Symbolic calculations

Some examples

```
In[1]:= x^2 + x - 4 x^2
Out[1]= x - 3 x^2
```

carry out basic algebraic simplifications

```
In[2]:= x y + 2 x^2 y + y^2 x^2 - 2 y x
Out[2]= -x y + 2 x^2 y + x^2 y^2
```

rearrange and combine terms

```
In[3]:= (x + 2y + 1)(x - 2)^2
Out[3]= (-2 + x)^2 (1 + x + 2 y)
In[4]:= Expand[%]
Out[4]= 4 - 3 x^2 + x^3 + 8 y - 8 x y + 2 x^2 y
```

Expand multiplies out products and powers

```
In[5]:= Sqrt[2]/9801 (4n)! (1103 + 26390 n) / (n!^4 396^(4n))
                 1/2 - 8 n  -2 - 4 n (1103 + 26390 n) (4 n)!
Out[5]=  2^(1/2 - 8 n) 99^(-2 - 4 n) (1103 + 26390 n) (4 n)! / (n!)^4
```

```
In[6]:= Expand[(1 + x + 3 y)^4]
Out[6]= 1 + 4 x + 6 x^2 + 4 x^3 + x^4 + 12 y + 36 x y + 36 x^2 y + 12 x^3 y +
 54 y^2 + 108 x y^2 + 54 x^2 y^2 + 108 y^3 + 108 x y^3 + 81 y^4
```

```
In[4]:= Factor[%]
Out[4]= (1 + x + 3 y)^4
```

Factor recovers the original form

Values for Symbols

One often needs to replace a symbol like x with a definite "value". Sometimes this value will be a number, it could be another expression

```
expr /. x -> value              replace x by value in the expression expr
expr /. {x -> xval, y -> yval}  perform several replacements
```

```
In[2]:= 1 + x + x^2 /. x -> 2 - y
Out[2]= 3 + (2 - y)^2 - y
```

replace x by 2-y

```
In[3]:= x -> 3 + y
Out[3]= x → 3 + y

In[4]:= x^2 - 9 /. %
Out[4]= -9 + (3 + y)^2
```

same in 2 steps

Simplifying Algebraic Expressions

```
In[1]:= Simplify[x^2 + 2x + 1]
```
Simplify writes a different form

$$\text{Out[1]}= (1+x)^2$$

Use Simplify to "clean up" complicated expressions that you get as the results of computations

```
In[3]:= Integrate[1/(x^4-1), x]
```
a more complicated calculation

$$\text{Out[3]}= -\frac{\text{ArcTan}[x]}{2}+\frac{1}{4}\text{Log}[-1+x]-\frac{1}{4}\text{Log}[1+x]$$

```
In[4]:= D[%, x]
```
differentiation should give the old form

$$\text{Out[4]}= \frac{1}{4(-1+x)}-\frac{1}{4(1+x)}-\frac{1}{2(1+x^2)}$$

```
In[5]:= Simplify[%]
```
need Simplify to get the easier form

$$\text{Out[5]}= \frac{1}{-1+x^4}$$

```
In[6]:= 2 a + 2 Sqrt[a - Sqrt[-b]] Sqrt[a + Sqrt[-b]]
```

$$\text{Out[6]}= 2a+2\sqrt{a-\sqrt{-b}}\,\sqrt{a+\sqrt{-b}}$$

```
In[7]:= Simplify[%, a > 0 && b > 0]
```
Simplifying with assumptions

$$\text{Out[7]}= 2\left(a+\sqrt{a^2+b}\right)$$

More algebraic transformations are tried by using “FullSimplify” .

See also the Online Help for a lot more functions to transform algebraic expressions.

Picking Out Pieces of Algebraic Expressions

```
In[1]:= e = Expand[(1 + 3x + 4y^2)^2]
```

$$\text{Out[1]}= 1+6x+9x^2+8y^2+24xy^2+16y^4$$

```
In[2]:= Coefficient[e, x]
```
coefficient of x in e

$$\text{Out[2]}= 6+24y^2$$

```
In[3]:= Exponent[e, y]
```
gives highest power of y that appears in expr

$$\text{Out[3]}= 4$$

```
In[4]:= r = (1 + x)/(2 (2 - y))
```

```
Out[4]=  1 + x
         -------
         2 (2 - y)

In[5]:=  Denominator[%]
Out[5]=  2 (2 - y)
```

Denominator picks out the denominator