Gluon and Gluon-Selfinteraction

3-Jet events at DELPHI

Seminar on key experiments in particle physics

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Outline

- 1. Introduction
  - Colour charge, Gluons, QCD, Asymptotic freedom, Jets
- 2. The experiment
  - Lep, DELPHI, detector acceptance simulation, event topologies
- 3. Measurements and Results
  - Goal: measurement of particle multiplicites in 3-jet events
- 4. Conclusion

1. Introduction

- Reminder: Existence of Δ^{++}: $|\Delta^{++}\rangle = |uuu\rangle\,|\uparrow\uparrow\uparrow\rangle$

  fermion (Spin 3/2) with symmetric wavefunction

  → contradiction to Pauli-Principle

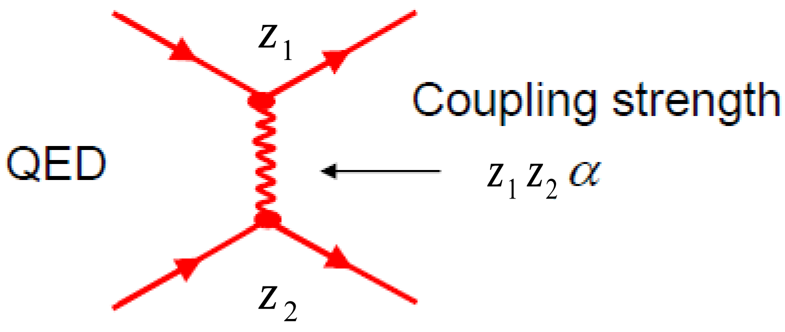
  → necessity for additional degree of freedom - „colour charge“ of quarks

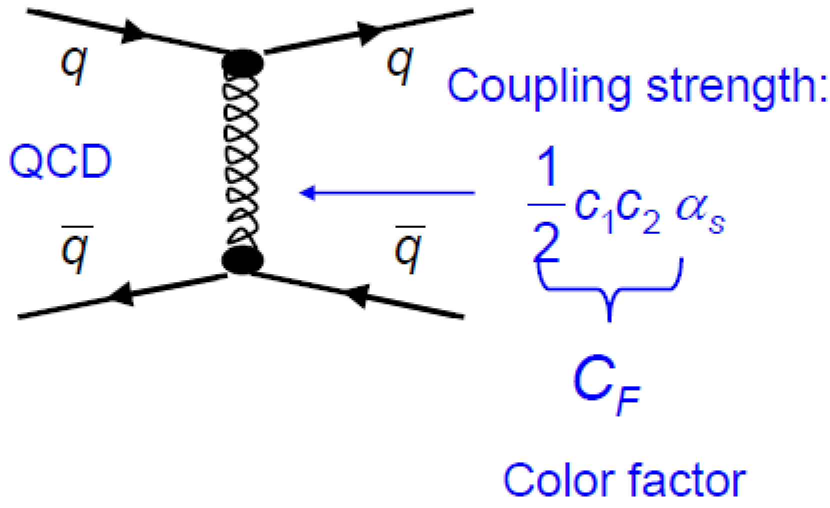
- Need 3 colour charges to construct asymmetric wavefunction
- Experimental evidence comes from determination of

$$\frac{\sigma(e^+e^- \rightarrow \mathrm{Hadron}\quad)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = N_{\mathrm{Colour}} \sum_f q_f^2$$ in $e^+e^- \rightarrow f\bar{f}$ reactions

QCD Theory - SU(3)

- Interactions are similar to QED:





- Quarks come in 3 colour states: R, G and B which form an orthonormalised basis in a 3-dimensional complex vector space
- Physical colour transformations are given by unitary matrices with det=1

  → These matrices form the group SU(3)
- Matrices do not commute → „non-abelian“ theory

Gluon states

- Physically colour exchanges are mediated by an exchange particle

  → the „Gluon“ (analogous to Photon in QED)

- SU(3) matrices have 8 independent parameters

  - these correspond to 8 gluon states:

$$|B\overline{G}\rangle \; ; \; |R\overline{B}\rangle \; ; \; -|G\overline{R}\rangle \; ; \; {}^{1}\!/\!{}_{\sqrt{2}}\left(|G\overline{G}\rangle - |R\overline{R}\rangle\right)$$

$$|\overline{B}G\rangle \; ; -|\overline{R}B\rangle \; ; \; |\overline{G}R\rangle \; ; \; {}^{1}\!/\!{}_{\sqrt{6}}\left(|R\overline{R}\rangle + |G\overline{G}\rangle - 2|B\overline{B}\rangle\right)$$

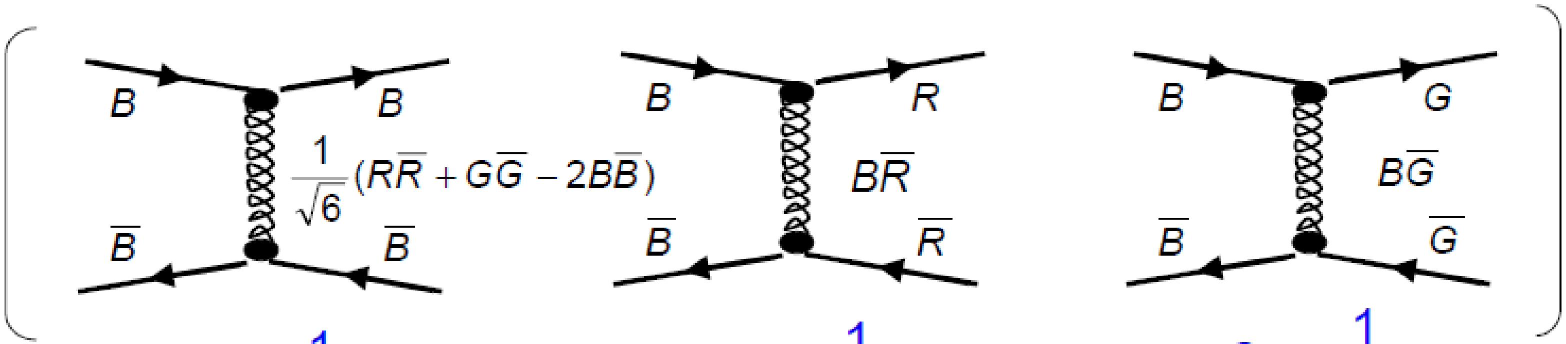
- Colour singlet (no gluon) state is given by

$$\frac{1}{\sqrt{3}}(R\overline{R} + G\overline{G} + B\overline{B})$$

Colour factor for quark anti-quark scattering

Color factor for qq color singlet state : $\frac{1}{\sqrt{3}}(R\overline{R}+G\overline{G}+B\overline{B})$

$3 \times$



$C_F = \frac{1}{3}$ $\qquad$ $C_F = \frac{1}{2}$ $\qquad$ $C_F = \frac{1}{2}$

$$\Rightarrow \quad C_F = 3 \cdot \left(\frac{1}{\sqrt{3}} \frac{1}{\sqrt{3}} \left(\frac{1}{3} + \frac{1}{2} + \frac{1}{2} \right) \right) = \frac{4}{3}$$

Color singlet is composed of 3 different possibilities

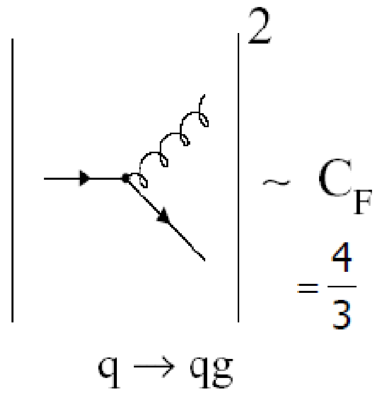
In the case of a color singlet, each initial and final state carries a factor $\frac{1}{\sqrt{3}}$

Important colour factors

- Gluon bremsstrahlung:

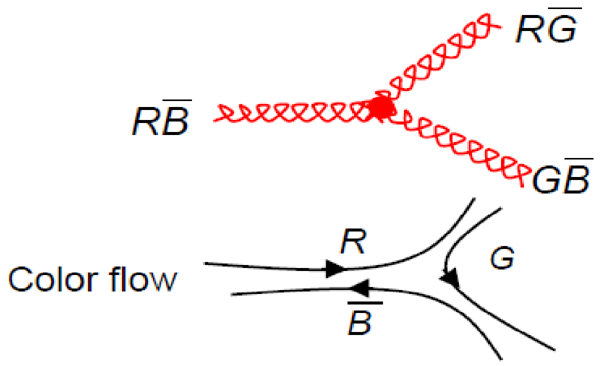
  (analoguous to singlet state

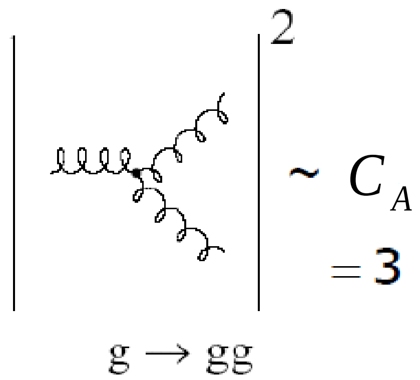
  colour factor)



- Important feature of QCD: <u>Gluon self-coupling</u>

  → gluons are also source of a gluon field !

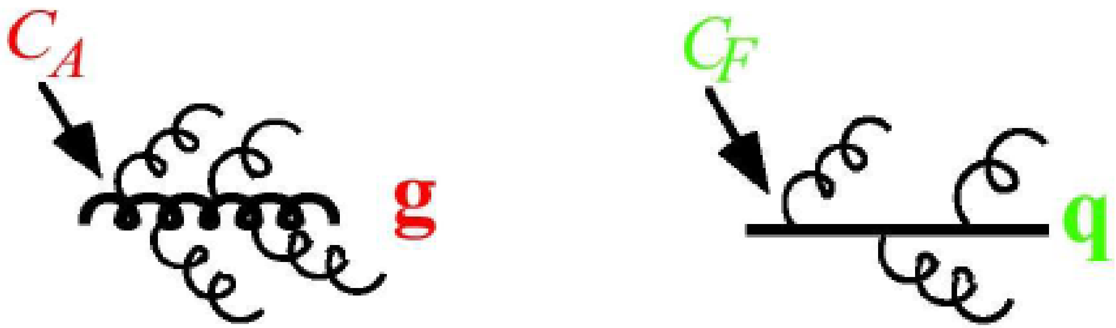




- Aim of the experiment will be to determine the colourfactor ratio C_A/C_F to test QCD

Determination of C_A/C_F

- Problem: Only colourless states appear in nature – How can we measure the coupling strengths of single quarks and gluons ?
- Idea: Gluon-Bremsstrahlung is proportional to the charge:



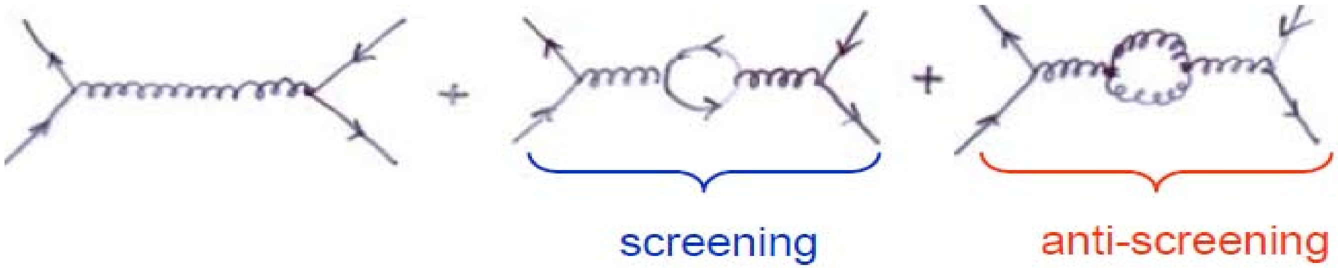
- Radiated gluons convert into observable (colour-neutral) particles

$\rightarrow$ naive expectation: $$\frac{N_{had}(g)}{N_{had}(q)} \approx \frac{C_A}{C_F} = \frac{3}{4/3} = \frac{9}{4}$$

But: How can we tell which hadrons descend from a primary quark or a gluon ?

Asymptotic Freedom

- Propagator corrections:



- Lead to effective coupling constant:

$$\boxed{\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1+\alpha_s(\mu^2)\underbrace{\frac{1}{12\pi}(33-2n_f)}_{\beta_0}\log\frac{Q^2}{\mu^2}}}$$

$$\beta_0 = \frac{1}{12\pi}(33-2n_f)$$

n_f = active quark flavors

μ^2 = renormalization scale

conventionally $\mu^2 = M_Z^2$

Introduce scale Λ_{QCD} at which perturbative solutions diverge:

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\beta_0 \log\frac{Q^2}{\mu^2}} \longleftarrow \mu^2 = \Lambda_{QCD}^2$$

$$\Lambda_{QCD} \approx 200 \text{ MeV}$$

(parameter, must be determined experimentally)

$$\Rightarrow \frac{1}{\alpha_s(Q^2)} = \underbrace{\frac{1}{\alpha_s(\Lambda_{QCD}^2)}}_{=0} + \beta_0 \ln(Q^2/\Lambda_{QCD}^2)$$

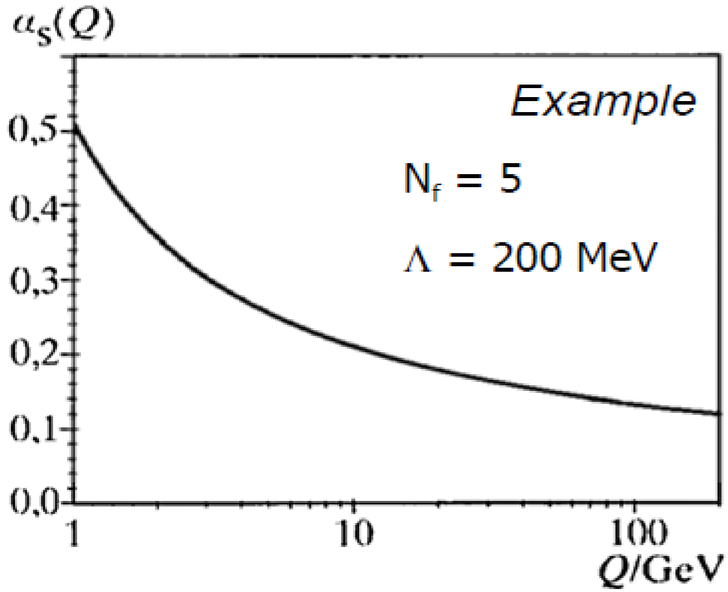
$$\Rightarrow \alpha_s(Q^2) = \frac{1}{\beta_0 \log(Q^2/\Lambda_{QCD}^2)}$$

→ **Confinement for small Q^2**

→ **Asymptotic freedom for large Q^2**

For large Q^2 quarks can be treated as free particles: → **Quark Parton Model**

Gross&Wilczek (1973), Politzer (1974)



- Important fo us: smaller coupling constant leads to weaker Bremsstrahlung-effects

Idea of the experiment

- Aim: Measurement of the colour factor ratio C_A/C_F in order to test SU(3) symmetry of QCD

  Problem: only colourless states appear in nature
  → quarks and gluons are not directly measureable

- Underlined "However:" Asymptotic freedom surpresses radiation of „hard“ gluons which can significantly redirect the original flow of energy and momentum, „soft“ gluons with low transverse momentum are not surpressed !

  → formation of Jets

Jets

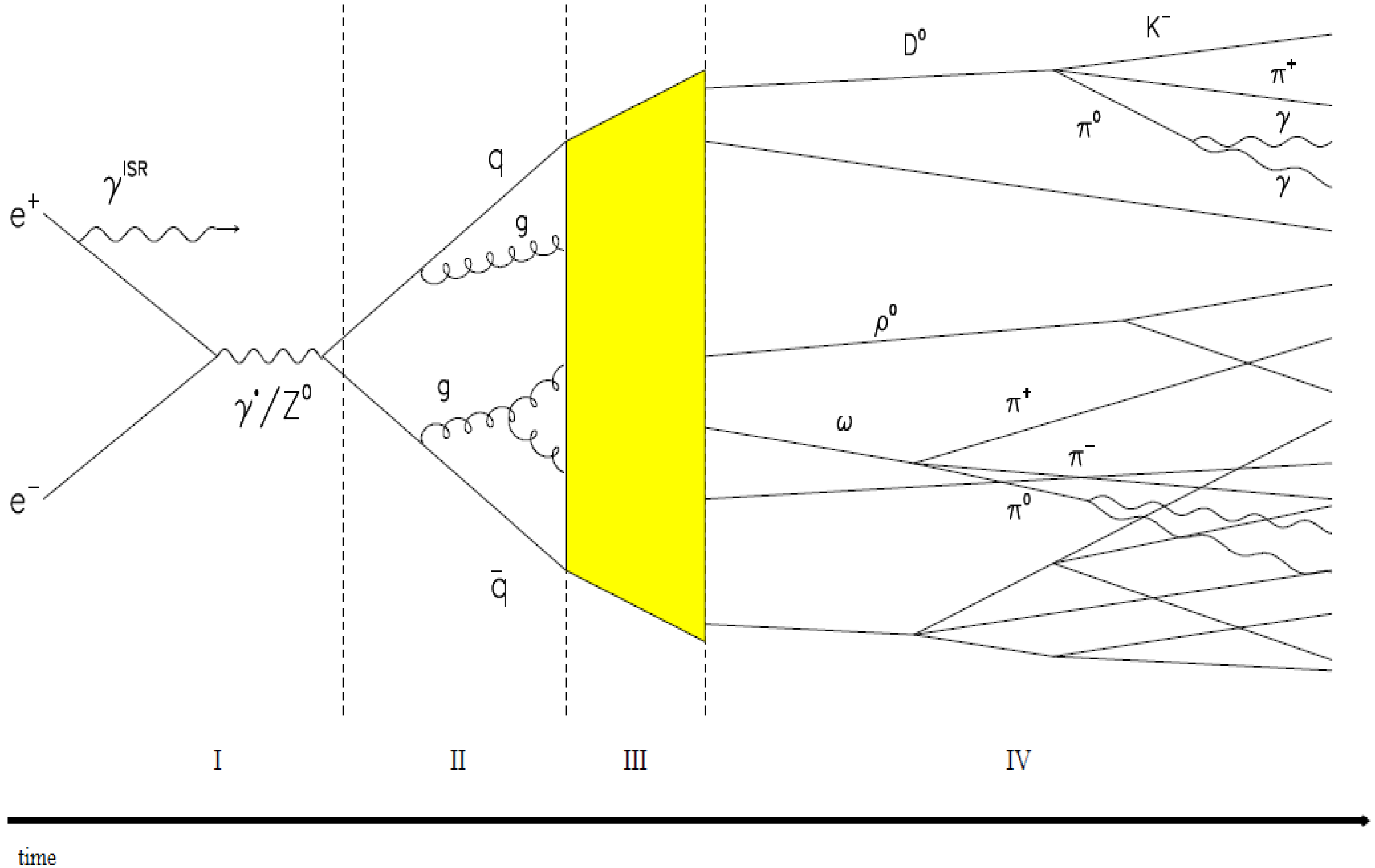
- bundles of particles traveling in the direction of the partons
- 4 phases:

  1. Annihilation

  2. Gluon radiation

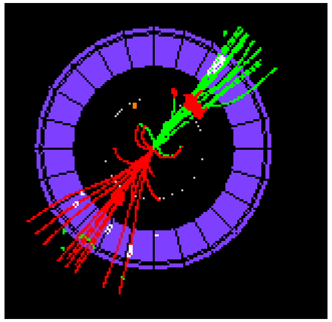
  3. Hadronisation

  4. Decay to stable particles

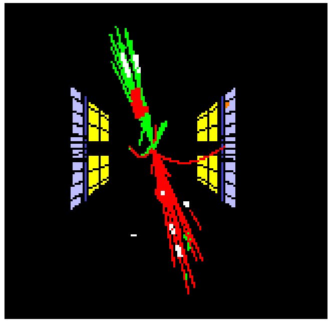


2- and 3-Jet events

- 2-Jet events occur primarily: quark + antiquark, emerging back-to- back because of momentum conservation



xy view



zy view

- 3-Jet events: one of the quarks radiates „hard“ gluon which significantly redirects flow of energy and momentum

  → led to discovery of gluon at PETRA 1977

$$\frac{\#3-\text{jet events}}{\#2-\text{jet events}} \approx 0.15$$

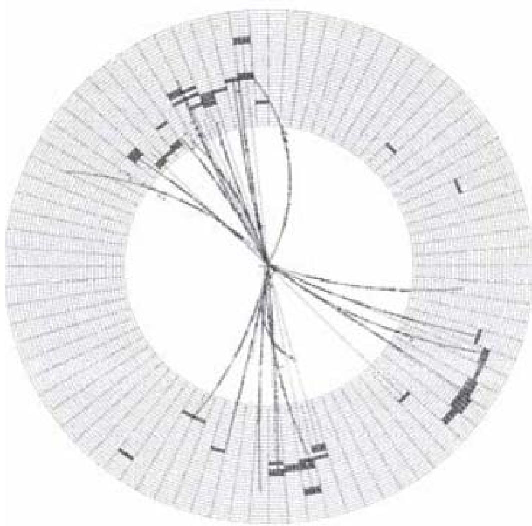
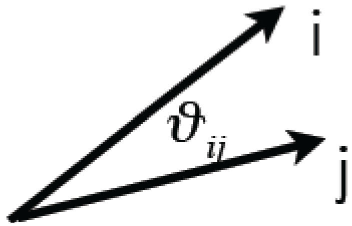


Fig. 11.12 A three-jet event observed by the JADE detector at PETRA.

Jet reconstructions – JADE algorithm

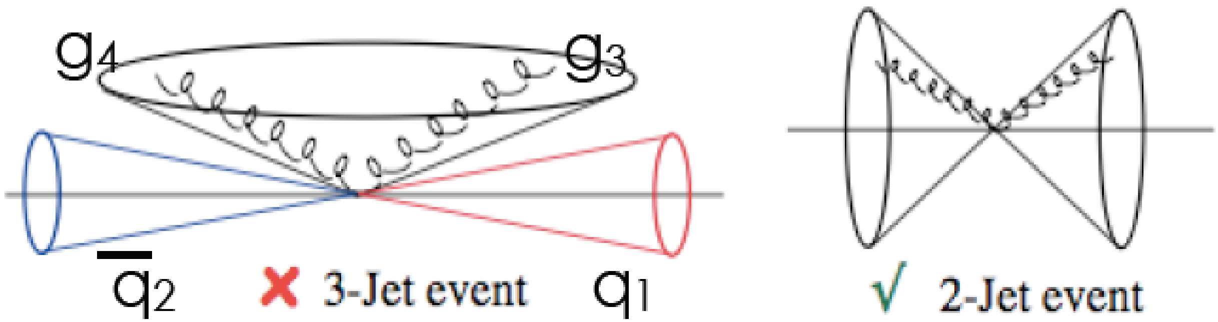
- for every pair of particles (i, j) compute a closeness measure y_{ij}

$$y_{ij} = \frac{2E_i E_j (1-\cos\vartheta_{ij})}{E_{cm}^{\;2}} \approx \frac{m_{ij}^{\;2}}{E_{cm}^{\;2}}$$



- the two particles are combined if $y_{ij} < y_{cut}$
- iterate until all particle pair satisfy $y_{ij} > y_{cut}$
- tendency to reconstruct phantom jets

$y_{34} < y_{13}, y_{24}$

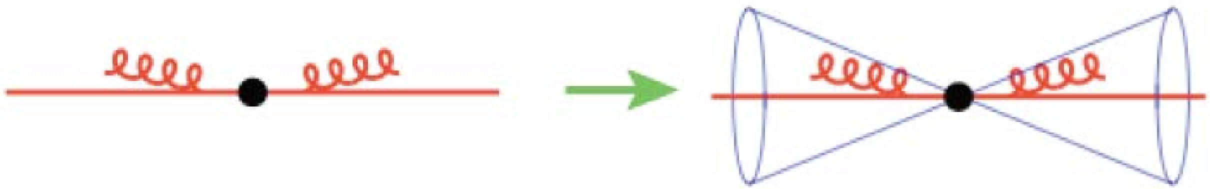


Durham algorithm

- introduced to reduce the JADE problem of phantom jets
- the metric is
$$y_{ij} = \frac{2\min(E_i^2, E_j^2)(1-\cos\vartheta_{ij})}{E_{cm}^2}$$
- for small emission angles what matters is the min transverse momentum of one particle w.r.t. the other

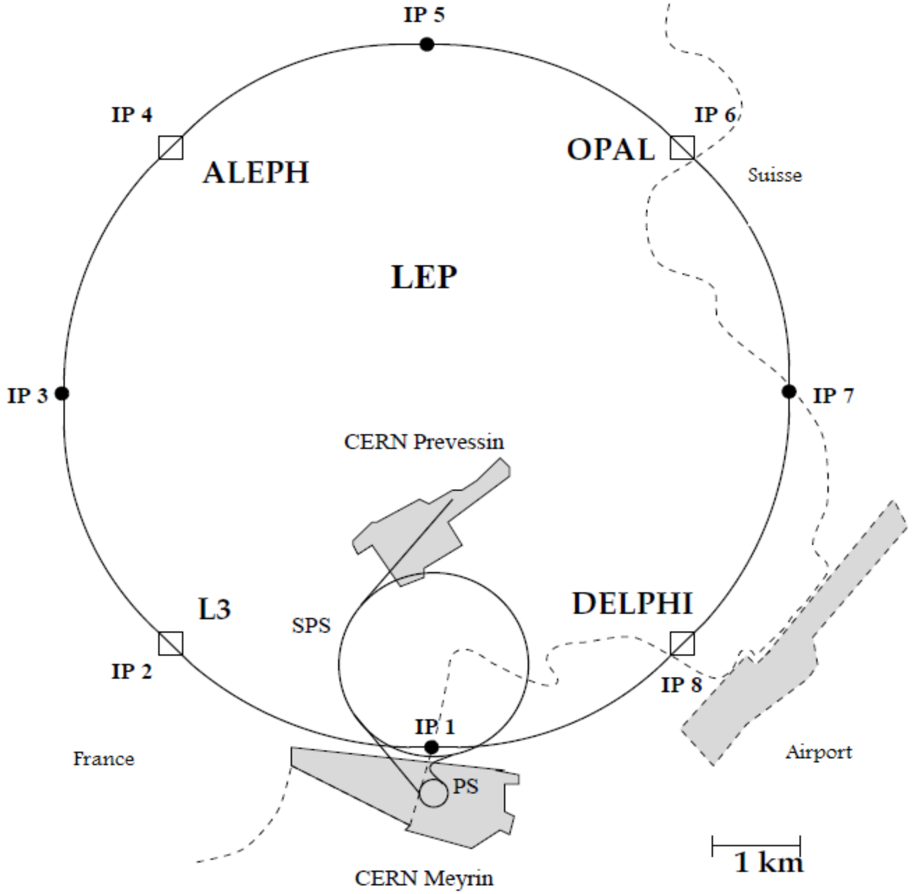
$$\vartheta_{ij} \to 0 \Rightarrow y_{ij} \approx \frac{2\min(E_i^2, E_j^2)[1-(1-\vartheta_{ij}^2/2+\ldots)]}{E_{cm}^2} \approx \frac{\min(E_i^2, E_j^2)\vartheta_{ij}^2}{E_{cm}^2} \approx \frac{\min(k_{ti}^2, k_{tj}^2)}{E_{cm}^2}$$

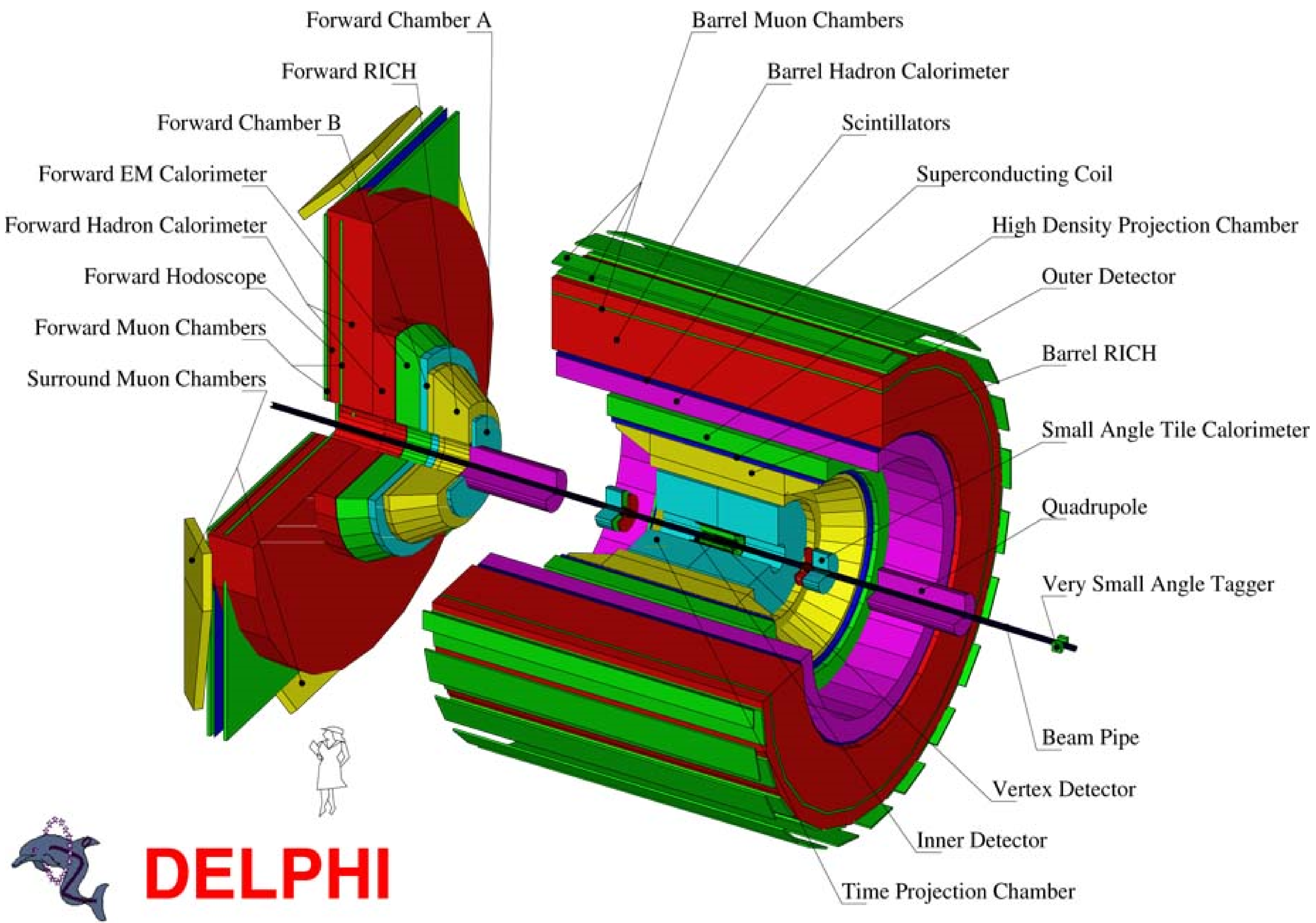
- soft, collinear radiations is attached to the quark jets



2. The experiment

- LEP: Large Electron Positron Collider, predecessor of LHC
- Running through 1989-2000
- Top energy 209 GeV




Forward Chamber A
Forward RICH
Forward Chamber B
Forward EM Calorimeter
Forward Hadron Calorimeter
Forward Hodoscope
Forward Muon Chambers
Surround Muon Chambers
Barrel Muon Chambers
Barrel Hadron Calorimeter
Scintillators
Superconducting Coil
High Density Projection Chamber
Outer Detector
Barrel RICH
Small Angle Tile Calorimeter
Quadrupole
Very Small Angle Tagger
Beam Pipe
Vertex Detector
Inner Detector
Time Projection Chamber
DELPHI

Data analysis

- Data cuts were applied in order to:
  - get well measured particles originating from the interaction point
  - select hadronic decays of the Z and surpress background from leptonic Z decays or beam-gas interactions

Table 1. Selection cuts applied to charged-particle tracks and to calorimeter clusters

variable	cut
p	≥ 0.4 GeV
ϑ_{polar}	$20^\circ - 160^\circ$
ϵ_{xy}	≤ 5 cm
ϵ_z	≤ 10 cm
L_{track}	≥ 30 cm
$\Delta p/p$	$\leq 100\%$
E_{HPC}	0.5 GeV – 50 GeV
E_{EMF}	0.5 GeV – 50 GeV
E_{HAC}	1 GeV – 50 GeV

Table 2. Selection cuts applied to general events and to three-jet events

variable	cut
general events	
$E_{\text{charged}}^{\text{hemisph.}}$	$\geq 0.03 \cdot \sqrt{s}$
$E_{\text{charged}}^{\text{total}}$	$\geq 0.12 \cdot \sqrt{s}$
N_{charged}	≥ 5
$\vartheta_{\text{sphericity}}$	$30^\circ - 150^\circ$
$p_{\max}$	45 GeV
three-jet events	
$\sum_{i=1}^{3} \theta_i$	$> 355^\circ$
$E_{\text{visible}}/\text{jet}$	≥ 5 GeV
$N_{\text{charged}}/\text{jet}$	≥ 2
ϑ_{jet}	$30^\circ - 150^\circ$

Event topologies

- 3-jet events have to be planar due to momentum conservation
- divided into two topologies:

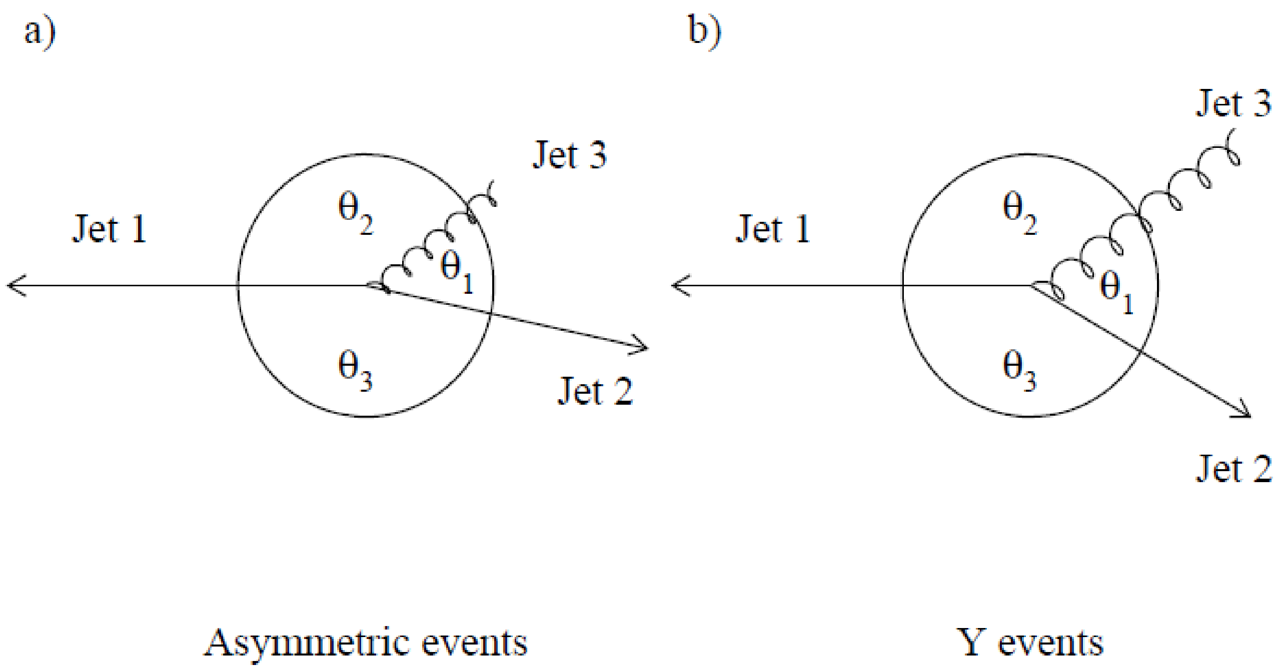


Figure 1: Definition of event topologies and angles used throughout this analysis. The lengths of the jet lines indicate the energies.

Preparation of the experiment

- What was measured was *the number of particles* in 3-jet events in dependance of the interjet angles
- The topology of the event is determined by two angles (θ_2 and θ_3) for asymmetric, resp. One angle (θ_1) for symmetric events
- Angles are divided into bins of 5° (as Monte-Carlo studies have shown that differences between the interjet angles in the partonic and in the hadronic state scatte with a standard deviation of 5°)
- For each bin a matrix correction is used
- The matrix M_{mn} is calculated by tracing in a Monte Carlo simulation how many particles *m* in an accepted event with *n* detected particles have been generated

Theoretical predictions

- Reminder: naive expectation was: $\frac{N_{had}(g)}{N_{had}(q)} \approx \frac{C_A}{C_F}$
- Problem: N_g/N_q is know to have large corrections in higher order and is supposed to be affected by non-perturbative effects during the hadronisation
- Ratio of the derivatives of the multiplicities with respect to the energy $N_g'(s)/N_q'(s)$ is a better calcuable observable
- Colour dipole model: $\left.\frac{dN_{gg}(L')}{dL'}\right|_{L'=L+c_g-c_q} = \frac{C_A}{C_F}\left(1-\frac{\alpha_0 c_r}{L}\right)\frac{d}{dL}N_{q\bar{q}}(L)$
- Two hypotheses: $N_{q\bar{q}g} = N_{q\bar{q}}(L_{q\bar{q}}, \kappa_{\mathrm{L}u}) + \frac{1}{2}N_{gg}(\kappa_{\mathrm{L}e})$ (A)

(Lu - Lund, Le - Leningrad-formalism) $N_{q\bar{q}g} = N_{q\bar{q}}(L, \kappa_{\mathrm{L}u}) + \frac{1}{2}N_{gg}(\kappa_{\mathrm{L}u})$ (B)

(A) $q\bar{q}$-contribution is determined mainly by the invariant mass of the $q\bar{q}$-system

(B) $q\bar{q}$-contribution given by the centre-of-mass energy of the whole event reflecting the phase-space available to the $q\bar{q}$-pair if no hard gluon had been emitted

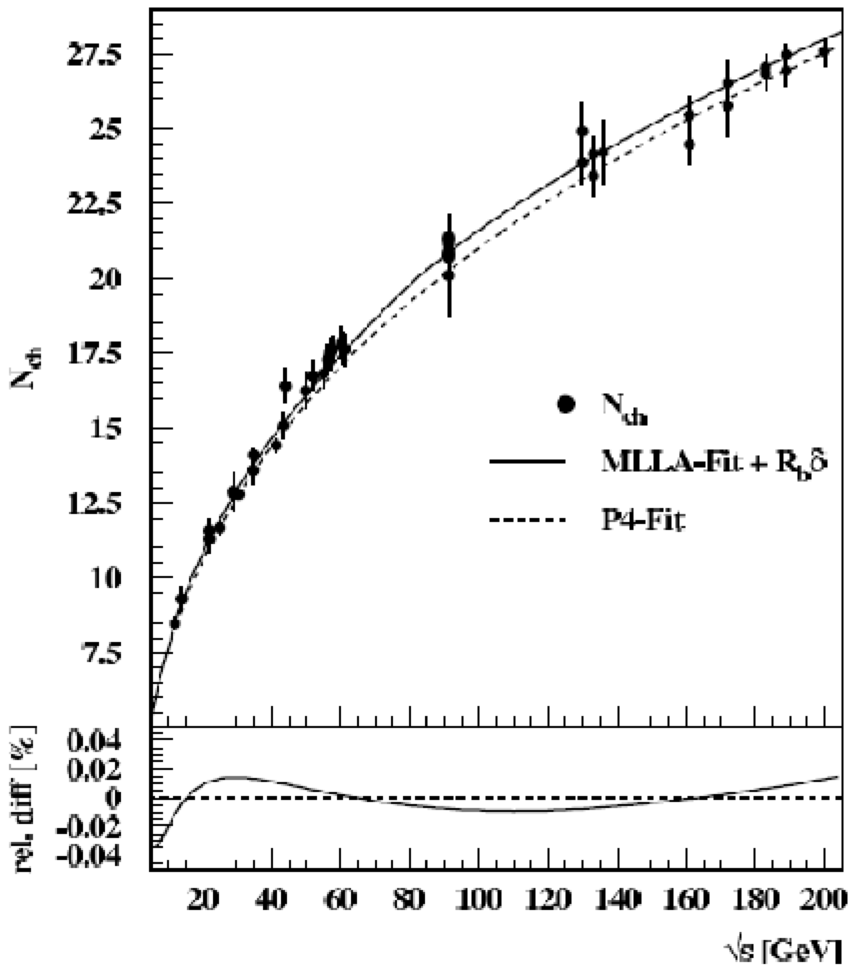
4. Measurements and results

- Mean multiplicities in 3-jet events at $\sqrt{s} = m_Z$:
  20.983 ±0.004 for *udscb*-events and
  20.353 ±0.004 for *udsc*-events
  $\rightarrow$
  $$N_0 \equiv \delta_{udscb-udsc} = N_{udscb} - N_{udsc}$$
  $$= 0.610 \pm 0.002_{\text{stat.}}$$

- $N_{q\bar{q}}(\sqrt{s})$ (without b-quarks) Taken out of other experiments:
  $$N_{q\bar{q}}(m_Z) = 20.2561$$

- To fix the constant of integration, a measurement of N_{gg} from χ' decays by the CLEO collaboration is used:

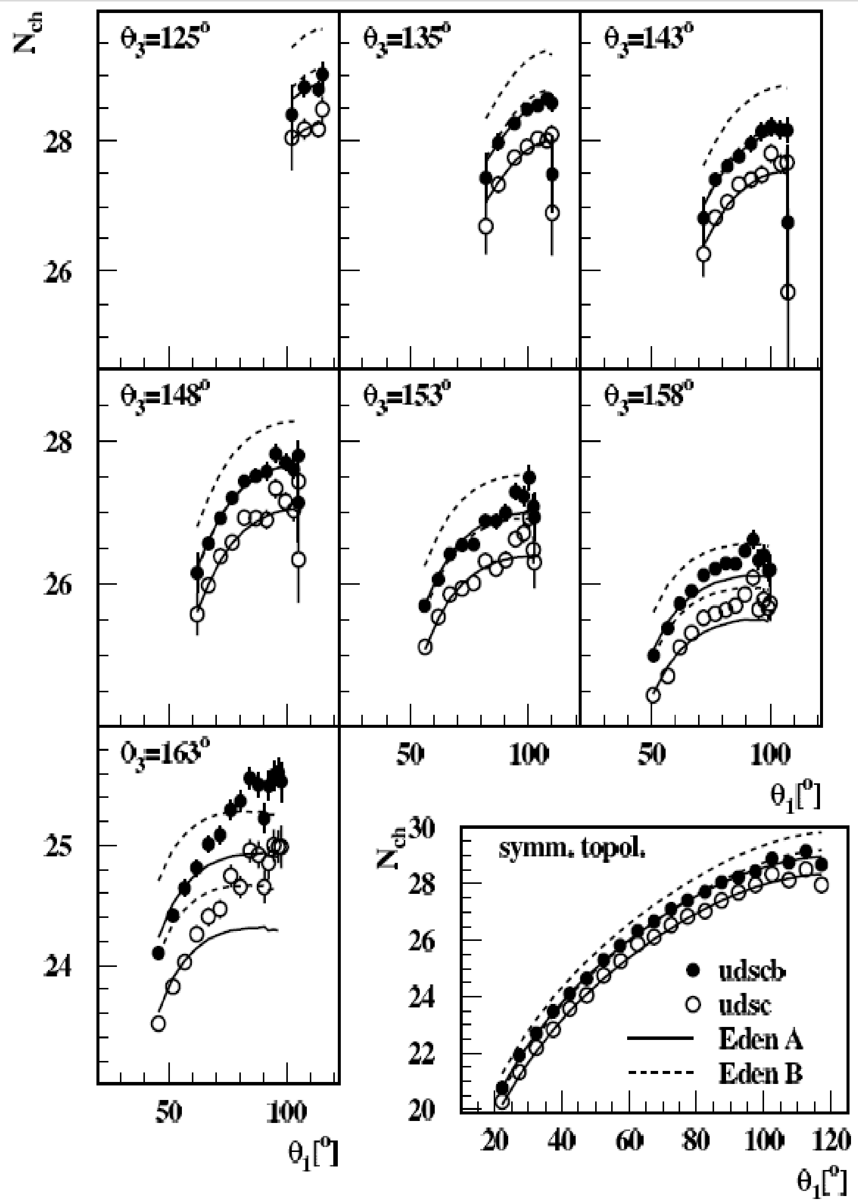
$$N_{gg}(9.9132\ \text{GeV}) = 9.339 \pm 0.090 \pm 0.045$$



Results

- Very good agreement with prediction (A)
- Prediction (B) overestimates the multiplicity by ~0.6 and also seems to overestimate the slope
- For large θ_3 and then especially for large θ_1 larger deviations occur, probably because of interactions between jet 2 and 3

→ (B) should be rejected in favour of (A)

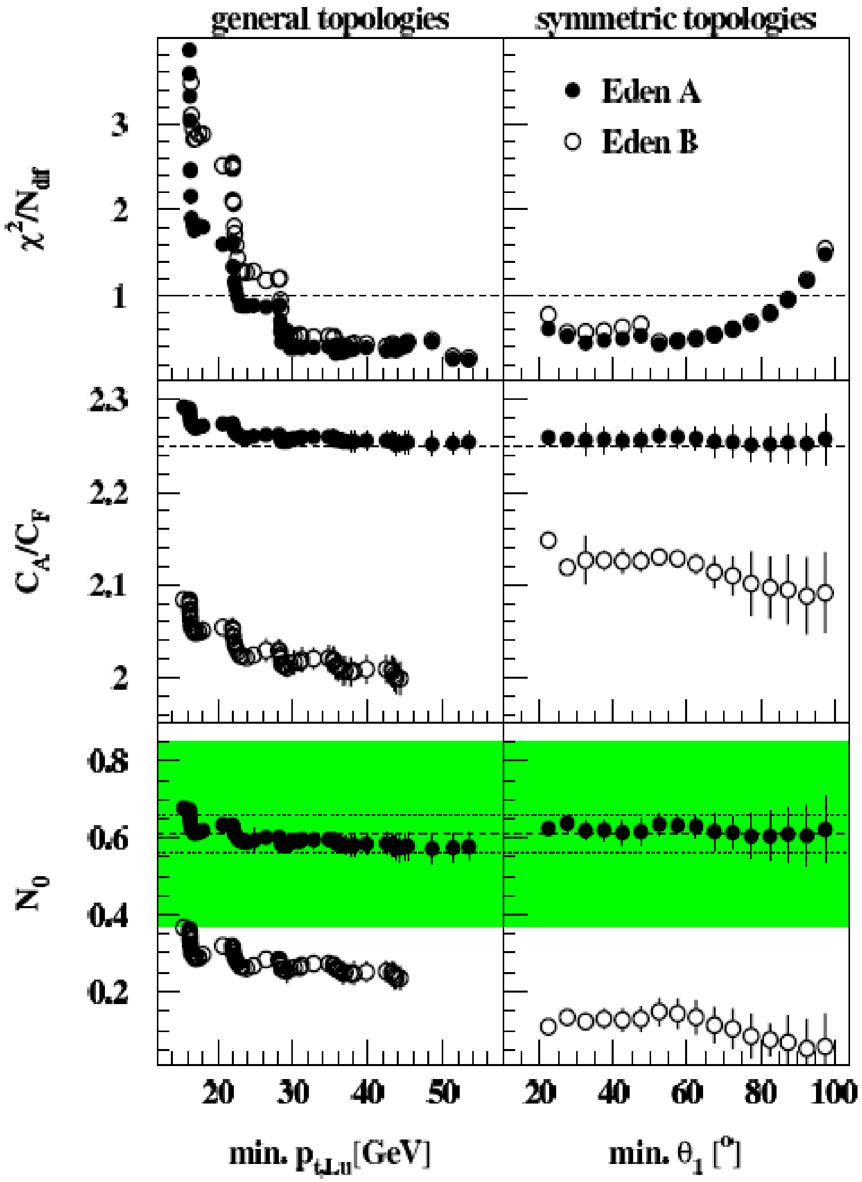


Fits

- C_A/C_F enters the prediction only via the derivatives of the multiplicities

  → additional constant N_0 is allowed to vary freely in the fit

- Again prediction (A) is in very good agreement with the expectation of N_0=0.61, prediction (B) fails to produce the expected result



Discussion of errors

- In order to estimate systematic errors, the following has been done :
  - Variation of the cuts on event- and jet-structure
  - The *udsc*-sample was used to estimate the uncertainty due to the b-quark multiplicity
  - The data set used for N_{qq} was varied
  - A different hadronisation correction simulation was used
  - Different cluster algorithms were used (Durham, Cambridge, Luclus)
  - The scale variable Λ has been varied between 200MeV and 300MeV
  - The constant c_r has been varied within theoretical uncertainties of about 10%
  - The CLEO measurement of N_{gg} has been varied within the given erros

Result

- Results obtained with Prediction (A):

$$\frac{C_A}{C_F} = 2.257 \pm 0.019_{\text{stat.}} \pm 0.056_{\text{exp.}} \pm 0.070_{\text{theo.}}$$

  for symmetric topologies and

$$\frac{C_A}{C_F} = 2.261 \pm 0.014_{\text{stat.}} \pm 0.036_{\text{exp.}} \pm 0.066_{\text{theo.}}$$

  for general topologies,

  - this is the most precise measurement of this quantity so far !

- The results agree well with the SU(3) ecpectation of QCD and within other measurements

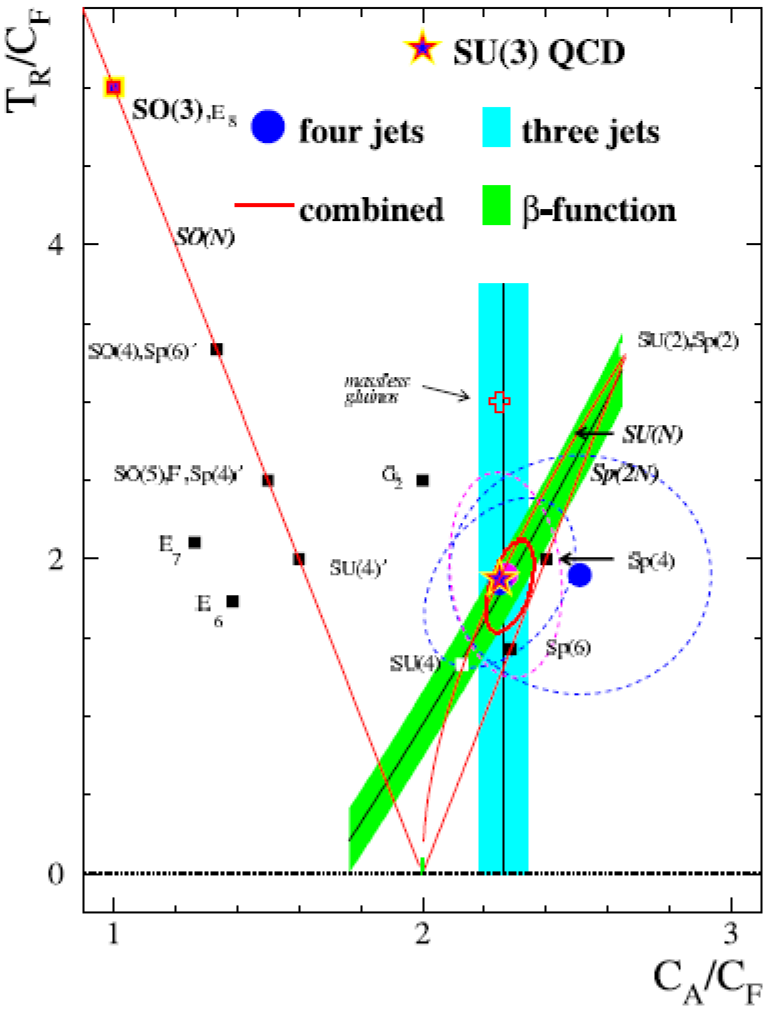


Fig. 9. The plot shows the Casimir eigenvalues expected for the special orthogonal (SO(N), straight line on the left) and the special unitary (SU(N), curved line on the right) groups as well as for the symplectic groups (SP(2N), straight line on the right). E_6, E_7, E_8, G_2 and F are the five exceptional Lie groups. The shaded bands and the dashed ellipses indicate the result of this and other [37,38] experimental analyses, which are combined in the solid ellipse

5. Conclusion

- Quarks carry colour charge
- Interaction between quarks is mediated by gluons which also carry charges and can therefore couple
- In order to test the SU(3) symmetry of QCD, a measurement of the colour factor ratio C_A/C_F has been performed
- C_A/C_F was extracted out of the charged particle multiplicities in 3 jet-events at DELPHI
- Thereby the most precise measurement of this quantity so far has been achieved, yielding

  $$\frac{C_A}{C_F} = 2.261 \pm 0.014_{\mathrm{stat.}} \pm 0.036_{\mathrm{exp.}} \pm 0.066_{\mathrm{theo.}}$$

  which is in excellent agreement with the SU(3) expectation of 2.25

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