrace{N_\nu \cdot \Gamma_\nu}_{\text{invisible} : \Gamma_{\text{inv}}} \rightarrow \begin{cases} e^+ e^- \rightarrow Z \rightarrow \nu_e \bar{\nu}_e \\ e^+ e^- \rightarrow Z \rightarrow \nu_\mu \bar{\nu}_\mu \\ e^+ e^- \rightarrow Z \rightarrow \nu_\tau \bar{\nu}_\tau \end{cases}$$

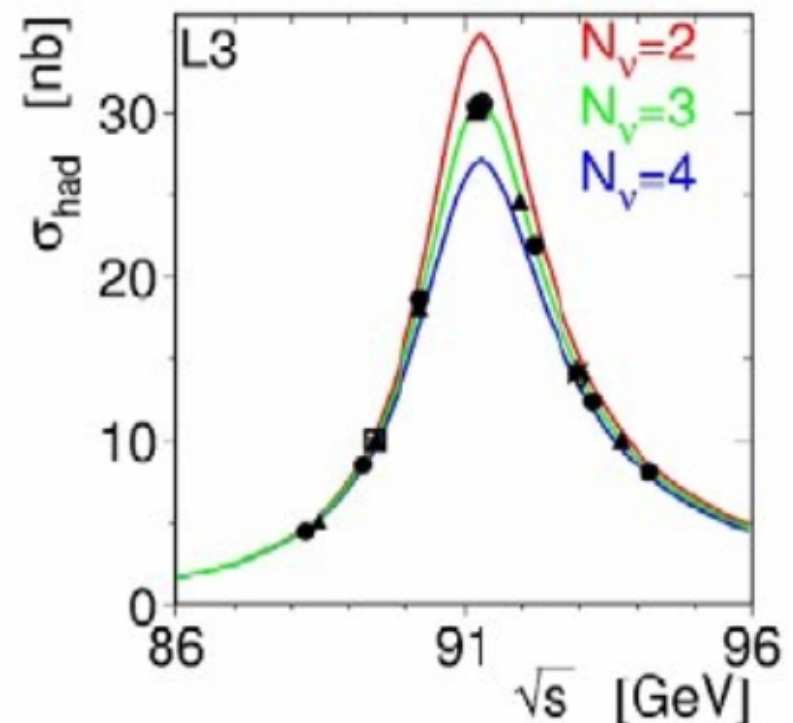
$$\Gamma_{\text{inv}} = 0.4990 \pm 0.0015 \text{ GeV}$$

To determine the number of light neutrino generations:

$$N_\nu = \underbrace{\left(\frac{\Gamma_{\text{inv}}}{\Gamma_\ell} \right)_{\text{exp}}}_{5.9431 \pm 0.0163} \cdot \underbrace{\left(\frac{\Gamma_\ell}{\Gamma_\nu} \right)_{\text{SM}}}_{=1.991 \pm 0.001 \text{ (small theo. uncertainties from } m_{\text{top}} M_H)}$$

$$N_\nu = 2.9840 \pm 0.0082$$

No room for new physics: $Z \rightarrow \text{new}$



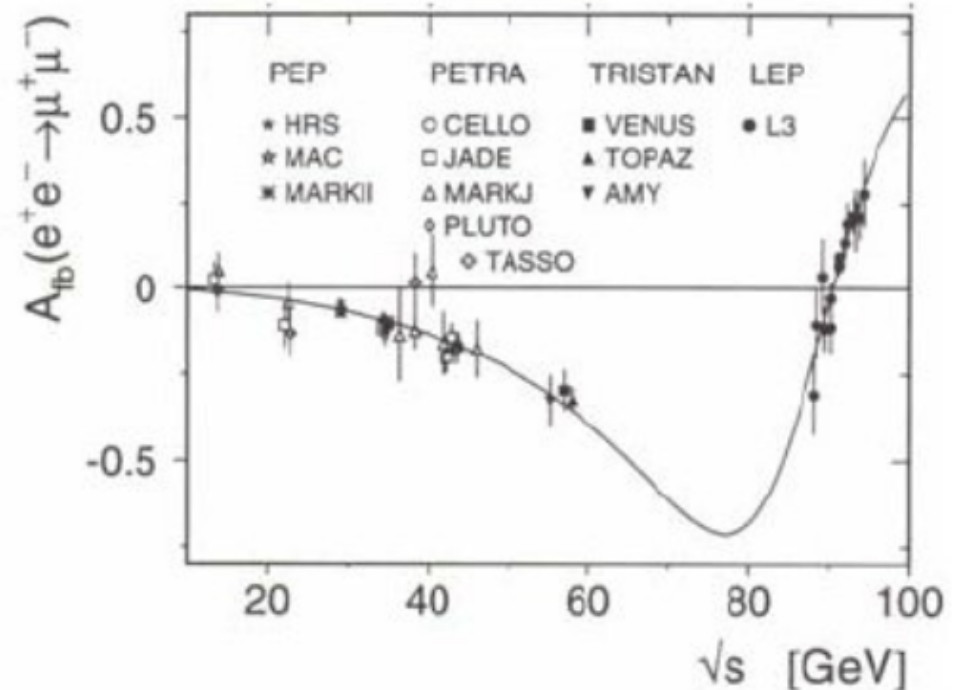
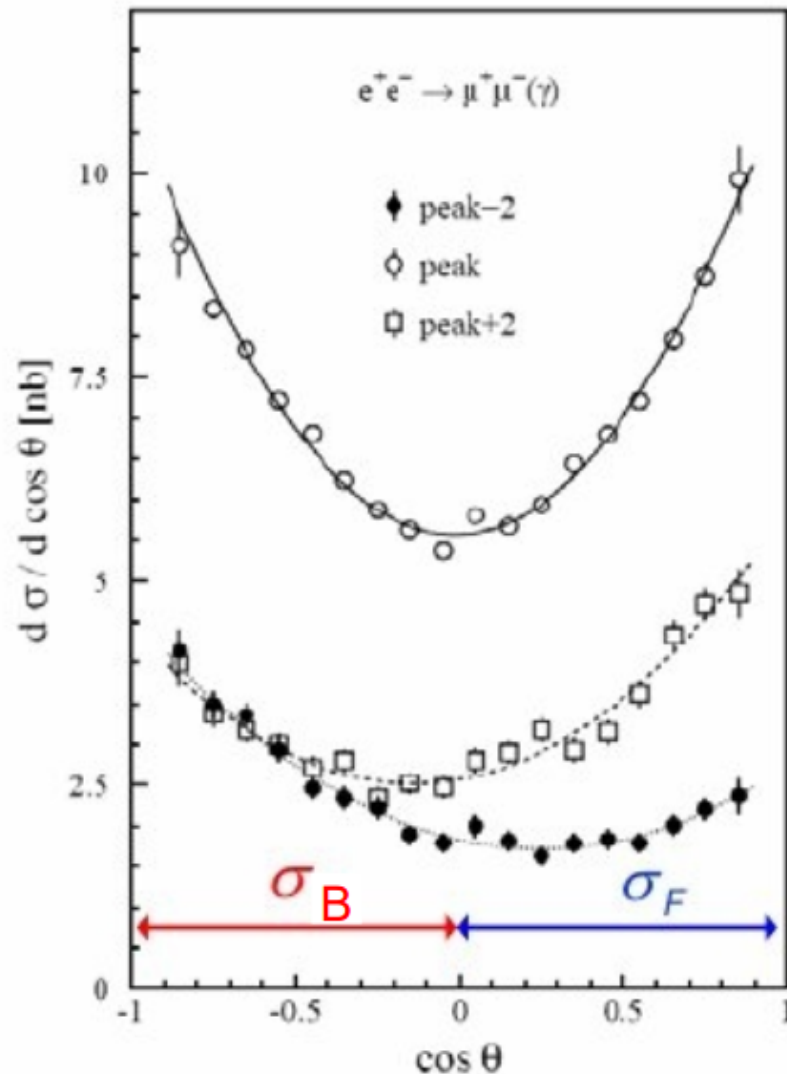
Forward-backward asymmetry and fermion couplings to Z

$$e^+ e^- \rightarrow Z \rightarrow \mu^+ \mu^-$$

$$\frac{d\sigma}{d\cos\theta} \sim (1 + \cos^2\theta) + \frac{8}{3} A_{FB} \cos\theta$$

$$\text{with } A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$$\sigma_{F(B)} = \int_{0(-1)}^{1(0)} \frac{d\sigma}{d\cos\theta} d\cos\theta$$



Fermion couplings

Forward-backward asymmetry

- Away from the resonance A_{FB} large
→ interference term dominates

$$A_{FB} \sim g_A^e g_A^f \cdot \frac{s(s - M_Z^2)}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

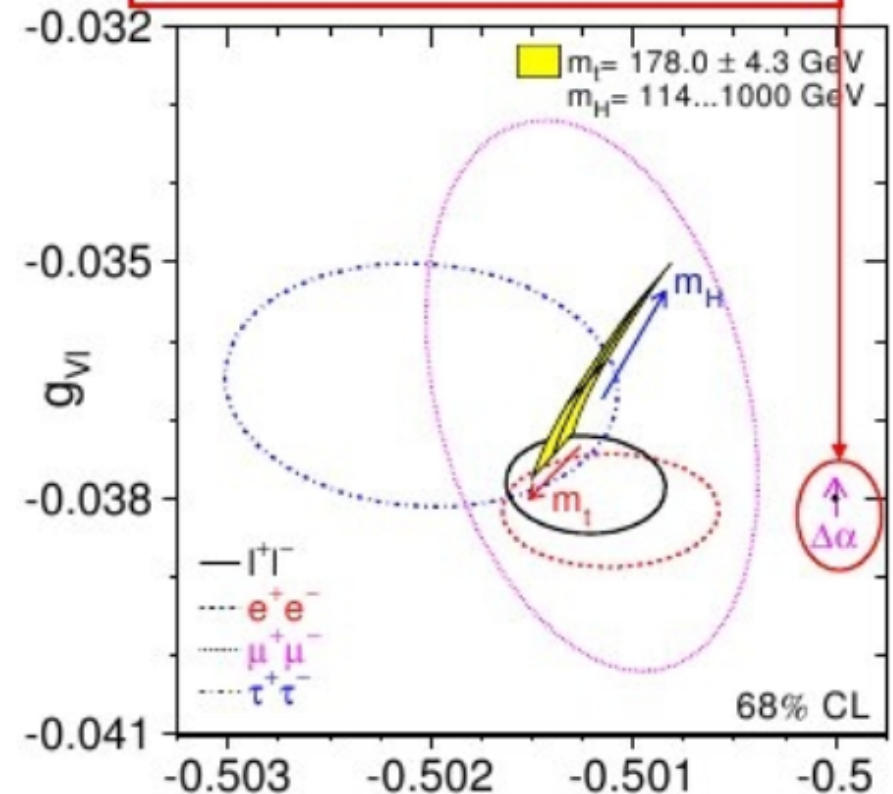
- At the Z pole: Interference = 0

$$A_{FB} \sim g_A^e g_V^e g_A^f g_V^f$$

→ very small because g_V^l small in SM

Lowest order SM prediction:

$$g_V = T_3 - 2q \sin^2 \theta_W \quad g_A = T_3$$



Asymmetries together with cross sections allow the determination of the fermion couplings g_A and g_V

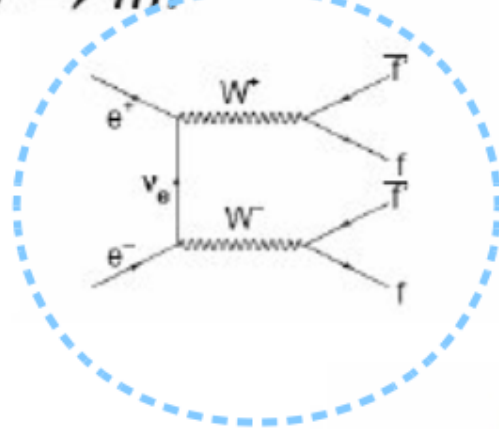
g_A

Confirms lepton universality
Higher order corrections seen

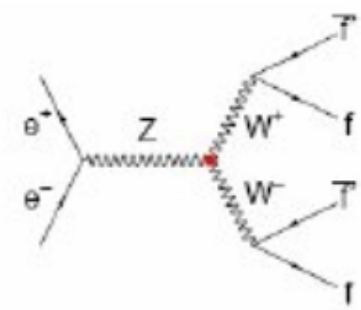
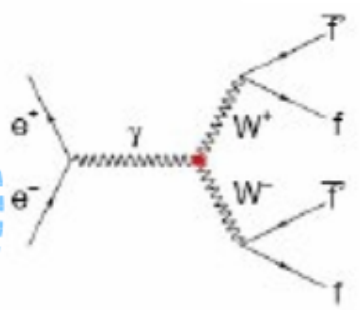
Precision measurements of W branching ratio at LEP (~10K WW events):

$$e^+e^- \rightarrow WW \rightarrow ffff$$

V-A theory:



additional SM vertices, triple gauge boson coupling



“Ugly” fine tuning!

06/07/2001

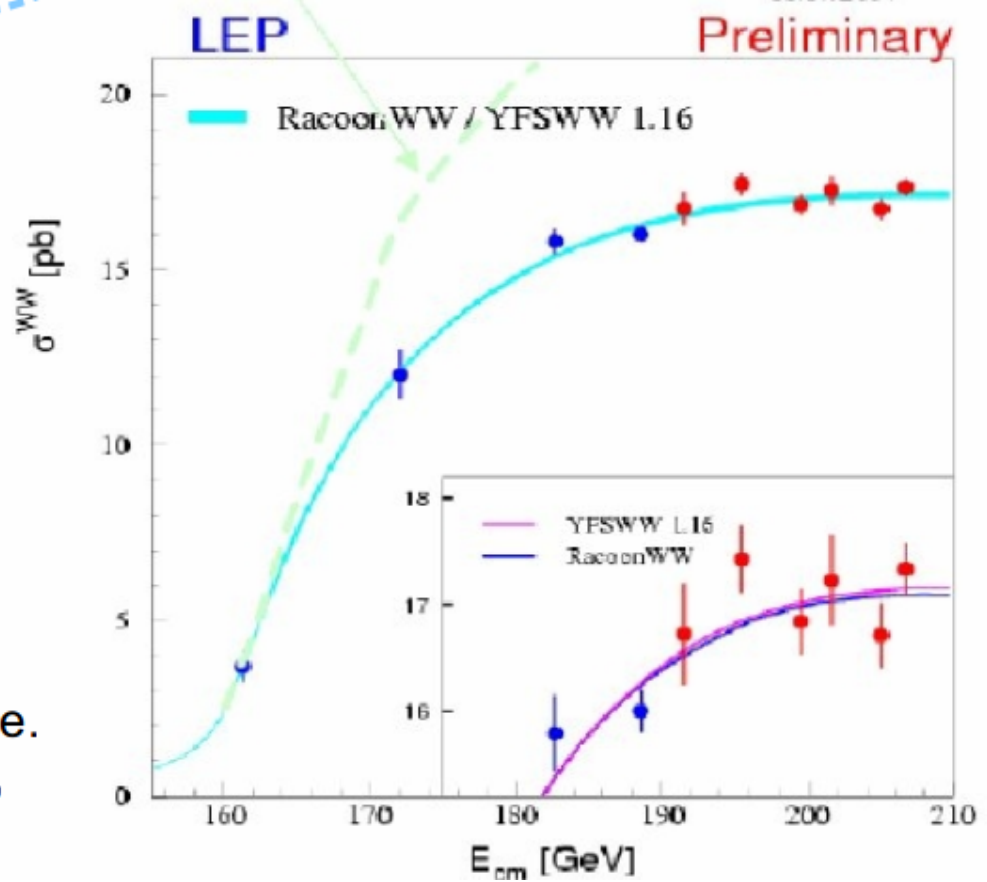
Threshold behavior of the cross section (phase space) for $ee \rightarrow WW$ production:



Phase space factor = $f(M_W, \sqrt{s})$:

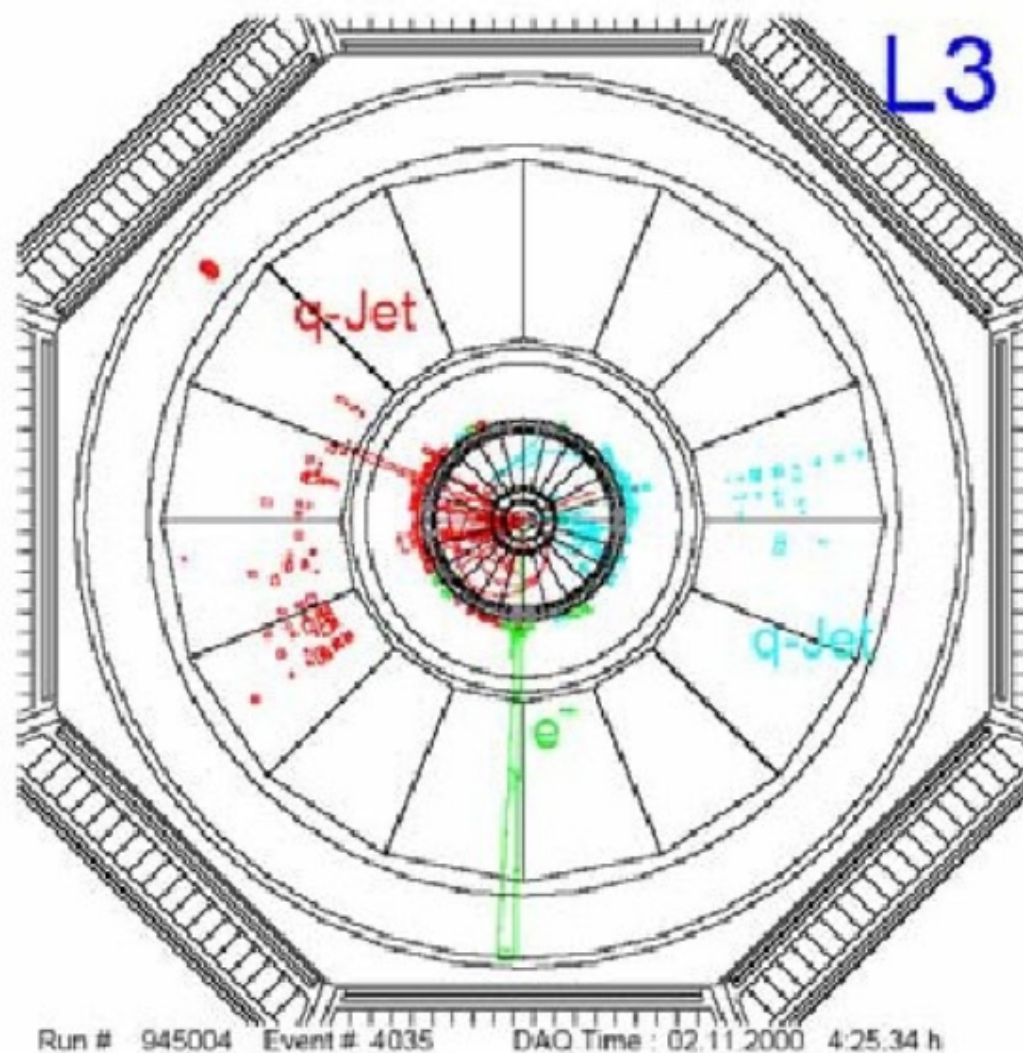
→ Allows determination of M_W

Jets and/or neutrinos in the final state.
Not easy to compute invariant mass,
but exploit that CME known in e^+e^-



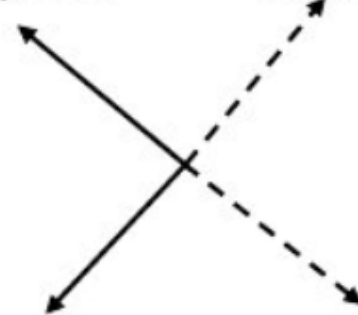
W decays

L3

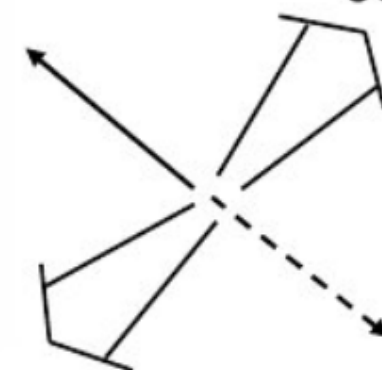


$$WW \rightarrow \begin{cases} qq\ell\nu & 44\% \\ qqqq & 45\% \\ \ell\nu\ell\nu & 11\% \end{cases}$$

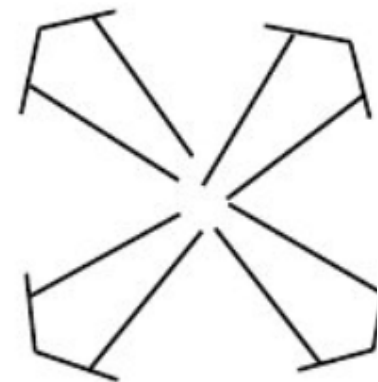
Lepton Neutrino



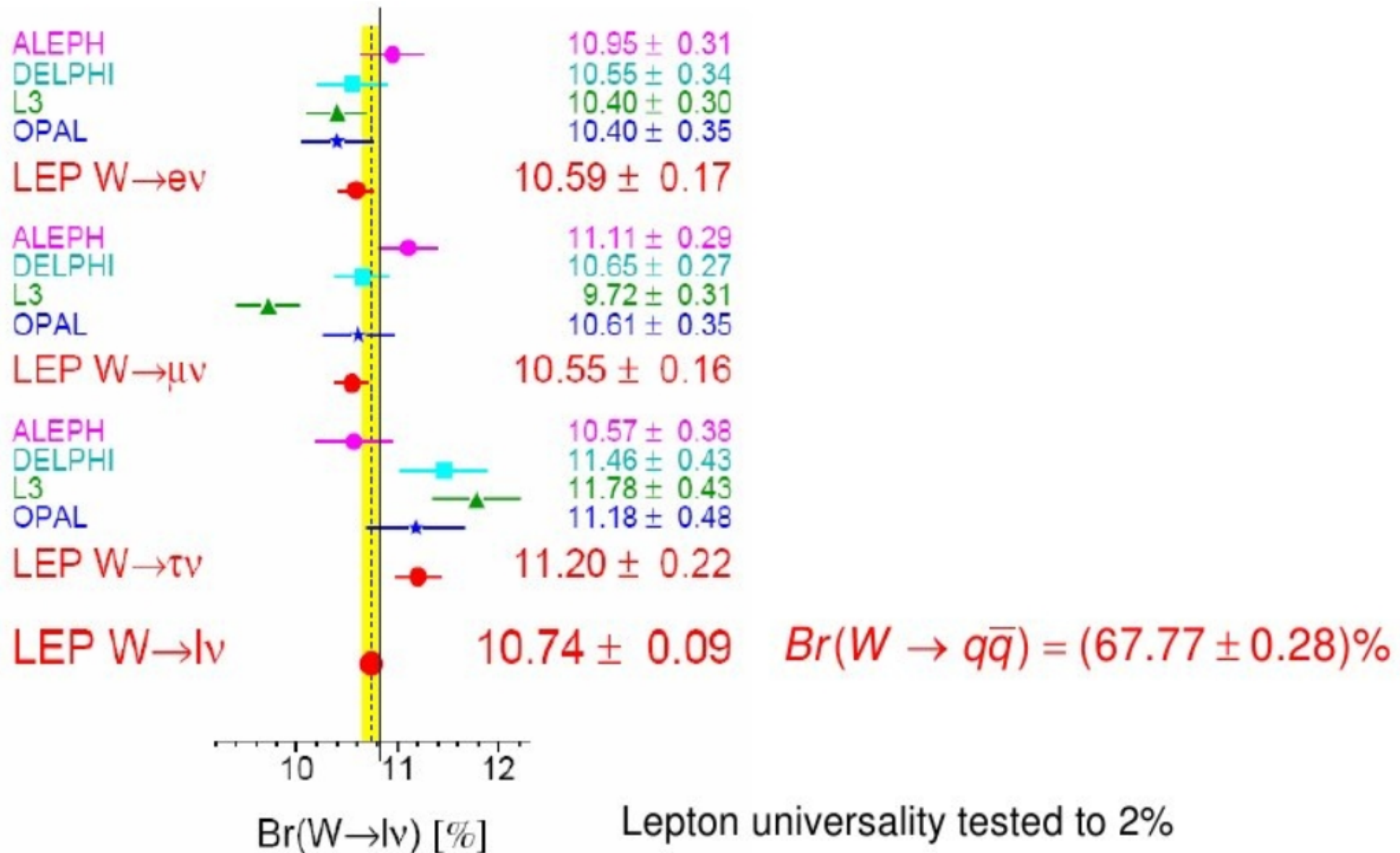
Jet



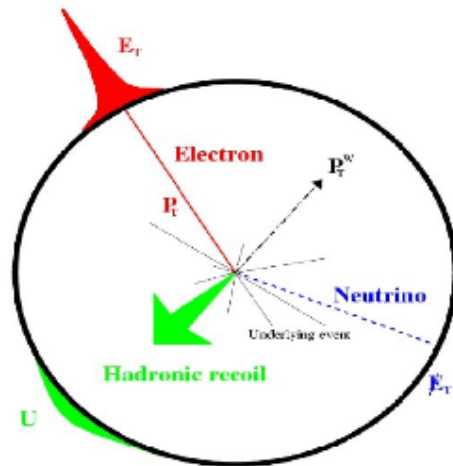
Jet



W Branching ratio at LEP



W Mass measurement at Tevatron:



Method of transverse mass:

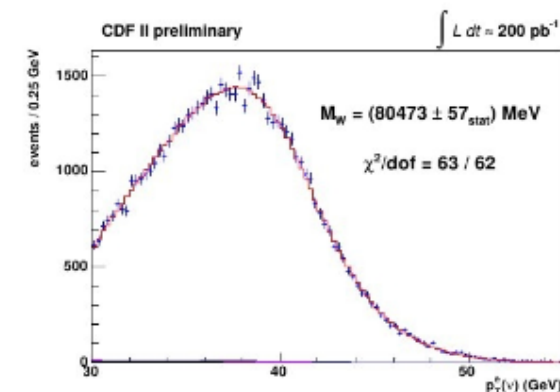
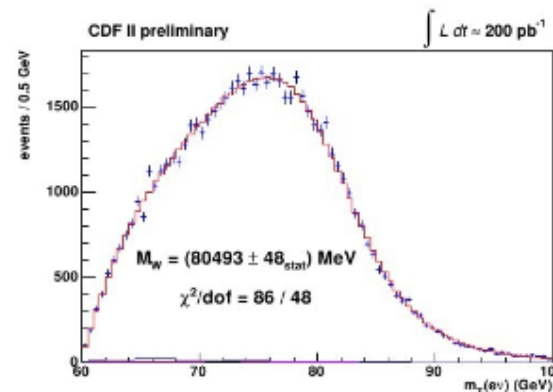
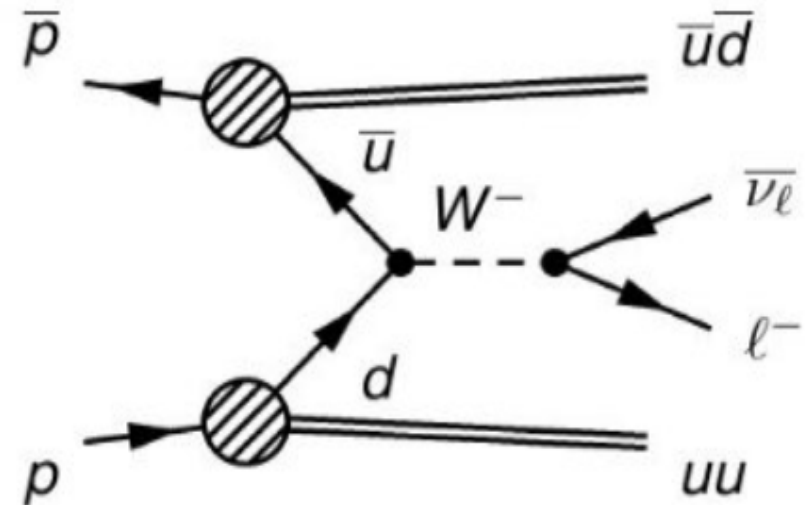
Idea: $\sum p_T = 0$

p_T^ℓ and p_T^{had} are measurable

$$p_T^\nu = -p_T^\ell - p_T^{had}$$

$$M_T^W = \sqrt{2p_T^\ell p_T^\nu - 2\vec{p}_T^\ell \vec{p}_T^\nu}$$

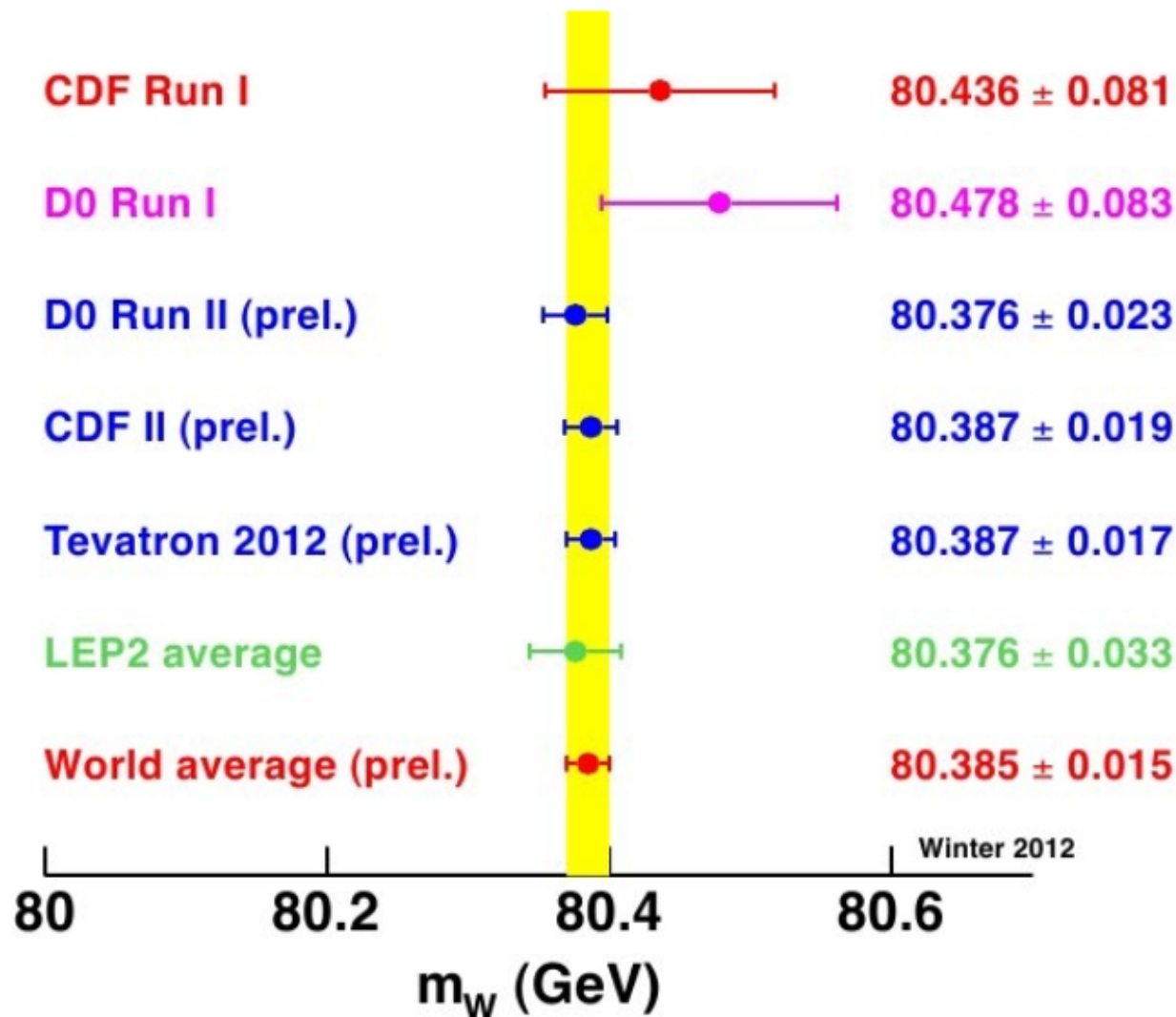
or method of p_T^ℓ , similar to UA1



Spectra not analytically computable;
both methods rely on **templates from MC**

main challenge: calibration of calorimeter

Precision measurement of W mass:



Mass measurements are completely systematically dominated:
calorimeter calibration, uncertainties on input from simulation ...

Many different methods with different source of systematics performed to get the uncertainties smaller.

Higher order corrections + the Higgs

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} = 1$$

$$\sin^2 \theta_W = 1 - \frac{m_W^2}{m_Z^2}$$

$$m_W^2 = \frac{\pi \alpha}{\sqrt{2} \sin^2 \theta_W G_F}$$

Lowest order
SM predictions

$\alpha(0)$

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

$$\bar{\rho} = 1 + \Delta\rho$$

$$\sin^2 \theta_{\text{eff}} = (1 + \Delta\kappa) \sin^2 \theta_W$$

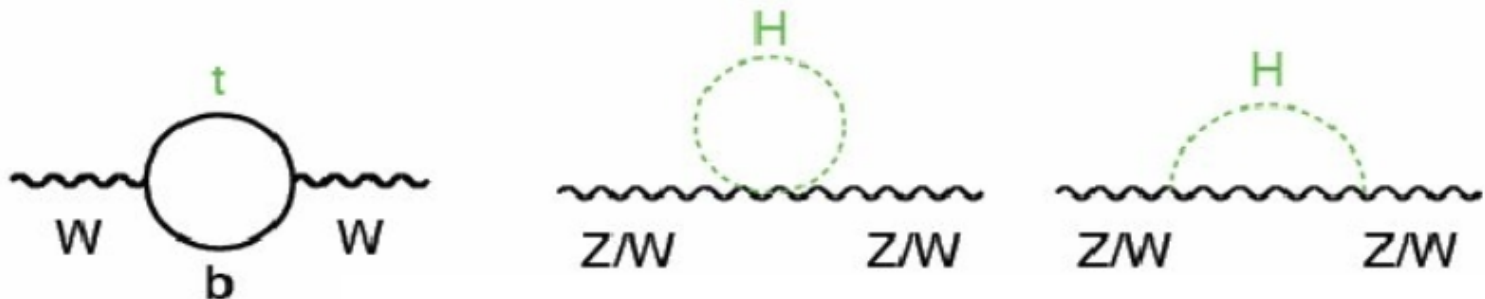
$$m_W^2 = \frac{\pi \alpha}{\sqrt{2} \sin^2 \theta_W G_F} (1 + \Delta r)$$

$$\alpha(m_Z^2) = \frac{\alpha(0)}{1 - \Delta\alpha}$$

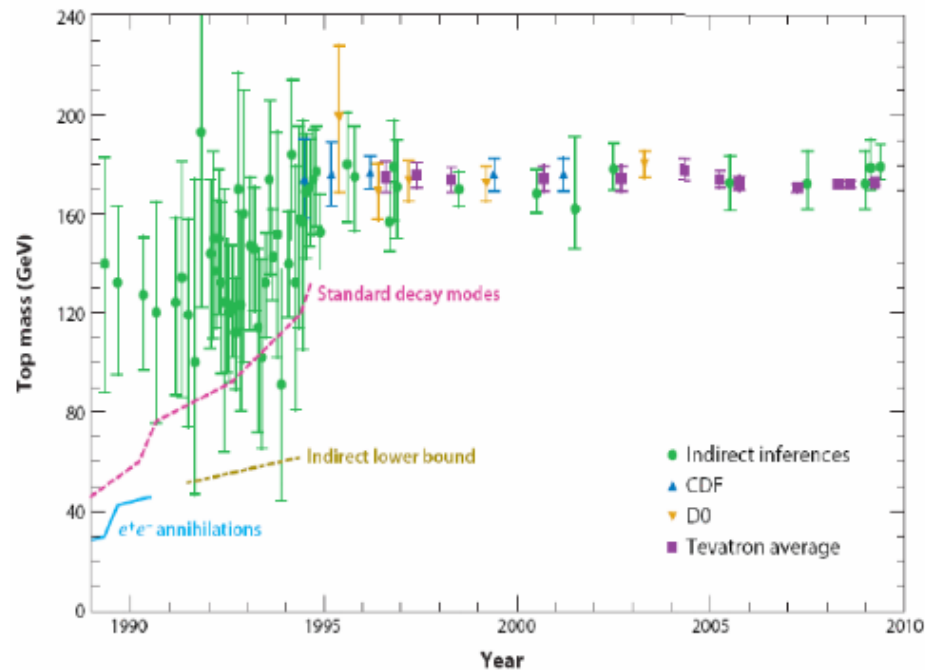
with : $\Delta\alpha = \Delta\alpha_{\text{lept}} + \Delta\alpha_{\text{top}} + \Delta\alpha_{\text{had}}^{(5)}$

Including radiative
corrections

$$\Delta\rho, \Delta\kappa, \Delta r = f(m_t^2, \log(m_H), \dots)$$



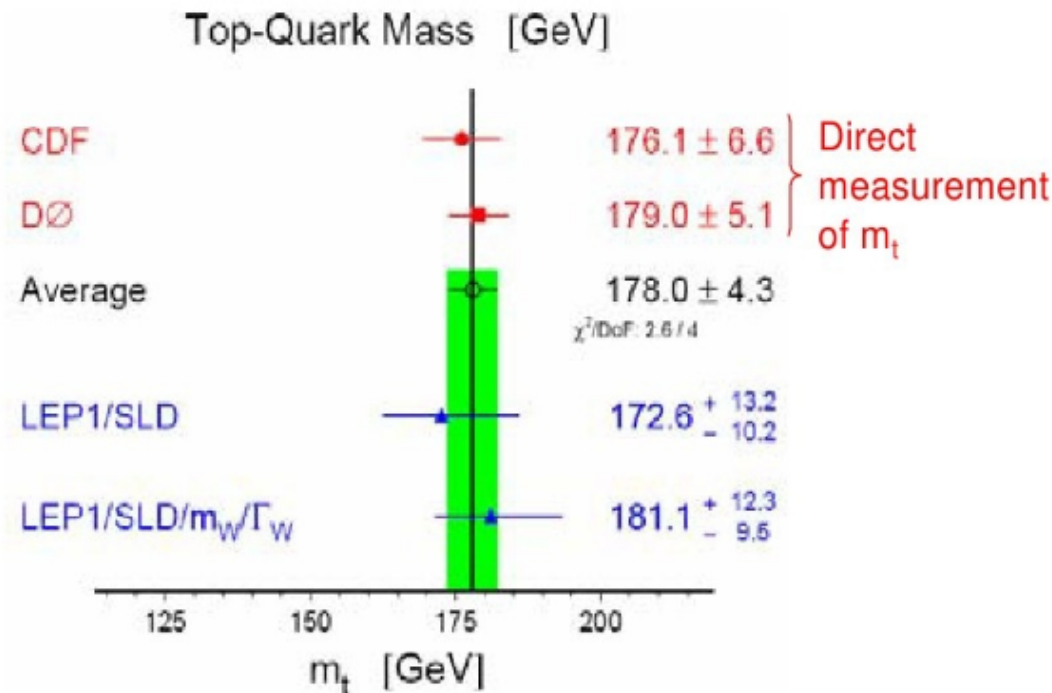
Prediction of the Top-Quark Mass



Today's best value:

$$m(\text{top}) = 173.2 \pm 0.9 \text{ GeV}$$

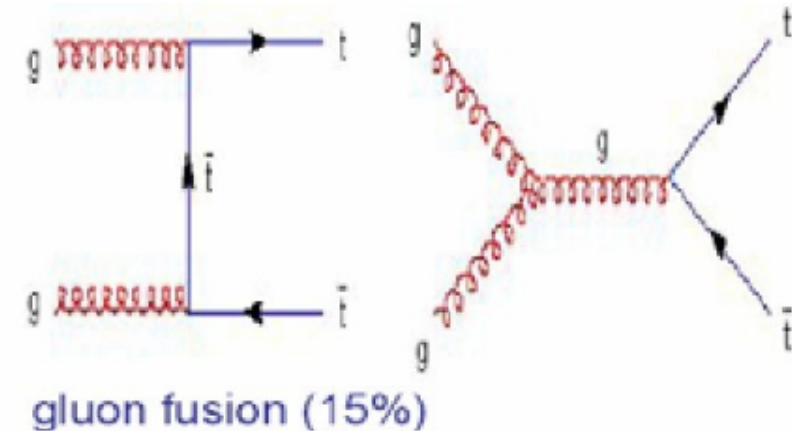
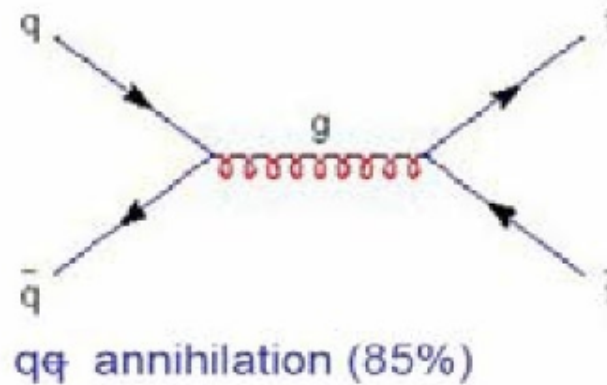
Very impressive proof of Standard Model predictive power!



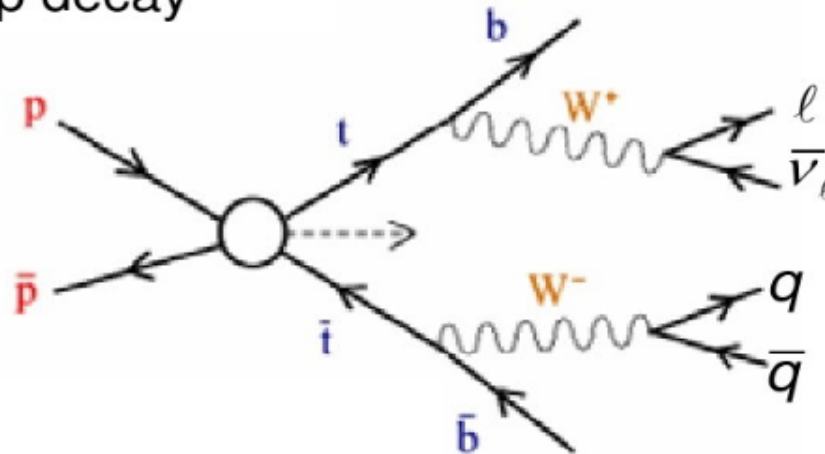
Prediction of m_t by LEP before the discovery of the top at TEVATRON.

Top discovery at the Tevatron 1995

$p\bar{p}$ @ 2 TeV

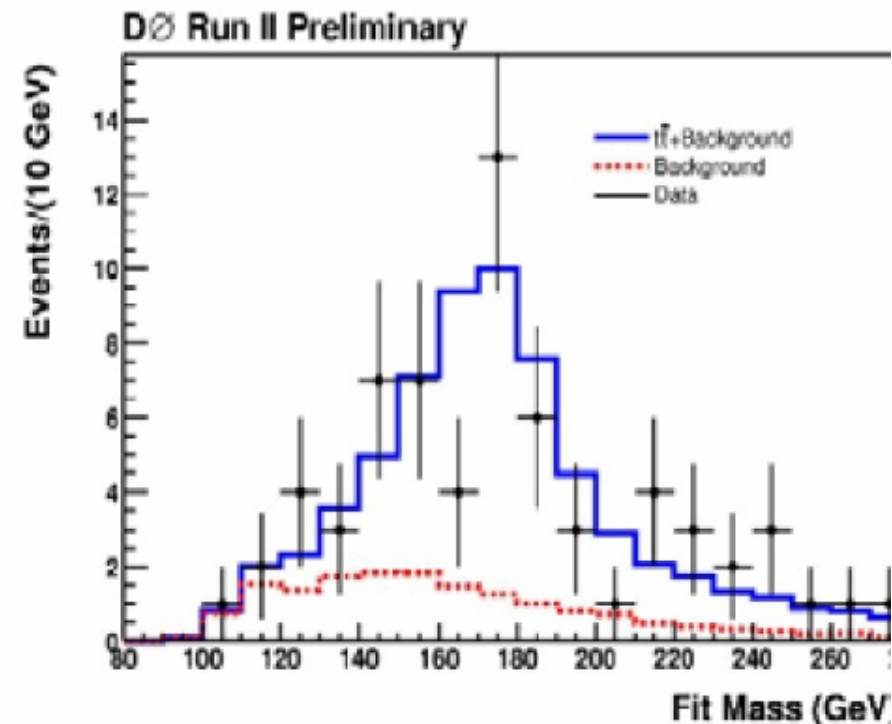


Top decay

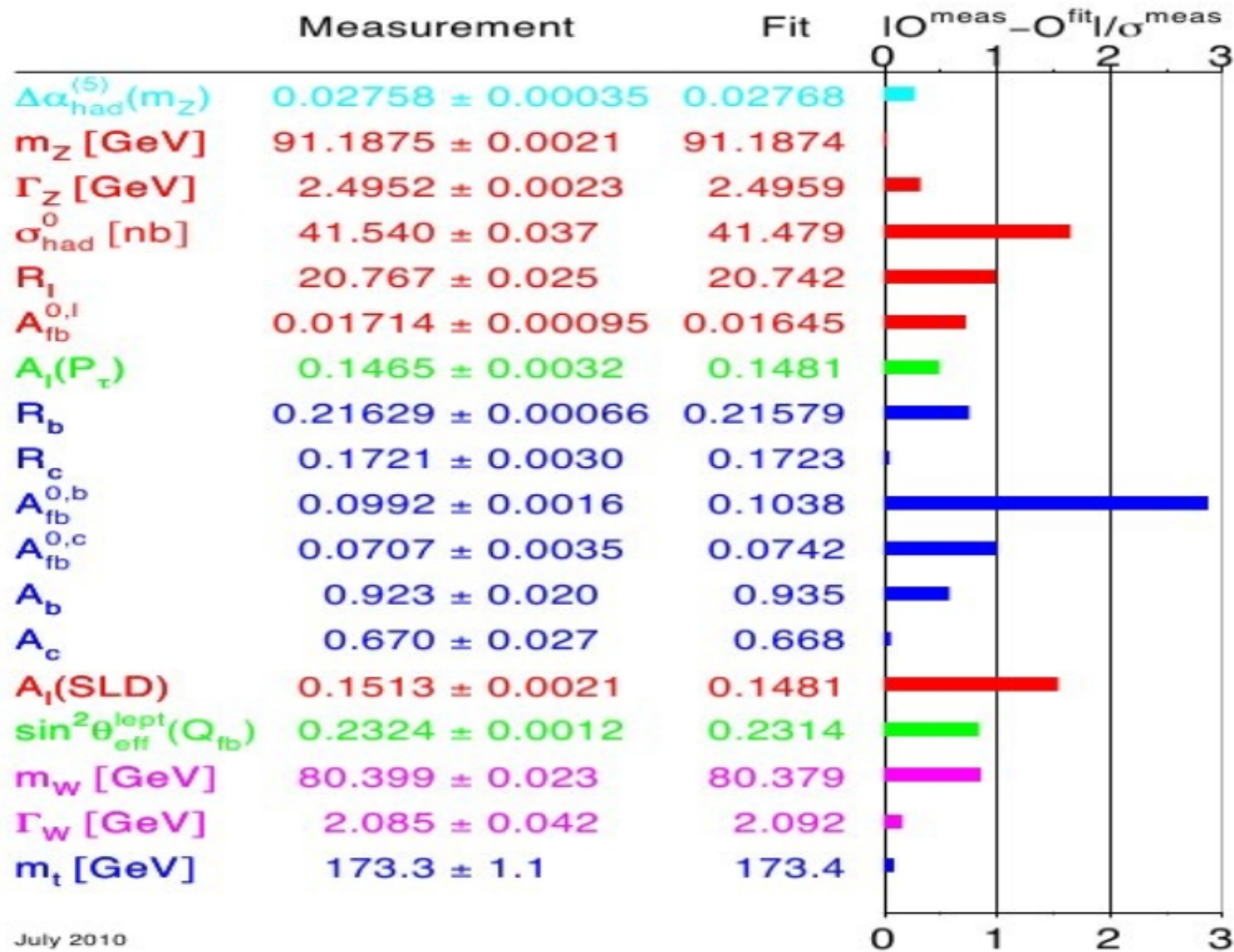


Channel used for mass reconstruction:

$$m_t = m_{inv}(b - jet, W \rightarrow jet + jet)$$



Standard Model precisely tested



July 2010

Everything is very consistent ... however Higgs boson is still missing.