

 $(Q^2 = -q^2)$

Elastic Scattering:

cross-section in lab frame: proton at rest before collision

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \frac{\theta}{2}} \frac{E_3}{E_1} \begin{pmatrix} \frac{G_E^2 + \tau G_M^2}{1 + \tau} \cos^2 \frac{\theta}{2} + 2\tau G_M^2 \sin^2 \frac{\theta}{2} \end{pmatrix}$$

$$f_2(Q^2) \qquad f_1(Q^2)$$

LI cross-section

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left(f_2(Q^2)(1 - y - \frac{M^2 y^2}{Q^2}) + \frac{1}{2}y^2 f_1(Q^2) \right)$$

for large Q²

Inelastic Scattering:

cross-section in lab frame: proton at rest before collision

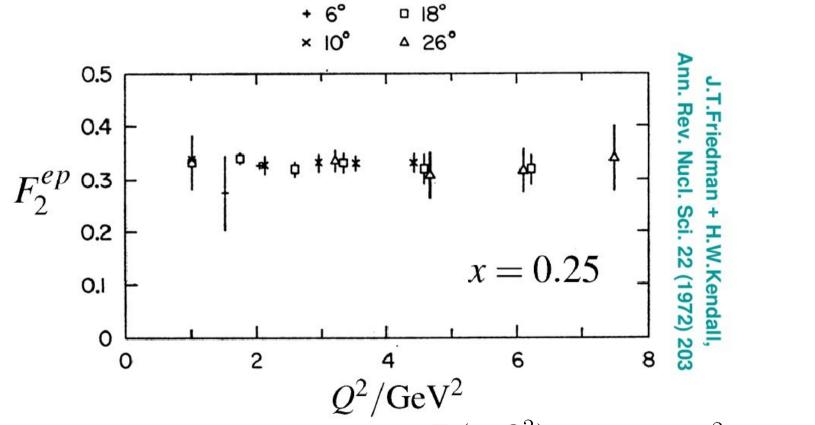
$$\frac{d\sigma^2}{dE_3 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \frac{\theta}{2}} \frac{1}{\nu} \left(F_2(x, Q^2) \cos^2 \frac{\theta}{2} + \frac{2\nu}{M} F_1(x, Q^2) \sin^2 \frac{\theta}{2} \right)$$

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4} \left((1 - y - \frac{M^2y^2}{Q^2}) \frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2) \right)$$

 $\tau = \frac{Q^2}{4M}$

Bjorken Scaling Hypothesis (1967)

"If scattering is caused by point-like constituents (partons), the structure functions for fixed x must be independent of Q^2 ."



experimental observation: structure functions $F_1(x,Q^2)$ and $F_2(x,Q^2)$ do not depend on Q^2

First evidence for point-like substructure of proton!

What is the spin of the partons?

Reminder elastic scattering: angluar dependence in Mott cross-section comes from "electron helicity conservation", thus is related to spin of incoming electron. Additional angular dependence of Dirac cross-section due to spin-spin IA of electron and proton. This term vanish in case of 0 spin of the target!

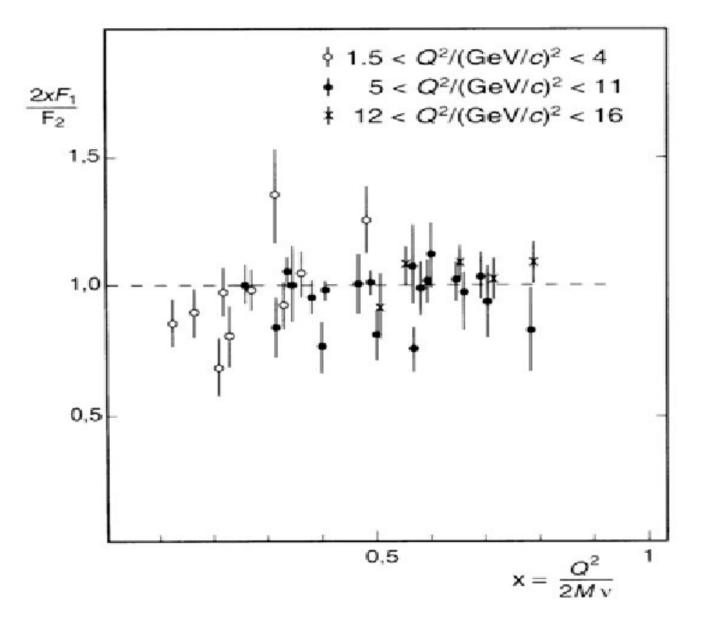
$$\left(\frac{d\sigma}{d\Omega}\right)_{Dirac} / \left(\frac{d\sigma}{d\Omega}\right)_{Mott} = 1 + \frac{Q^2}{2M^2} \tan^2 \frac{\theta}{2}$$

Inelastic scattering:

$$\left(\frac{d^2\sigma}{d\Omega dE_3}\right) / \left(\frac{d\sigma}{d\Omega Mott}\right) = \frac{1}{\nu} \left(F_2(x) + \frac{2\nu}{M}F_1(x)\tan^2\frac{\theta}{2}\right)$$
$$\mu = Mx$$
$$F_2(x) \left(1 + Q^2 xF_1(x) + Q^2 \theta\right)$$

$$= \frac{F_2(x)}{\nu} \left(1 + \frac{Q^2}{\mu^2} \frac{xF_1(x)}{F_2(x)} \tan^2 \frac{\theta}{2} \right)$$

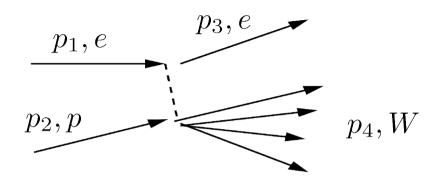
If parton spin = 0 \rightarrow $F_1(x) = 0$ If parton spin = $\frac{1}{2}$ \rightarrow $F_2(x) = 2xF_1(x)$ Callan-Gross relation



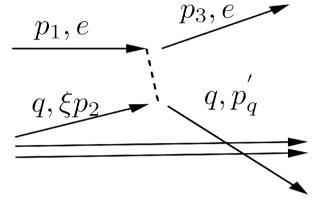
partons have spin 1/2 !

Quark-Parton Model

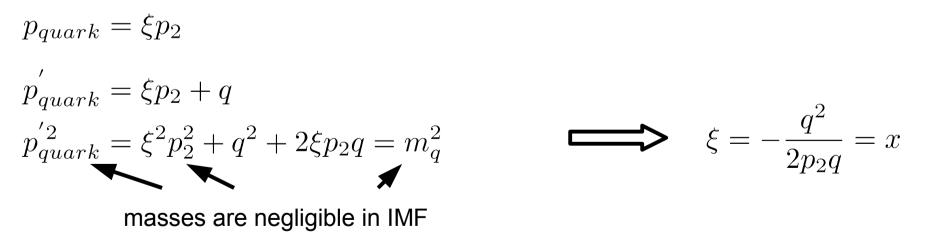
Inelastic scattering from proton



Quark-Parton-Model: elastic scattering from point-like quark within proton



quark in quark-parton model as free-particle which is only true in "infinite momentum frame", Thus assuming all masses and transverse momentum components are negligible.



Bjorken variable x can (in IMF) be identified as fraction of four momentum carried by quark involved in scatter process.

Cross section of electron with one quark which carries the momentum fraction x of the proton:

$$\left(\frac{d\sigma}{d\Omega}\right)_{quark,x} = \frac{\alpha^2 e_i^2}{4E_1^2 \sin^4 \frac{\theta}{2}} \frac{E_3}{E_1} (\cos^2 \frac{\theta}{2} + \frac{Q^2}{2M_Q^2} \sin^2 \frac{\theta}{2})$$

 $e_i: \ \ {\rm charge} \ {\rm of} \ {\rm quark} \ {\rm in} \ {\rm units} \ {\rm of} \ {\rm e} \ M_Q^2 = x^2 p_2^2 \qquad \mbox{(not the "real" quark mass)}$

Lorentz invariant form:

$$\left(\frac{d\sigma}{dQ^2}\right)_{quark,x} = \frac{4\pi\alpha^2 e_i^2}{Q^4} \left[(1-y) + \frac{y^2}{2} \right]$$

To get the complete cross-section, need to sum over all quarks in the proton and To integrate about their x-distributions.

Quark momentum distribution: $q^p(x)$

$$\left(\frac{d\sigma}{dQ^2}\right)_{quark} = \left(\frac{d\sigma}{dQ^2}\right)_{quark,x} q^p(x) dx$$
$$\left(\frac{d^2\sigma}{dxdQ^2}\right)_{quark} = \left(\frac{d\sigma}{dQ^2}\right)_{quark,x} q^p(x)$$

$$\left(\frac{d^2\sigma}{dxdQ^2}\right)_{proton} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y) + \frac{y^2}{2}\right] \times \sum_i e_i^2 q^p(x)$$

sum over all quarks in proton

compare with electron proton cross-section in terms of structure functions

$$\left(\frac{d^2\sigma}{dxdQ^2}\right)_{proton} = \frac{4\pi\alpha^2}{Q^4} \left[(1-y)\frac{F_2(x)}{x} + y^2F_1(x)\right]$$

$$F_2(x) = 2xF_1(x) = x\sum_i e_i^2 q^p(x)$$

Can related structure functions (in IMF) to quark momentum distribution!

Sum rules for quark parton distributions

$$\begin{split} u(x) &= u_v(x) + u_s(x) & u \\ d(x) &= d_v(x) + d_s(x) \\ \int_0^1 u_v^p(x) &= 2 & \text{number of u valence quarks in proton} \\ \int_0^1 d_v^p(x) &= 1 & \text{number of d valence quarks in proton} \\ \int_0^1 u_v^n(x) &= 1 & \text{number of u valence quarks in neutron} \\ \int_0^1 d_v^n(x) &= 2 & \text{number of d valence quarks in neutron} \\ \int_0^1 d_v^n(x) &= 2 & \text{number of d valence quarks in neutron} \\ \int_0^1 x(u(x) + d(x) + \overline{u(x)} + \overline{d(x)}) dx \stackrel{?}{=} 1 & \text{momentum conservation} \\ d_s(x) &= \overline{d_s(x)} & u_s(x) = \overline{u_s(x)} \end{split}$$

(heavier sea quarks strongly suppressed)

Structure function for electron proton scattering:

$$\frac{F_2^{ep}(x)}{x} = \sum_i e_i^2 q_i(x)$$
$$= \frac{4}{9} (u^p(x) + \overline{u^p(x)}) + \frac{1}{9} (d^p(x) + \overline{d^p(x)})$$

heavier sea quarks are strongly suppressed!

Structure function for electron neutron scattering:

$$\frac{F_2^{en}(x)}{x} = \sum_i e_i^2 q_i(x)$$

= $\frac{4}{9}(u^n(x) + \overline{u^n(x)}) + \frac{1}{9}(d^n(x) + \overline{d^n(x)})$
= $\frac{4}{9}(d(x) + \overline{d(x)}) + \frac{1}{9}(u(x) + \overline{u(x)})$

Isospin symmetrie: $u \Leftrightarrow d$ $p \Leftrightarrow n$

$$\frac{u(x)}{u(x)} \equiv \frac{u^p(x)}{u^p(x)} = \frac{d^n(x)}{d^n(x)} \qquad \qquad \frac{d(x)}{d(x)} \equiv \frac{d^p(x)}{d^p(x)} = \frac{u^n(x)}{u^n(x)}$$

$$\int_0^1 F_2^{ep}(x)dx = \int x\frac{4}{9}(u(x) + \overline{u(x)}) + \frac{1}{9}(d(x) + \overline{d(x)})dx = \frac{4}{9}f_u + \frac{1}{9}f_d$$

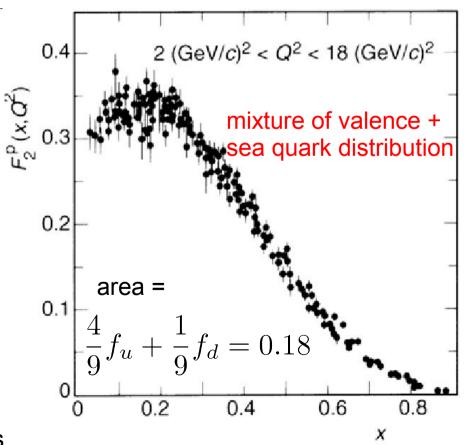
$$\int_0^1 F_2^{en}(x)dx = \int x\frac{4}{9}(d(x) + \overline{d(x)}) + \frac{1}{9}(u(x) + \overline{u(x)})dx = \frac{4}{9}f_d + \frac{1}{9}f_u$$

$$f_u = \int_{0}^{1} x(u(x) + \overline{u(x)}) dx$$
$$f_d = \int_{0}^{1} x(d(x) + \overline{d(x)}) dx$$

Experimentally found:

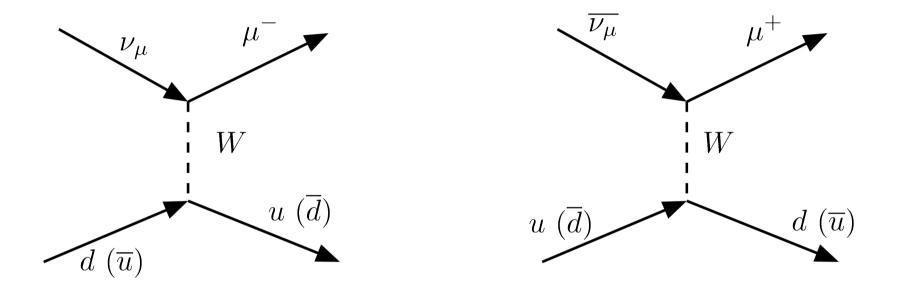
$$\int_0^1 F_2^{ep}(x) dx \sim 0.18$$
$$\int_0^1 F_2^{en}(x) dx \sim 0.12$$
$$\implies f_u = 0.36 \qquad f_d = 0.18$$

~ 50% of proton momentum is carried by quarks



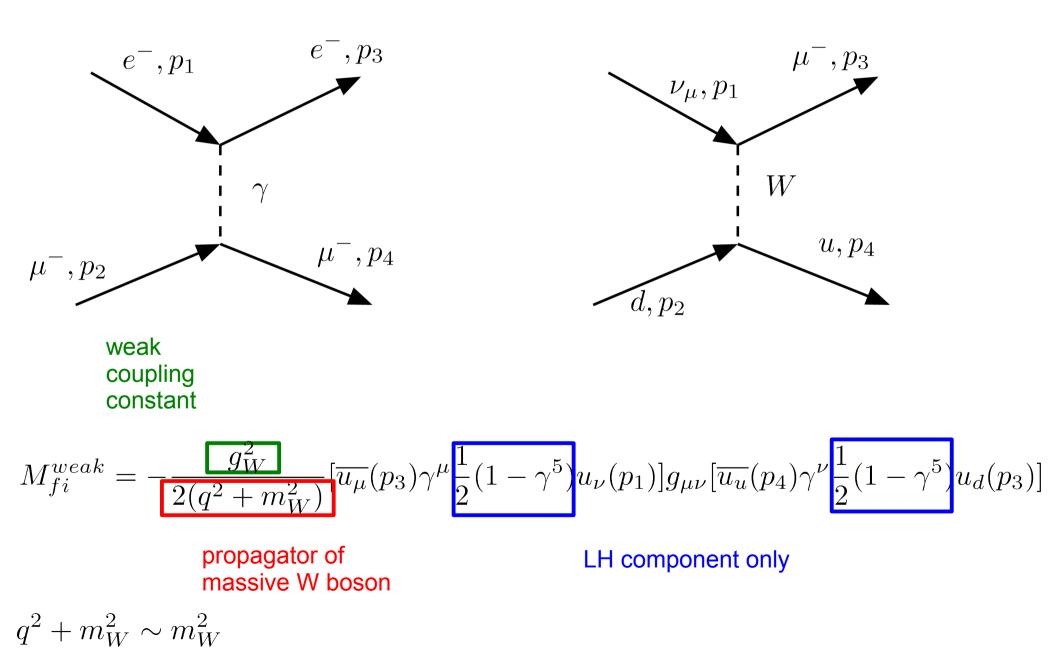
How does valence & sea quark momentum distributions look like?

Neutrino-Nucleon scattering



Property of weak IA: W boson couples only to LH particles (will be discussed in detail later)

 $M_{fi}^{QED} = -\frac{e^2}{q^2} [\overline{u_e}(p_2)\gamma^{\mu}u_e(p_1)]g_{\mu\nu}[\overline{u_{\mu}}(p_2)\gamma^{\nu}u_{\mu}(p_4)]$

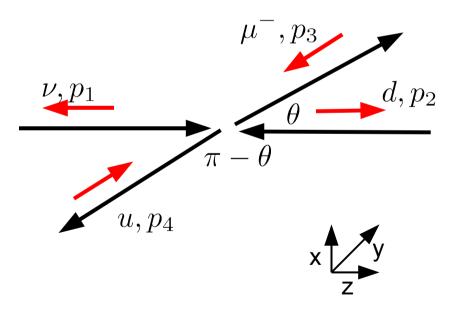


Cross-section for: $\nu + d \rightarrow \mu^- + u$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{|\vec{p_i}|}{|\vec{p_f}|} |M^{fi}|^2$$

 $\vec{p_i}$ momentum of one incoming particle $\vec{p_f}$ momentum of one outgoing particle in the following assume E >> m!

$$\begin{bmatrix} 0,0 \\ [\pi,0] \\ [\pi,0] \\ [\theta,0] \\ [\pi-\theta,\pi] \\ p_{4} = (E,-E\sin\theta,0,-E\cos\theta) \\ p_{4} = (E,-E\sin\theta,0,-E\cos\theta) \\ p_{5} = |\vec{p}| \begin{pmatrix} \sin\theta\cos\phi\\\sin\theta\sin\phi\\\cos\theta \end{pmatrix} \\ \end{bmatrix}$$



in CMS

 θ

4 common eigenstates for energy, momentum and helicity:

$$p = |p| \begin{pmatrix} \sin\theta\cos\phi\\ \sin\theta\sin\phi\\ \cos\theta \end{pmatrix}$$

particles:

$$u_{h=+1} = \sqrt{E+m} \begin{pmatrix} \cos(\theta/2) \\ e^{i\phi}\sin(\theta/2) \\ \frac{|\vec{p}|}{E+m}\cos(\theta/s) \\ \frac{|\vec{p}|}{E+m}e^{i\phi}\sin(\theta/2) \end{pmatrix} \quad u_{h=-1} = \sqrt{E+m} \begin{pmatrix} -\sin(\theta/2) \\ e^{i\phi}\cos(\theta/2) \\ \frac{|\vec{p}|}{E+m}\sin(\theta/s) \\ \frac{-|\vec{p}|}{E+m}e^{i\phi}\cos(\theta/2) \end{pmatrix}$$

antiparticles:

$$v_{h=1} = \sqrt{E+m} \begin{pmatrix} \frac{|\vec{p}|}{E+m} \sin(\theta/2) \\ -\frac{|\vec{p}|}{E+m} e^{i\phi} \cos(\theta/2) \\ -\sin(\theta/2) \\ e^{i\phi} \cos(\theta/2) \end{pmatrix}, v_{h=-1} = \sqrt{E+m} \begin{pmatrix} \frac{|\vec{p}|}{E+m} \cos(\theta/2) \\ -\frac{|\vec{p}|}{E+m} e^{i\phi} \sin(\theta/2) \\ -\cos(\theta/2) \\ e^{i\phi} \sin(\theta/2) \end{pmatrix}$$

neutrino

$$u_{h=-1}(p_1) = \sqrt{E} \begin{pmatrix} 0\\1\\0\\-1 \end{pmatrix}$$

muon

$$u_{h=-1}(p_3) = \sqrt{E} \begin{pmatrix} -\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} \\ -\cos\frac{\theta}{2} \end{pmatrix}$$

d quark $u_{h=-1}(p_2) = \sqrt{E} \begin{pmatrix} -1\\ 0\\ 1\\ 0 \end{pmatrix}$

u quark

$$u_{h=-1}(p_4) = \sqrt{E} \begin{pmatrix} -\cos\frac{\theta}{2} \\ -\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} \end{pmatrix}$$

Compute particle current:

$$\begin{split} \overline{\Psi}\gamma^{\mu}\phi &= \begin{pmatrix} \Psi^{T*}\gamma^{0}\gamma^{0}\phi \\ \Psi^{T*}\gamma^{0}\gamma^{2}\phi \\ \Psi^{T*}\gamma^{0}\gamma^{2}\phi \\ \Psi^{T*}\gamma^{0}\gamma^{3}\phi \end{pmatrix} \\ \gamma^{0}\gamma^{0} &= \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} \qquad \gamma^{0}\gamma^{1} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \\ \gamma^{0}\gamma^{2} &= \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \qquad \gamma^{0}\gamma^{3} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \\ \overline{u_{h=-1}(p_{3})}\gamma^{\mu}u_{h=-1}(p_{1}) &= 2E(\cos\frac{\theta}{2}, \sin\frac{\theta}{2}, -i\sin\frac{\theta}{2}, \cos\frac{\theta}{2}) \\ \overline{u_{h=-1}(p_{4})}\gamma^{\mu}u_{h=-1}(p_{2}) &= 2E(\cos\frac{\theta}{2}, -\sin\frac{\theta}{2}, -i\sin\frac{\theta}{2}, -\cos\frac{\theta}{2}) \end{split}$$

$$M_{fi} = \frac{g_W^2 E^2}{2m_W^2} (\cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2}) = \frac{4g_W^2 E^2}{m_W^2}$$

$$\overline{|M_{fi}|^2} = \frac{1}{2} \left| \frac{g_W^2 4E^2}{m_W^2} \right| = \frac{1}{2} \left| \frac{g_W^2 s}{m_W^2} \right|$$

neutrinos are always LH; Incoming d quarks are in 50% of the case LH, 50% RH

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{1}{2} \left(\frac{g_W^2 s}{m_W^2}\right)^2 = \left(\frac{g_W^2}{8\sqrt{2}\pi m_W^2}\right)^2 s$$

 π

 G_{F}^2s

no angular dependence

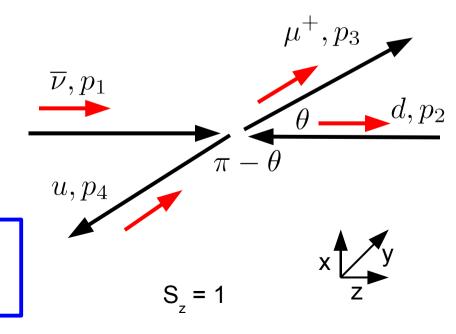
$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2} \qquad \rightarrow \frac{d\sigma}{d\Omega} = \frac{G_F^2}{4\pi^2}s$$
$$\sigma_{\nu q} = \int \frac{G_F^2}{4\pi^2} d\Omega = \frac{G_F^2s}{\pi}$$

now consider $\overline{\nu}q$ scattering,

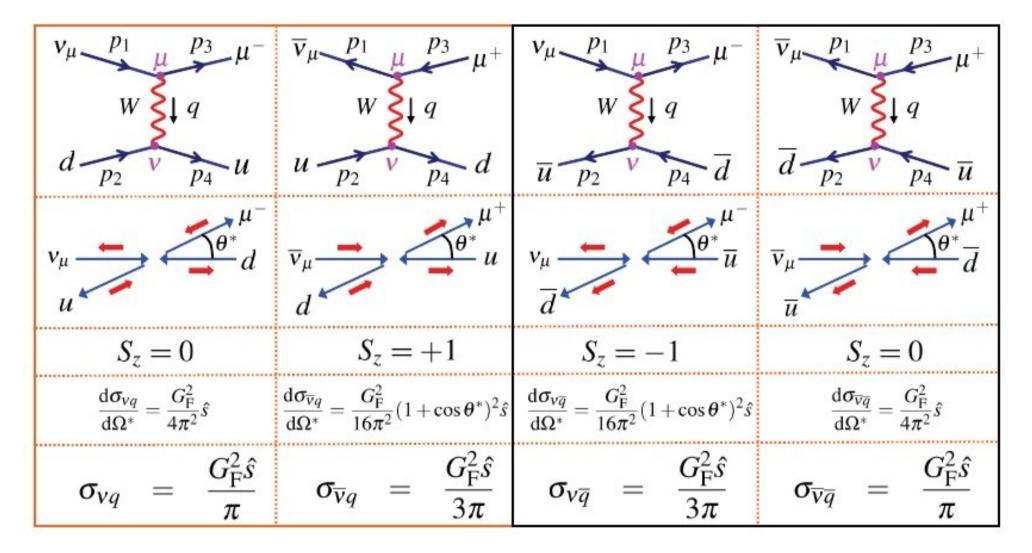
 $\overline{4\pi^2}$

same computation, but this time one LH particle current and one RH antiparticle current

$$\frac{d\sigma_{\overline{\nu}q}}{d\Omega} = \frac{G_F^2}{16\pi^2} (1 + \cos\theta)^2 s$$
$$\int (1 + \cos\frac{\theta}{2})^2 d\Omega = \frac{16\pi}{3} \quad \longrightarrow \sigma_{\overline{\nu}q}$$



Summary of (anti-)neutrino IA with valence and sea quarks



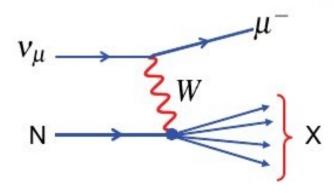
Differential cross sections still given in CMS system, transform in LI notation

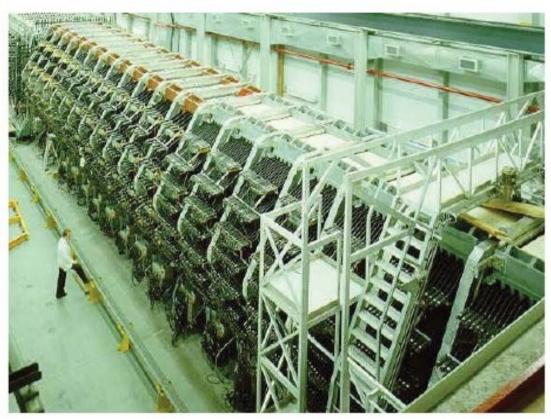
$$\frac{d\sigma_{\nu q}}{dy} = \frac{G_F^2}{4\pi^2}s \qquad \qquad \frac{d\sigma_{\overline{\nu}q}}{dy} = \frac{d\sigma_{\nu\overline{q}}}{dy} = \frac{G_F^2}{4\pi^2}(1-y)^2s \qquad \qquad \frac{d\sigma_{\overline{\nu}q}}{dy} = \frac{G_F^2}{4\pi^2}s$$

CDHS Experiment at CERN (1976 – 84) (CERN-Dortmund-Heidelberg-Saclay - Experiment)

- •1250 tons
- Magnetized iron modules
- Separated by drift chambers

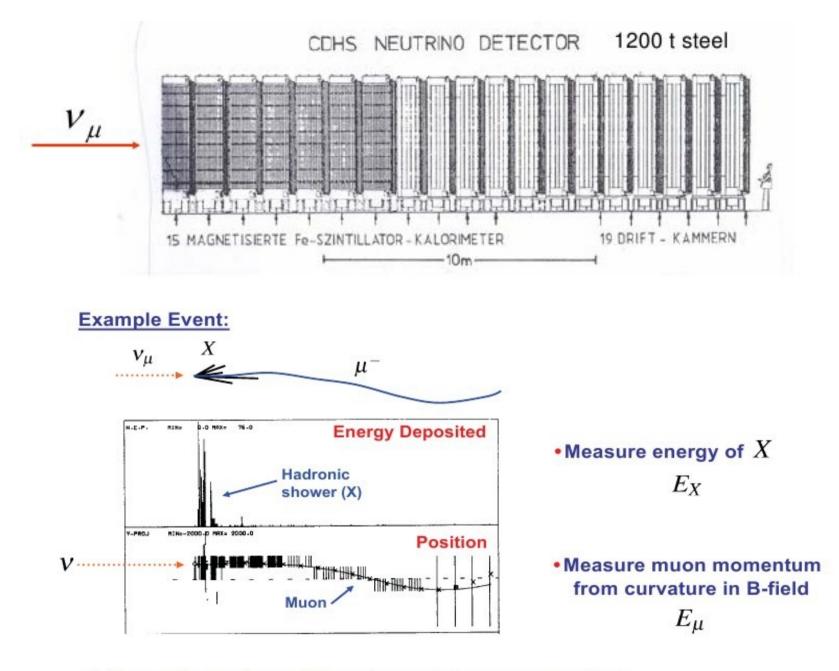
Study Neutrino Deep Inelastic Scattering





Experimental Signature:

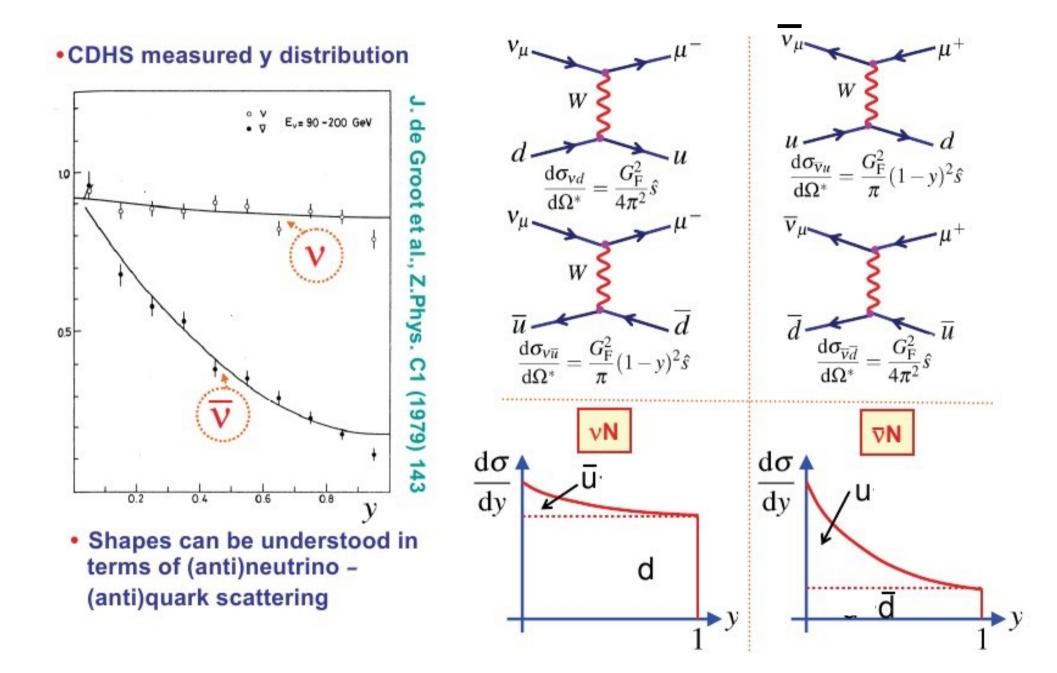
Х



* For each event can determine neutrino energy and y !

$$E_{\nu} = E_X + E_{\mu}$$
$$E_{\mu} = (1 - y)E_{\nu} \implies y = \left(1 - \frac{E_{\mu}}{E_{\nu}}\right)$$

Measured y distribution

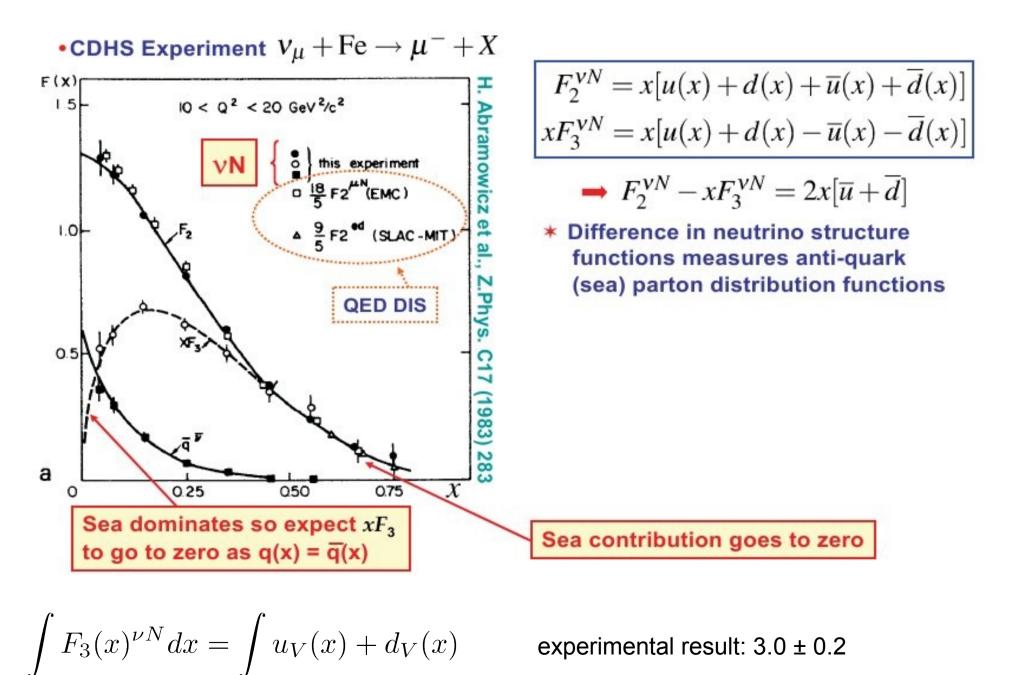


$$\frac{d\sigma^{\nu p}}{dy} = \frac{d\sigma^{\nu q}}{dy} + \frac{d\sigma^{\nu \overline{q}}}{dy} = \frac{G_F^2 sx}{\pi} \left(d(x) + (1-y)^2 \overline{u(x)} \right) dx$$

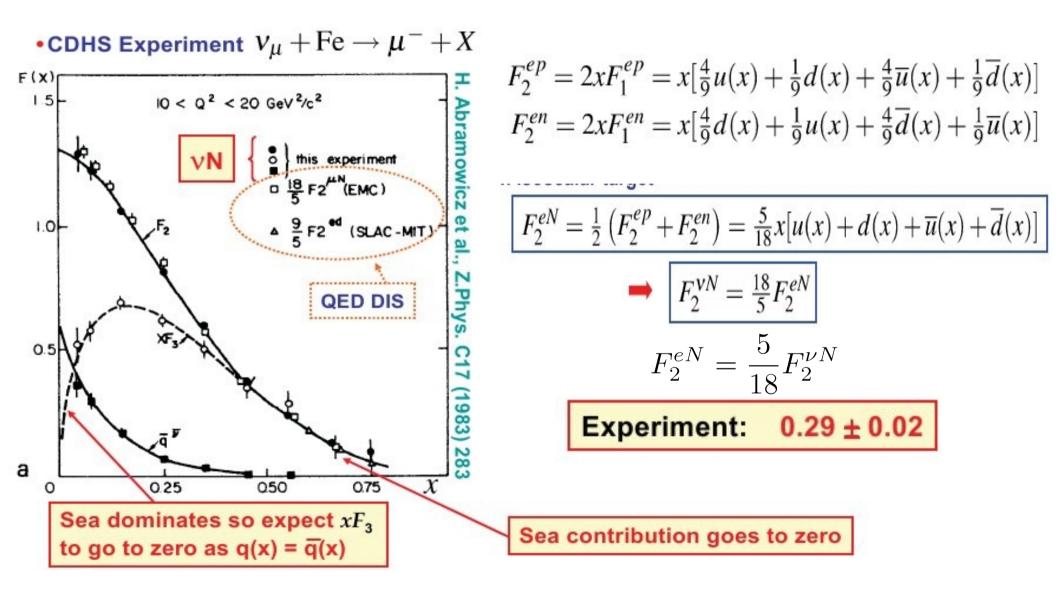
use notation with structure functions

$$\frac{d^{2}\sigma^{\nu p}}{dxdy} = \frac{G_{F}^{2}s}{2\pi} \left((1-y)F_{2}^{\nu p}(x) + y^{2}xF_{1}^{\nu p}(x) + y(1-\frac{y}{2})xF_{3}^{\nu p}(x) \right)$$
Exploit y dependence to fit for structure functions
$$1 + \frac{1}{2} + \frac{1}{$$

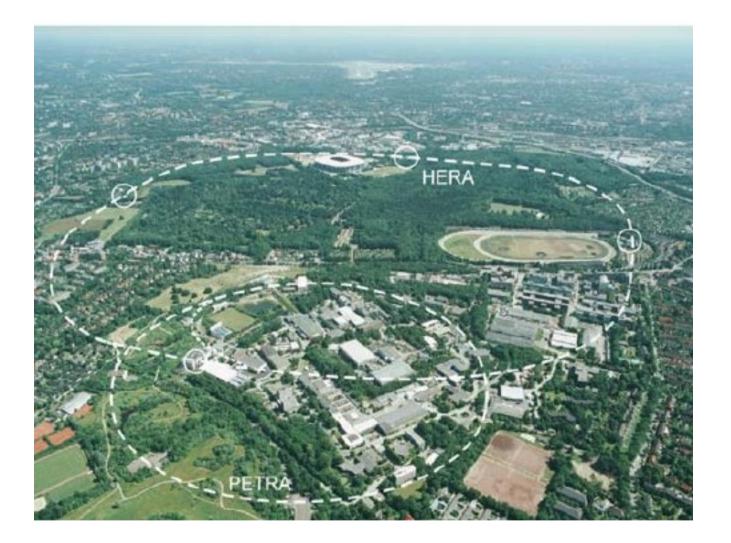
Measurement of neutrino structure functions

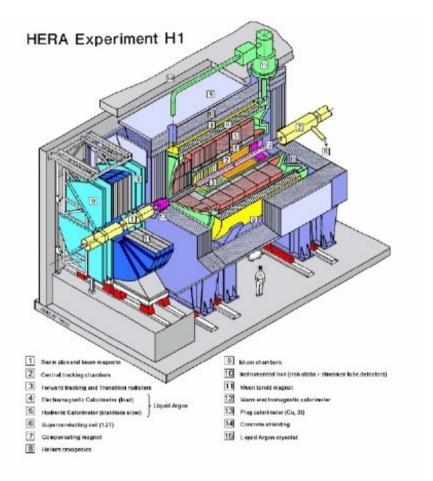


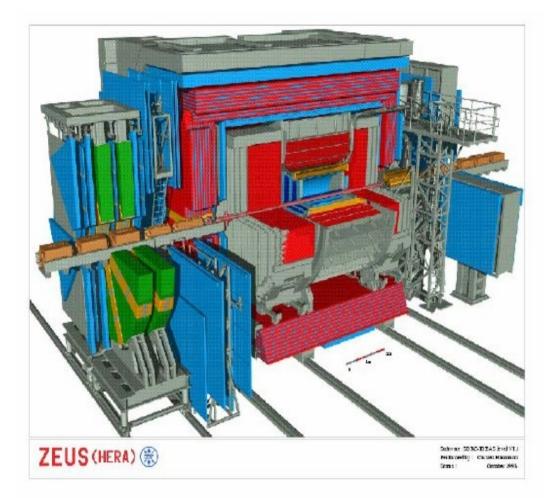
Measurement of neutrino structure functions

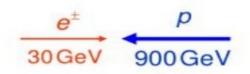


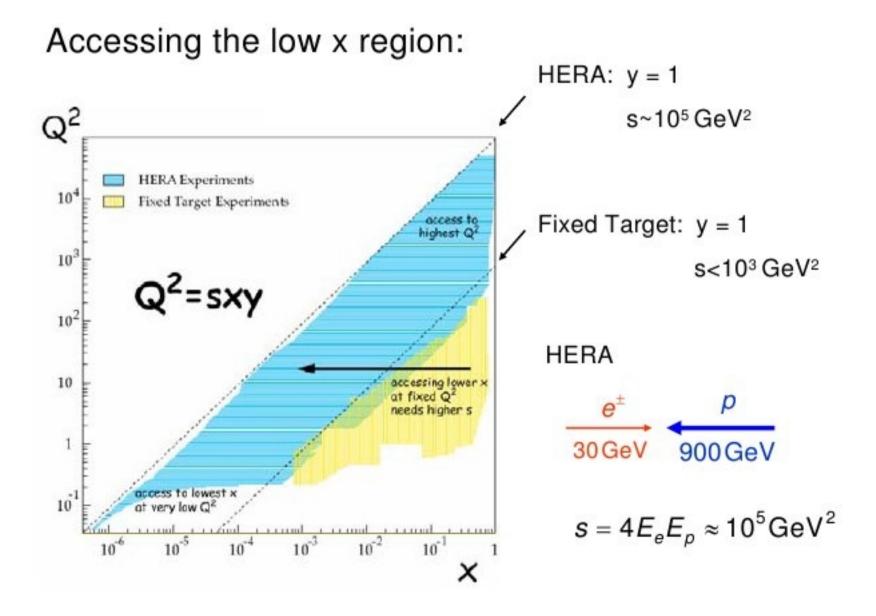
HERA Collider at DESY

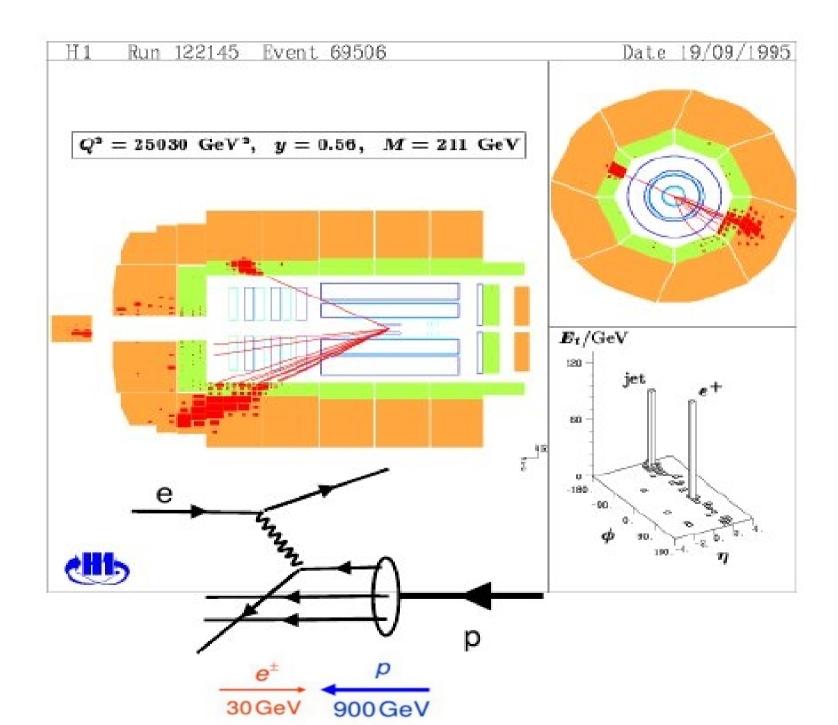




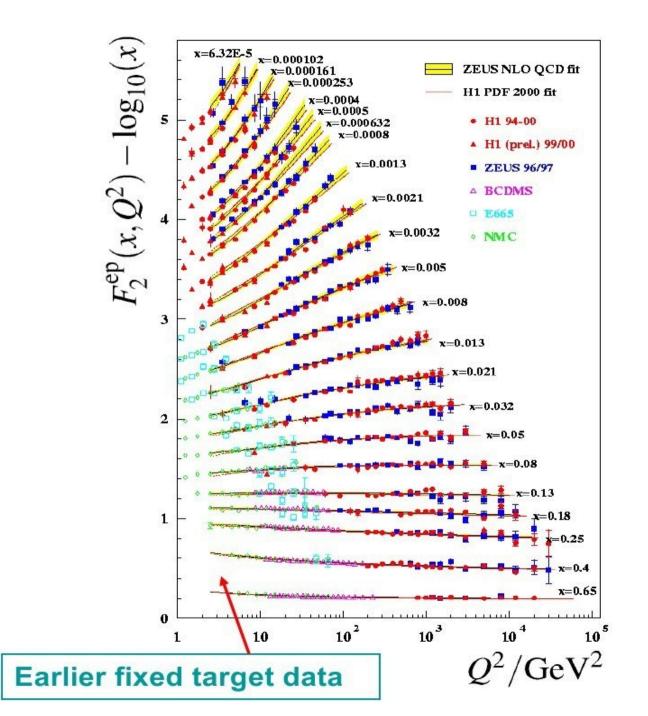








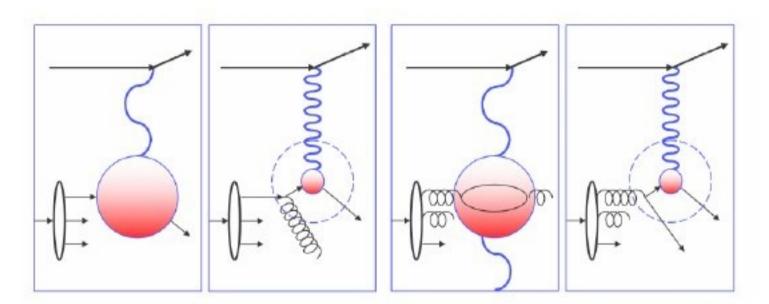
Scaling violation



QCD explains observed scaling violation

Large x: valence quarks

Small x: Gluons, sea quarks

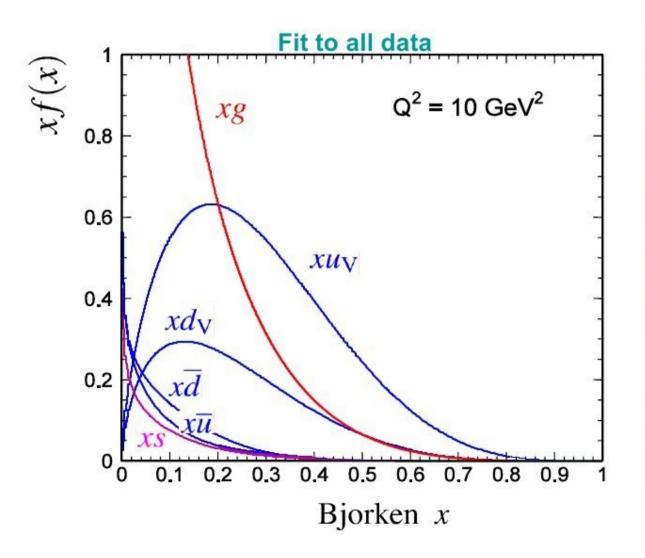


 $Q^2 \uparrow \Rightarrow F_2 \downarrow$ for fixed x

 $Q^2 \uparrow \Rightarrow F_2 \uparrow \text{ for fixed x}$

only understandable if gluon self IA are taken into account, however exactly predicted by QCD low x range exploited to measure gluon momentum functions

Parton density distribution in protons



Note: • Apart from at large *x* $u_{\rm V}(x) \approx 2d_{\rm V}(x)$ •For x < 0.2gluons dominate In fits to data assume $u_s(x) = \overline{u}(x)$ • $\overline{d}(x) > \overline{u}(x)$ not understood exclusion principle? Small strange quark component s(x)

Summary of structure of protons

- Protons consist of
 - point-like particles \rightarrow structure functions depend only on x not on x and Q²
 - with spin $\frac{1}{2} \rightarrow F_2(x) = 2xF_1(x)$
 - number of valence quarks = $3 \rightarrow$ neutrino scattering
- (valence + sea) quarks carry 50% of the proton momentum
- momentum distribution of valence and sea quarks and gluons are measured