Mixing Probabilities

★ **Hence the two-flavour oscillation probability is:**

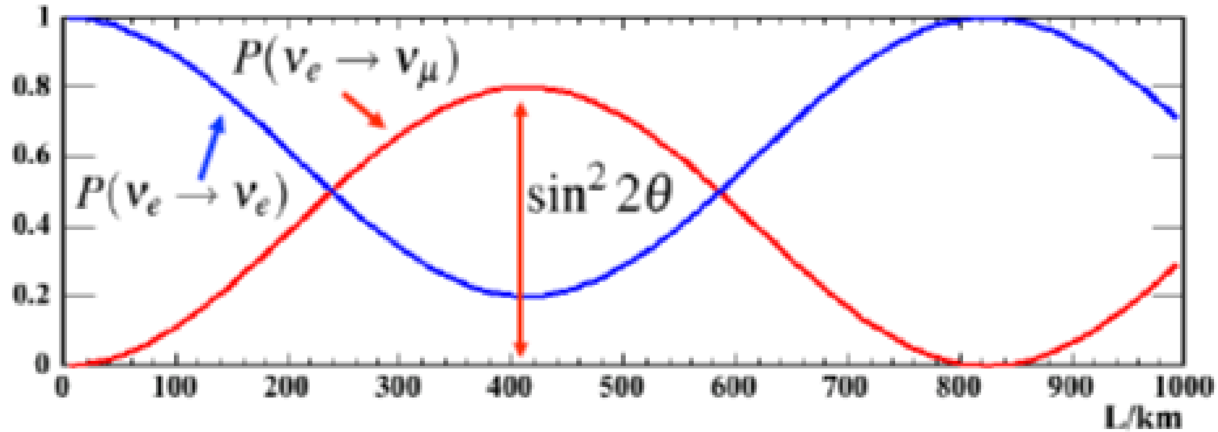
$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

with $\Delta m_{21}^2 = m_2^2 - m_1^2$

★ **The corresponding two-flavour survival probability is:**

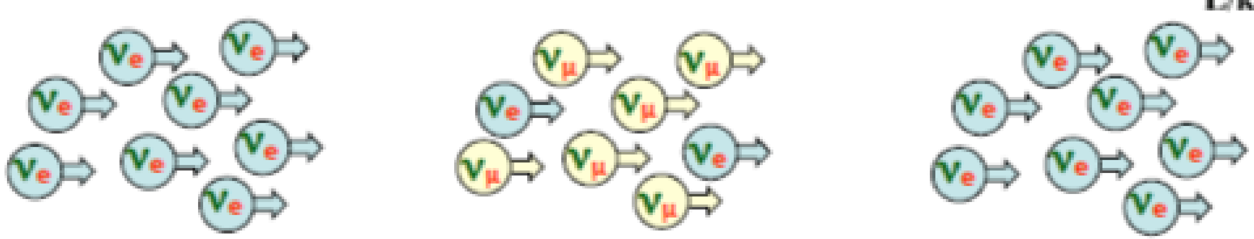
$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

- e.g. $\Delta m^2 = 0.003\,\mathrm{eV}^2, \quad \sin^2 2\theta = 0.8, \quad E_\nu = 1\,\mathrm{GeV}$



- wavelength

$$\lambda_{osc} = \frac{4\pi E}{\Delta m^2}$$

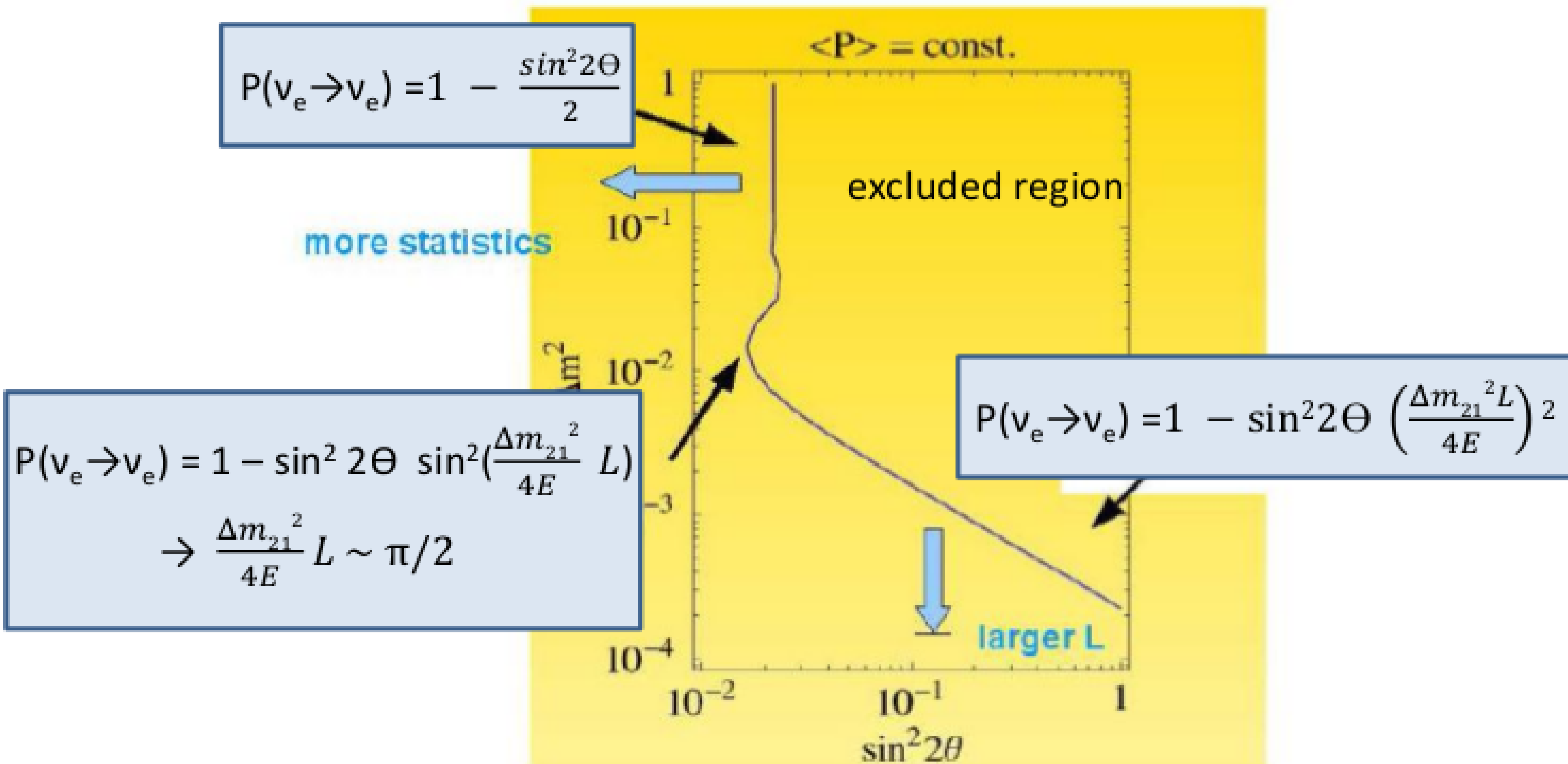


Sensitivity to Mixing

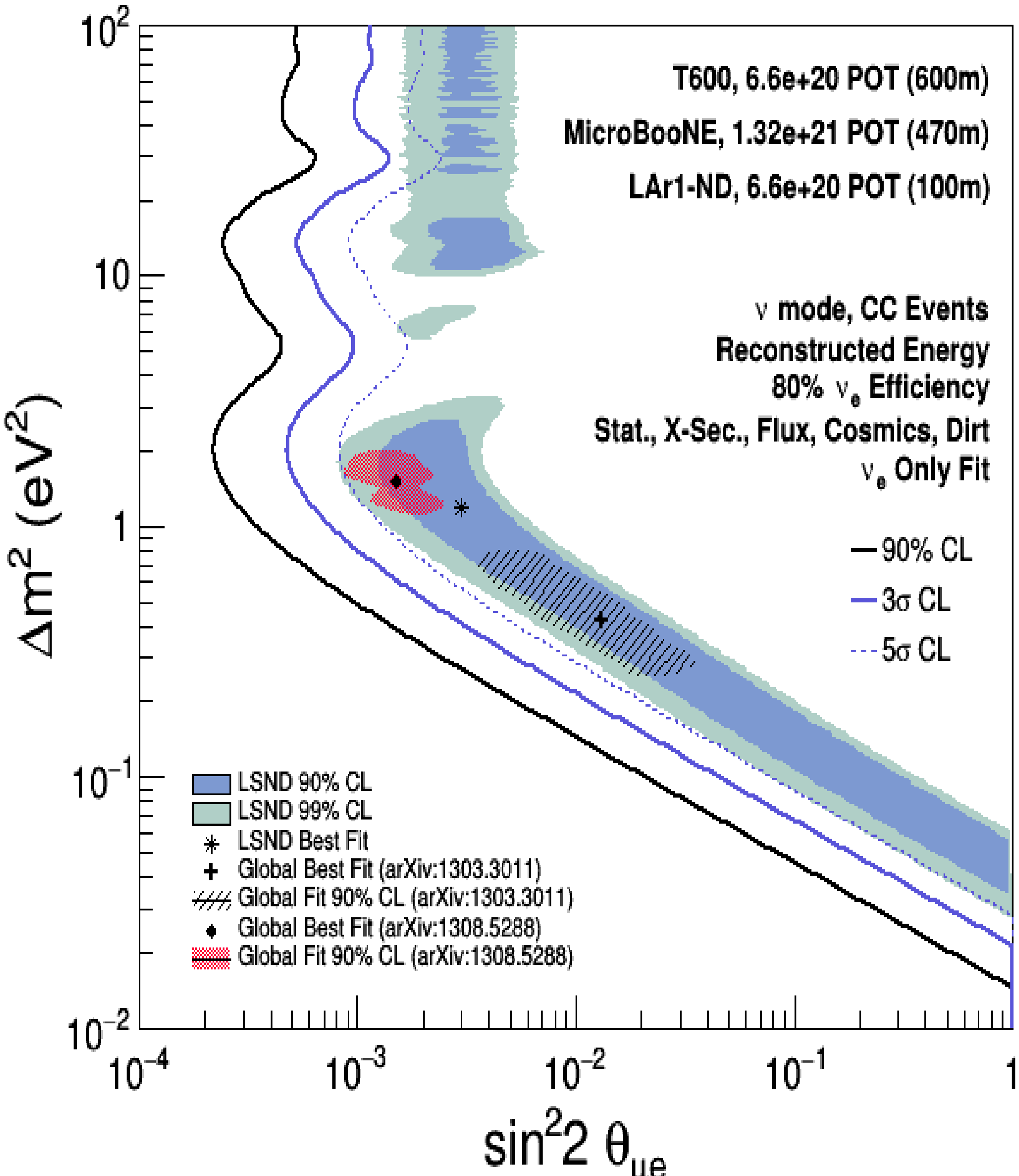
$L_{detector} >> L_{osc}$ fast oscillation $P(\nu_e \rightarrow \nu_e) = 1 - \frac{\sin^2 2\Theta}{2}$

$L_{detector} << L_{osc}$ hardly oscillate before measurement: $P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\Theta \left(\frac{\Delta m_{21}^2 L}{4E}\right)^2$

In case of no mixing observed, we can make statements like $P(\nu_e \rightarrow \nu_e) > 0.9$ @ 90% CL



Just one example of exclusion plot for reactor neutrinos …... note LSND claimed to have found oscillation signal in 2003, in a region whichi is excluded by other experiments (blue/gray area), for example MicroBooNE in this plot.



Neutrino Mass Hierachy

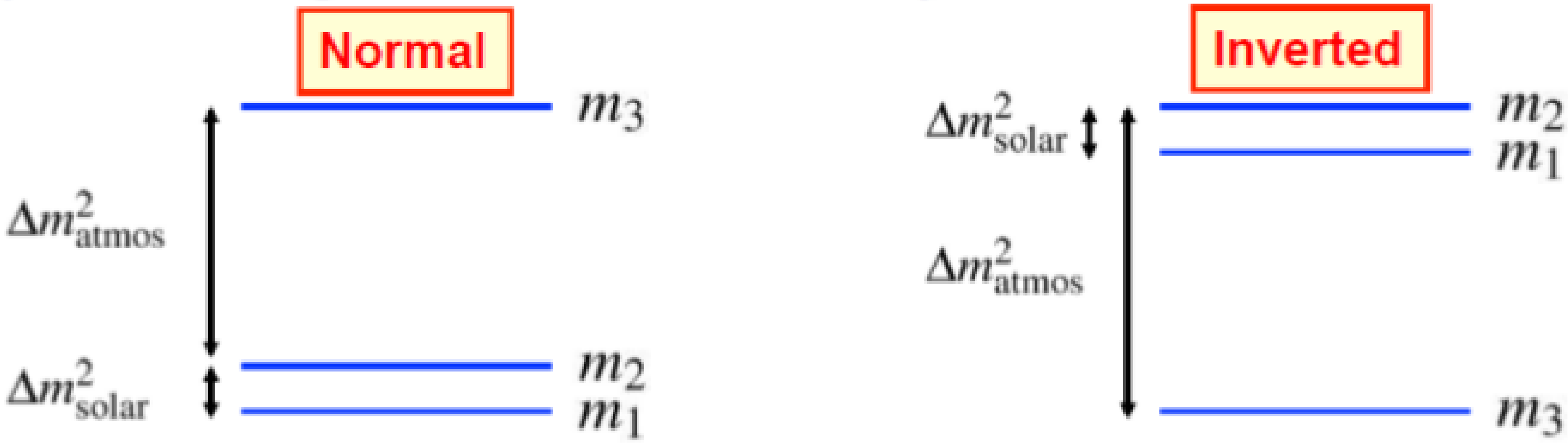
★ To date, results on neutrino oscillations only determine

$$|\Delta m^2_{ji}| = |m^2_j - m^2_i|$$

★ Two distinct and very different mass scales:

- Atmospheric neutrino oscillations : $|\Delta m^2|_{\mathrm{atmos}} \sim 2.5 \times 10^{-3}\,\mathrm{eV}^2$
- Solar neutrino oscillations: $|\Delta m^2|_{\mathrm{solar}} \sim 8 \times 10^{-5}\,\mathrm{eV}^2$

• Two possible assignments of mass hierarchy:



• In both cases: $\Delta m^2_{21} \sim 8 \times 10^{-5}\,\mathrm{eV}^2$ (solar)

$|\Delta m^2_{31}| \approx |\Delta m^2_{32}| \sim 2.5 \times 10^{-3}\,\mathrm{eV}^2$ (atmospheric)

• Hence we can approximate $\Delta m^2_{31} \approx \Delta m^2_{32}$

PNMS Matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \qquad U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13}e^{-i\delta} \\ & 1 & \\ -s_{13}e^{i\delta} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}$$

With $\Delta m_{12} << \Delta m_{23} \sim \Delta m_{13}$ and Θ_{13} small

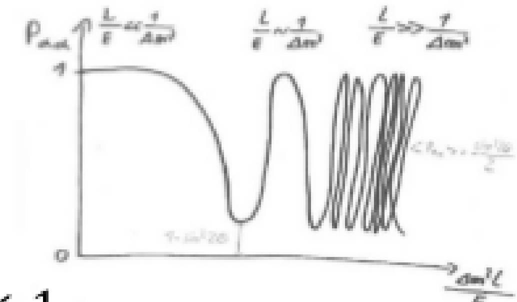
For atmospheric and accelerator neutrions (E large, L medium): $\frac{\Delta m_{12}^2 L}{E} \ll 1$

$$P(\nu_\mu \rightarrow \nu_\tau) = 1 - \sin^2 2\Theta_{23} \sin^2(\frac{\Delta m_{23}^2 L}{4E}) \qquad P(\nu_e \rightarrow \nu_e) \sim 1$$

For solar neutrinos and very long baseline neutrinos (L large, E small): $\frac{\Delta m_{13}^2 L}{E} \gg 1$

(almost like two neutrino system, only one mass difference)

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\Theta_{12} \sin^2(\frac{\Delta m_{12}^2 L}{4E})$$


logarithmic x axis

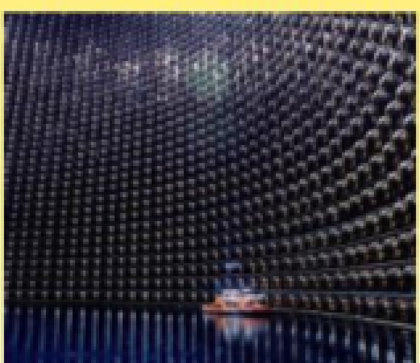
For short baseline reactor neutrinos (E small, L small): $\frac{\Delta m_{12}^2 L}{E} \ll 1$

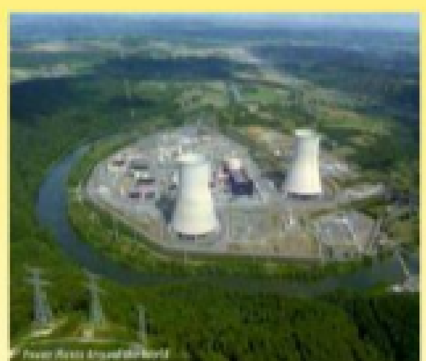
$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\Theta_{13} \sin^2(\frac{\Delta m_{13}^2 L}{4E})$$

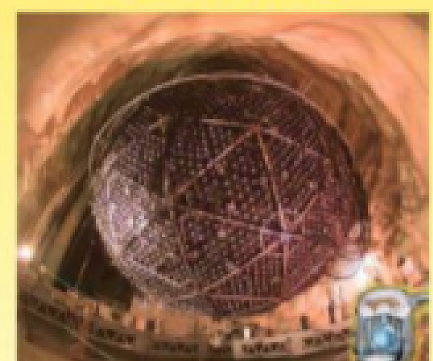
Sensitivity

$$U = \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{i\delta} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{-i\delta} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{-i\delta} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{-i\delta} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{-i\delta} & c_{23}\,c_{13} \end{pmatrix} =$$

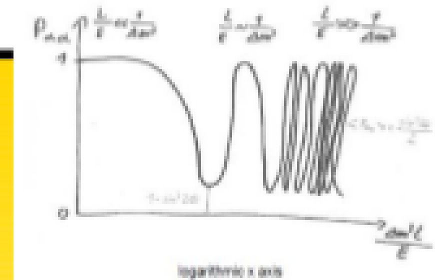
$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{atmospheric and LBL accelerator}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}\,e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\,e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{SBL reactor}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{solar and LBL reactor}}$$







Sensitivity



Characteristics of typical oscillation experiments

Source	Flavor	E [GeV]	L [km]	$(\Delta m^2)_{\mathrm{min}}$ [eV^2]
Atmosphere	$\overset{(-)}{\nu}_e, \overset{(-)}{\nu}_\mu$	$10^{-1} \ldots 10^{2}$	$10 \ldots 10^{4}$	10^{-6}
Sun	ν_e	$10^{-3} \ldots 10^{-2}$	10^{8}	10^{-11}
Reactor SBL	$\bar{\nu}_e$	$10^{-4} \ldots 10^{-2}$	10^{-1}	10^{-3}
Reactor LBL	$\bar{\nu}_e$	$10^{-4} \ldots 10^{-2}$	10^{2}	10^{-5}
Accelerator LBL	$\overset{(-)}{\nu}_e, \overset{(-)}{\nu}_\mu$	$10^{-1} \ldots 1$	10^{2}	10^{-1}
Accelerator SBL	$\overset{(-)}{\nu}_e, \overset{(-)}{\nu}_\mu$	$10^{-1} \ldots 1$	1	1

SBL: Short Base Line; LBL: Long Base Line

Some Neutrino Experiments

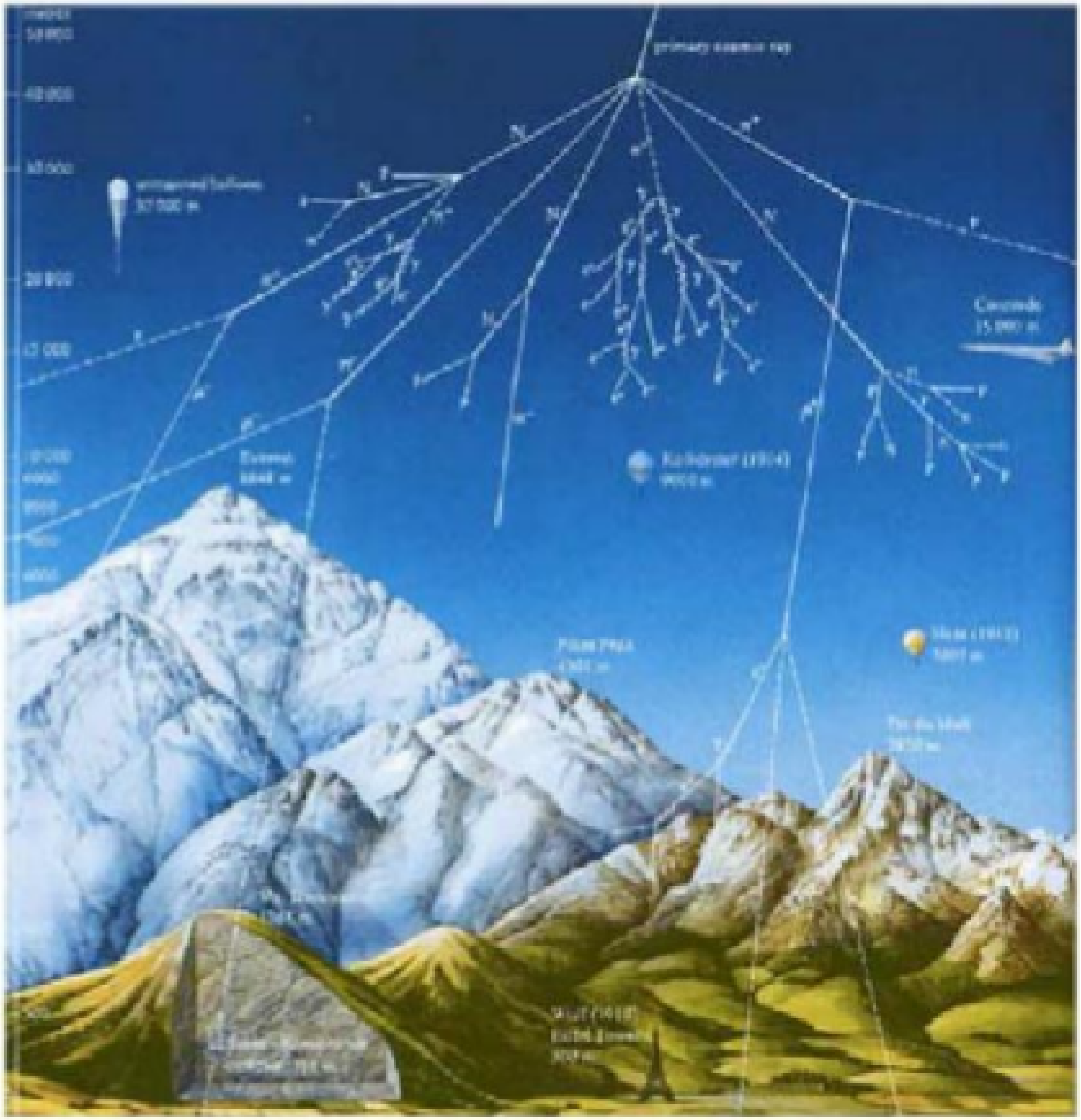
Neutrino Oscillation Experiments:

neutrino source	experiment	comments
solar neutrinos	radiochemical exp Homestake Cl, GALEX, SAGE	First observation of "neutrino disappearance" more than 20 years ago "solar neutrino problem"
	water experiments (Super)Kamiokande	confirm disappearance
	"heavy water": SNO	proves ν oscillation appearance signal
atmospheric neutrinos	(Super)Kamiokande	oscillation (appearance) signal
accelerator	LSDN K2K OPERA	not confirmed clear disappearances signal fake result: ν travel faster than light
reactor	KamLAND, CHOOZ	first disappearance evidence $\theta_{13} \rightarrow$ CP violation

many more experiments exist,
will just discuss a small selection
in this lecture.

Neutrino absolute mass measurements: e.g. Katrin
(measurement of endpoint of betas spectrum)

Atmospheric Neutrinos



Cosmic radiation: Air shower

$$p + N \rightarrow \pi^{\pm}, K^{\pm}$$

$$\pi^{\pm}, K^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}(\overline{\nu}_{\mu})$$

$$\mu^{\pm} \rightarrow e^{\pm} + \nu_{e}(\overline{\nu}_{e}) + \overline{\nu}_{\mu}(\nu_{\mu})$$

$$R = \frac{\nu_{\mu} + \overline{\nu}_{\mu}}{\nu_{e} + \overline{\nu}_{e}} = 2$$

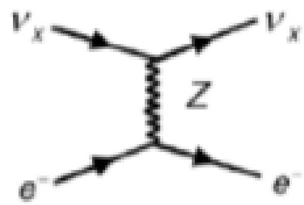
Exact calculation: R=2.1 (E_{ν}<1GeV)

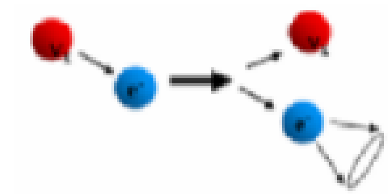
(For larger energies R>2.1)

(Super-)Kamiokande

Water = "active target"

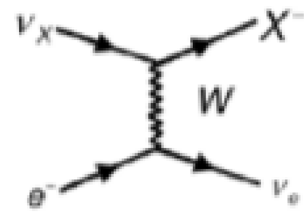
Elastic scattering





Cherenkov Light

Charged current



Detection of Cherenkov photons: Photo multiplier

Experiments: (Super)-Kamiokande

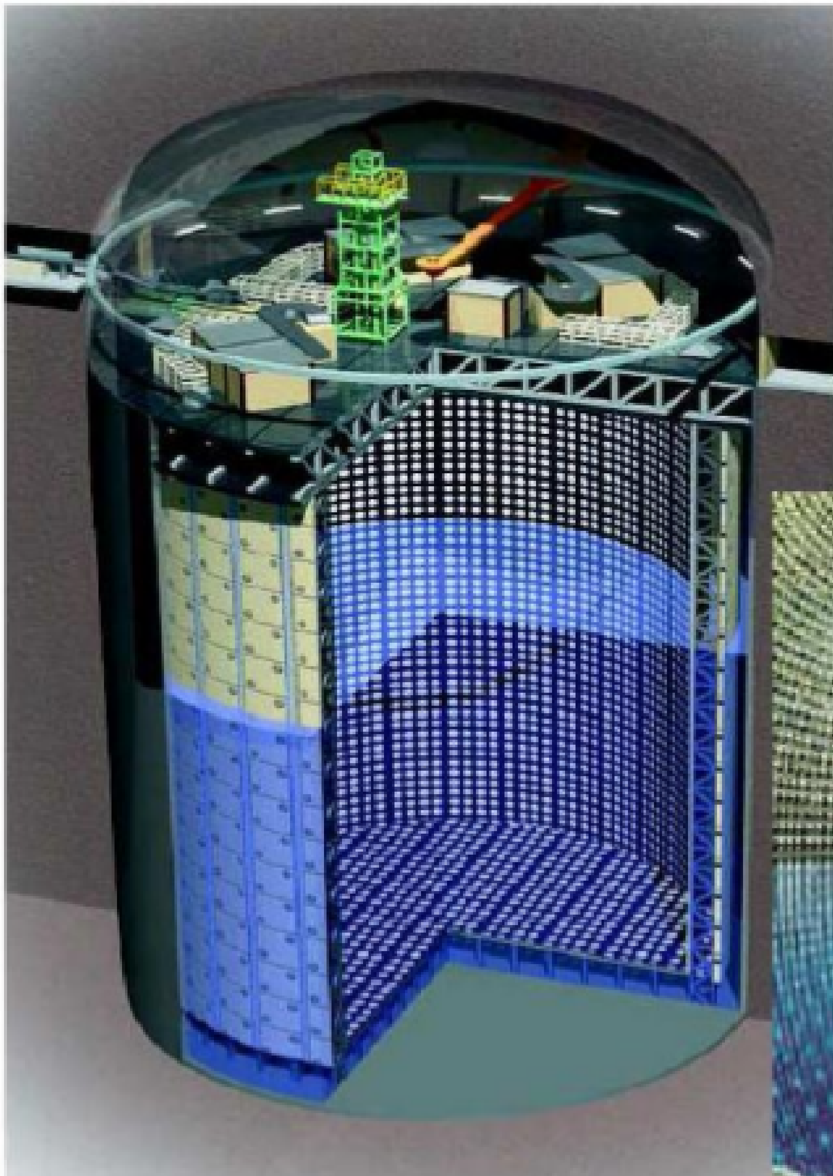
IMB

Soudan-2

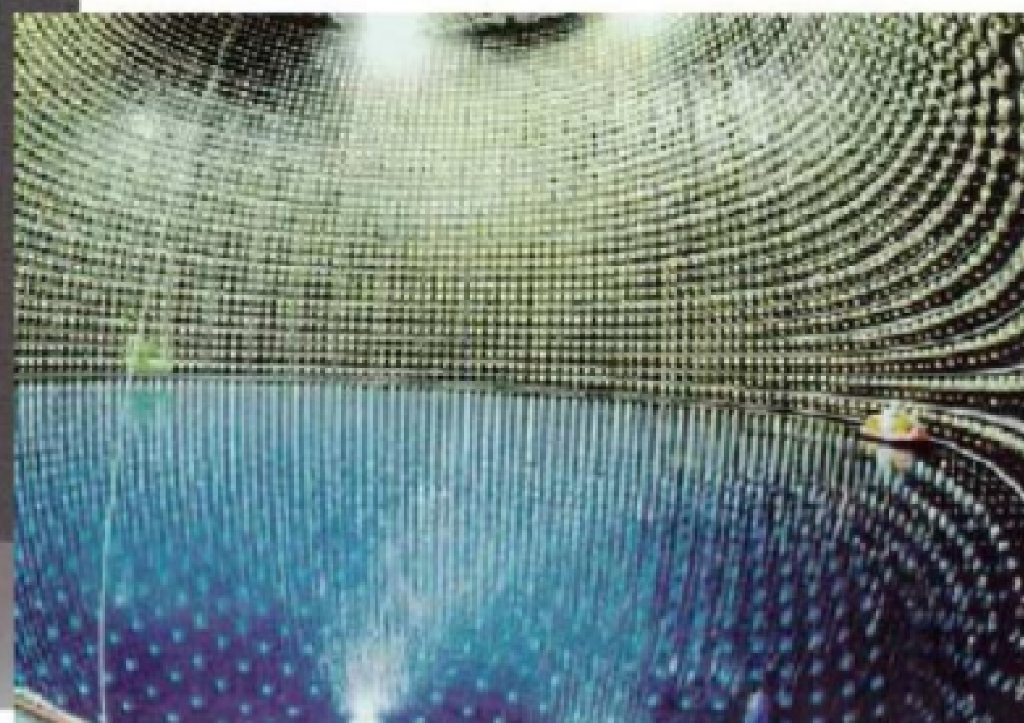
treshold: ~ 5 MeV (to produce sufficiently high energetic electrons to produce Cherenkov light, no issue for atmospheric neutrinos, but for solar neutrinos)

Advantage: - scattered electron/muons carry direction information
- in-situ detection
- measurement of neutrino energy

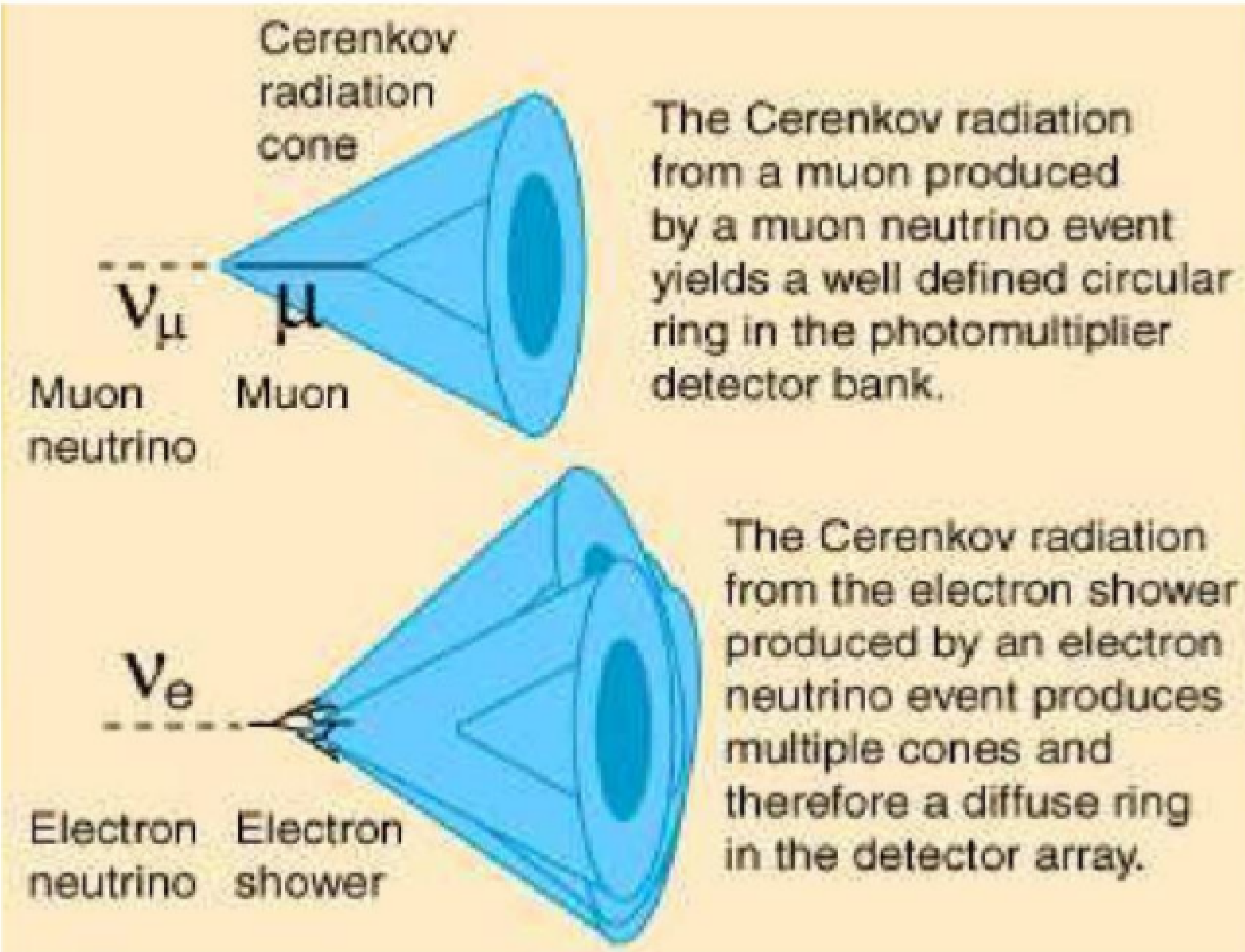
(Super-)Kamiokande



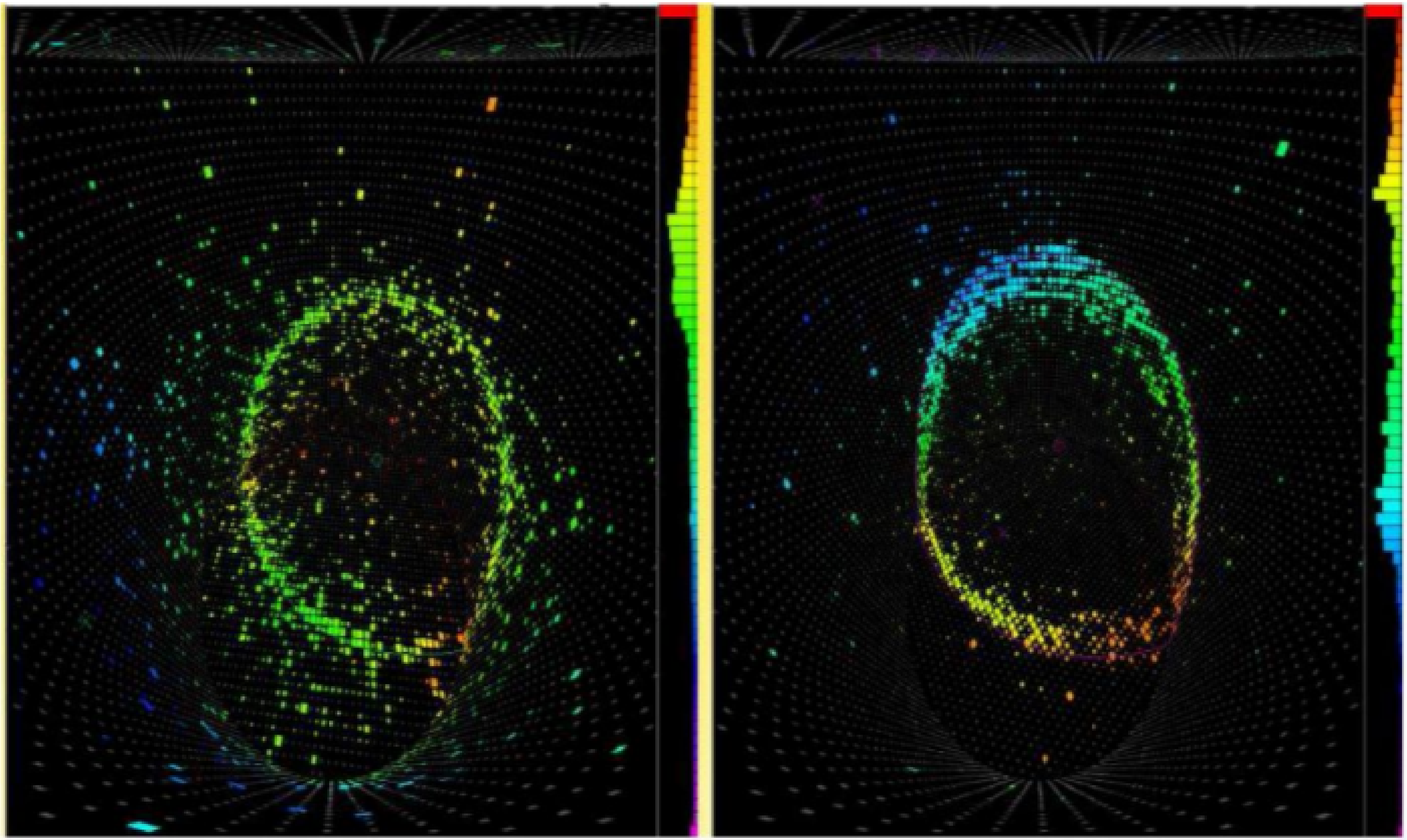
- Largest artificial water detector (50 kt)
- Until the 2001 accident:
  11000 PMTs (50 cm tubes!): 40% of surface covered with photo-cathode
- Back in operation since 2003



Detection of Cherenkov-Light



$\nu_e \leftrightarrow \nu_\mu$



Energy measurement:
Thickness of ring is related to travel distance of electron/muon in the detector. This is related to the energy of the electron/muon thus (with some uncertainties) to the original neutrino energy.

Ratio of Muon to Electron Neutrinos

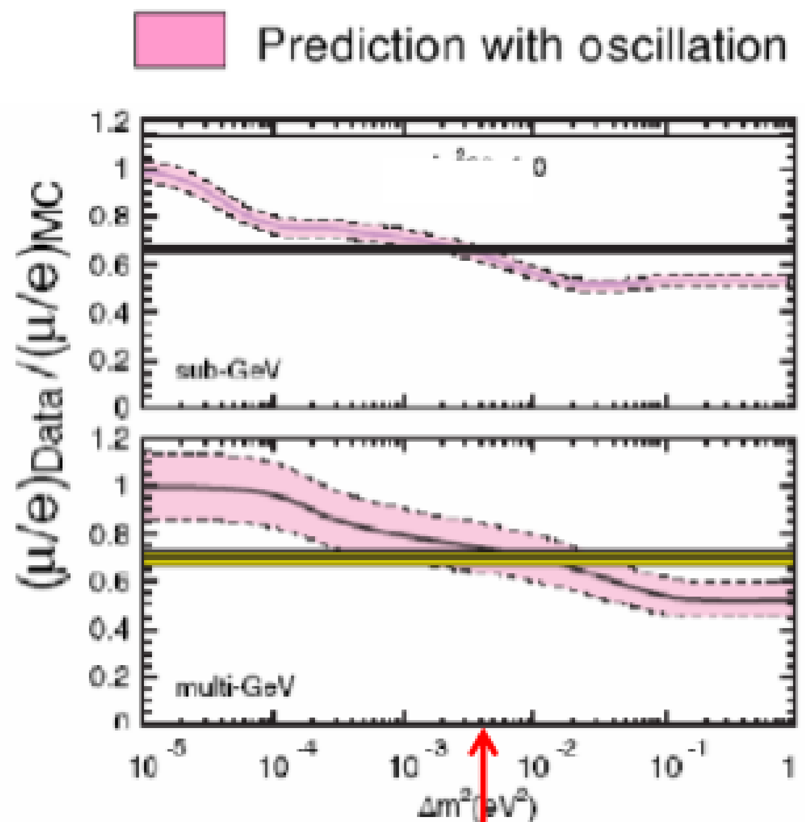
oscillation depend on E, thus split up
sample in high and low momentum neutrinos

$$R = \frac{(\mu/e)_{DATA}}{(\mu/e)_{M.C.}}$$

$$R_{sub-GeV} = 0.658 \pm 0.016(stat) \pm 0.032(sys)$$

$$R_{multi-GeV} = 0.702^{+0.032}_{-0.030}(stat) \pm 0.099(sys)$$

assuming no oscillation
in simulation



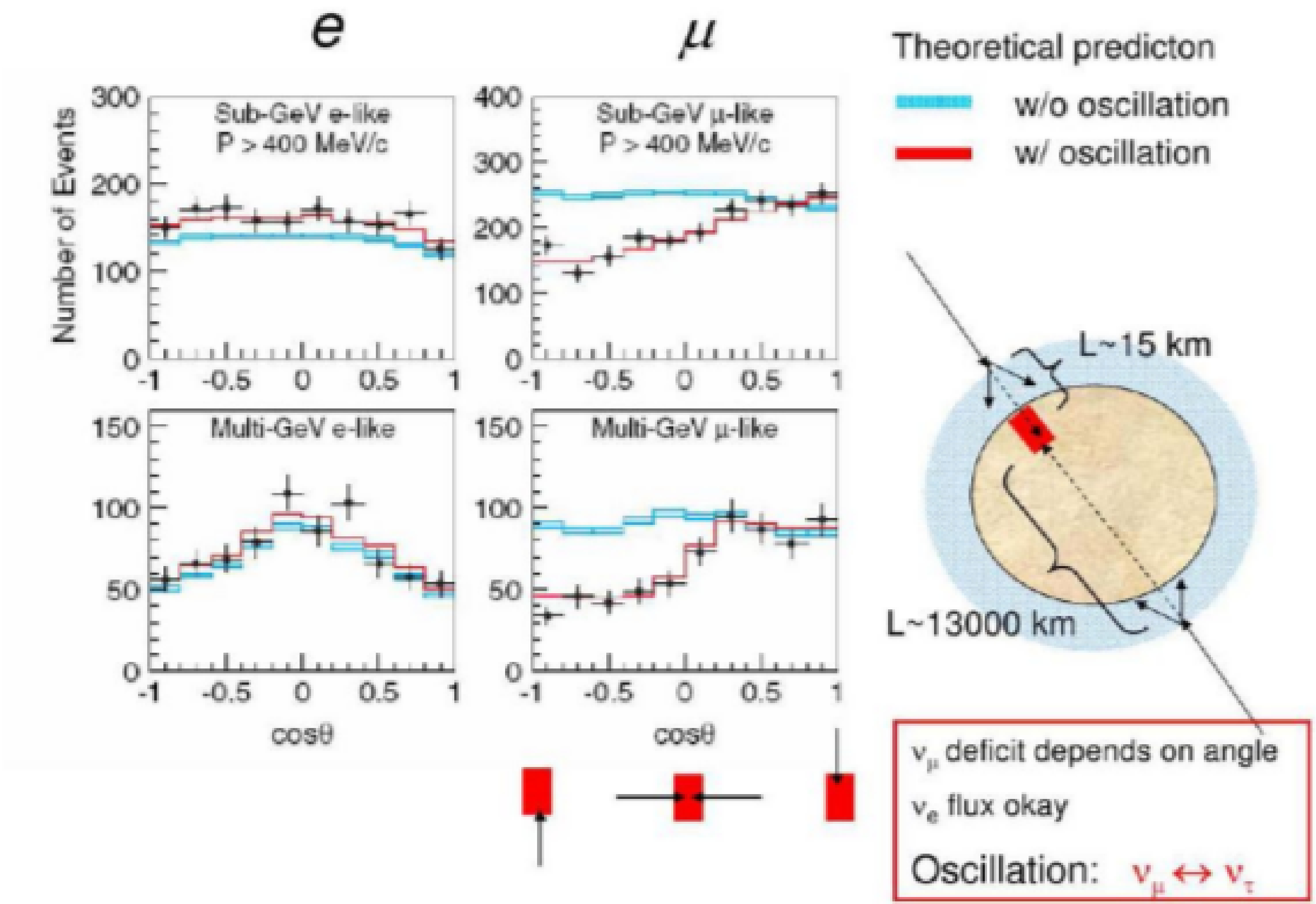
Δm²(atmospheric) ~ 2-5 10^{-3}

To few muon neutrinos observed → can be explained by oscillation!

For atmospheric and accelerator neutrions (E large, L medium): $\frac{\Delta m_{12}^2 L}{E} \ll 1$

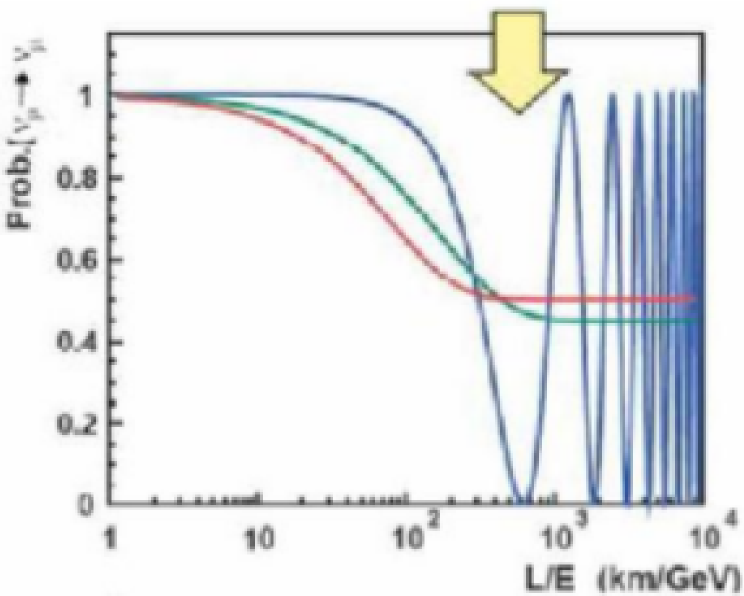
$P(\nu_\mu \rightarrow \nu_\tau) = 1 - \sin^2 2\Theta_{23} \sin^2(\frac{\Delta m_{23}^2 L}{4E})$ $\quad$ $P(\nu_e \rightarrow \nu_e) \sim 1$

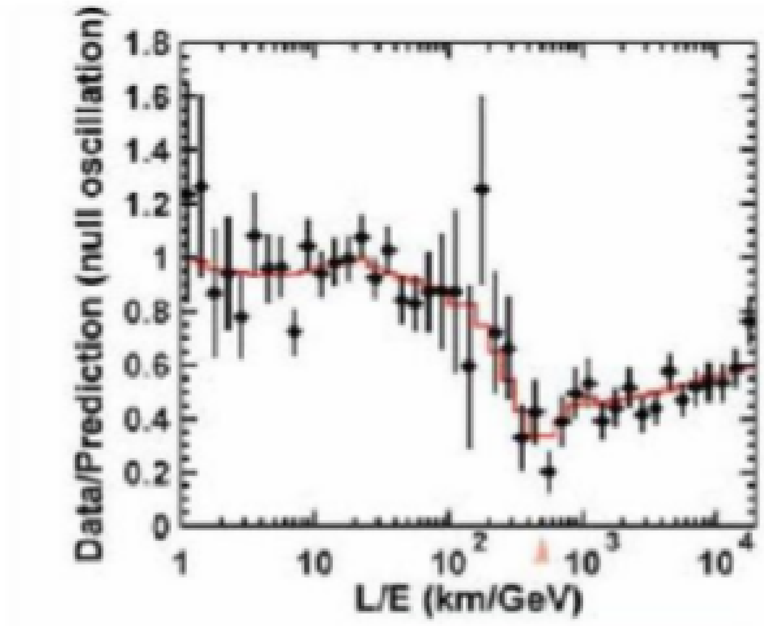
Zenith Angle Dependence of Neutrino Flux



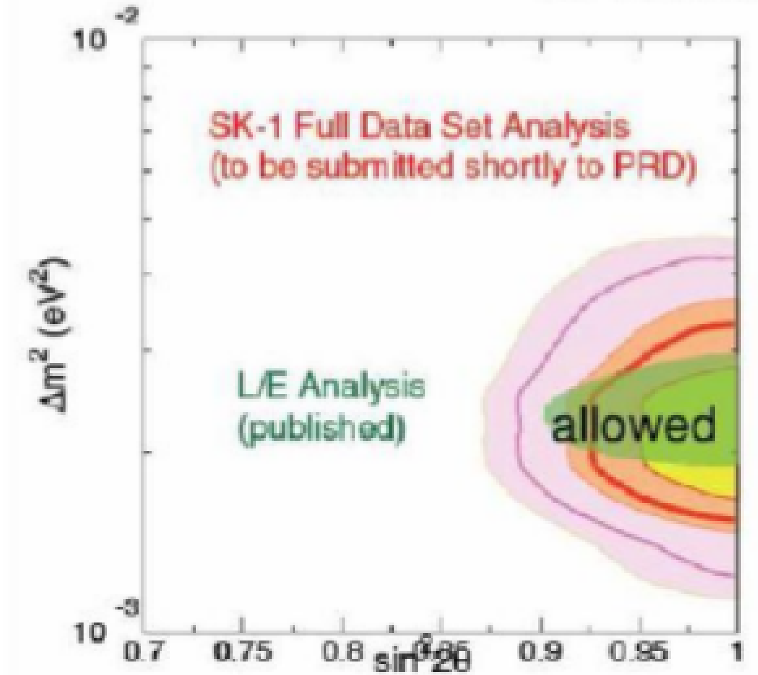
Electron neutrion seem to consist mainly of matter eigenstate, which has a significant longer oscillation distance both compared to Distance atmosphere earth and to diameter of the earth.

Oscillating Pattern of Atmospheric Neutrinos





oscillation dip seen at ~500 km/GeV



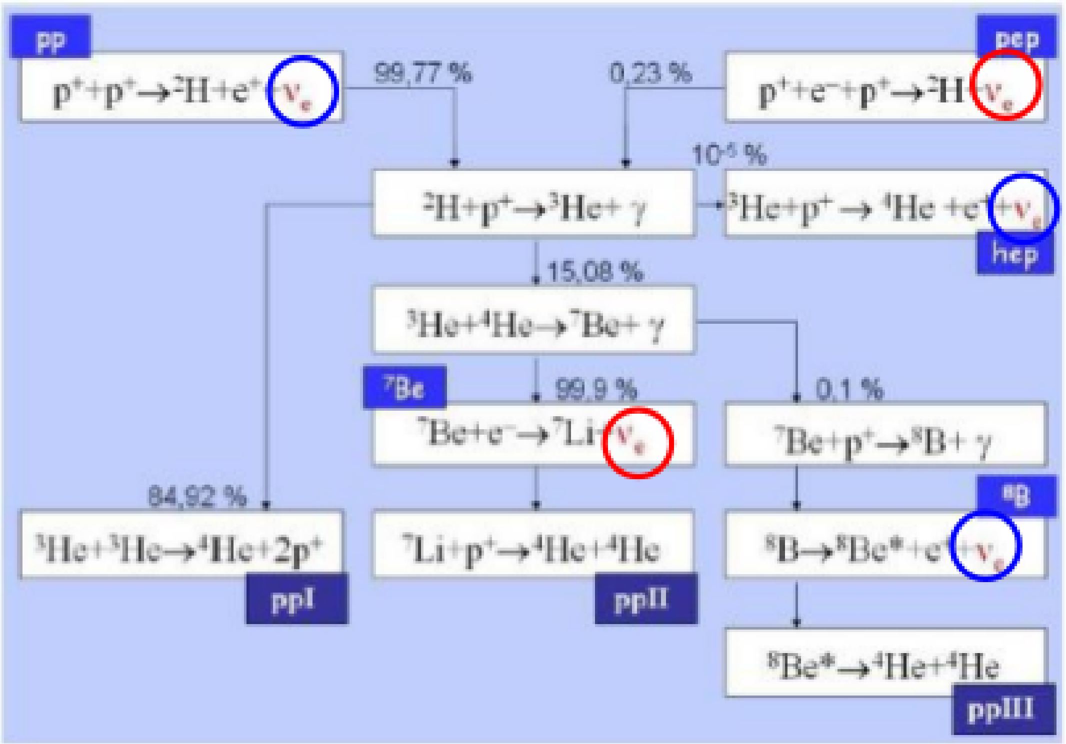
$\nu_\mu \leftrightarrow \nu_\tau$ mixing of atmos. neutrinos

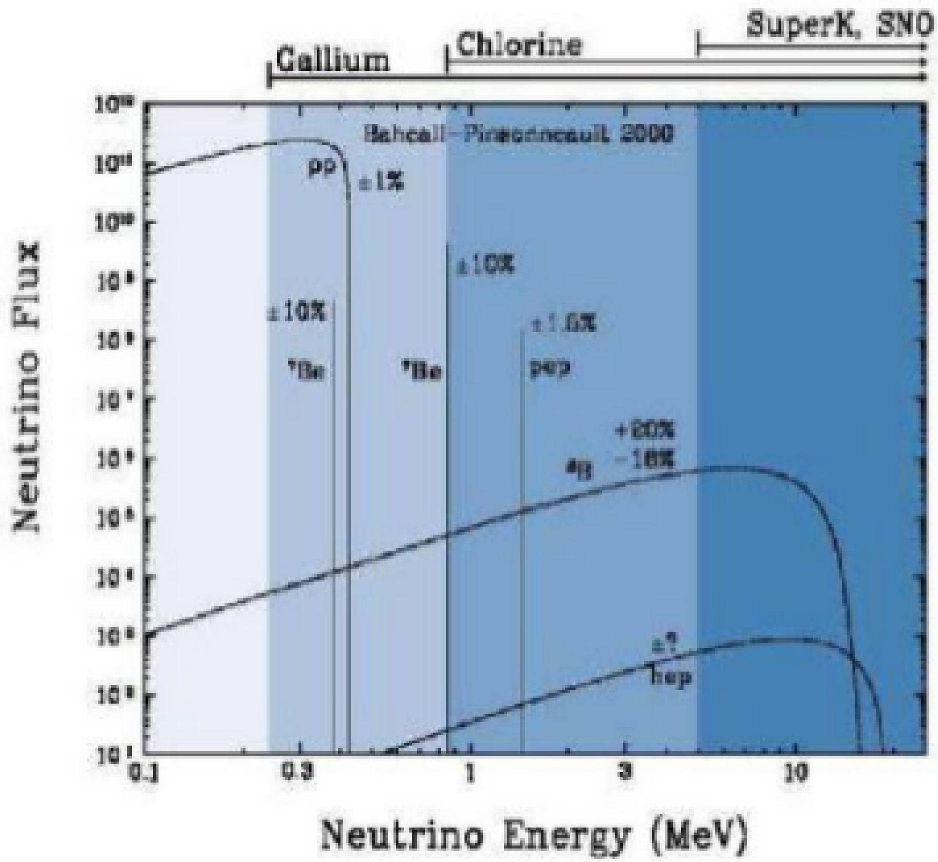
$$\Delta m^2 = (2.4 \pm 0.4) \times 10^{-3}\,\mathrm{eV}$$

$$\sin^2 2\theta > 0.92 \; @ \; 90\% \; C.L.$$

Slide taken from Neutrino 2004 conference

Solar Neutrinos





5 sources of solar neutrinos:

fixed energy from 2 body decay, continious spectrum from multi-body decay

measured flux depends strongly on detection treashold

Homestake Experiment

Pioneering experiment, started in 1970:

Raymond Davids Jr. (NP 2002)

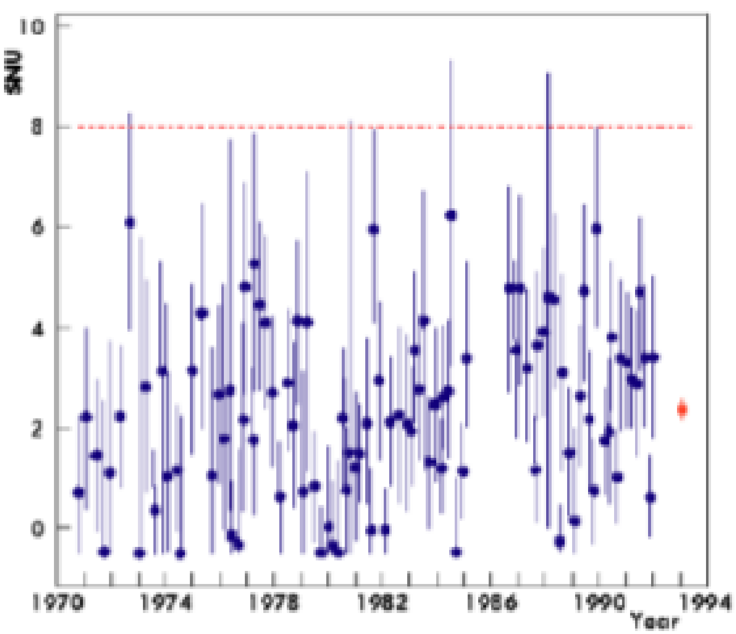
Homestake mine, 1400 m underground

615 t of $C_2CL_4 \sim 2.2 \times 10^{30}$ atoms of ^{37}Cl

Detection via: $\nu_e +^{37} Cl \rightarrow^{37} Ar + e$, treshold (0.8 MeV)

Homestake Experiment

- after 60-70 days ^{37}Ar is extracted by blowing helium gas over a thin layer of ^{37}Cl (^{37}Ar mean decay time: 35 days)
- measured ^{37}Ar decays in helium gas in proportional counters
- no "in-situ" measurement
- average count: one neutrino event every 2 days
- expected number of neutrino events: 1.6 per day



First indication of solar neutrino problem

→ initially results were ignored
„maybe" a mistake in the measurement method

Further Solar Neutrino Experiment

- GALLEX/GNO/SAGE, radiochemic experiments:

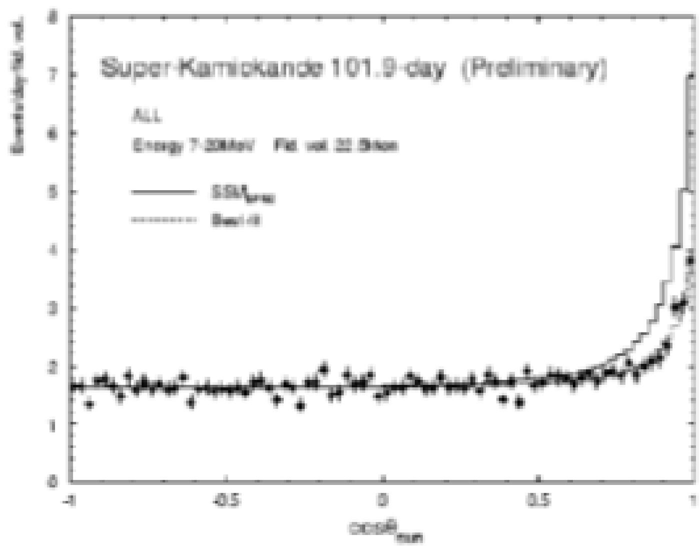
  $\nu_e +^{71} Ga \rightarrow^{71} Ge + e^-$, threshold: 0.2 MeV

  - delayed read out

  - no information on neutrino energy

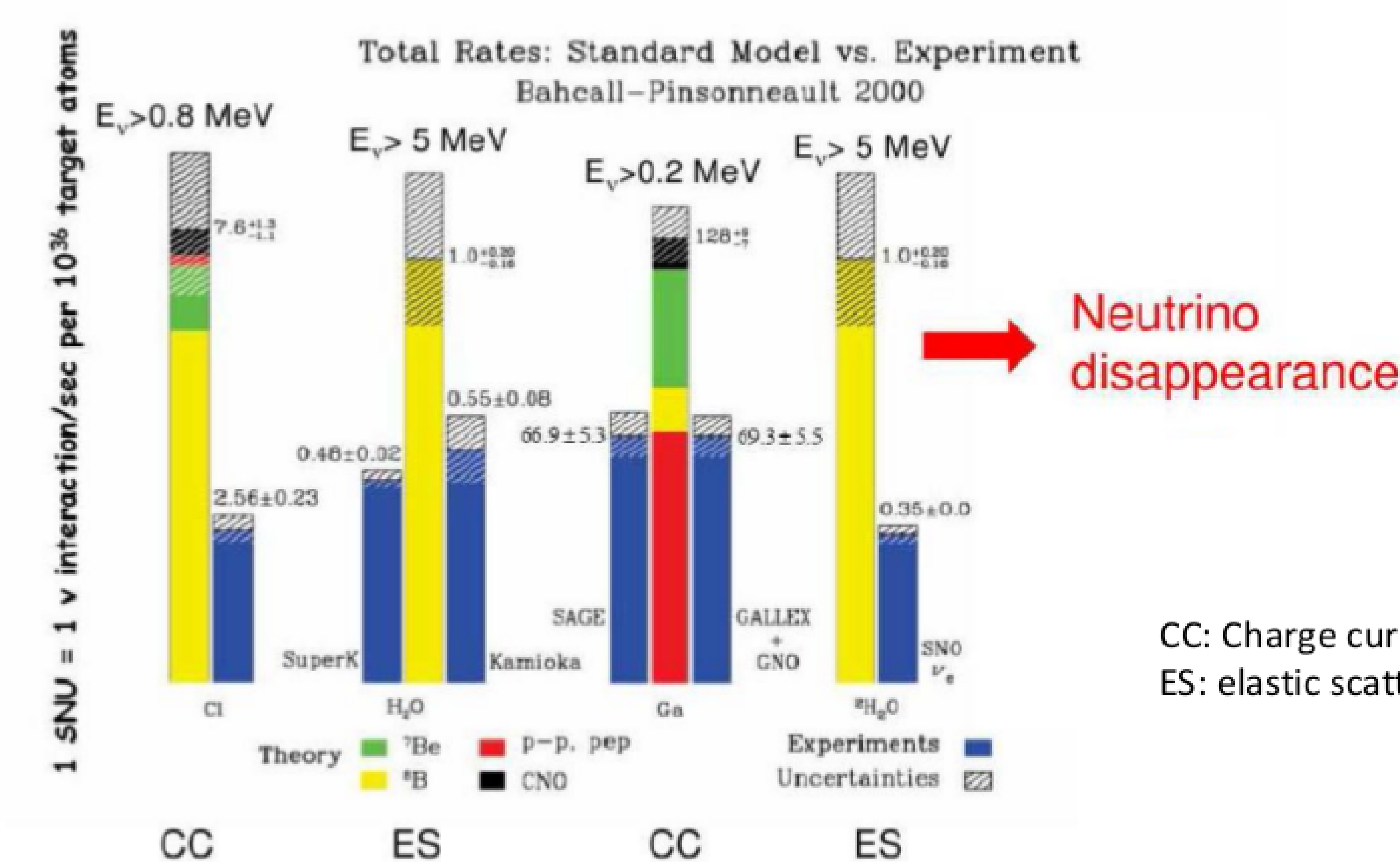
  - no direction information

- (Super)Kammiokande: $\nu_e + e^- \rightarrow \nu_e + e^-$



elastic scattering: direction of electron related to direction of initial neutrino, thus usefull to separate background from neutrinos from the sun.

Solar Neutrino Problem



CC: Charge current IA
ES: elastic scattering

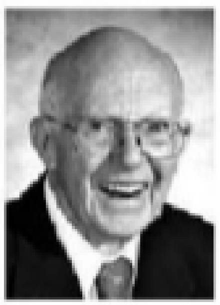
To establish neutrino mixing, need to measure additionally appearance effects!

Nobel Prize in 2002

Homestake experiment

The Nobel Prize in Physics 2002

spokesperson Kamiokande



Raymond Davis Jr.	**Masatoshi Koshiba**	**Riccardo Giaconi**

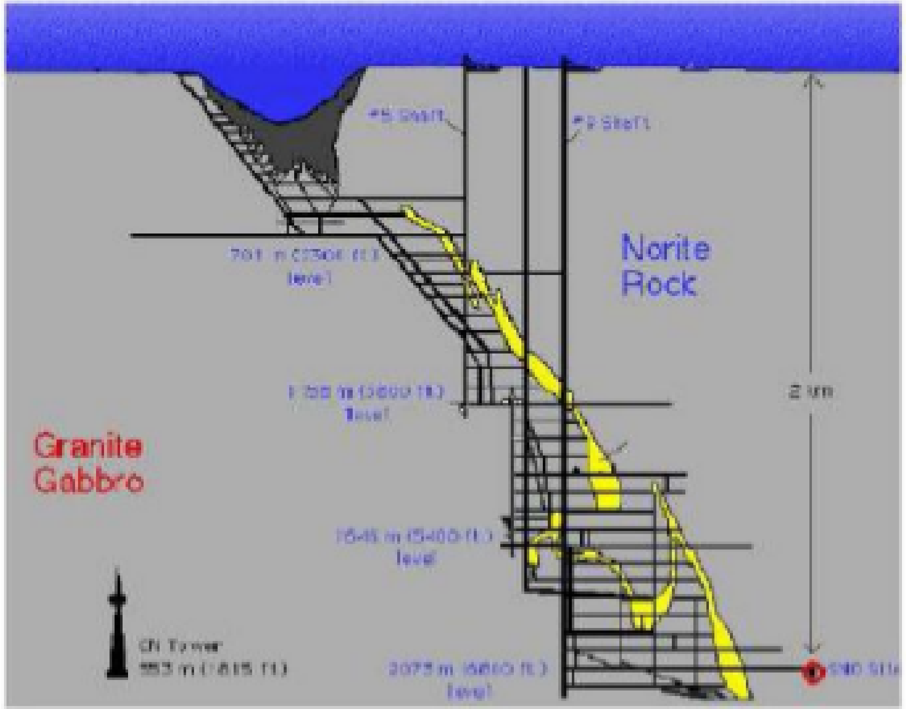
"for pioneering contributions to astrophysics, in particular for the detection of cosmic neutrinos"

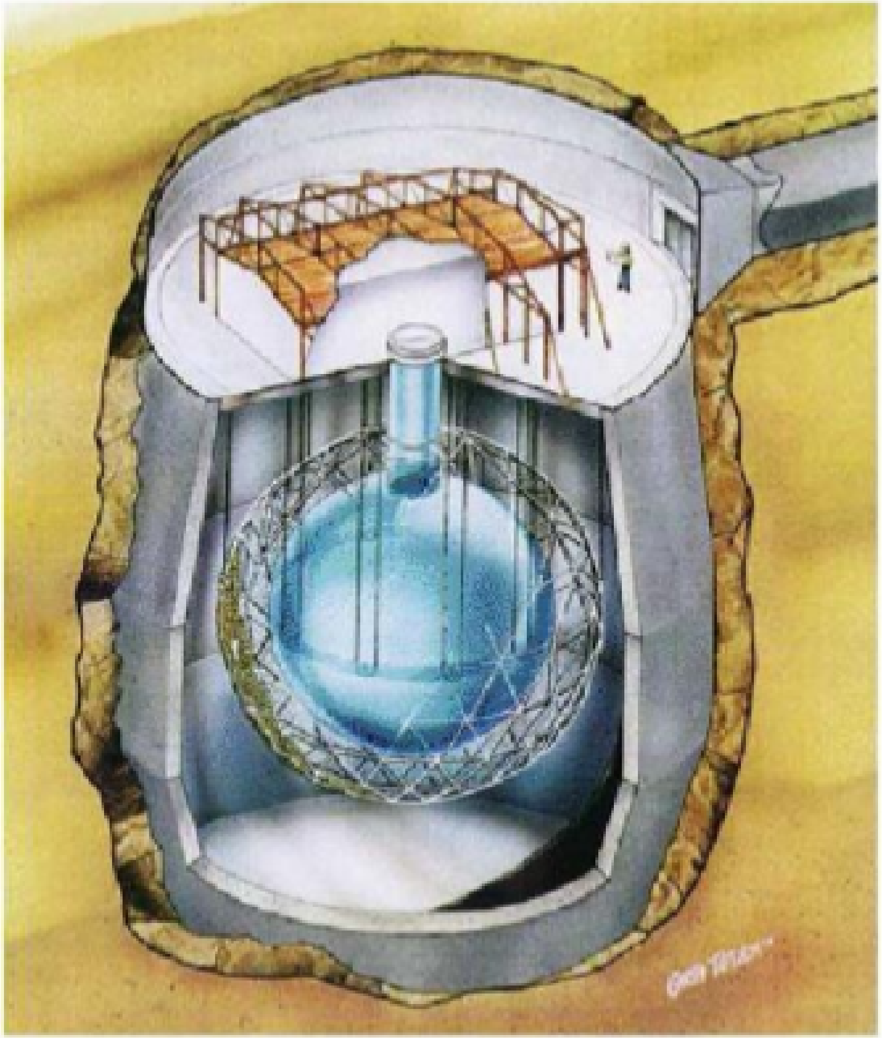
"for pioneering contributions to astrophysics, which have led to the discovery of cosmic X-ray sources"

SNO Experiment

Sudbury Neutrino Observatory:

- 6 m radius transparent acrylic vessel
- 1000 t of heavy water (D_2O)
- 9456 inward looking photo multipliers
- Add 2 t of NaCl to detect neutrons



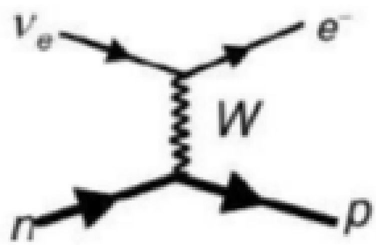


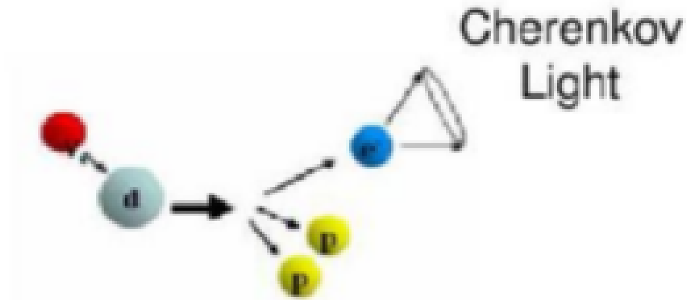
SNO Experiment

Charged current

$\sigma(\nu_\mu) = \sigma(\nu_\tau) = 0$

$\phi_\nu = \phi_{\nu_e}$



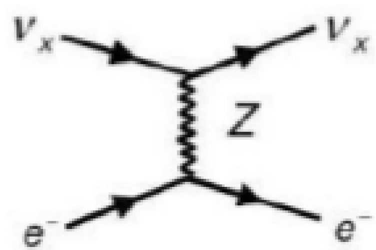


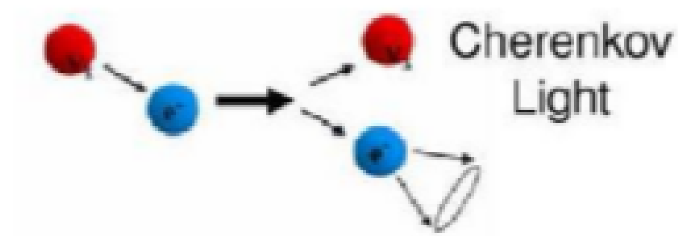
Elastic scattering

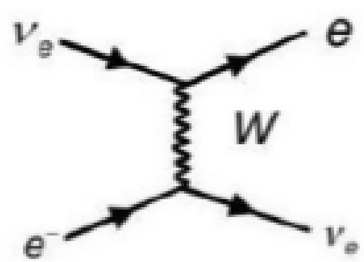
$0.154 \cdot \sigma(\nu_e) =$

$\sigma(\nu_\mu) = \sigma(\nu_\tau)$

$\phi_\nu = \phi_{\nu_e} + (\phi_{\nu_\mu} + \phi_{\nu_\tau})/6$



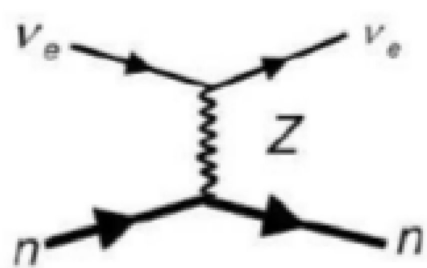


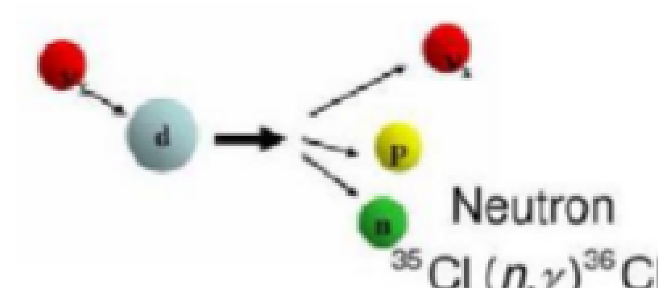


Neutral current

$\sigma(\nu_e) = \sigma(\nu_\mu) = \sigma(\nu_\tau)$

$\phi_\nu = \phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau}$



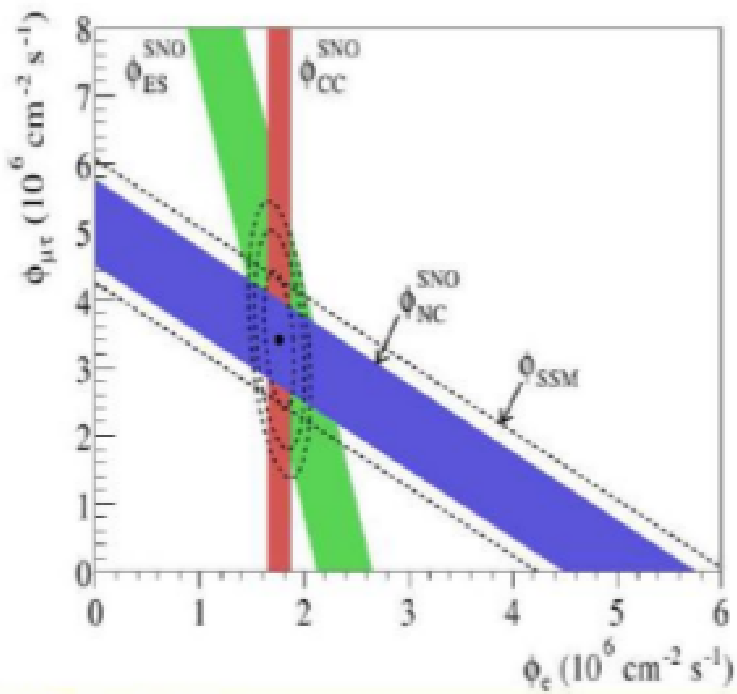


SNO Experiment

$\phi_{CC} < \phi_{ES} < \phi_{NC}$!

appearance of μ/τ neutrinos detected!

$$\phi_{CC}^{SNO} = 1.76^{+0.06}_{-0.05}(stat)^{+0.09}_{0.09}(syst)$$
$$\phi_{ES}^{SNO} = 2.39^{+0.24}_{-0.23}(stat)^{+0.12}_{0.12}(syst)$$
$$\phi_{NC}^{SNO} = 5.09^{+0.44}_{-0.43}(stat)^{+0.46}_{0.43}(syst)$$



Electron neutrino flux is too low: $P_{\nu_e \to \nu_e} = (35 \pm 2)\%$

Total flux of neutrinos is correct $\longrightarrow$ Interpreted as $\nu_e \leftrightarrow \nu_\mu$ or $\nu_e \leftrightarrow \nu_\tau$ oscillation

But in case of simple "vacuum oszillation":

$$P_{\nu_e \to \nu_e} = 1 - \frac{1}{2}\sin^2 2\theta = 50\%$$