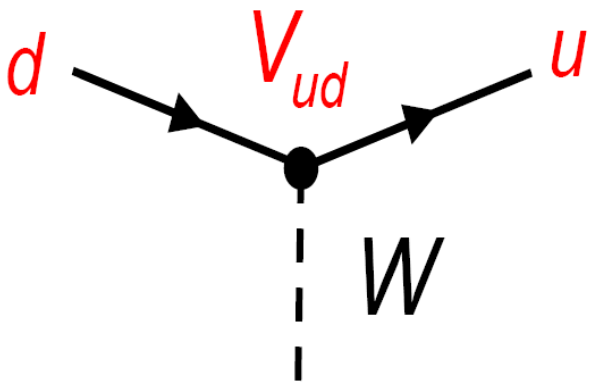
CKM Matrix

Charged currents: $J_\mu^+ \propto \left(\bar{u}, \bar{c}, \bar{t}\right)\left(1-\gamma_5\right)\gamma_\mu V_{CKM}\begin{pmatrix} d \\ s \\ b \end{pmatrix}$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \times \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

flavour CKM matrix mass



18 parameters (9 complex elements)

-5 relative quark phases (unobservable)

-9 unitarity conditions

________________-

= 4 independent parameters 3 Euler angles and 1 Phase

Phase is only source of CPV in SM, requires third quark family (Nobel Prize 2008)

5 relative phases

Charged currents: $J^+_\mu \propto \left(\bar{u}, \bar{c}, \bar{t}\right)\left(1-\gamma_5\right)\gamma_\mu V_{CKM}\begin{pmatrix} d \\ s \\ b \end{pmatrix}$

Lagrangian insensitive to phases of left-handed fields, possible redefinition:

$$u_L \to e^{i\phi_u} u_L \quad c_L \to e^{i\phi_c} c_L \quad t_L \to e^{i\phi_t} t_L$$
$$d_L \to e^{i\phi_d} d_L \quad s_L \to e^{i\phi_s} s_L \quad b_L \to e^{i\phi_b} b_L$$

$$V_{CKM} \to \begin{pmatrix} e^{i\phi_u} & 0 & 0 \\ 0 & e^{i\phi_c} & 0 \\ 0 & 0 & e^{\phi_t} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{-i\phi_d} & 0 & 0 \\ 0 & e^{-i\phi_s} & 0 \\ 0 & 0 & e^{-i\phi_b} \end{pmatrix}$$

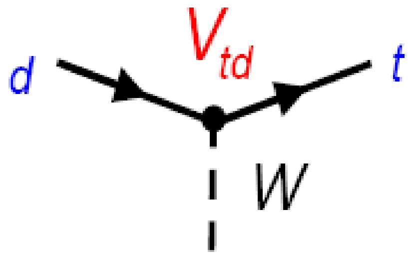
or $V_{\alpha\beta} \to e^{\phi_\beta - \phi_\alpha} V_{\alpha\beta}$

5 unobservable phase differences $\phi_\beta - \phi_\alpha$.

CKM under CP Transformation

Quarks

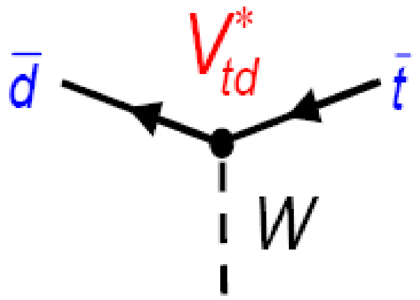
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



CP

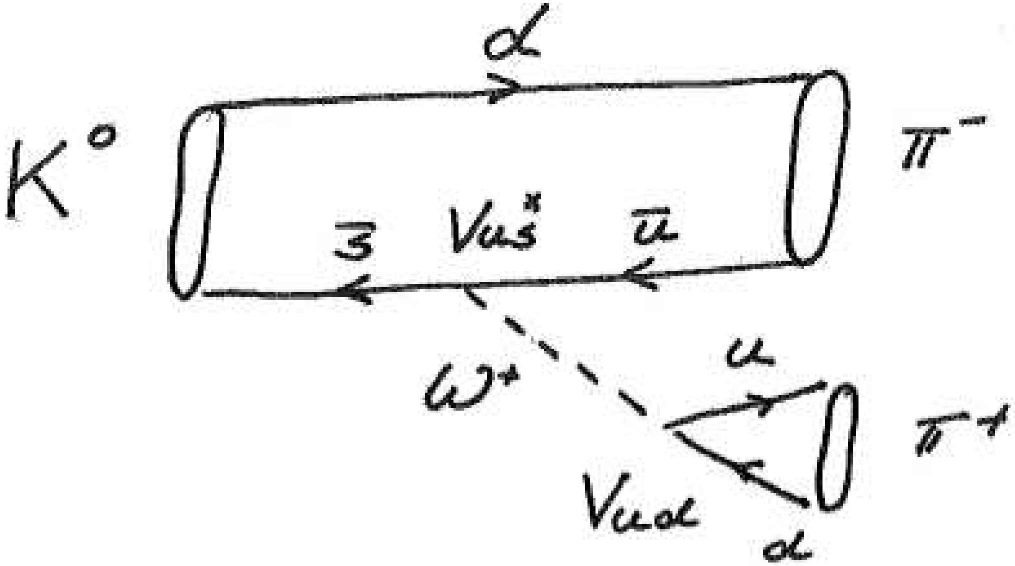
Anti-quarks:

$$\begin{pmatrix} \bar{d}' \\ \bar{s}' \\ \bar{b}' \end{pmatrix} = \begin{pmatrix} V^*_{ud} & V^*_{us} & V^*_{ub} \\ V^*_{cd} & V^*_{cs} & V^*_{cb} \\ V^*_{td} & V^*_{ts} & V^*_{tb} \end{pmatrix} \begin{pmatrix} \bar{d} \\ \bar{s} \\ \bar{b} \end{pmatrix}$$



Weak (CKM) phases change sign under CP transformation!

Weak and Strong Phases



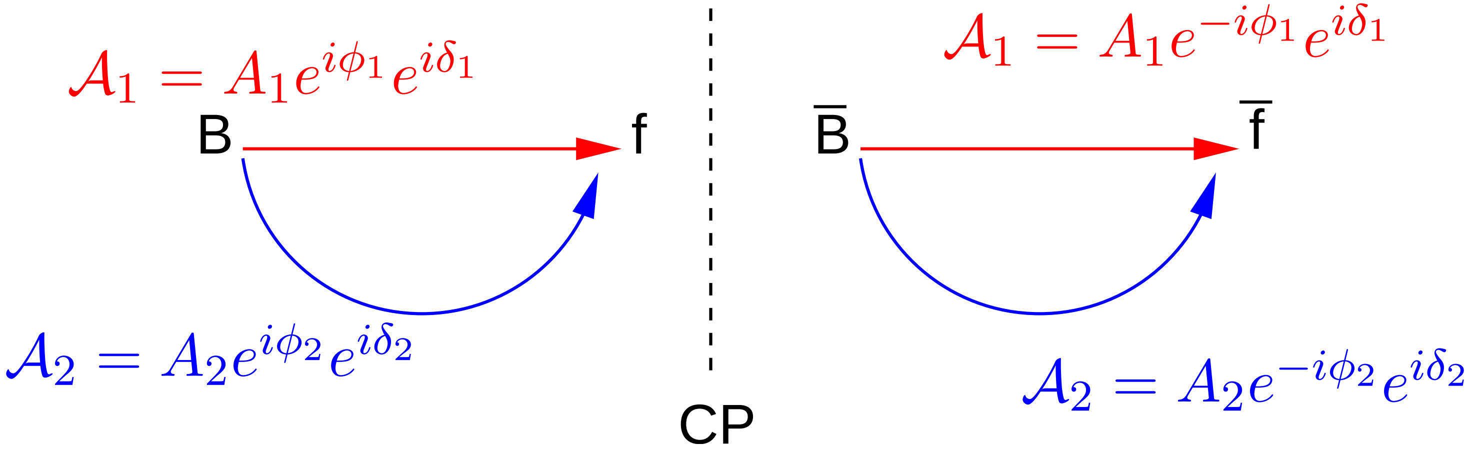
Weak phases are related to involved CKM elements: $\phi_{weak} = arg(V_{us}^{*} V_{ud})$

Strong phases δ comes often (but not always) from the hadronisation.

Definition of strong phase:

phase which doesn't change sign under CP transformation.

CP Violation



$$|\mathcal{A}|^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\Delta\phi + \Delta\delta)$$

$$|\mathcal{A}|^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(-\Delta\phi + \Delta\delta)$$

$\mathcal{A}_1$ and $\mathcal{A}_2$ need to have different weak phases ϕ and different strong phases δ.

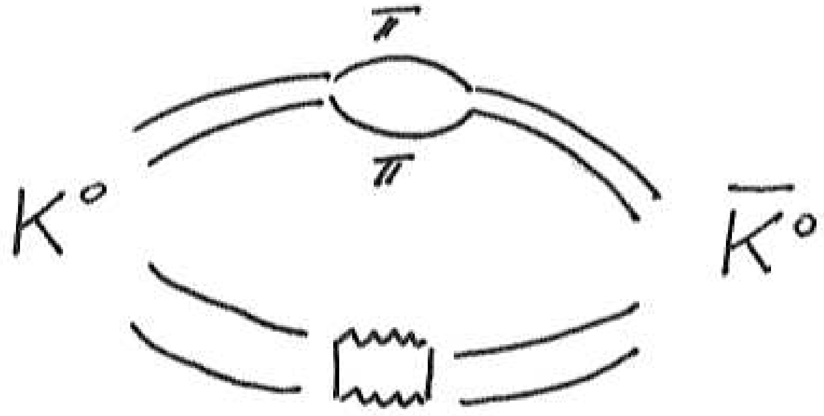
For sizable (measurable) effects both amplitudes should have about same size,

and both phase differences have to be sizable.

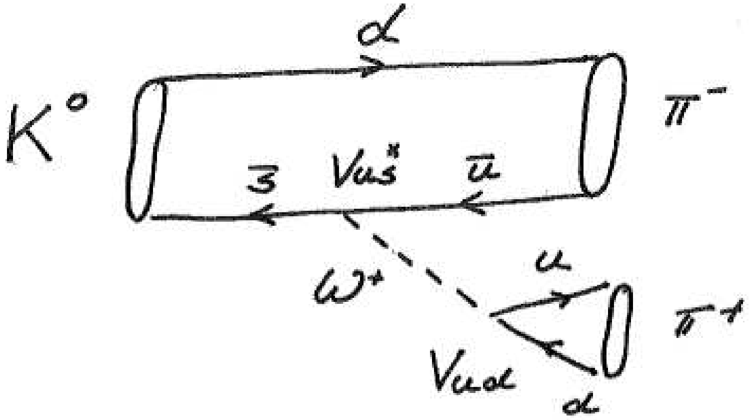
To conclude on weak phases, strong phases need to be known/measured.

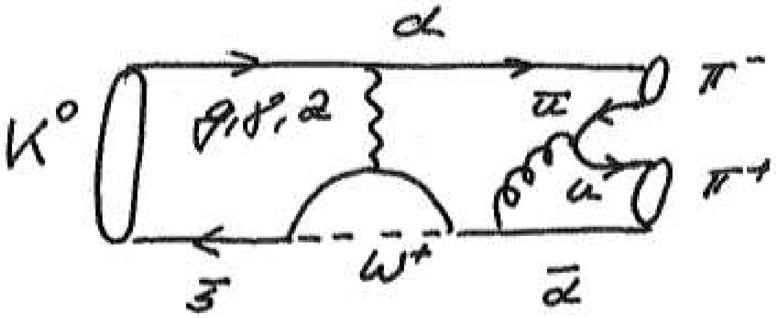
CPV in Kaon System

Interfering amplitudes which cause CPV in mixing:

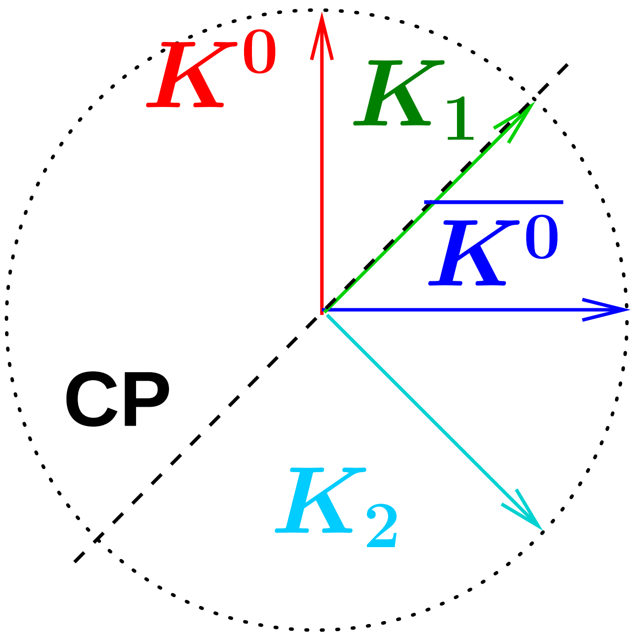


Interfering amplitudes which cause CPV in decay:





Neutral Meson Mixing



$$CP(K^0) = \overline{K^0}$$
$$CP(\overline{K^0}) = K^0$$

$$K_1 = \frac{1}{\sqrt{2}}(K^0 + \overline{K^0})$$
$$CP(K_1) = +K_1$$

$$K_2 = \frac{1}{\sqrt{2}}(K^0 - \overline{K^0})$$
$$CP(K_2) = -K_2$$

K^0, $\overline{K^0}$: flavour eigenstates; clear defined quark content ($K^0 = |d\bar{s} >$, $\overline{K^0} = |\bar{d}s >$)

K_1, K_2: CP eigenstates

K_S, K_L: mass eigenstates (τ_S = 89 ps; τ_L = 51 ns)

(with clear defined mass and lifetime, $\psi_{S/L}(t) = e^{-im_{S/L}t} e^{-\Gamma_{S/L}t/2}$)

in absence of CPV: $K_S = K_1, K_L = K_2$

Kaon Mixing

$$|\mathbf{K_S}> = p|\mathbf{K^0}> + q|\overline{\mathbf{K^0}}>, \quad |\mathbf{K_S(t)}> = |\mathbf{K_S}> e^{-\frac{\Gamma_S}{2}t} e^{-im_S t}$$

$$|\mathbf{K_L}> = p|\mathbf{K^0}> - q|\overline{\mathbf{K^0}}>, \quad |\mathbf{K_L(t)}> = |\mathbf{K_L}> e^{-\frac{\Gamma_L}{2}t} e^{-im_L t}$$

$|p|^2 + |q|^2 = 1$ complex coefficients; $q = p = \frac{1}{\sqrt{2}} \Leftrightarrow \mathbf{K_S} = \mathbf{K_1}, \mathbf{K_L} = \mathbf{K_2}$

Flavour eigenstates:

$$|\mathbf{K^0}> = \frac{1}{2p}(|\mathbf{K_S}> + |\mathbf{K_L}>)$$

$$|\overline{\mathbf{K^0}}> = \frac{1}{2q}(|\mathbf{K_L}> - |\mathbf{K_S}>)$$

time development of originally (at t=0) pure $\mathbf{K^0}$ and $\overline{\mathbf{K^0}}$ states:

$$|\mathbf{K^0(t)}> = \frac{1}{2p}(|\mathbf{K_S(t)}> + |\mathbf{K_L(t)}>)$$

$$|\overline{\mathbf{K^0}}\mathbf{(t)}> = \frac{1}{2q}(|\mathbf{K_L(t)}> - |\mathbf{K_S(t)}>)$$

Kaon Mixing

$$P(\mathbf{K^0} \rightarrow \overline{\mathbf{K^0}}) = | < \mathbf{K^0(t)} | \overline{\mathbf{K^0}} > |^2 =$$
$$\frac{1}{4}|\frac{q}{p}|^2 \left(e^{-\Gamma_L t} + e^{-\Gamma_S t} - 2e^{-(\Gamma_L + \Gamma_S)t/2} \cos \Delta m t \right)$$

$$P(\overline{\mathbf{K^0}} \rightarrow \mathbf{K^0}) = | < \overline{\mathbf{K^0}}\mathbf{(t)} | \mathbf{K^0} > |^2 =$$
$$\frac{1}{4}|\frac{p}{q}|^2 \left(e^{-\Gamma_L t} + e^{-\Gamma_S t} - 2e^{-(\Gamma_L + \Gamma_S)t/2} \cos \Delta m t \right)$$

CP conserved: $P(\mathbf{K^0} \rightarrow \overline{\mathbf{K^0}}) = P(\overline{\mathbf{K^0}} \rightarrow \mathbf{K^0})$

$$\Leftrightarrow$$

$$|\frac{q}{p}| = 1$$

(+ normalisation $q^2 + p^2 = 1$)

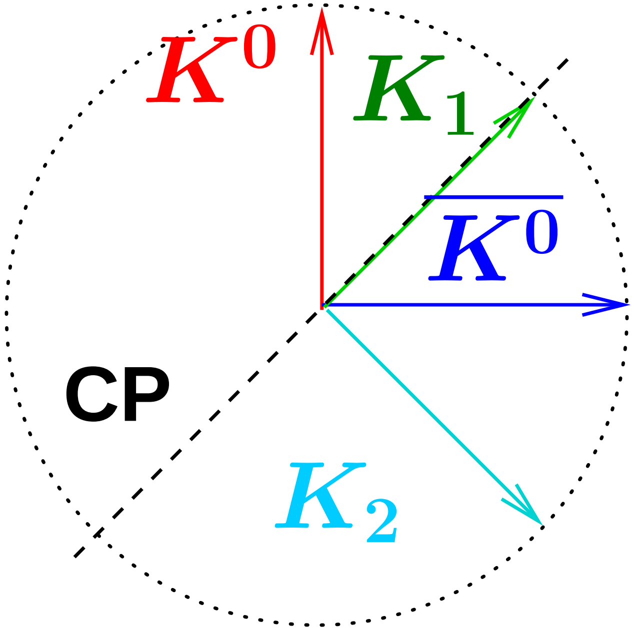
$$\Leftrightarrow$$

$$q = p = \frac{1}{\sqrt{2}}$$

$$\Leftrightarrow$$

$$K_S = K_1, K_L = K_2$$

Neutral Meson Mixing



$$CP(K^0) = \overline{K^0}$$
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$$K_1 = \frac{1}{\sqrt{2}}(K^0 + \overline{K^0})$$
$$CP(K_1) = +K_1$$

$$K_2 = \frac{1}{\sqrt{2}}(K^0 - \overline{K^0})$$
$$CP(K_2) = -K_2$$

$$P(\Psi(\pi)) = P(\Psi(q)) \cdot P(\Psi(\overline{q})) \cdot (-1)^{L=0} = 1 \cdot -1 \cdot 1 \cdot \Psi(\pi) = -\Psi(\pi)$$
$$C(\Psi(\pi)) = C(\Psi(q\overline{q})) = (-1)^{L+S} \cdot \Psi(q\overline{q}) = +\Psi(\pi)$$

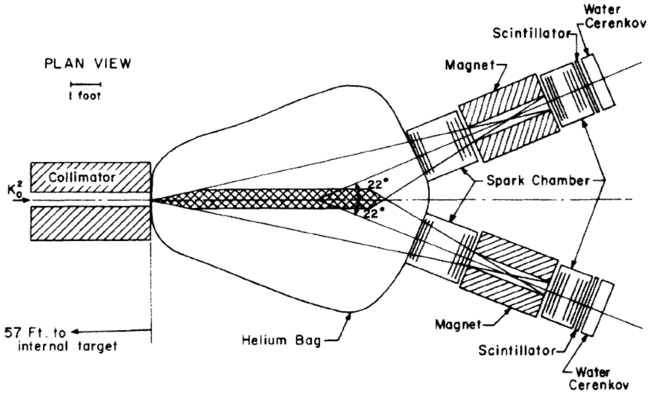
$$CP(\Psi(\pi^+\pi^-)) = CP(\Psi(\pi^+)) \cdot CP(\Psi(\pi^-)) \cdot (-1)^{L=0} = +\Psi(\pi^+\pi^-)$$
$L = 0$ in $K^0 \to \pi^+\pi^-$

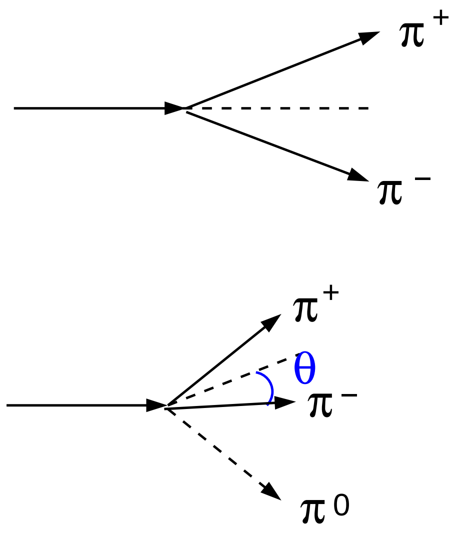
$$CP(\Psi(\pi^+\pi^-\pi^0)) = CP(\Psi(\pi^-))^3 \cdot (-1)^L = -\Psi(\pi^+\pi^-\pi^0)$$
$L = 0$ in $K^0 \to \pi^+\pi^-\pi^0$

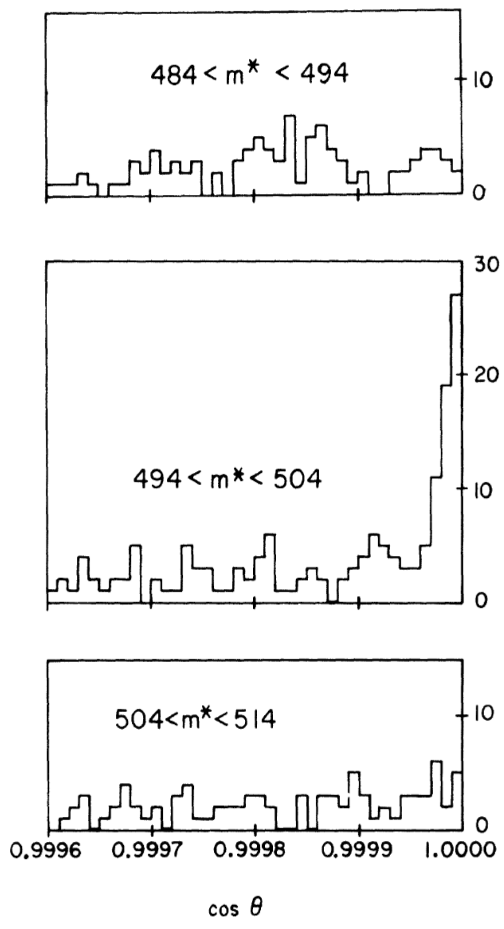
If there is no CPV in decay, then: $K_1 \to \pi^+\pi^-$; $K_2 \to \pi^+\pi^-\pi^0$

1964: Discovery of CPV

- produce K^0, wait long enough for K_S component to decay away $\rightarrow$ pure K_L beam
- search for CP violation: $K_L \rightarrow \pi^+\pi^-$
  $\rightarrow$ excess of 56 events: BR($K_L \rightarrow \pi^+\pi^-$) $\sim 2 \times 10^{-3}$







mass eigenstates $\neq$ CP eigenstates: $|\mathbf{K_L}>= \frac{1}{\sqrt{1+|\epsilon^2|}}(|\mathbf{K_2}> +\epsilon|\mathbf{K_1}>)$

CP=-1 CP=+1

Nobel prize for Cronin and Fitch in 1980

After 40 years ...

$$|\mathbf{K_L}> = \frac{1}{\sqrt{1+|\epsilon^2|}}(|\mathbf{K_2}> + \epsilon|\mathbf{K_1}>$$

$\downarrow$ ε'

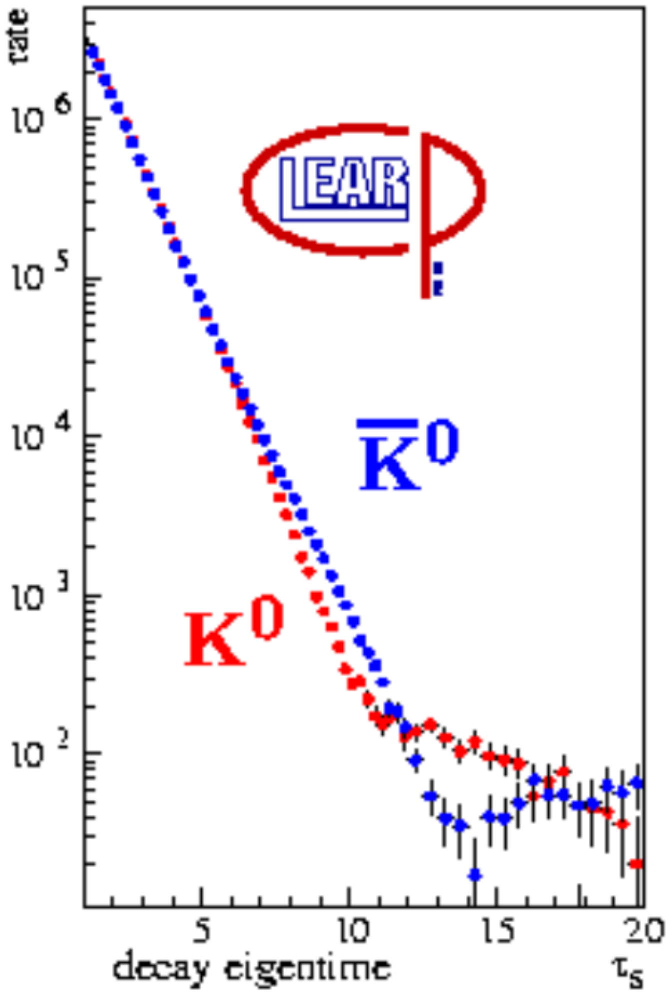
ππ

$\mathbf{K_L}$ mainly CP odd, a bit (ϵ) CP even ("CP in mixing")

CP odd state can decay in $\pi\pi$ with a tiny probability of $\epsilon^{'}$

$\rightarrow$ CPV in decay

$$|\epsilon| = (2.284 \pm 0.014) \times 10^{-3}$$
$$Re(\epsilon^{'}/\epsilon) = (1.67 \pm 0.26) \times 10^{-3}$$



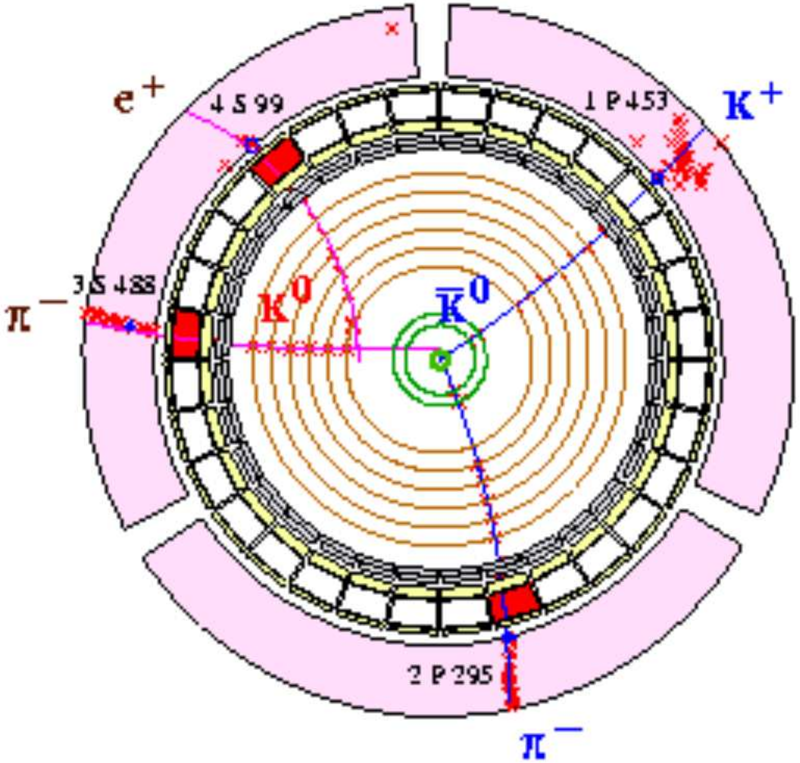
$p\bar{p} \rightarrow K^- + \pi^+ + K^0$

Kaon Mixing

CPLear:

tag of initial state: $p\bar{p} \rightarrow K^+\pi^-\overline{K^0}$

self-tagging semileptonic final state $K^0 \rightarrow \pi^- e^+ \nu_e$

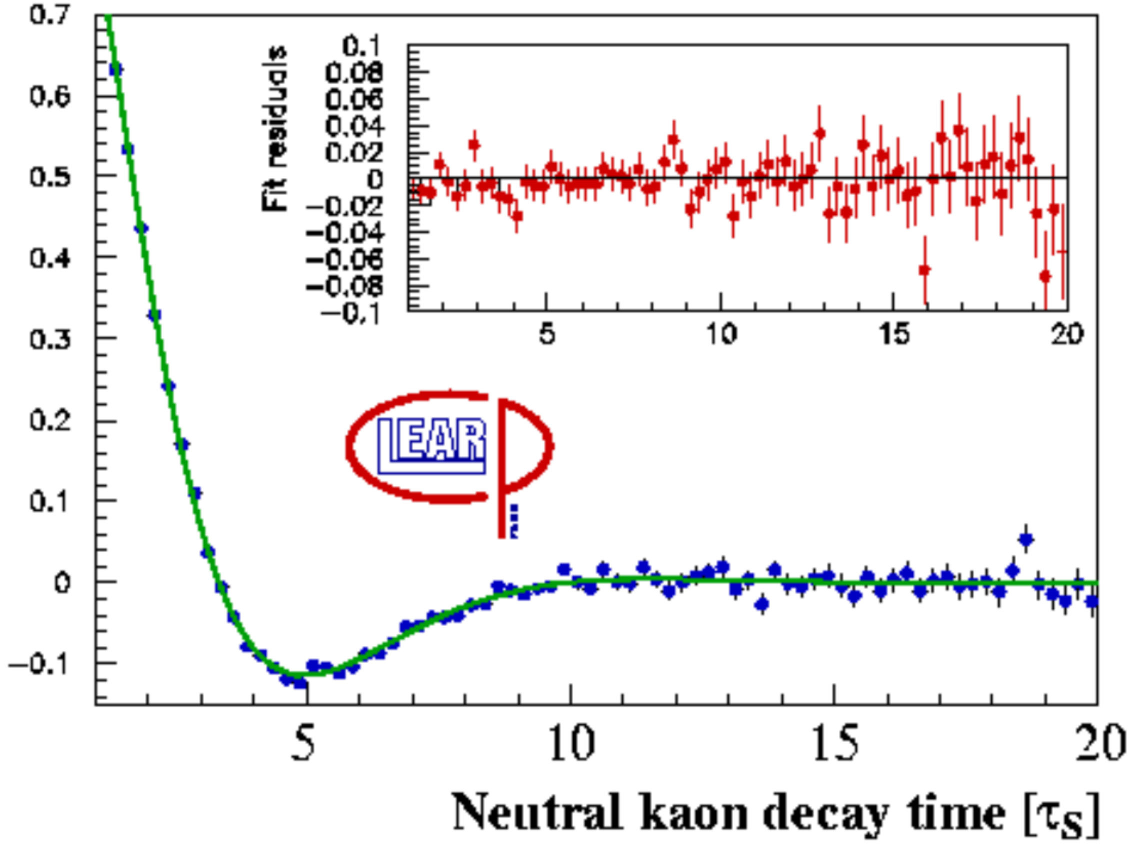


unmixed : $N(K^0_{\tau=0} \rightarrow e^+\pi^-\nu_e)(t) + N(\overline{K^0}_{\tau=0} \rightarrow e^-\pi^+\overline{\nu}_e)(t)$

mixed : $N(K^0_{\tau=0} \rightarrow e^-\pi^+\overline{\nu}_e)(t) + N(\overline{K^0}_{\tau=0} \rightarrow e^+\pi^-\nu_e)(t)$

Kaon Mixing

$$\mathcal{A}_{\Delta m}(t) = \frac{N_{unmixed}(t) - N_{mixed}(t)}{N_{unmixed}(t) + N_{mixed}(t)} = 2\frac{e^{-0.5(\frac{1}{\tau_S}+\frac{1}{\tau_L})t} \cdot \cos(\Delta m)}{e^{-\frac{t}{\tau_S}} + e^{-\frac{t}{\tau_L}}}$$



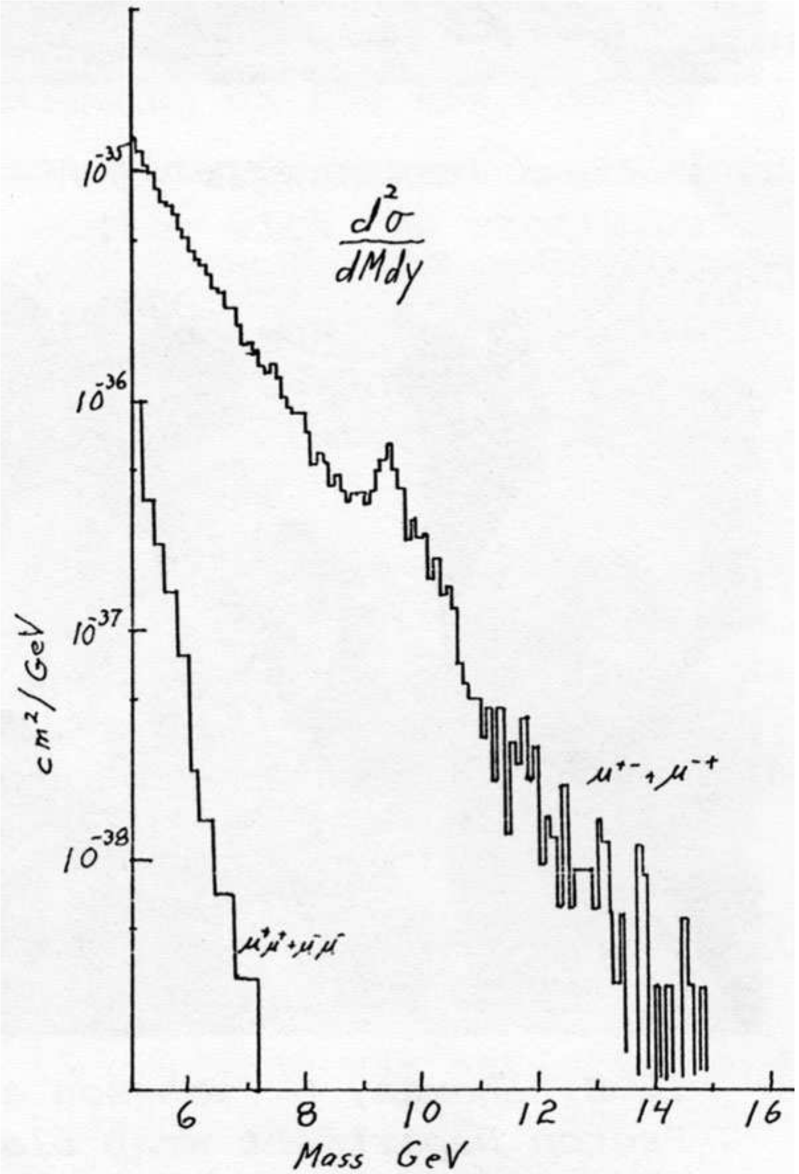
$$\Delta m = (529.5 \pm 2.0 \pm 0.3) \times 10^{-7} \hbar s^{-1}$$

Nobel Prize 2008

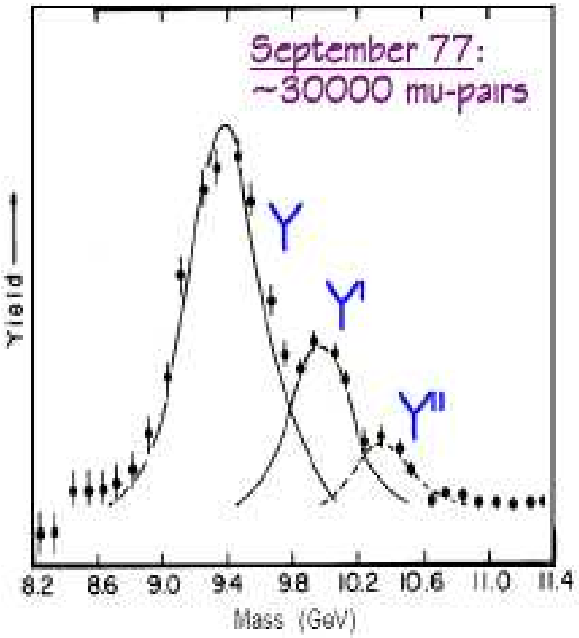
Kobayashi & Maskawa:

"for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature"

1977: Discovery of beauty



Leo Lederman



First surprises with B Lifetime

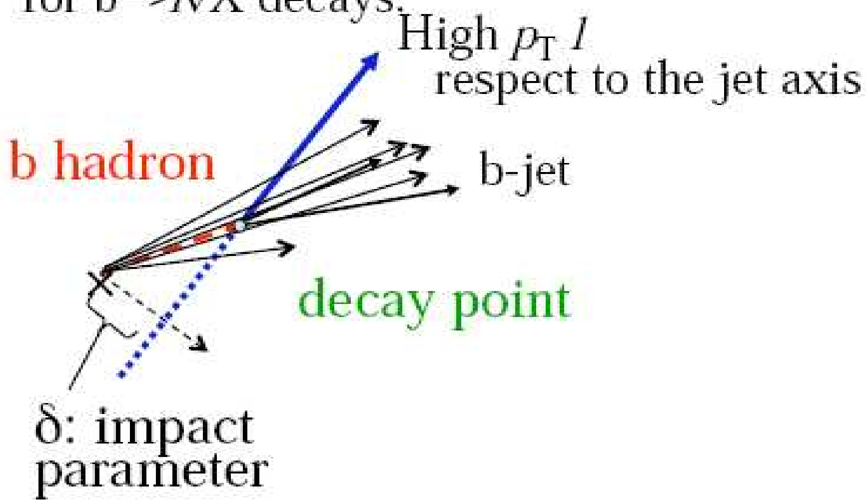
MAC

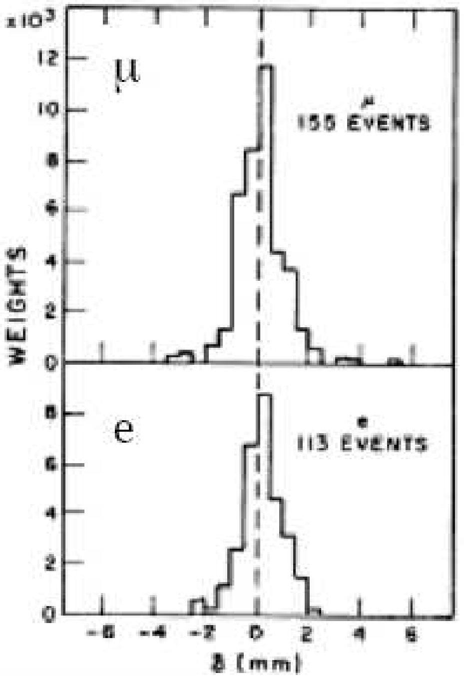
Phys. Rev. Lett. 51, (1983) 1022

Lifetime of Particles Containing *b* Quarks

From a sample of hadronic events produced in e^+e^- collisions, semileptonic decays of heavy particles have been isolated and used to obtain a measurement for the bottom-quark lifetime of $[1.8\pm0.6(\text{stat.})\pm0.4(\text{syst.})]\times10^{-12}$ *sec.*

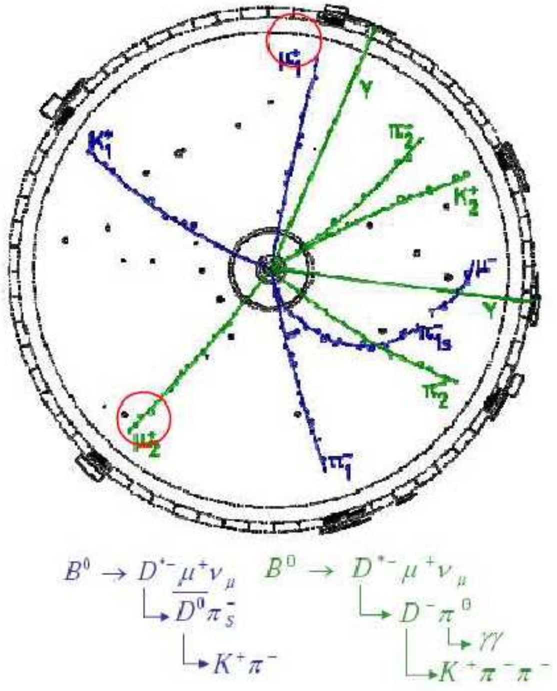
Impact parameter distributions for $b \to l\nu X$ decays.



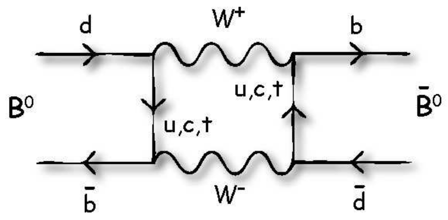


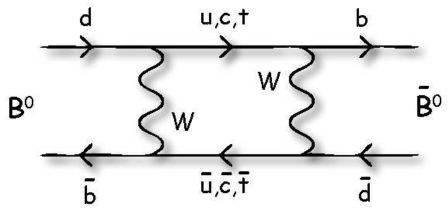
Relative long lifetime, opens up interesting possibilities for B mesons, e.g. oscillations, CP violation

1986: B^0 Oscillation at ARGUS



$$e^+e^- \rightarrow Y(4S) \rightarrow B^0\overline{B^0}$$





Time integrated mixing rate: $\chi_d = \int P_{mixed}(t) \cdot e^{-t/\tau} dt = 0.17 \pm 0.05$

25 mixed events:

$B^0\overline{B^0} \rightarrow \ell^-\ell^-$

$B^0\overline{B^0} \rightarrow \ell^+\ell^+$

250 unmixed events:

$B^0\overline{B^0} \rightarrow \ell^+\ell^-$

First indication for a heavy top quark $m_t > 40$ GeV!

New physics in B mixing?

- $P(B \to \overline{B}) \neq P(\overline{B} \to B)$

semileptonic asymmetry

$(B^0 + B_s)$

$B \longrightarrow \overline{B} \longrightarrow \mu^-$

$\overline{B} \longrightarrow \mu^-$

$\overline{B} \longrightarrow B \longrightarrow \mu^+$

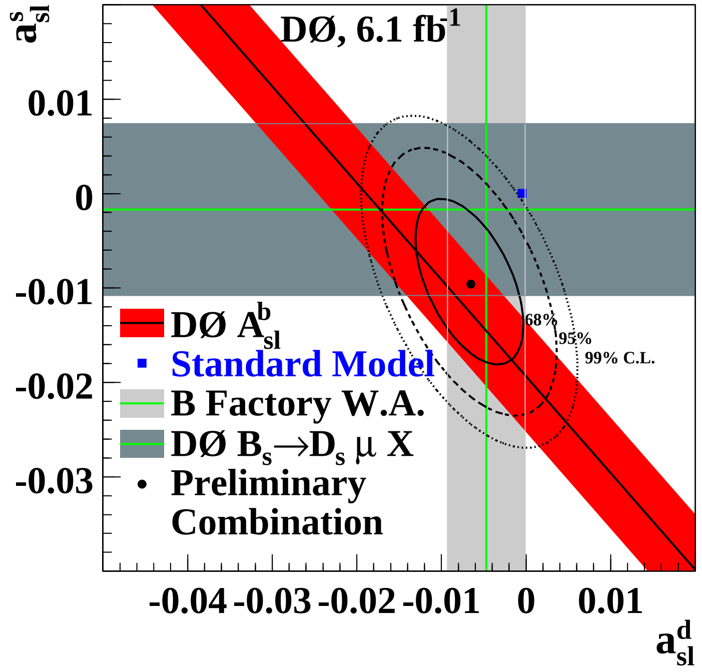
$B \longrightarrow \mu^+$

$$A = \frac{N(\mu^+\mu^+) - N(\mu^-\mu^-)}{N(\mu^+\mu^+) + N(\mu^-\mu^-)}$$

$$a = \frac{N(\mu^+) - N(\mu^-)}{N(\mu^+) + N(\mu^-)}$$

SM: A^b_{sl} = (-0.20 $\pm$ 0.03) $\times$ 10^{-3}

A. Lenz, U. Nierste, (2006/2011)



A^b_{sl} = -0.957 $\pm$ 0.251 (stat) $\pm$ 0.14 (syst) %

(Phys. Rev. Lett 105, 081802 (2010))

$\rightarrow$ 3.2σ deviation from SM

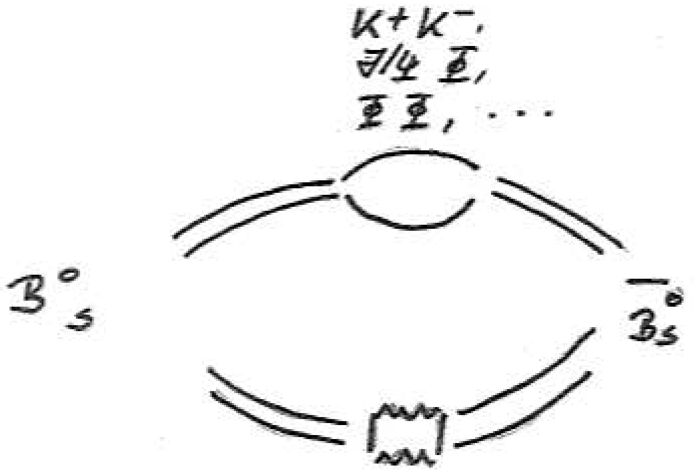
CPV in B Mixing

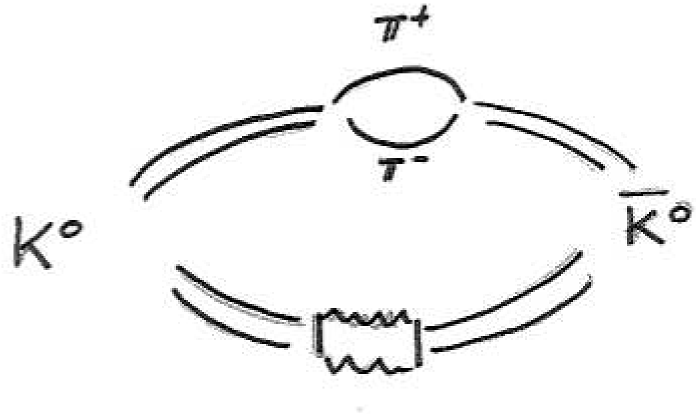
What are the interfering amplitudes?

Why is CPV in B mixing so much smaller than CPV in K^0 mixing?

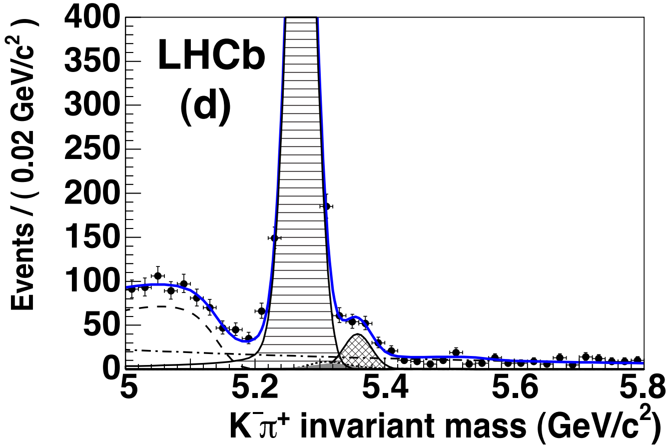
branching ratio into non-flavour specific decays

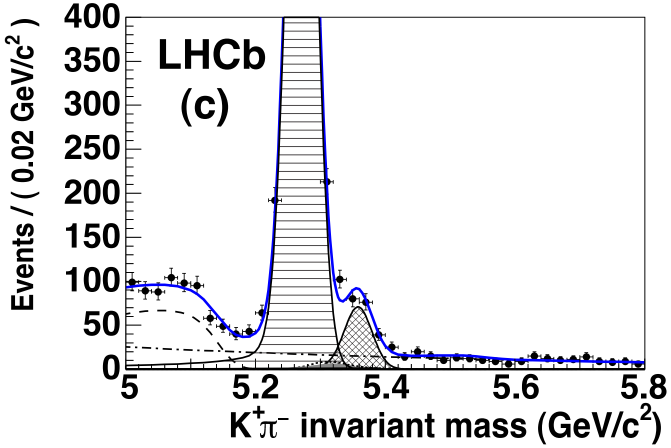
$\sim 10^{-4}$ $\qquad$ $> 0.95\%$





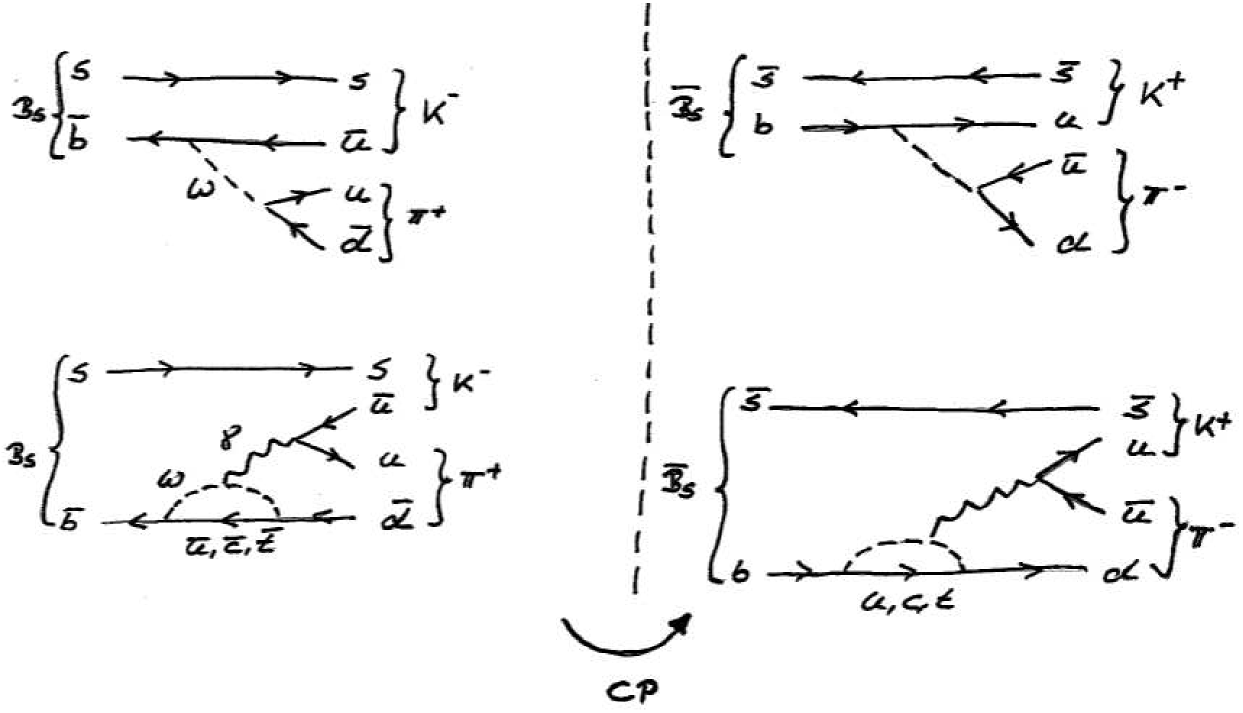
Example of CPV in decay



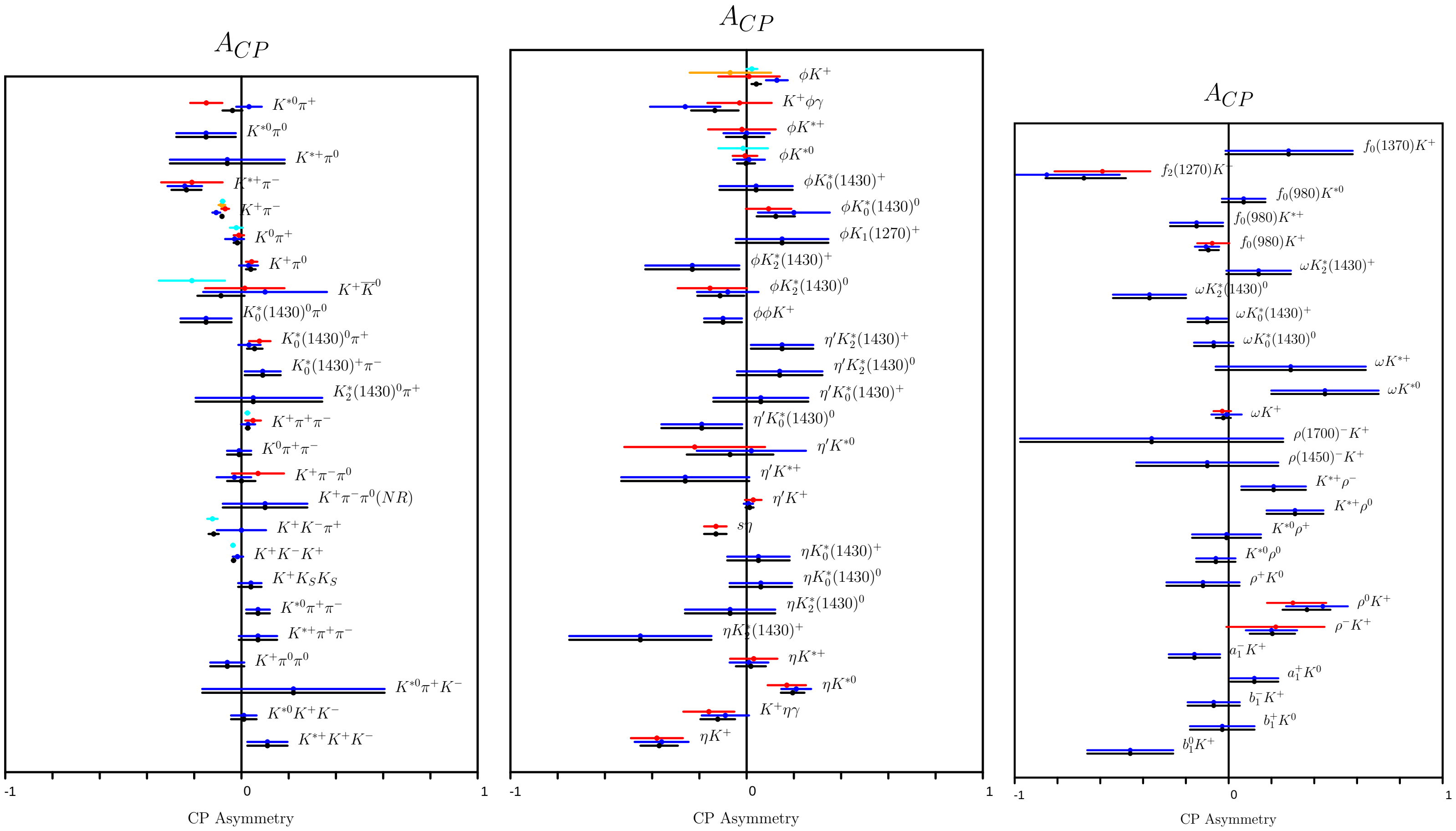


$$A_{CP}(B_s \rightarrow K\pi) = 27 \pm 8(stat) \pm 2(sys)\%$$

Phys. Rev. Lett 108 (2012) 201601



Lot's of CPV in decays ...



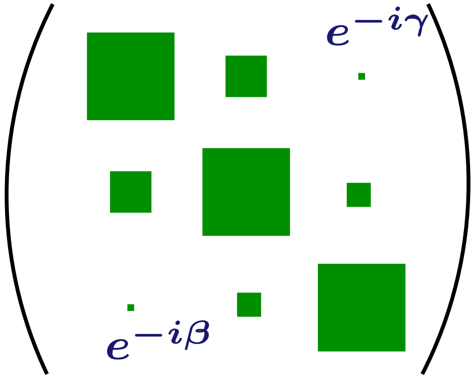
Due to unknown strong phases, hard to relate CPV directly to CKM parameters :-(.

"The strong interaction can be seen either as the unsung hero or the villain in the story of quark flavour physics"; I. Bigi.

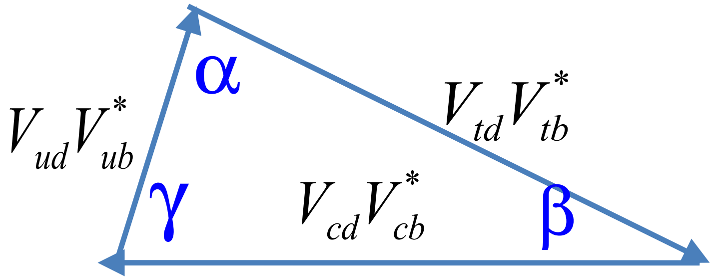
CKM Angles

size of box, illustrates absolute value

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim$$



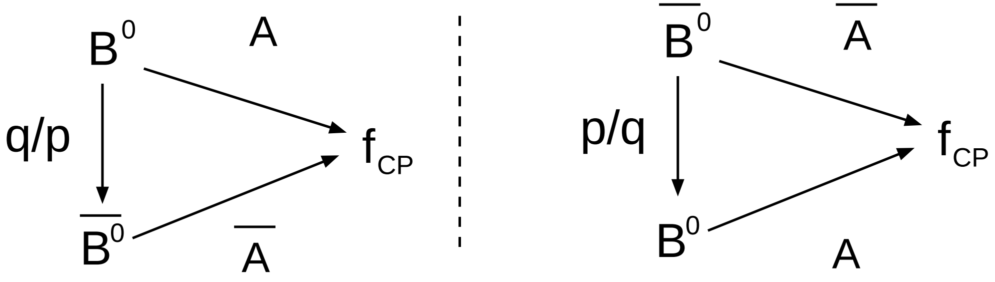
CKM triangle

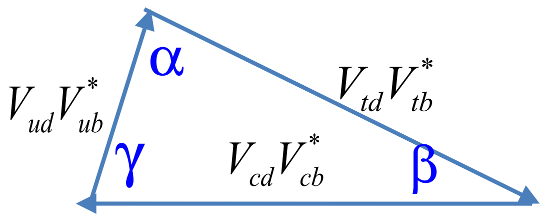


CPV in interference of mixing and decay

Measurement of $\sin(2\beta)$: golden channel $B_d \to J/\psi K_s$

“Golden”: large statistics, easy to detect, (almost) no CPV in decay





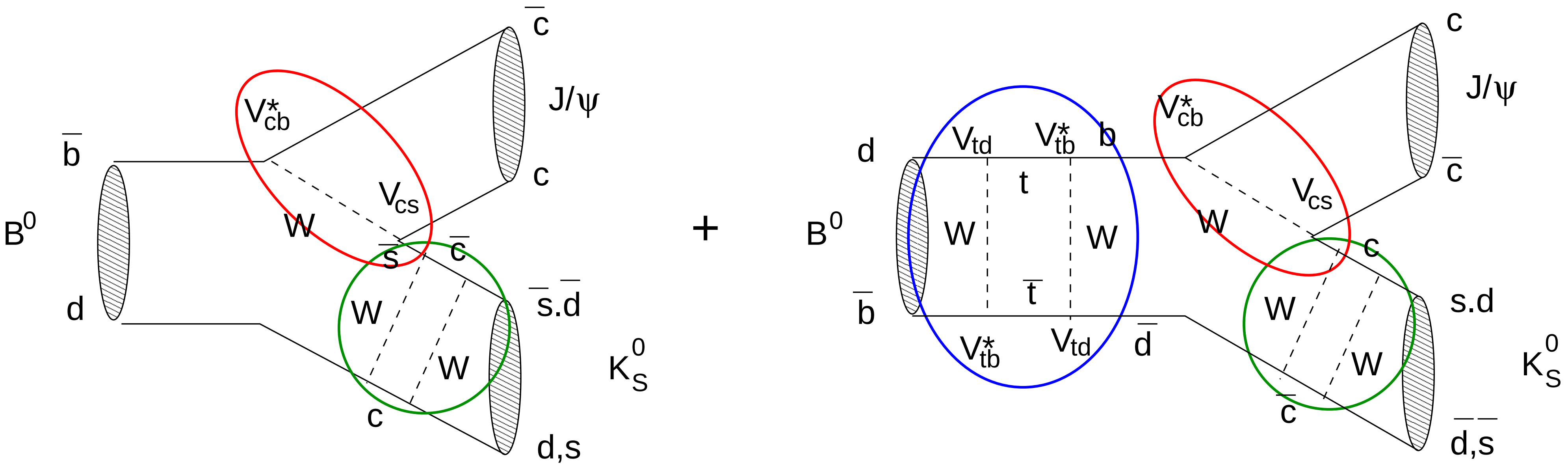
Weak phase: $Im(\frac{q}{p}\frac{\overline{A}}{A})$

$$\beta = \arg \frac{V_{cb}V_{cd}^*}{V_{tb}V_{td}^*}$$

$B_d \to J/\Psi K^0$

Reach same final state through decay & mixing + decay

(assume no CPV in mixing and no CPV in decay)



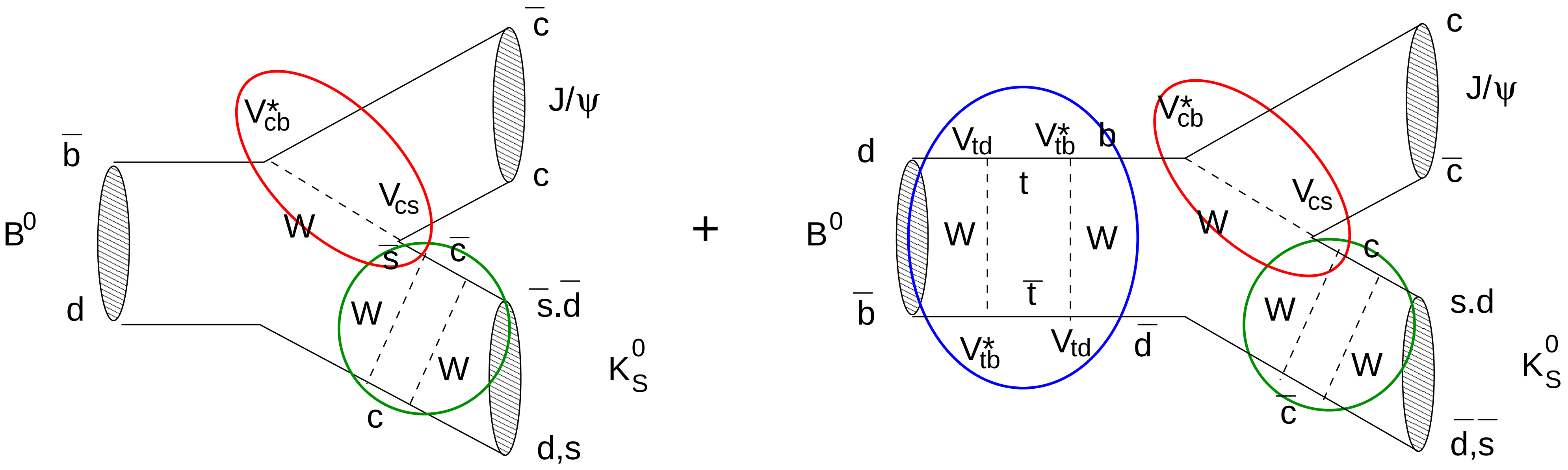
$$\mathcal{A}_1 = \mathcal{A}_{mix}(B^0 \to B^0) * \mathcal{A}_{decay}(B^0 \to J/\Psi K^0) = \cos(\frac{\Delta mt}{2}) * A * e^{i\omega} * A_K$$

$$\mathcal{A}_2 = \mathcal{A}_{mix}(B^0 \to \overline{B^0}) * \mathcal{A}_{decay}(\overline{B^0} \to J/\Psi K^0) = i\sin(\frac{\Delta mt}{2}) * e^{+i\phi} * A * e^{-i\omega} A_K * e^{+i\xi}$$

weak phase difference $\mathcal{A}_2 - \mathcal{A}_1$: $\Delta\phi = \phi - 2\omega + \xi = 2\beta$

strong phase difference $\Delta\delta = \pi/2 \Leftarrow$ mixing introduces second phase difference

$B_d \to J/\Psi K^0$



$$\Delta\phi = \phi - 2\omega + \xi = arg[\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}\frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}]$$

$$= arg[\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\frac{V_{cb}V_{cd}^*}{V_{cb}^*V_{cd}}] = 2arg[\frac{V_{cb}V_{cd}^*}{V_{tb}V_{td}^*}] = 2\beta$$

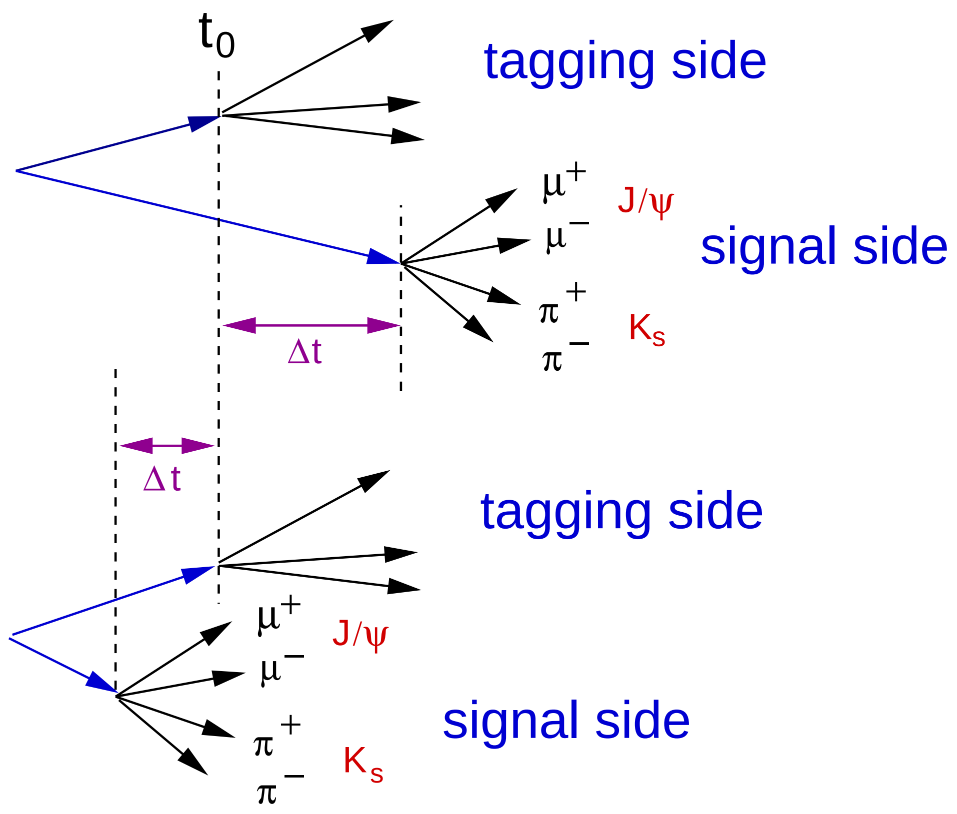
t quark dominates B^0 mixing box diagram

c quark dominates K^0 mixing box diagram

Correlated B Production

$$A(t) = \frac{N(\overline{B}\to J/\psi K_s)(t) - N(B\to J/\psi K_s)(t)}{N(\overline{B}\to J/\psi K_s)(t) + N(B\to J/\psi K_s)(t)} = \eta_{CP} \sin(2\beta) \sin \Delta m_d t$$

(for K_s η_{CP} = -1, for K_L η_{CP} = +1 ... neglecting CP in kaon mixing)

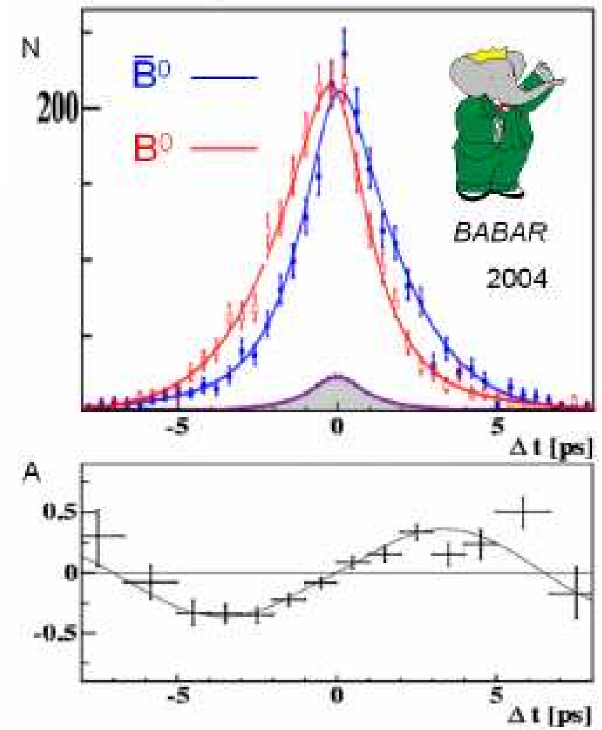


This is how it works at e^+e^- B factories

$B - \overline{B}$ pair produced on Y(4S) resonance with well defined quantum numbers.

$\to$ Correlated $B - \overline{B}$ state till the time of the decay of the first B.

$B_d \to J/\psi K_s$



$$\mathcal{A}(t) = \frac{N(B^0)(t) - N(\overline{B^0})(t)}{N(B^0)(t) - N(\overline{B^0})(t)} = -\sin(2\beta)\sin(\Delta m_d t)$$

Babar:

$\sin(2\beta) = 0.722 \pm 0.040 \pm 0.023$

Belle:

$\sin(2\beta) = 0.652 \pm 0.039 \pm 0.020$