Reminder of Parity

particle solution $\Psi_i = u_i(E,\vec{p})\ e^{i(\vec{p}\vec{x}-Et)}$

$$u_1 = \sqrt{E+m}\begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \end{pmatrix} \quad u_2 = \sqrt{E+m}\begin{pmatrix} 0 \\ 1 \\ \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix}$$

antiparticle solution $\Psi_i = v_i(E,\vec{p})\, e^{-i(\vec{p}\vec{x}-Et)}$

$$v_1 = \sqrt{E+m}\begin{pmatrix} \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \\ 1 \\ 0 \end{pmatrix} \quad v_2 = \sqrt{E+m}\begin{pmatrix} \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \\ 0 \\ 1 \end{pmatrix}$$

These solutions have positive energies.

Partity operator P:

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

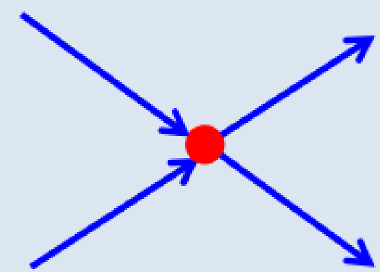
Spin ½ particles AT REST have intrinsic partiy P = +1

Spin ½ particles AT REST have intrinsic parity P=-1

Connection to Fermi Theory

1934: Fermi proposed in analogy to QED following matrix element for the β-decay:

$$|M_{fi}| = |G_F\, g_{\mu\nu}\, [\bar{\Psi}\gamma^{\mu}\Psi][\bar{\Psi}\gamma^{\nu}\Psi]|$$



here Fermi constant: $G_F = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$
(very well determined from muon lifetimes)

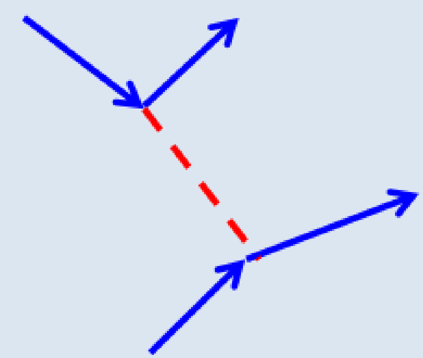
no propagator, IA at a point

after discovery of partiy violation in 1975, this was modfied to:

$$|M_{fi}| = |\frac{G_F}{\sqrt{2}}\, g_{\mu\nu}\, [\bar{\Psi}\gamma^{\mu}(1-\gamma^5)\Psi][\bar{\Psi}\gamma^{\nu}(1-\gamma^5)\Psi]|$$

Compare to the prediction for W-boson exchange:

$$|M_{fi}| = |\frac{g_W}{\sqrt{2}}\, \bar{\Psi}\frac{1}{2}\gamma^{\mu}(1-\gamma^5)\Psi\, \frac{g_{\mu\nu} - q_{\mu}q_{\nu}/m_W^2}{q^2 - m_W^2}\, \frac{g_W}{\sqrt{2}}\, \bar{\Psi}\frac{1}{2}\gamma^{\mu}(1-\gamma^5)\Psi|$$



for $q^2 \ll m_W^2$

$$M_{fi} = |\frac{g_W^2}{8}\, \bar{\Psi}\gamma^{\mu}(1-\gamma^5)\Psi\, \frac{g_{\mu\nu}}{-m_W^2}\, \bar{\Psi}\gamma^{\mu}(1-\gamma^5)\Psi|$$

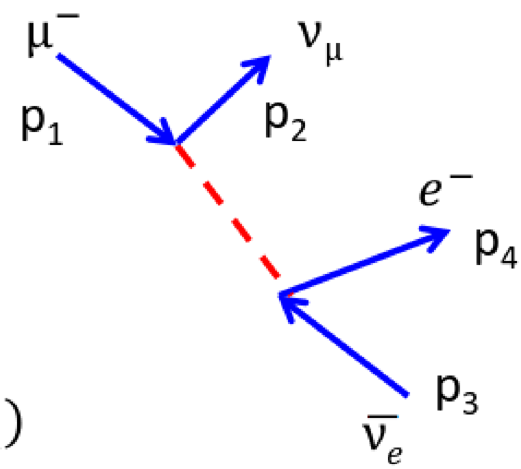
⇒ $\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8 m_W^2}$ normally use G_F to express strength of weak IA, as it is precisely determined in muon decays

Matrix Element and Coupling Strength

$-i\,M_{fi} = \frac{g_W}{\sqrt{2}}\,\overline{u(p_2)}\,\frac{1}{2}\gamma^\mu\,(1-\gamma^5)\,u(p_1)\,\left(-i\,\frac{g_{\mu\nu}-q_\mu q_\nu/m_W{}^2}{q^2-m_W{}^2}\right)\frac{g_W}{\sqrt{2}}\,\overline{u(p_4)}\,\frac{1}{2}\gamma^\mu\,(1-\gamma^5)\,v(p_3)$

In muon decays $q^2 < m_\mu{}^2 \ll m_W{}^2$ to very good approximation

$$-i\,\frac{g_{\mu\nu}-q_\mu q_\nu/m_W{}^2}{q^2-m_W{}^2} \sim i\frac{g_{\mu\nu}}{m_W{}^2} \Rightarrow \frac{G_F}{\sqrt{2}} = \frac{g_W{}^2}{8m_W{}^2}$$



Analoguous to the QED calculations one finds after lengthly calculation: $M^2 = \frac{1}{2}\sum_{spins}|M|^2 = 64\,G_F{}^2(p_2 p_4)(p_3 p_1)$

$d\Gamma = \frac{1}{2E}\,|M|^2\,d\rho$ LI phase space

due to choice of LI phase space and ME

For given electron energie E': $\frac{d\Gamma}{dE'} = \frac{G_F{}^2}{12\pi^3}\,m_\mu{}^2\,E'^2(3-\frac{4E'}{m_\mu})$

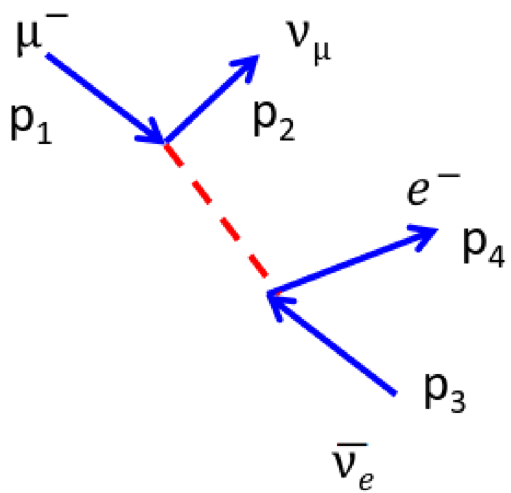
Integral over all possible electron energies : $\frac{1}{\tau} = \Gamma = \int_0^{m_\mu/2}\frac{d\Gamma}{dE'}\,dE' = \frac{G_F{}^2\,m_\mu{}^5}{192\,\pi^3}$

Measurements of the muon lifetime determines fundamental coupling G_F (sometimes calle G_μ)

Matrix Element and Coupling Strength

$\tau_\mu = (2.19703 \pm 0.00004)\ 10^{-6}$s

$G_\mu = (1.116639 \pm 0.00002)\ 10^{-5}\ \text{GeV}^{-2}$



To directly compare couling strength to QED interaction, need to plug in $m_W = 80.4$ GeV

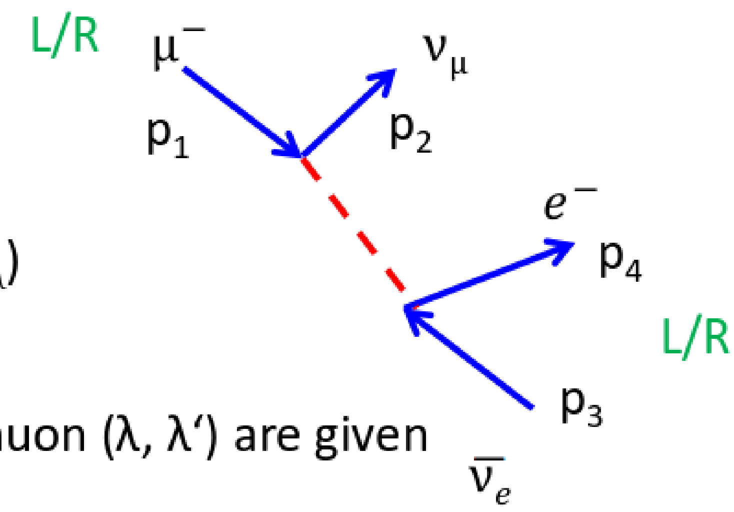
$$\frac{G_F}{\sqrt{2}} = \frac{g_W{}^2}{8m_W{}^2}$$

$$\Rightarrow \quad \alpha_W = \frac{g^2{}_W}{4\pi} = \frac{8m_W{}^2 G_F}{4\sqrt{2}\pi} = \frac{1}{30} > \frac{1}{137} = \alpha_{elm}$$

The intrinsic strength of the weak interaction is similar to, but actually greater than, the EM Interaction! It is the massive W-boson in the propagator which makes it appear weak. for $q^2 \gg m_W{}^2$ weak IAs are more likely than EM.

Experimental Probe of V-A structure

Most general form of matrix element, include scalar (S), vector (V) and tensor (T) currents.



$$M = \frac{G_F}{\sqrt{2}} \sum_{\substack{i=S,V,T \\ \lambda,\lambda'=R,L}} g^i_{\lambda\lambda'} \left(\overline{u(p_4)_{\lambda'}}\;\; \Gamma^i\, v(p_3)_m\right) \left(\overline{u(p_2)_n}\;\; \Gamma^i\, v(p_1)_\lambda\right)$$

n,m = R/L given if coupling i and handiness of electron and muon (λ, λ‘) are given

Possible current-current couplings

i / λλ‘	RR	RL	LR	LL
S	x	x	x	x
V	x	x	x	x
T		x	x	

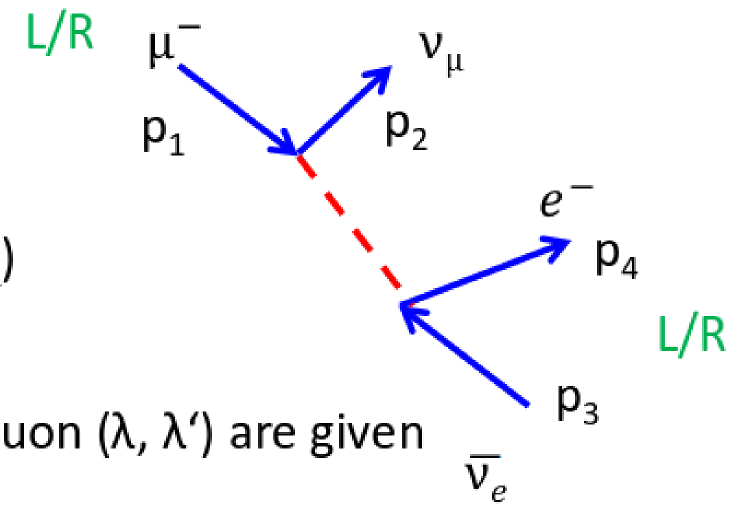
There are in general 10 complex amplitudes $g^i_{\lambda\lambda'}$

pure V-A couling: $g^V_{LL} = 1$, all others 0

Experimental idea: measure polarization of electron for a given polarization of initial state.
Determine energy and angular distribution of electron.

Experimental Probe of V-A structure

Most general form of matrix element, include scalar (S), vector (V) and tensor (T) currents.



$$M = \frac{G_F}{\sqrt{2}} \sum_{\substack{i=S,V,T \\ \lambda,\lambda'=R,L}} g^{i}_{\lambda\lambda'} \left(\overline{u(p_4)_{\lambda'}}\ \Gamma^i\ v(p_3)_m\right)\left(\overline{u(p_2)_{n}}\ \Gamma^i\ v(p_1)_\lambda\right)$$

n,m = R/L given if coupling i and handiness of electron and muon (λ, λ‘) are given

Possible current-current couplings

i / λλ‘	RR	RL	LR	LL
S	x	x	x	x
V	x	x	x	x
T		x	x	

There are in general 10 complex amplitudes $g^{i}_{\lambda\lambda'}$

pure V-A couling: $g^{V}_{LL} = 1$,
all others 0

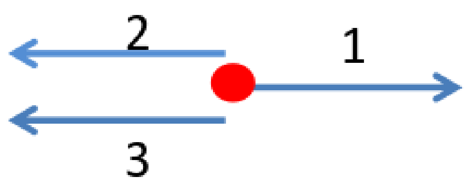
Experimental idea: measure polarization of electron for a given polarization of initial state.
Determine energy and angular distribution of electron.

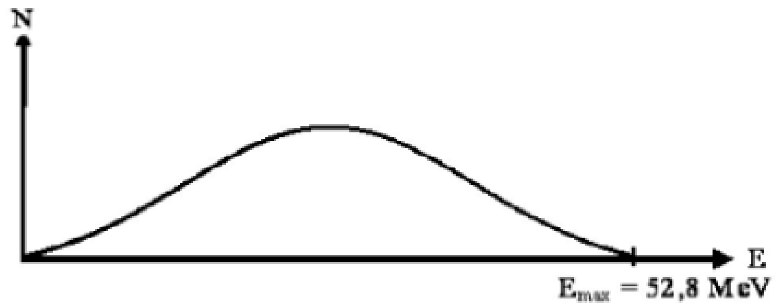
Experimental Probe of V-A structure: Muon Decay

Experimental idea: measure polarization of electron for a given polarization of initial state.
Determine energy and angular distribution of electron.

Consider muon rest system: m(μ) ~ 105 MeV; m(e)~m(ν)~0

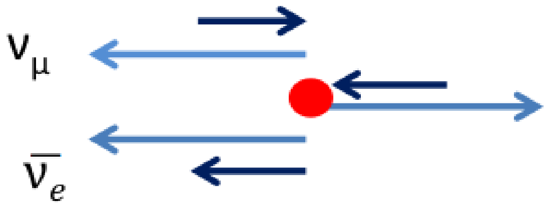
Maximum energy (m(μ)/2) of particle 1, if particle 2, 3 fly in opposite direction in muon CMS.





pure kinematics

E.g. V-A theory



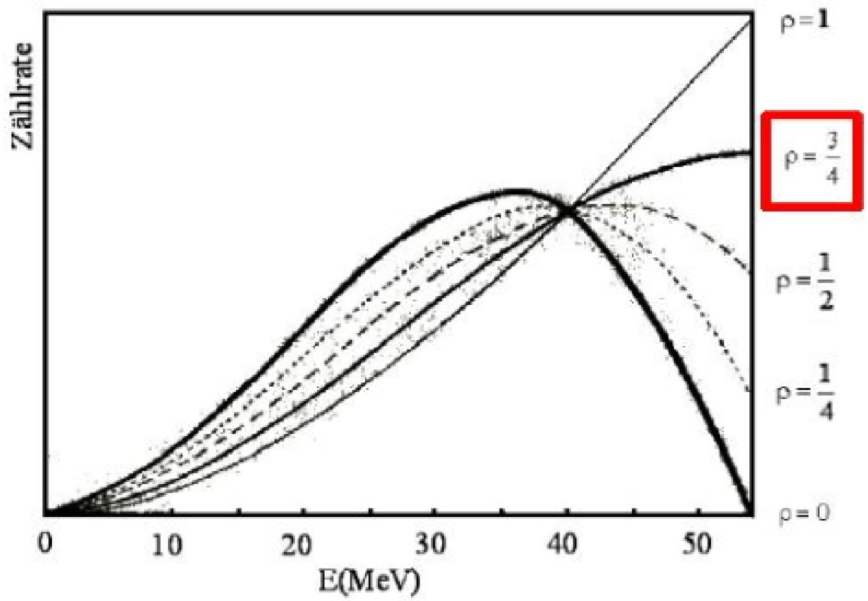
in approximation of zero mass, this is only possible configuration (despite it is kinematically unlikely)

Experimental Probe of V-A structure: Muon Decay

Energy spectrum of emitted electron:

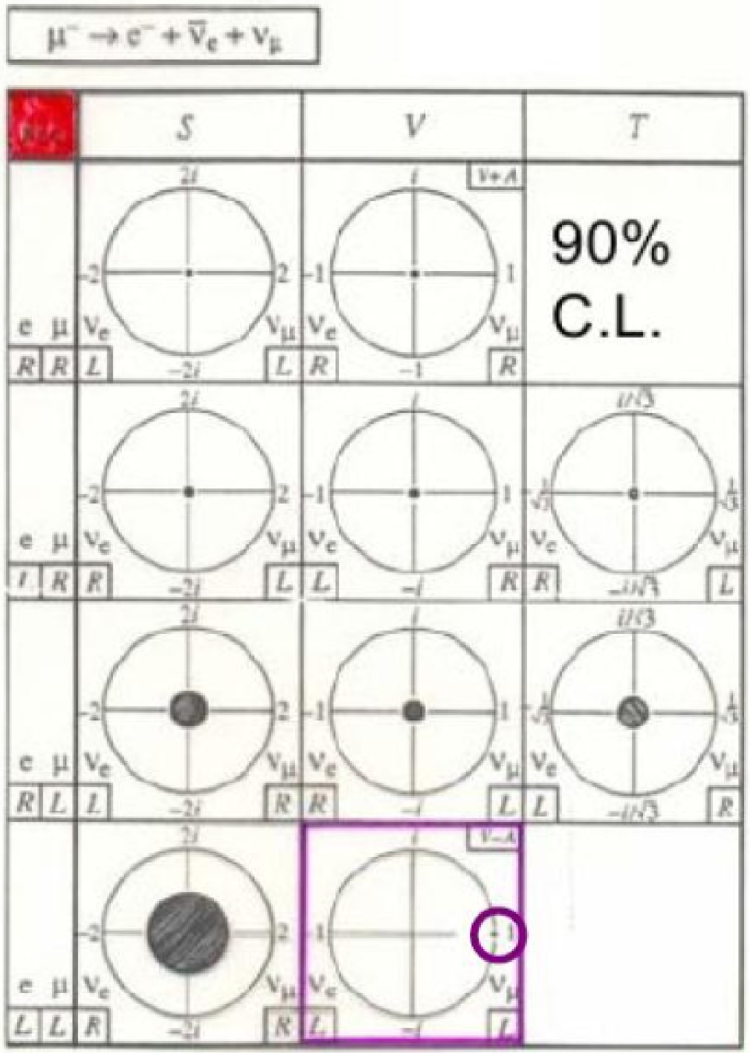
$$dN(E) = \frac{4E^2 dE}{\tau_\mu}\left[3(1-E) + \frac{2}{3}\rho\,(4E-3)\right]$$

Michelparameter: ρ



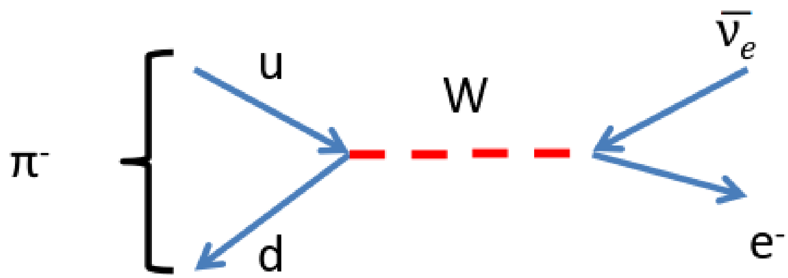
V-A theory: $\rho = 0.75$

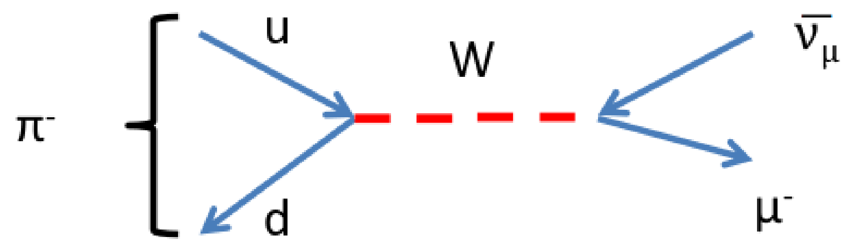
Couplings in muon decay



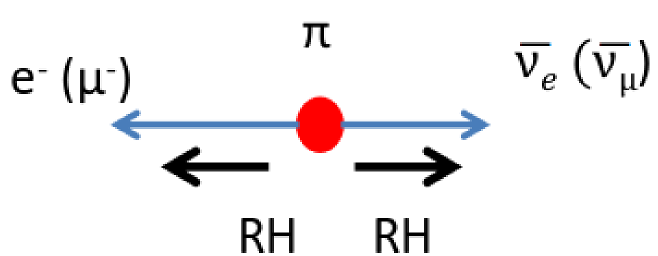
V-A theory is confirmed

Experimental Probe of V-A structure: Pion Decay





momentum and angular momentum conservation (pion CMS):



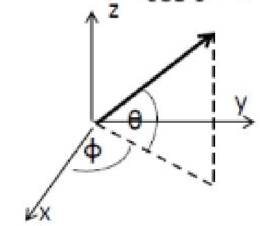
Phase space favors electron channel: m(π) ~140 MeV, m(μ)~105 MeV, m(e) ~ 511 keV

However:
Since anti-neutrino is (almost) massless, CC weak ineraction can only occur in RH state, thus electron/muon has to be in RH helicity state as well.

Weak IA cuples to LH chirality component of RH helicity state.

Definition of Polarization

$$\vec{p} = |\vec{p}| \begin{pmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{pmatrix}$$



Right handed helicity spinor: $u_{h=+1} = N \begin{pmatrix} \cos\Theta/2 \\ e^{i\varphi}\, sin\Theta/2 \\ \frac{|\vec{p}|}{E+m} cos\Theta/2 \\ \frac{|\vec{p}|}{E+m}\, e^{i\varphi} sin\Theta/2 \end{pmatrix}$

projector on left handed chirality: $P_L = \frac{1}{2}\,(1-\gamma^5) = \frac{1}{2} \begin{pmatrix} +1 & 0 & -1 & 0 \\ 0 & +1 & 0 & -1 \\ -1 & 0 & +1 & 0 \\ 0 & -1 & 0 & +1 \end{pmatrix}$

$$P_L\, u_{h=+1} = \frac{1}{2}\, N \left(1 - \frac{|\vec{p}|}{E+m}\right) \begin{pmatrix} \cos\Theta/2 \\ e^{i\varphi}\, sin\Theta/2 \\ -cos\Theta/2 \\ -e^{i\varphi} sin\Theta/2 \end{pmatrix}$$

Right handed helicity spinor has left handed chirality component.

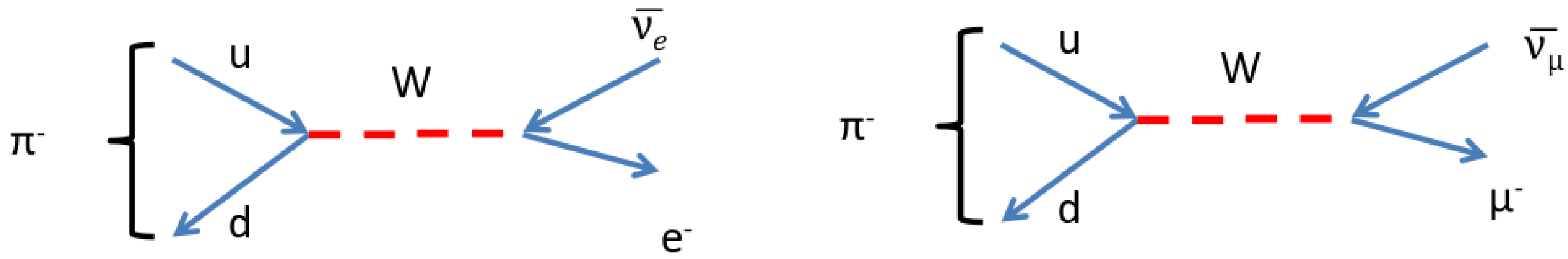
$$u_{h=+1} = P_R\, u_{h=+1} + P_L\, u_{h=+1} = \frac{1}{2}\, N\left(1 + \frac{|\vec{p}|}{E+m}\right) u_R + \frac{1}{2}\, N\left(1 - \frac{|\vec{p}|}{E+m}\right) u_L$$

right handed helicity — right handed chirality — left handed chirality

$$Pol = \frac{<P_R> - <P_L>}{<P_R> + <P_L>} = -\,\beta\ \left(= -\frac{v}{c}\right)$$

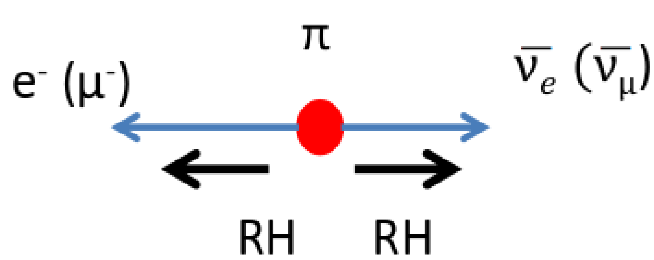
for lighter particles left handed chirality component is smaller!

Experimental Probe of V-A structure: Pion Decay



Measurement :

$$\frac{\Gamma(\pi^- \to e^- \ \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \ \bar{\nu}_\mu)} = (1.230 \ \pm 0.004)\ 10^{-4}$$



Electron decay is strong helicity suppressed.
Force the lepton in the „wrong“ helicity state, suppressed by v/c.

(For complete derivation of relative production rate, see homeworks):

$$\frac{\Gamma(\pi^- \to e^- \ \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \ \bar{\nu}_\mu)} = \left(\frac{m_e^{\ 2}}{m_\mu^{\ 2}}\right)\left(\frac{m_\pi^{\ 2} - m_e^{\ 2}}{m_\pi^{\ 2} - m_\mu^{\ 2}}\right) = 1.275\ 10^{-4}$$

excellent agreement with experiment