

# PNMS Matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & s_{13}e^{-i\delta} & 0 \\ -s_{13}e^{i\delta} & c_{13} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

With  $\Delta m_{12}^2 \ll \Delta m_{23}^2 \sim \Delta m_{13}^2$  and  $\Theta_{13}$  small

For atmospheric and accelerator neutrinos (E large, L medium):  $\frac{\Delta m_{12}^2 L}{E} \ll 1$

$$P(\nu_\mu \rightarrow \nu_\tau) = 1 - \sin^2 2\Theta_{23} \sin^2\left(\frac{\Delta m_{23}^2 L}{4E}\right) \quad P(\nu_e \rightarrow \nu_e) \sim 1$$

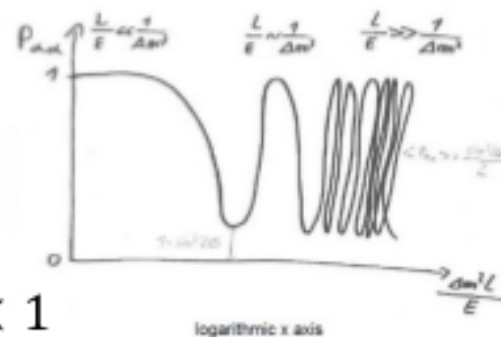
For solar neutrinos and very long baseline neutrinos (L large, E small):  $\frac{\Delta m_{13}^2 L}{E} \gg 1$

(almost like two neutrino system, only one mass difference)

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\Theta_{12} \sin^2\left(\frac{\Delta m_{12}^2 L}{4E}\right)$$

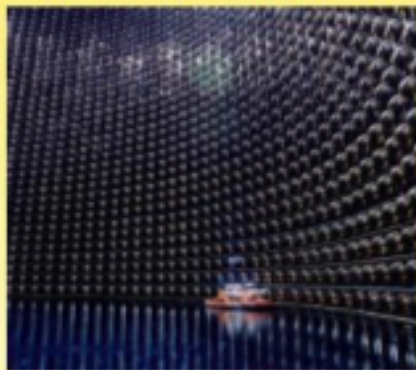
For short baseline reactor neutrinos (E small, L small):  $\frac{\Delta m_{12}^2 L}{E} \ll 1$

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\Theta_{13} \sin^2\left(\frac{\Delta m_{13}^2 L}{4E}\right)$$

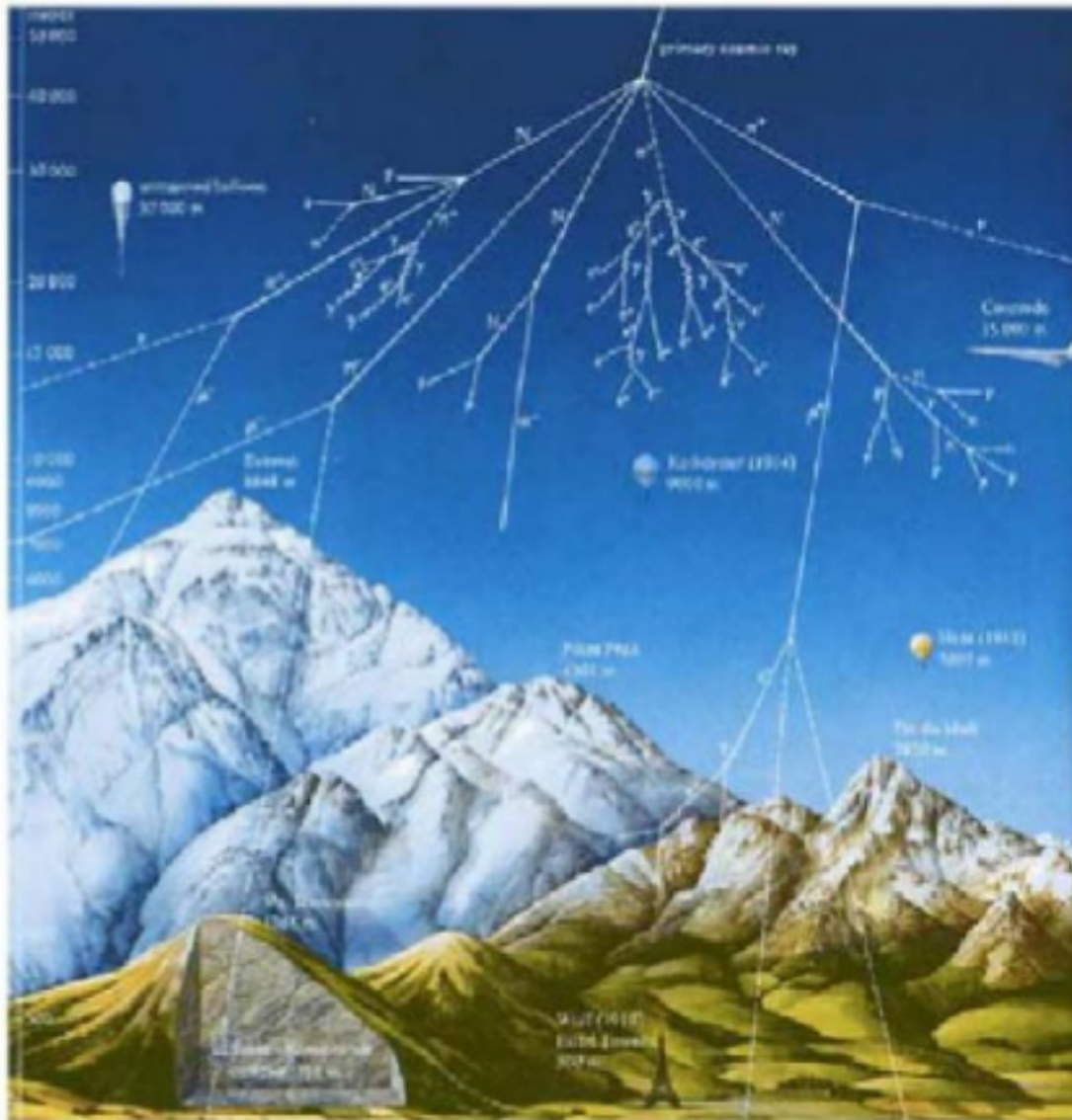


# Sensitivity

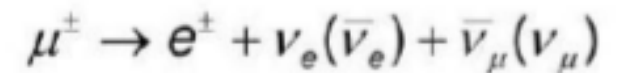
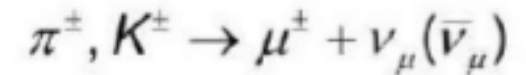
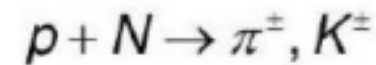
$$U = \underbrace{\begin{pmatrix} c_{12} c_{13} & -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{-i\delta} & s_{12} s_{23} - c_{12} c_{23} s_{13} e^{-i\delta} \\ 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric and LBL accelerator}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{SBL reactor}} \underbrace{\begin{pmatrix} s_{13} e^{i\delta} & s_{23} c_{13} & c_{23} c_{13} \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{solar and LBL reactor}} =$$



# Atmospheric Neutrinos



Cosmic radiation: Air shower



$$R = \frac{v_\mu + \bar{v}_\mu}{v_e + \bar{v}_e} = 2$$

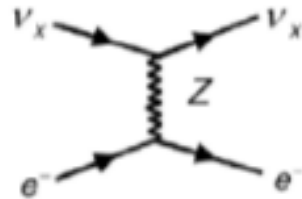
Exact calculation:  $R=2.1$   
( $E_\nu < 1\text{GeV}$ )

(For larger energies  $R > 2.1$ )

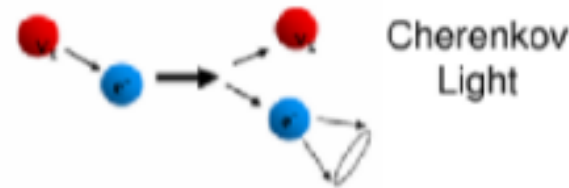
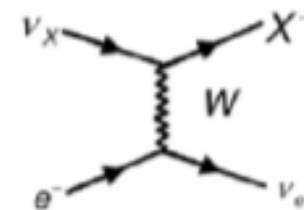
# (Super-)Kamiokande

Water = "active target"

Elastic scattering



Charged current



Detection of Cherenkov photons: Photo multiplier

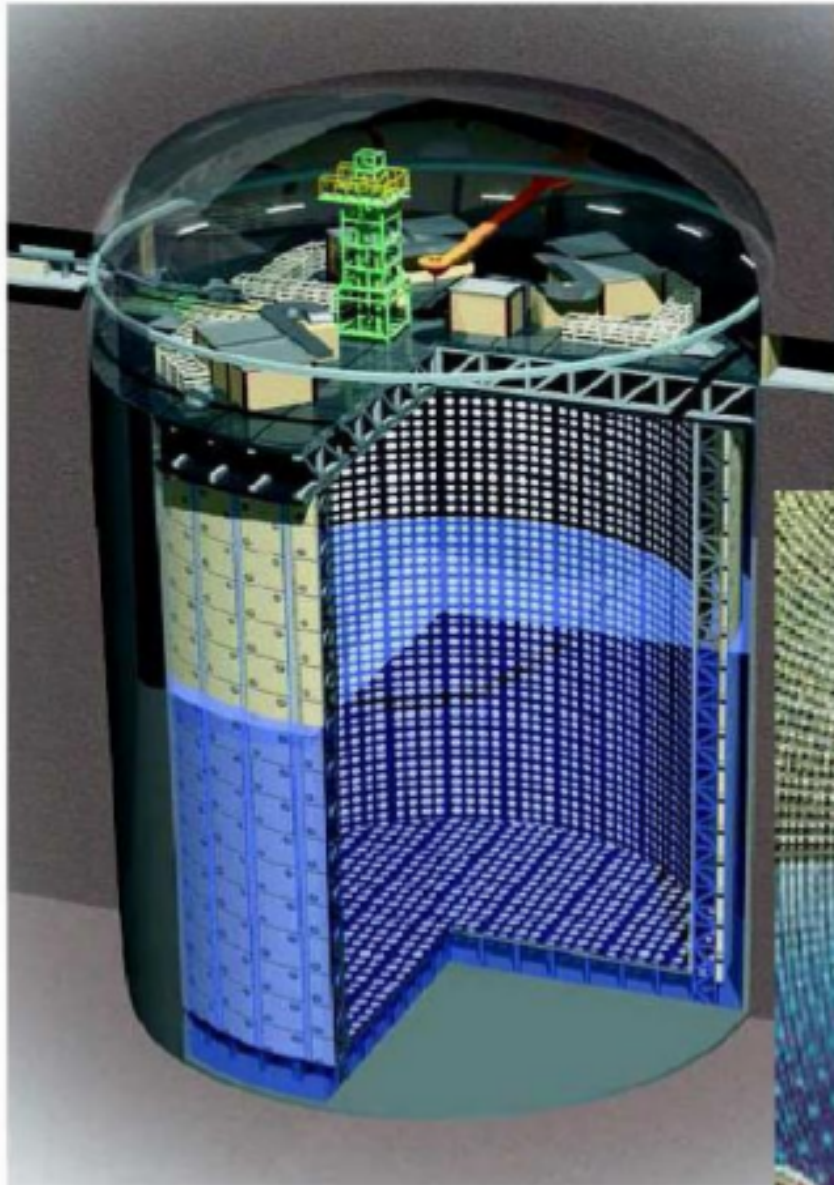
Experiments:

(Super)-Kamiokande  
IMB  
Soudan-2

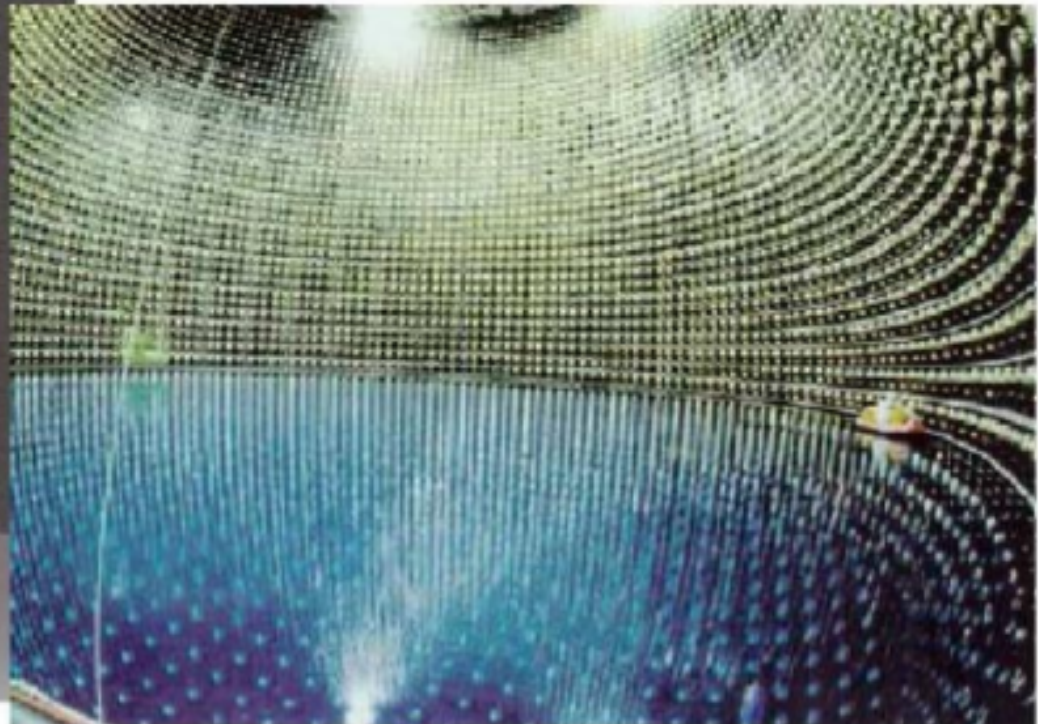
threshold:  $\sim 5$  MeV (to produce sufficiently high energetic electrons to produce Cherenkov light, no issue for atmospheric neutrinos, but for solar neutrinos)

Advantage: - scattered electron/muons carry direction information  
- in-situ detection  
- measurement of neutrino energy

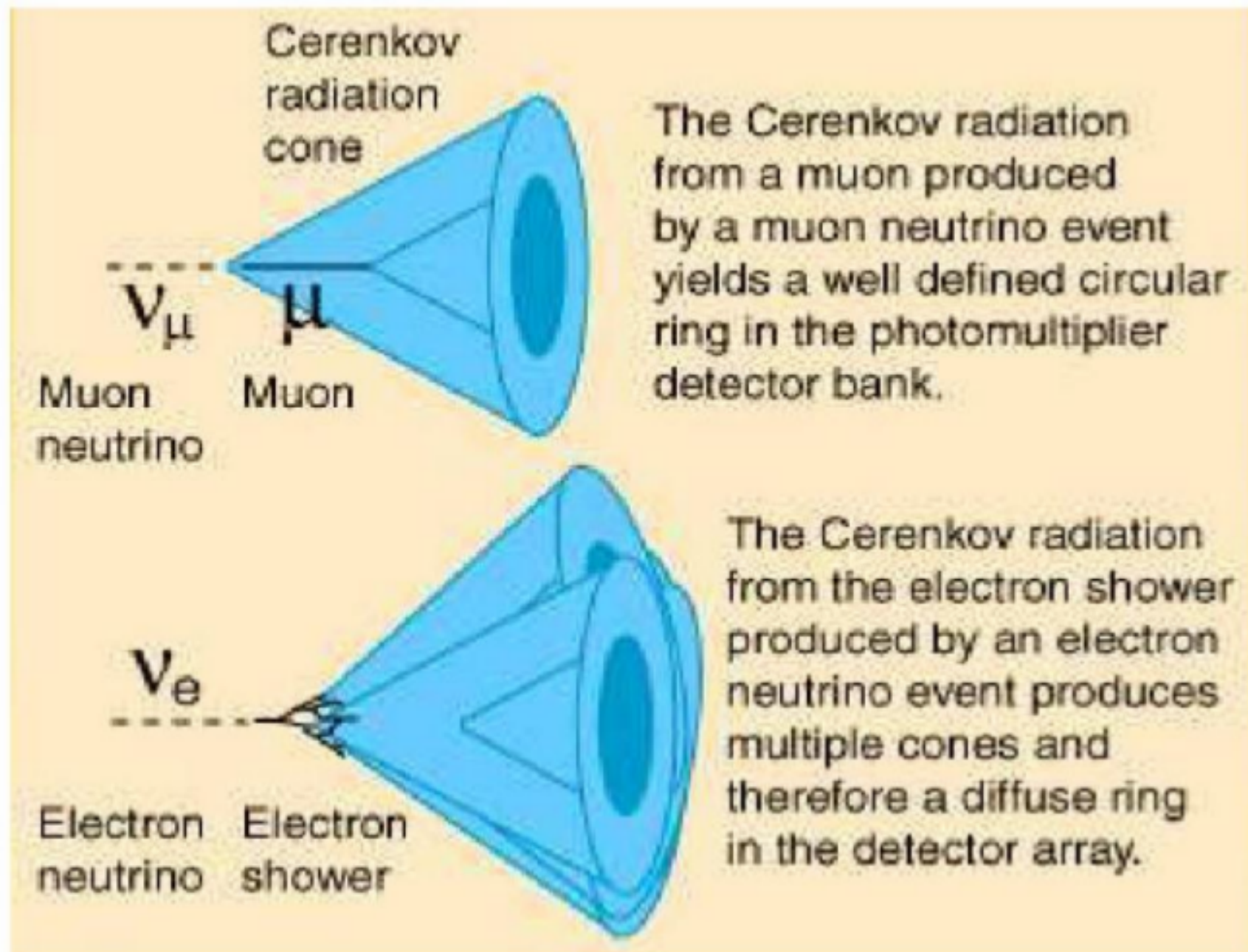
# (Super-)Kamiokande



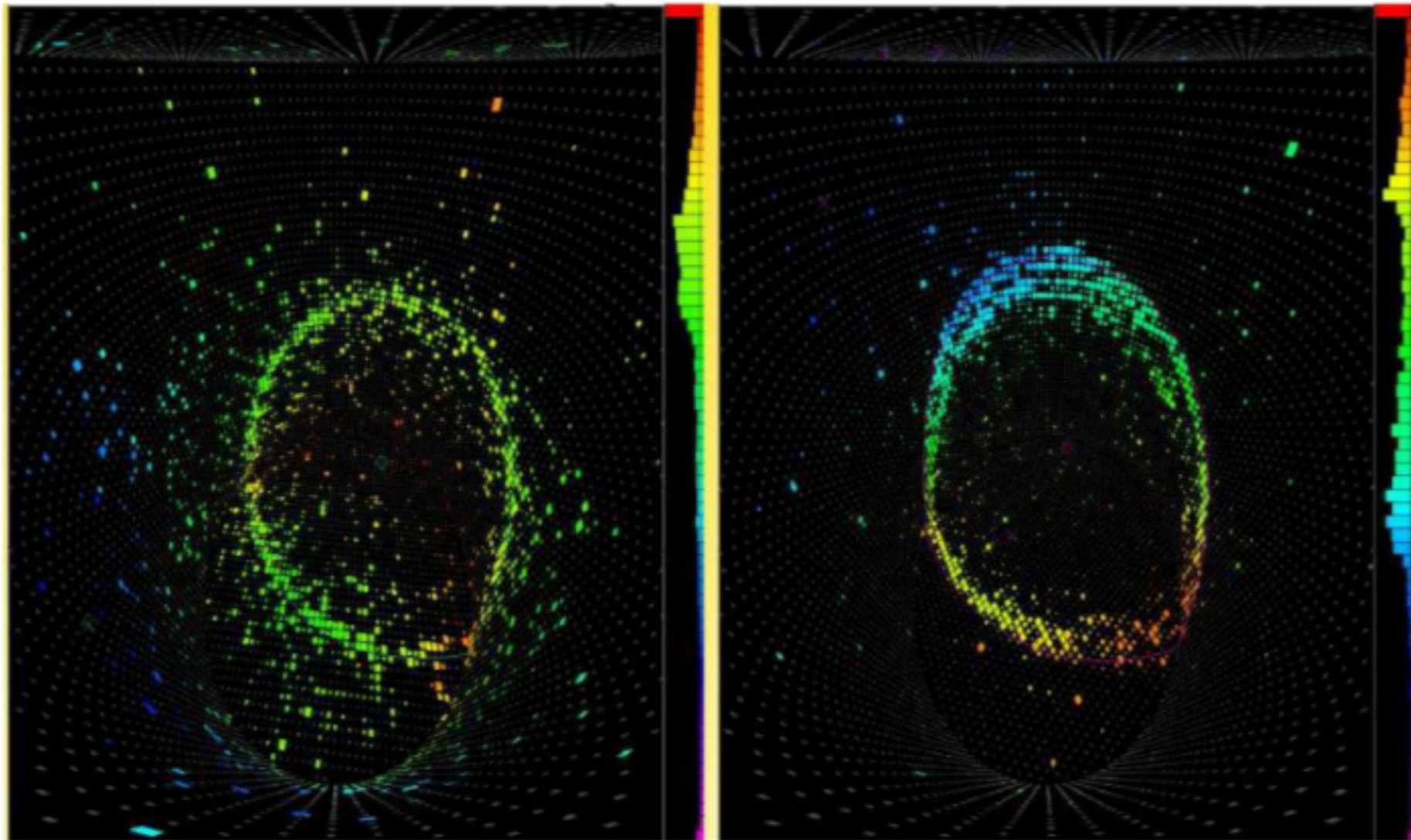
- Largest artificial water detector (50 kt)
- Until the 2001 accident:  
11000 PMTs (50 cm tubes!): 40% of  
surface covered with photo-cathode
- Back in operation since 2003



# Detection of Cherenkov-Light



$$\nu_e \leftrightarrow \nu_\mu$$



### Energy measurement:

Thickness of ring is related to travel distance of electron/muon in the detector.  
This is related to the energy of the electron/muon thus (with some uncertainties)  
to the original neutrino energy.

# Ratio of Muon to Electron Neutrinos

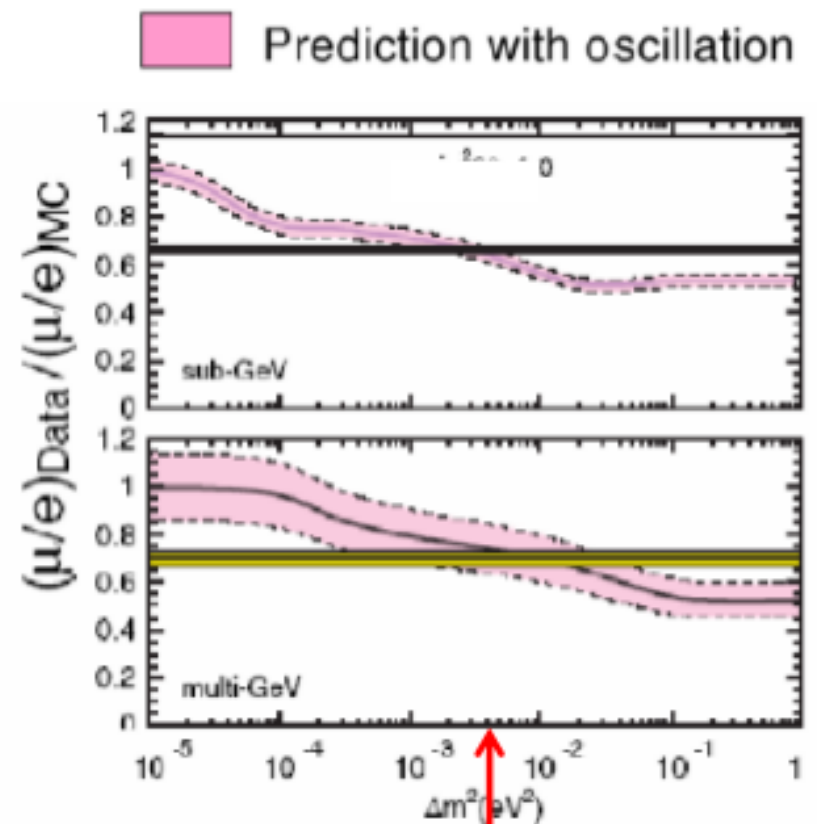
oscillation depend on E, thus split up sample in high and low momentum neutrinos

$$R_{\text{sub-GeV}} = 0.658 \pm 0.016(\text{stat}) \pm 0.032(\text{sys})$$

$$R = \frac{(\mu/e)_{\text{DATA}}}{(\mu/e)_{\text{M.C.}}}$$

$$R_{\text{multi-GeV}} = 0.702^{+0.032}_{-0.030}(\text{stat}) \pm 0.099(\text{sys})$$

assuming no oscillation  
in simulation



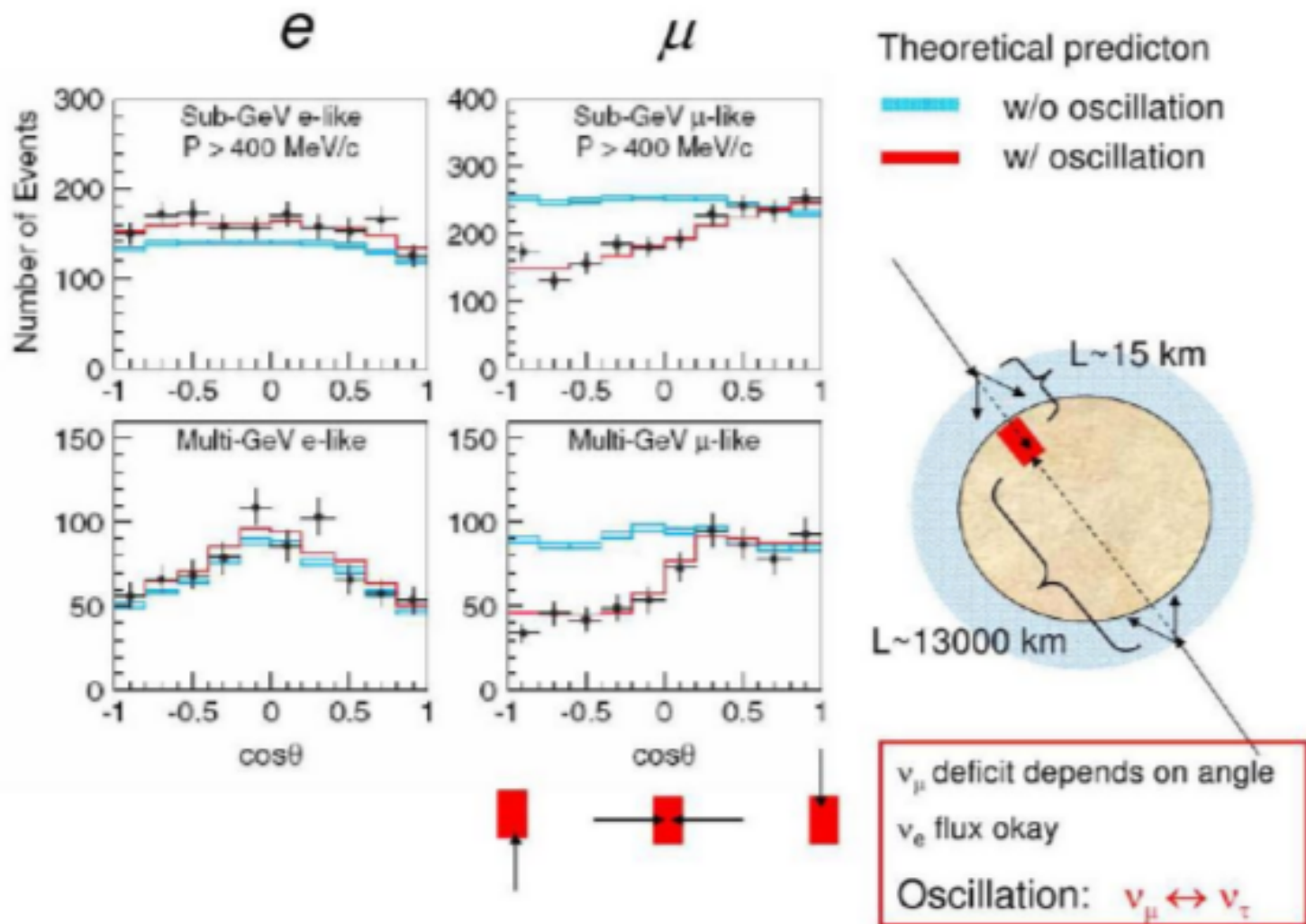
$\Delta m^2(\text{atmospheric}) \sim 2-5 \cdot 10^{-3}$

To few muon neutrinos observed  $\rightarrow$  can be explained by oscillation!

For atmospheric and accelerator neutrinos (E large, L medium):  $\frac{\Delta m_{12}^2 L}{E} \ll 1$

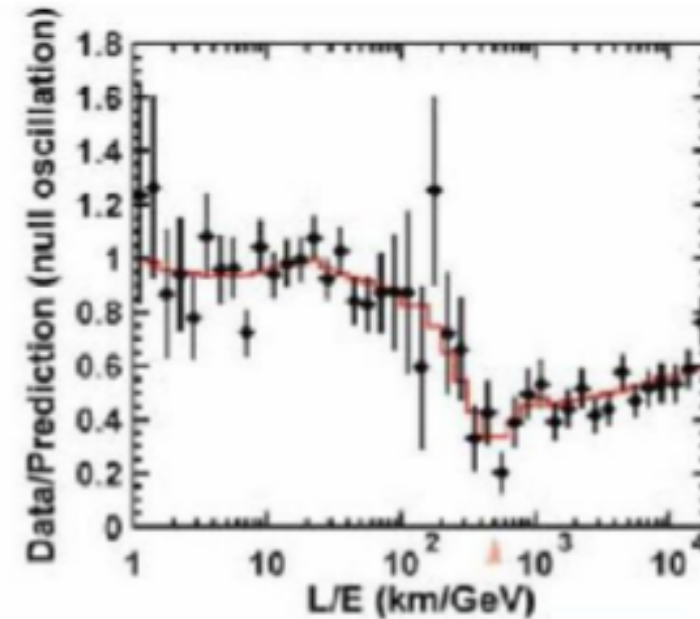
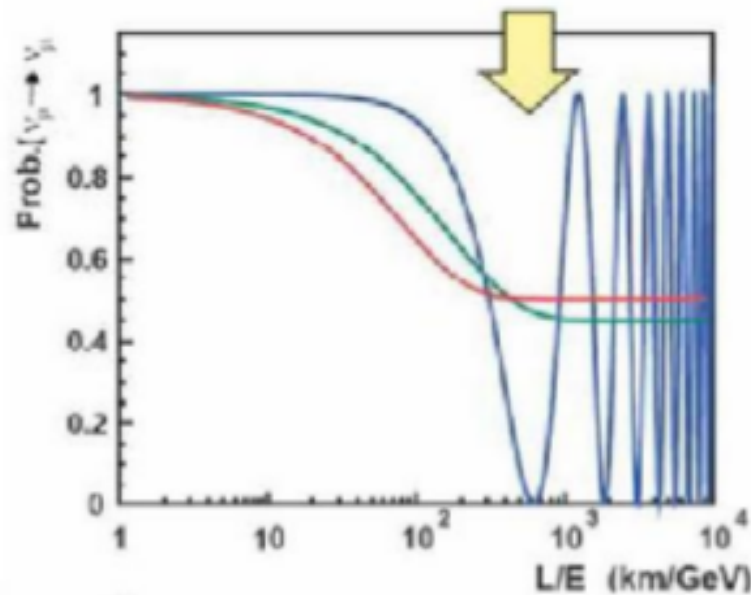
$$P(\nu_\mu \rightarrow \nu_\tau) = 1 - \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m_{23}^2 L}{4E}\right) \quad P(\nu_e \rightarrow \nu_e) \sim 1$$

# Zenith Angle Dependence of Neutrino Flux

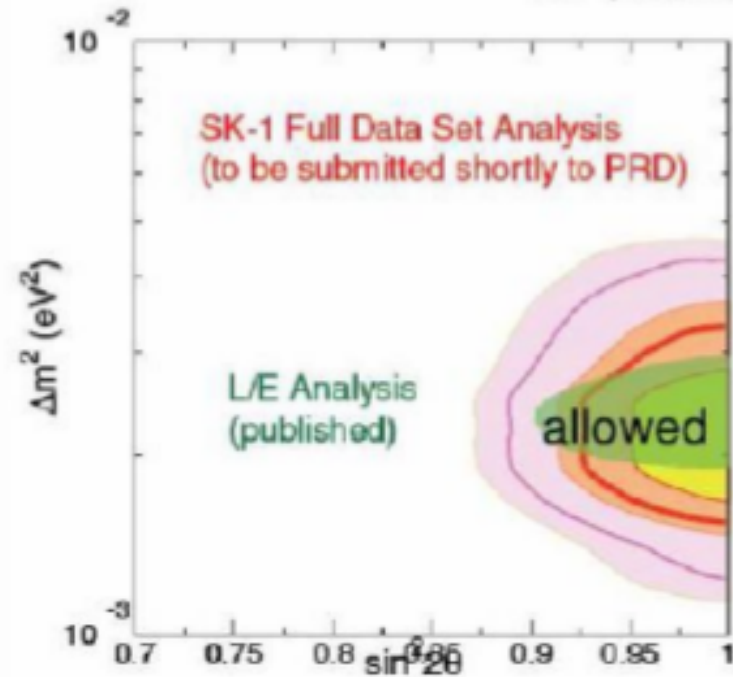


Electron neutrino seem to consist mainly of matter eigenstate, which has a significant longer oscillation distance both compared to Distance atmosphere earth and to diameter of the earth.

# Oscillating Pattern of Atmospheric Neutrinos



*oscillation dip seen  
at  $\sim 500$  km/GeV*

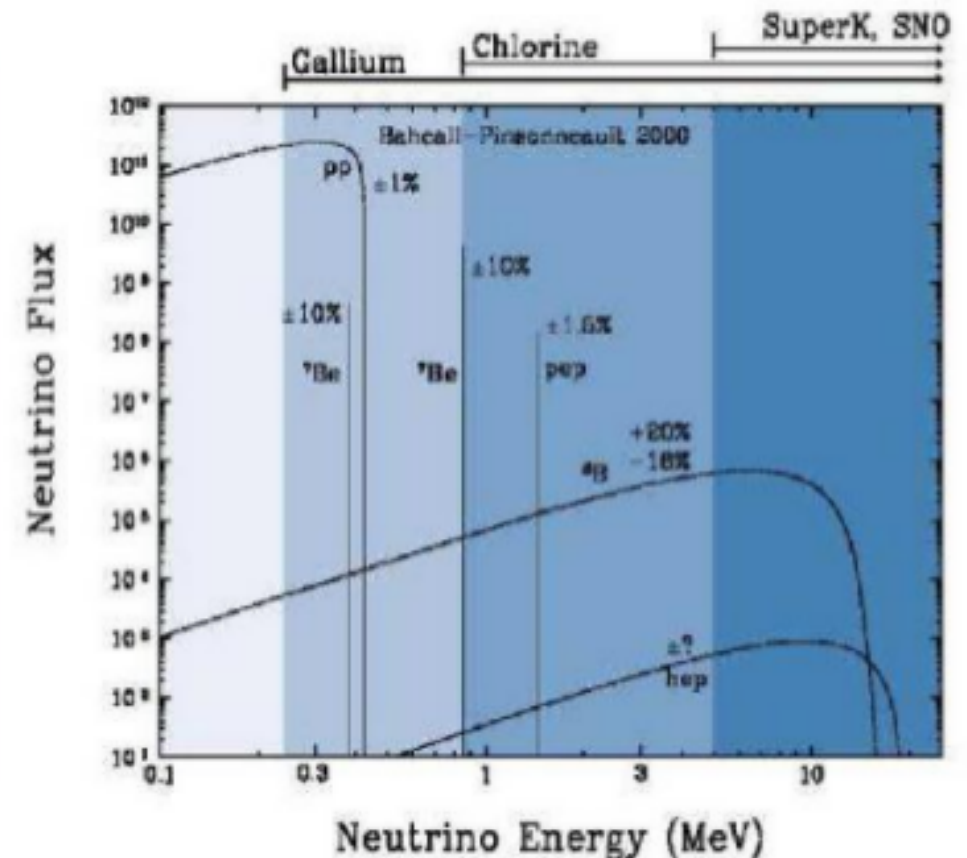
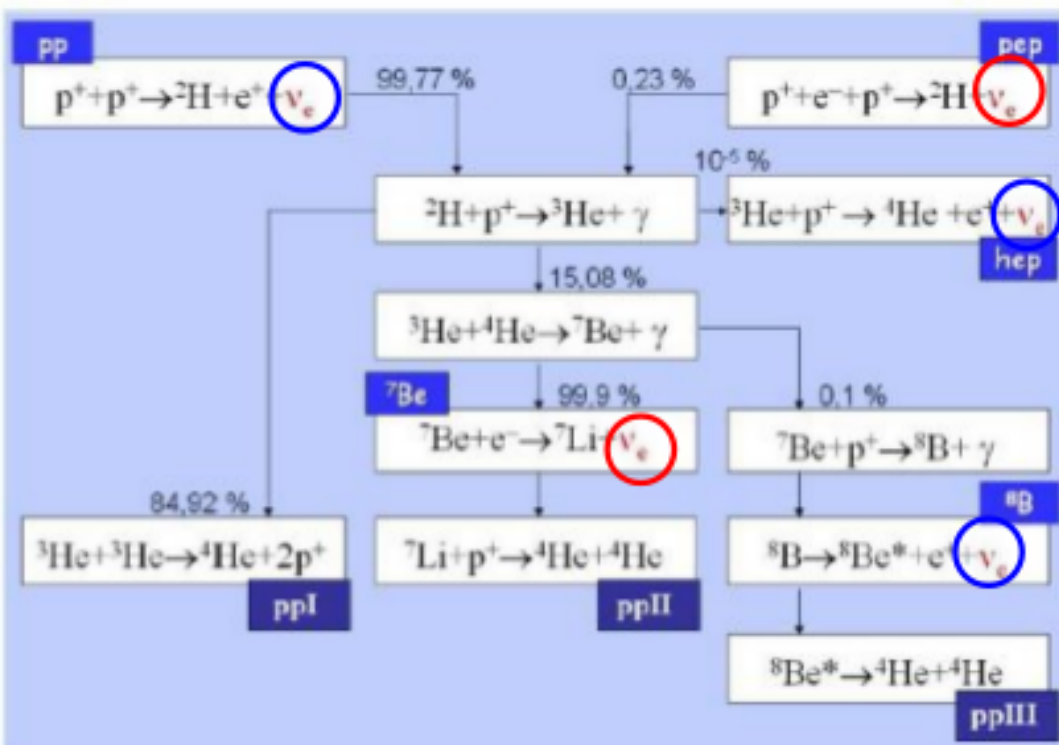


$\nu_\mu \leftrightarrow \nu_\tau$  mixing of atmos. neutrinos

$$\Delta m^2 = (2.4 \pm 0.4) \times 10^{-3} \text{ eV}^2$$

$$\sin^2 2\theta > 0.92 \text{ @ } 90\% \text{ C.L.}$$

# Solar Neutrinos



5 sources of solar neutrinos:

fixed energy from 2 body decay, continuous spectrum from multi-body decay

measured flux depends strongly on detection threshold

# Homestake Experiment

## Pioneering experiment, started in 1970:

Raymond Davids Jr. (NP 2002)

Homestake mine, 1400 m underground

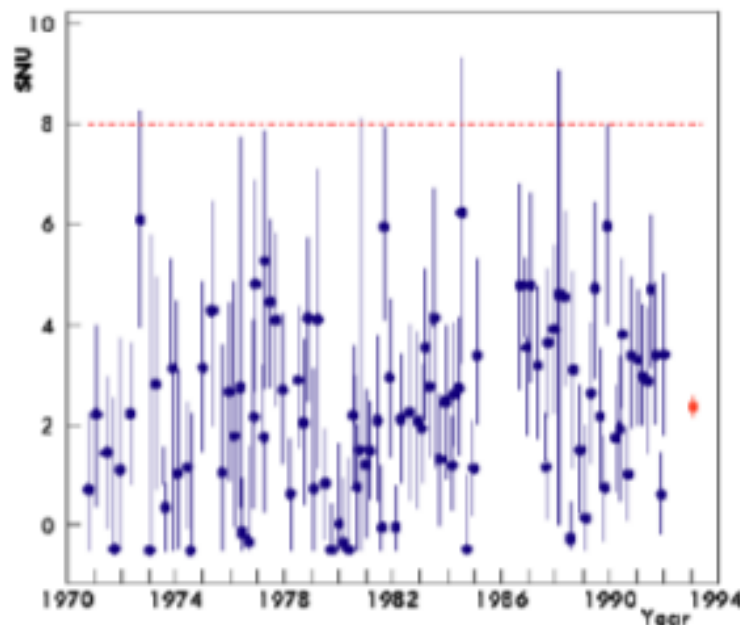


615 t of  $C_2CL_4 \sim 2.2 \times 10^{30}$  atoms of  $^{37}Cl$

Detection via:  $\nu_e + ^{37}Cl \rightarrow ^{37}Ar + e$ , threshold (0.8 MeV)

# Homestake Experiment

- after 60-70 days  $^{37}\text{Ar}$  is extracted by blowing helium gas over a thin layer of  $^{37}\text{Cl}$  ( $^{37}\text{Ar}$  mean decay time: 35 days)
- measured  $^{37}\text{Ar}$  decays in helium gas in proportional counters
- no “in-situ” measurement
- average count: one neutrino event every 2 days
- expected number of neutrino events: 1.6 per day

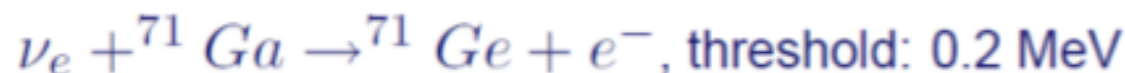


First indication of  
solar neutrino problem

→ initially results were ignored  
„maybe“ a mistake in the measurement method

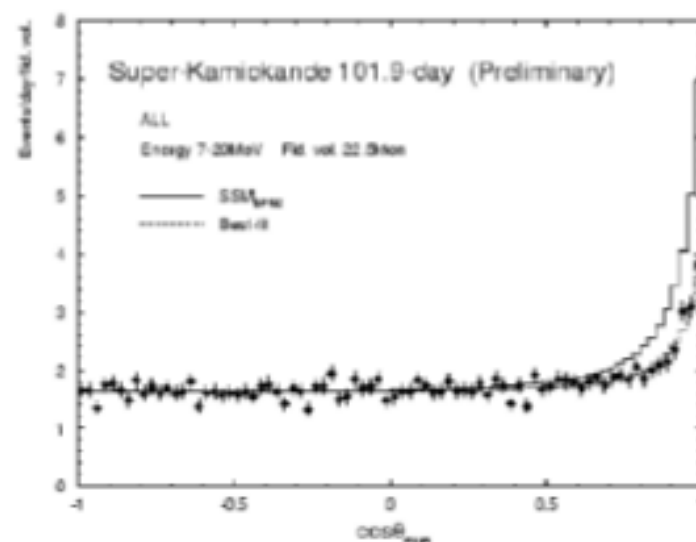
# Further Solar Neutrino Experiment

- GALLEX/GNO/SAGE, radiochemic experiments:



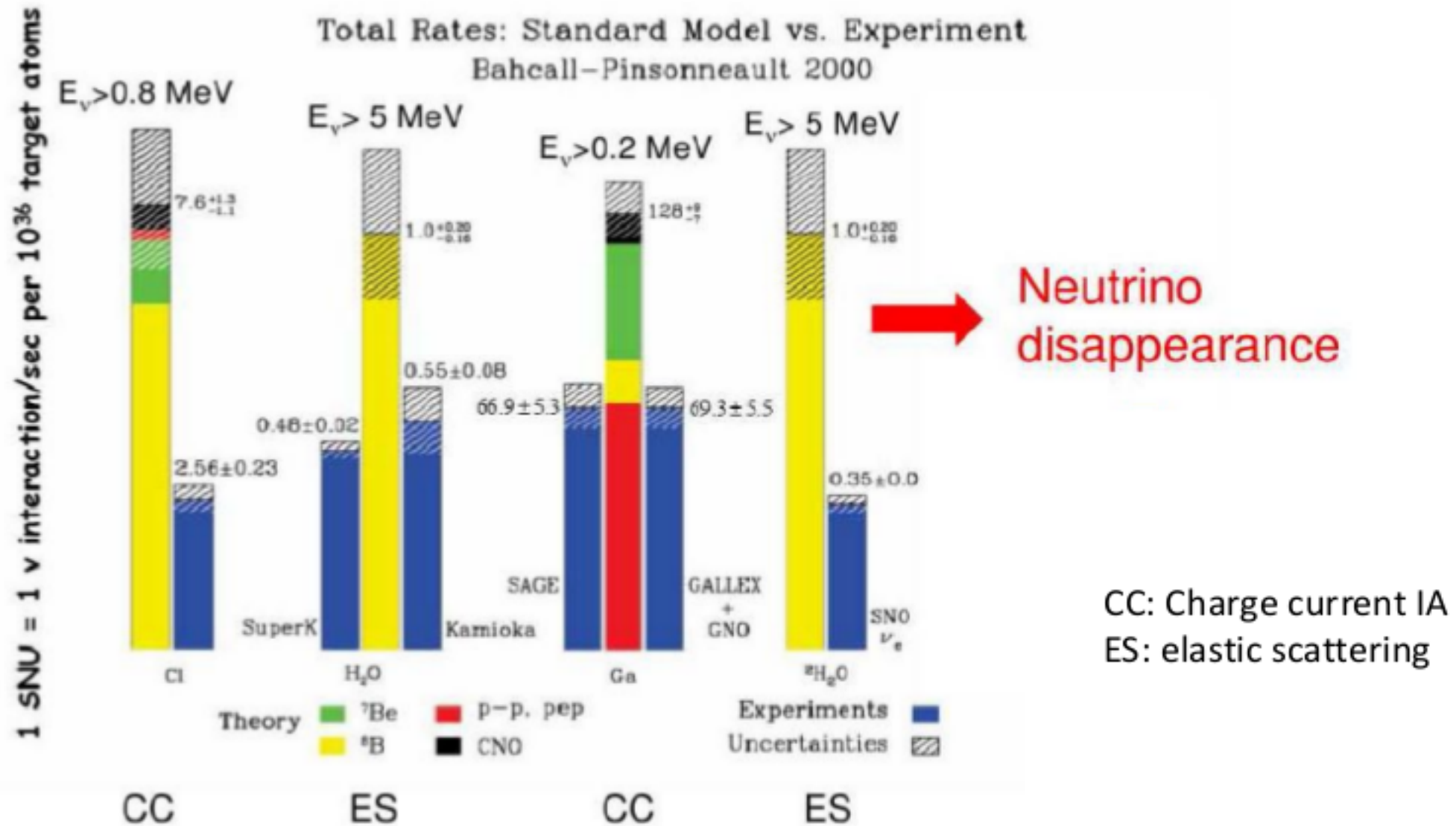
- delayed read out
- no information on neutrino energy
- no direction information

- (Super)Kammiokande:  $\nu_e + e^- \rightarrow \nu_e + e^-$



elastic scattering: direction of electron related to direction of initial neutrino, thus usefull to separate background from neutrinos from the sun.

# Solar Neutrino Problem



To establish neutrino mixing, need to measure additionally **appearance effects**!

# Nobel Prize in 2002

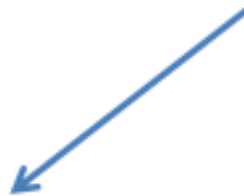
Homestake experiment



## The Nobel Prize in Physics 2002



spokesperson Kamiokande



**Raymond Davis Jr.**

**Masatoshi Koshiro**

**Riccardo Giacconi**

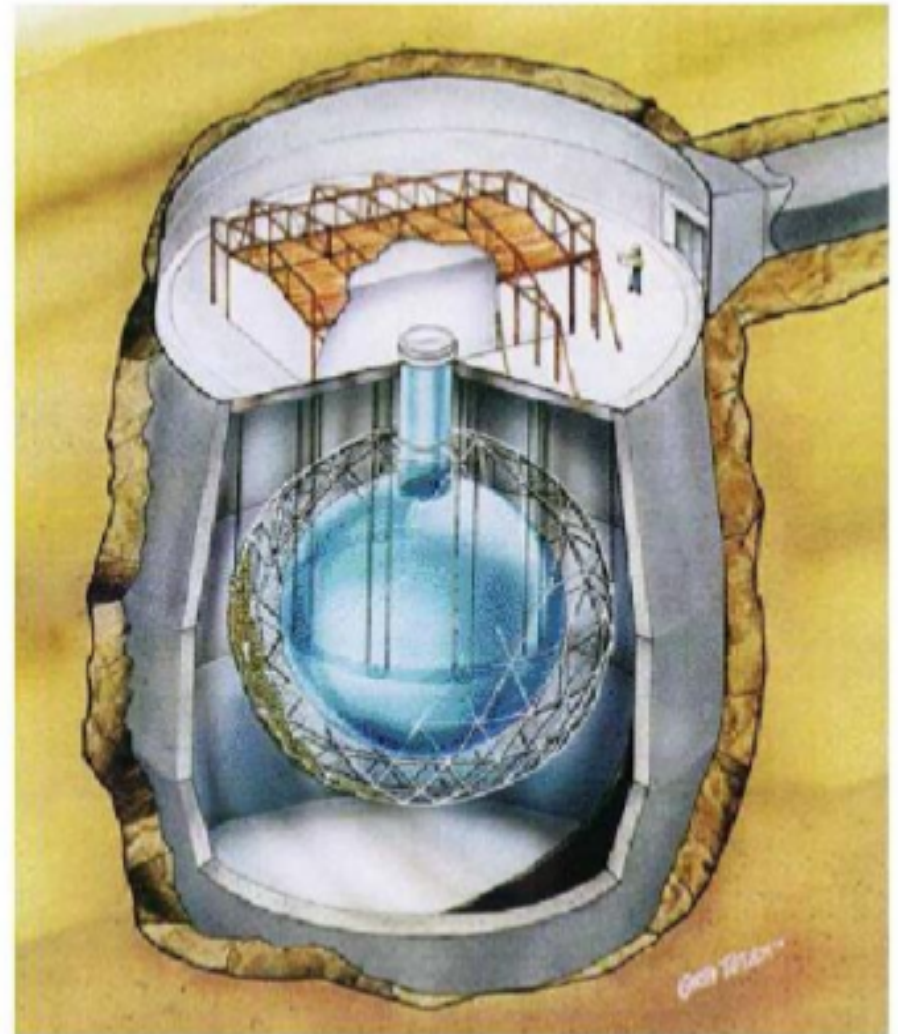
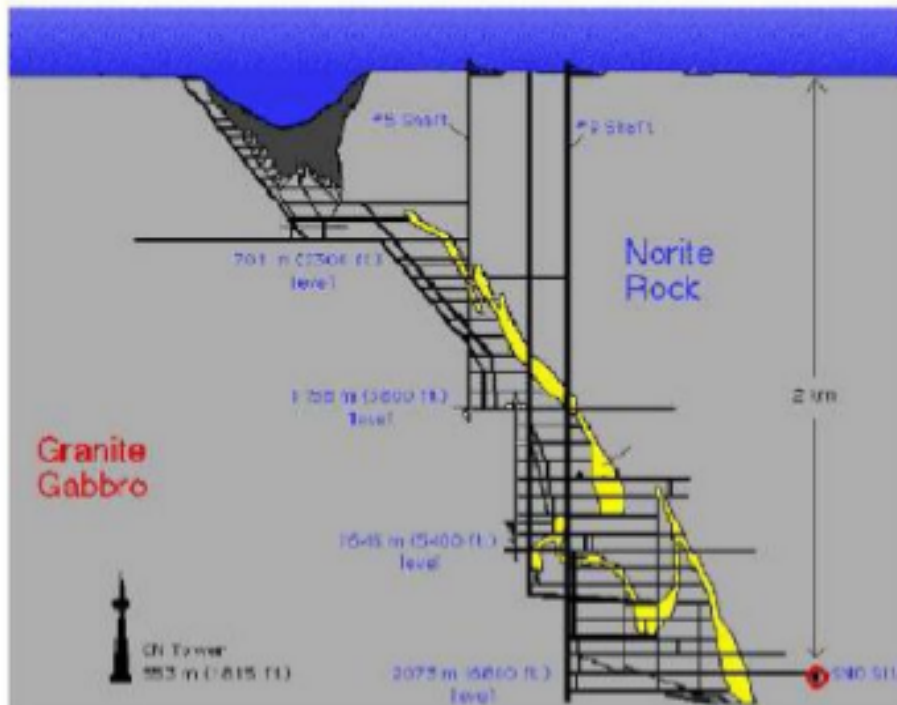
"for pioneering contributions to astrophysics, in particular for the detection of cosmic neutrinos"

"for pioneering contributions to astrophysics, which have led to the discovery of cosmic X-ray sources"

# SNO Experiment

## Sudbury Neutrino Observatory:

- 6 m radius transparent acrylic vessel
- 1000 t of heavy water ( $D_2O$ )
- 9456 inward looking photo multipliers
- Add 2 t of NaCl to detect neutrons

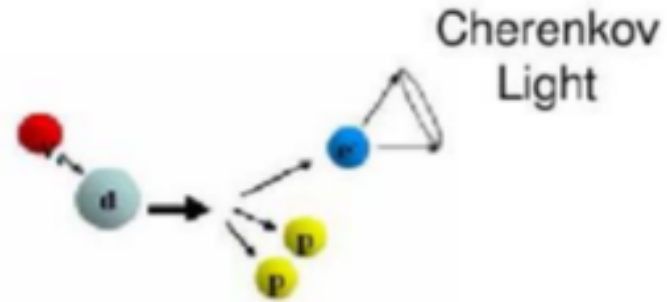
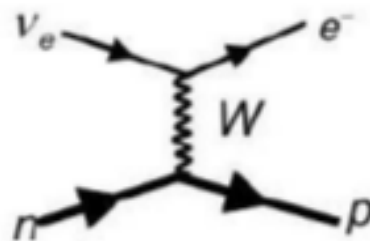


# SNO Experiment

## Charged current

$$\sigma(\nu_\mu) = \sigma(\nu_\tau) = 0$$

$$\phi_\nu = \phi_{\nu_e}$$

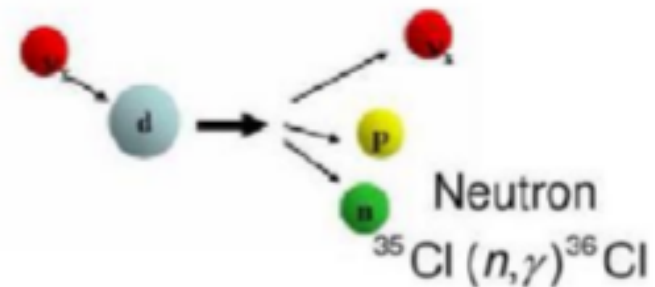
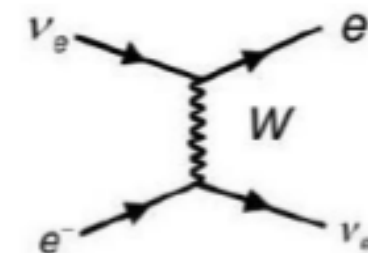
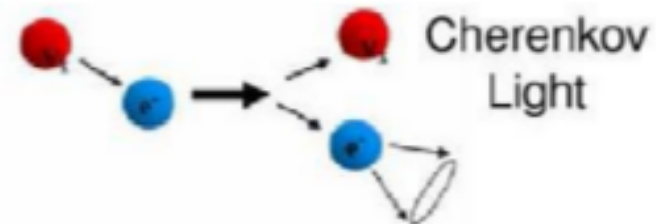
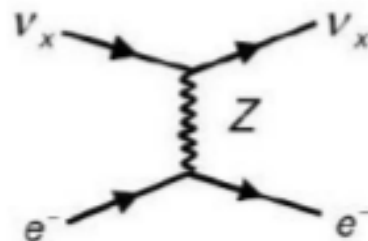


## Elastic scattering

$$0.154 \cdot \sigma(\nu_e) =$$

$$\sigma(\nu_\mu) = \sigma(\nu_\tau)$$

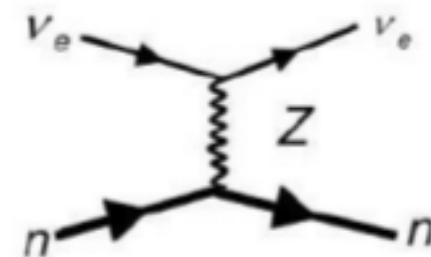
$$\phi_\nu = \phi_{\nu_e} + (\phi_{\nu_\mu} + \phi_{\nu_\tau})/6$$



## Neutral current

$$\sigma(\nu_e) = \sigma(\nu_\mu) = \sigma(\nu_\tau)$$

$$\phi_\nu = \phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau}$$



# SNO Experiment

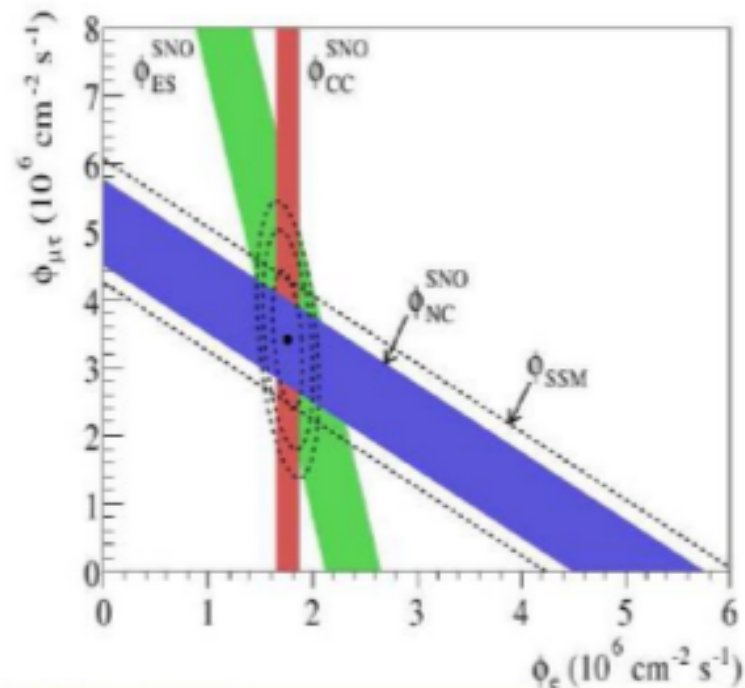
$$\phi_{CC} < \phi_{ES} < \phi_{NC}$$

appearance of  $\mu/\tau$  neutrinos detected!

$$\phi_{CC}^{SNO} = 1.76^{+0.06}_{-0.05} (stat)^{+0.09}_{0.09} (syst)$$

$$\phi_{ES}^{SNO} = 2.39^{+0.24}_{-0.23} (stat)^{+0.12}_{0.12} (syst)$$

$$\phi_{NC}^{SNO} = 5.09^{+0.44}_{-0.43} (stat)^{+0.46}_{0.43} (syst)$$

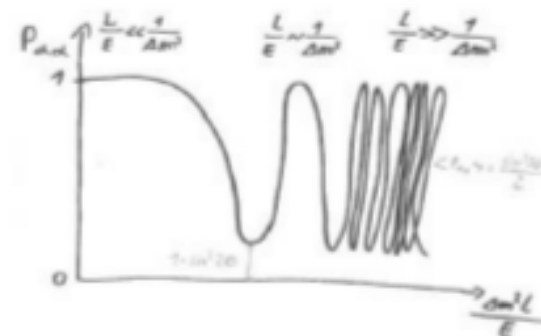


Electron neutrino flux is too low:  $P_{\nu_e \rightarrow \nu_e} = (35 \pm 2)\%$

Total flux of neutrinos is correct  $\rightarrow$  Interpreted as  $\nu_e \leftrightarrow \nu_\mu$  or  $\nu_e \leftrightarrow \nu_\tau$  oscillation

But in case of simple "vacuum oscillation":

$$P_{\nu_e \rightarrow \nu_e} = 1 - \frac{1}{2} \sin^2 2\theta = 50\%$$



# V-A Coupling for Quarks

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} u & d & s & b \\ \text{red square} & \text{red square} & \text{red square} & \text{red square} \\ c & \text{red square} & \text{red square} & \text{red square} \\ t & \text{red square} & \text{red square} & \text{red square} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Most preferred transitions with in the same quark family.

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.975 & 0.225 & 0.00364 \\ 0.225 & 0.975 & 0.0405 \\ 0.00820 & 0.0405 & 1 \end{pmatrix}$$

In first order only  $V_{td}$  and  $V_{ub}$  are complex numbers.

CKM matrix and their complex elements important for meson mixing and CP violation

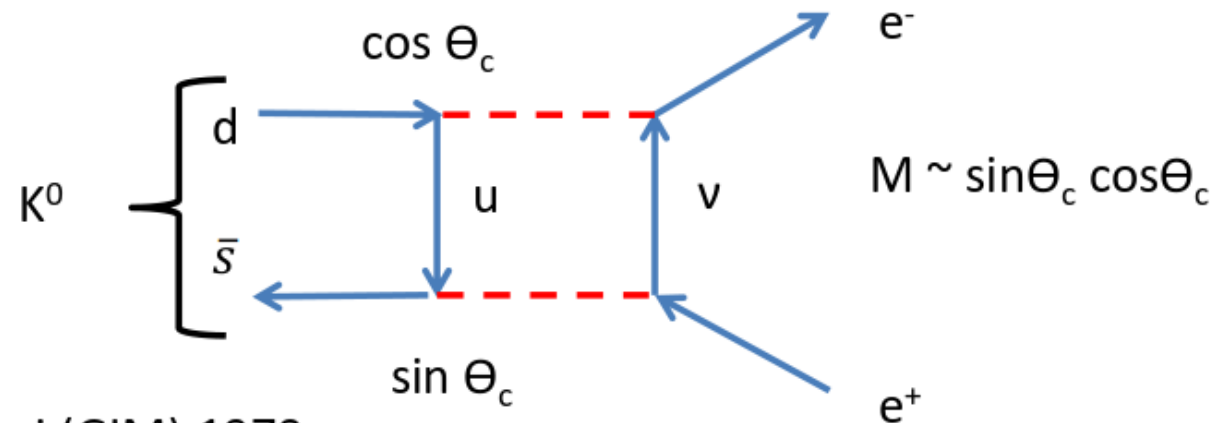
# Missing FCNC GIM mechanism

In early 70s only 3 quarks were known (u,d,s)

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ \cos\Theta_c d + \sin\Theta_c s \end{pmatrix}$$

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\Theta_c & \sin\Theta_c \\ -\sin\Theta_c & \cos\Theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

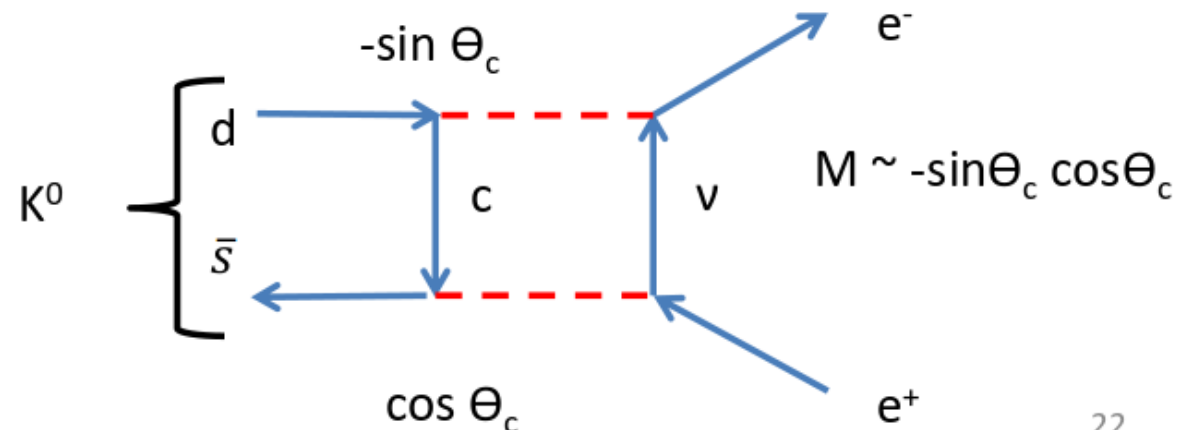
$$\frac{BR(K \rightarrow \mu\mu)}{BR(K \rightarrow \text{all})} = 7.2 \pm 0.5 \cdot 10^{-9}$$



Proposal by Glashow, Iliopoulos, Miani (GIM) 1970:

➡ There is a fourth quark which build together with the s quark a second doublet

$$\begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ -\sin\Theta_c d + \cos\Theta_c s \end{pmatrix}$$



➡ Prediction of a fourth quark!

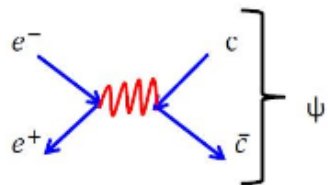
# Discovery of J/ψ Resonance

- SLAC-Mark-I experiment (see tau discovery)  
group leader Burton Richter

$$e^+ + e^- \rightarrow \Psi$$

- ↳ multi hadrons
- ↳  $e^+e^-$
- ↳  $\mu^+\mu^- + K^+K^- + \pi^+\pi^-$

$$m(\psi) = 3.105 \pm 0.003 \text{ GeV}, \quad \Gamma(\psi) < 1.3 \text{ MeV}$$



performed scan of  $E_{\text{CMS}}(e^+ + e^-)$

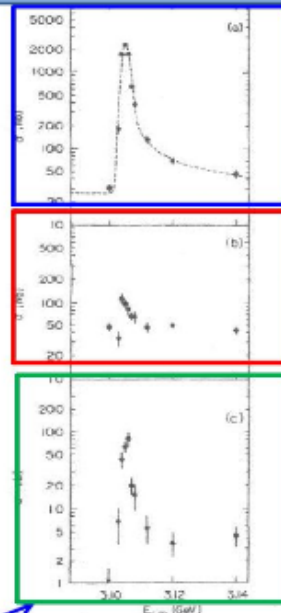


FIG. 1. Cross section versus energy for (a) multi-hadron final states, (b)  $e^+e^-$  final states, and (c)  $\mu^+\mu^-$ ,  $K^+K^-$ , and  $\pi^+\pi^-$  final states. The curve in (a) is the total cross section of  $e^+e^-$  annihilation into hadrons.

- BNL (Berkly National Laboratory): proton beam on fix target (Berilium)  
group leader Samuel Ting

$$p (28 \text{ GeV}) + \text{Be} \rightarrow J + X$$

$$\searrow e^+e^-$$

$$m(J) = 3.1 \text{ GeV}$$

$\Gamma$ : consistent with detector resolution (20 MeV)



丁肇中簡介

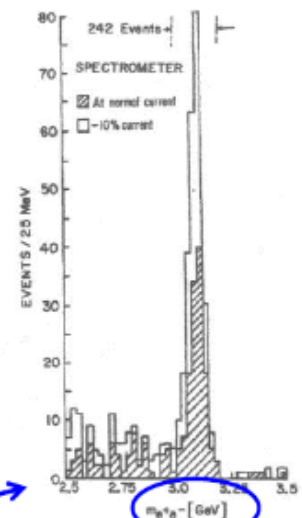


FIG. 1. Mass spectrum showing the existence of  $J/\psi$ . Results from two spectrometer settings are plotted showing that the peak is independent of spectrometer currents. The run at reduced current was taken two months later than the normal run.

**Direct discovery  
of charm quark 5  
years later than  
indirect  
prediction**

reconstructed invariant mass of final state particles

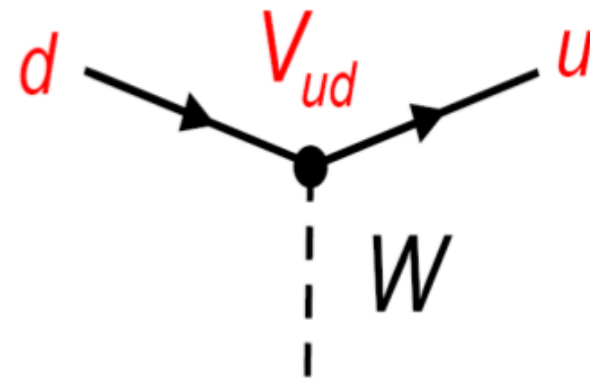
# CKM Matrix

CKM matrix is consequence of introduction of Yukawa term to Lagrangian:

Charged currents:  $J_\mu^+ \propto (\bar{u}, \bar{c}, \bar{t}) (1 - \gamma_5) \gamma_\mu V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \times \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

flavour                      CKM matrix                      mass



18 parameters (9 complex elements)

-5 relative quark phases (unobservable)

-9 unitarity conditions

---

= 4 independent parameters 3 Euler angles and 1 Phase

Phase is only source of CPV in SM, requires third quark family (Nobel Prize 2008)

# 5 relative phases

Charged currents:  $J_\mu^+ \propto (\bar{u}, \bar{c}, \bar{t}) (1 - \gamma_5) \gamma_\mu V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$

Lagrangian insensitive to phases of left-handed fields, possible redefinition:

$$u_L \rightarrow e^{i\phi_u} u_L \quad c_L \rightarrow e^{i\phi_c} c_L \quad t_L \rightarrow e^{i\phi_t} t_L$$

$$d_L \rightarrow e^{i\phi_d} d_L \quad s_L \rightarrow e^{i\phi_s} s_L \quad b_L \rightarrow e^{i\phi_b} b_L$$

$$V_{CKM} \rightarrow \begin{pmatrix} e^{i\phi_u} & 0 & 0 \\ 0 & e^{i\phi_c} & 0 \\ 0 & 0 & e^{i\phi_t} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{-i\phi_d} & 0 & 0 \\ 0 & e^{-i\phi_s} & 0 \\ 0 & 0 & e^{-i\phi_b} \end{pmatrix}$$

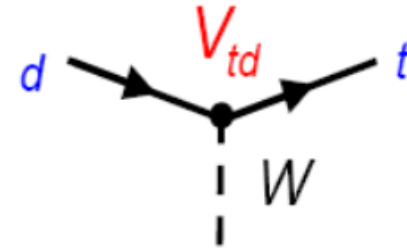
or  $V_{\alpha\beta} \rightarrow e^{\phi_\beta - \phi_\alpha} V_{\alpha\beta}$

5 unobservable phase differences  $\phi_\beta - \phi_\alpha$ .

# CKM under $CP$ Transformation

Quarks

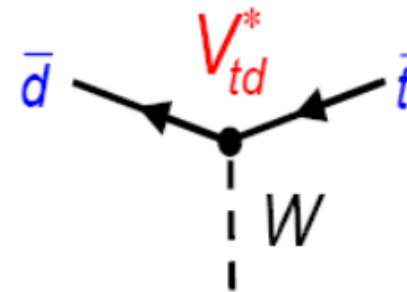
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



-----  $CP$  -----

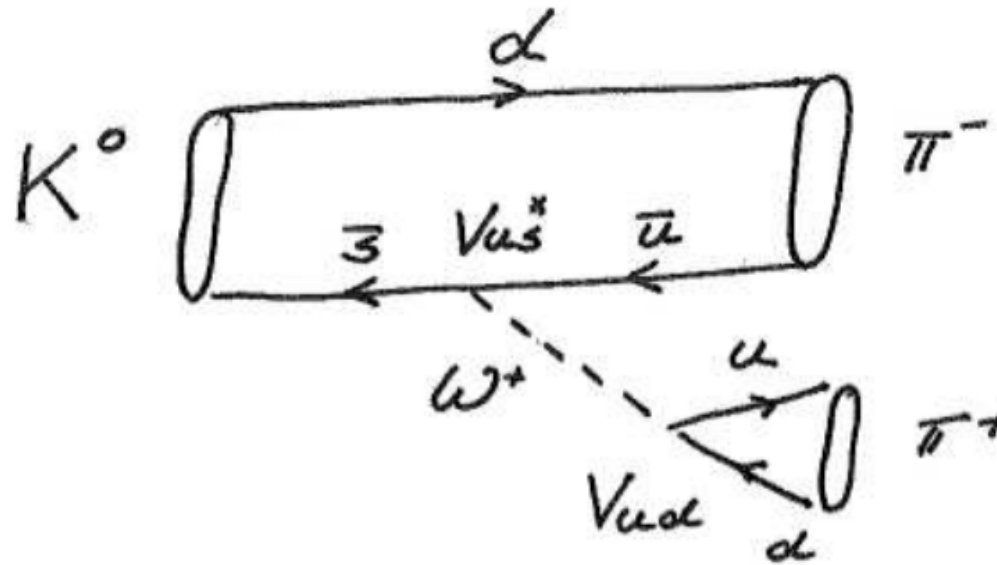
Anti-quarks:

$$\begin{pmatrix} \bar{d}' \\ \bar{s}' \\ \bar{b}' \end{pmatrix} = \begin{pmatrix} V_{ud}^* & V_{us}^* & V_{ub}^* \\ V_{cd}^* & V_{cs}^* & V_{cb}^* \\ V_{td}^* & V_{ts}^* & V_{tb}^* \end{pmatrix} \begin{pmatrix} \bar{d} \\ \bar{s} \\ \bar{b} \end{pmatrix}$$



Weak (CKM) phases change sign under  $CP$  transformation!

# Weak and Strong Phases



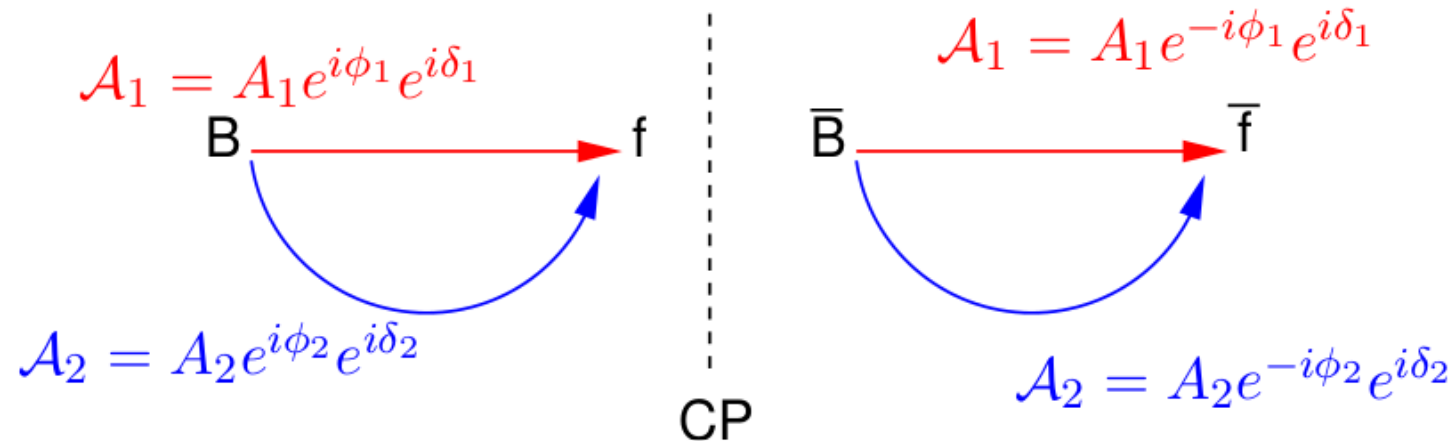
Weak phases are related to involved CKM elements:  $\phi_{weak} = \arg(V_{us}^* V_{ud})$

Strong phases  $\delta$  comes often (but not always) from the hadronisation.

Definition of strong phase:

*phase which doesn't change sign under CP transformation.*

# CP Violation



$$|\mathcal{A}|^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos(\Delta\phi + \Delta\delta)$$

$$|\mathcal{A}|^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos(-\Delta\phi + \Delta\delta)$$

$\mathcal{A}_1$  and  $\mathcal{A}_2$  need to have **different weak phases  $\phi$**  and **different strong phases  $\delta$** .

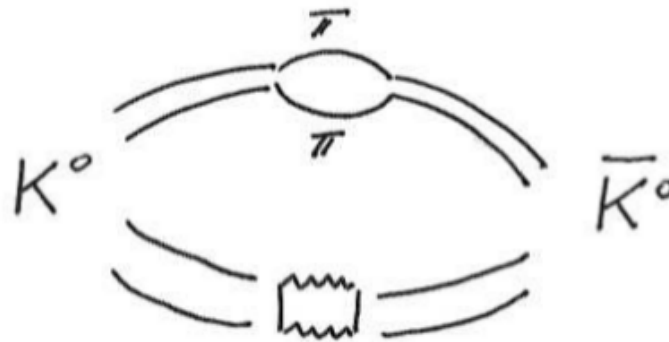
For sizable (measurable) effects both amplitudes should have about same size, and both phase differences have to be sizable.

To conclude on weak phases, strong phases need to be known/measured.

# CPV in Kaon System

Interfering amplitudes which cause CPV in mixing:

long range contribution  $\Delta\Gamma$



short range contribution  $\Delta m$

Interfering amplitudes which cause CPV in decay:

