Statistical Hadron Production from AGS to Collider Energies

Johanna Stachel

Physikalisches Institut, Universität Heidelberg

- Statistical Model
- Fixed Target Data
- RHIC Data
- Chemical Freeze-Out and the Phase Boundary
- Chemical Freeze-Out vis-a-vis Inital Condition and Thermal Freeze-Out
- Outlook and Open Questions

Choice of Statistical Ensemble

- Grand Canonical Emsemble (GC): in large system, with large number of produced particles, conservation of additive quantum numbers (B, S, I_3) can be implemented on average by use of chemical potential μ; asymptotic realization of exact canonical approach
- Canonical Ensemble (C): in small system, with small particle multiplicity, conservation laws must be implemented locally on event-by-event basis (Hagedorn 1971, Shuryak 1972, Rafelski/Danos 1980, Hagedorn/Redlich 1985)
  → severe phase space reduction for particle production "canonical suppression"
- C relevant in
  - low energy HI collisions (Cleymans/Redlich/Oeschler 1998/1999)
  - high energy hh or e^+e^- collisions (Becattini/Heinz 1996/1997)
  - peripheral HI collisions (Cleymans/Oeschler/Redlich 1998, Hamieh/Redlich/Tounsi 2000)

- **Connection C and GC:** recently shown by Tounsi/Redlich hep-ph/0111159 and 0111261:

$$n_k^C = n_k^{GC} F_s$$

where k stands for all hadrons with a given strangeness S

- for strangeness 1: [1]

$$F_s = \frac{I_1(x)}{I_0(x)}$$

x describes size of thermal phase space available for strange particles

$$x = 2S_1 = 2\sum_k Z_k^1 \propto V = (V_0/2)N_{part}$$

ignoring multistrange hadrons, x is total of all particles with S=1 in GC

- for multistrange:

$$F_s = \frac{I_s(x)}{I_0(x)}$$

- limiting cases:

$$\lim_{x\to\infty} \frac{I_1(x)}{I_0(x)} = 1 \qquad C \to GC$$

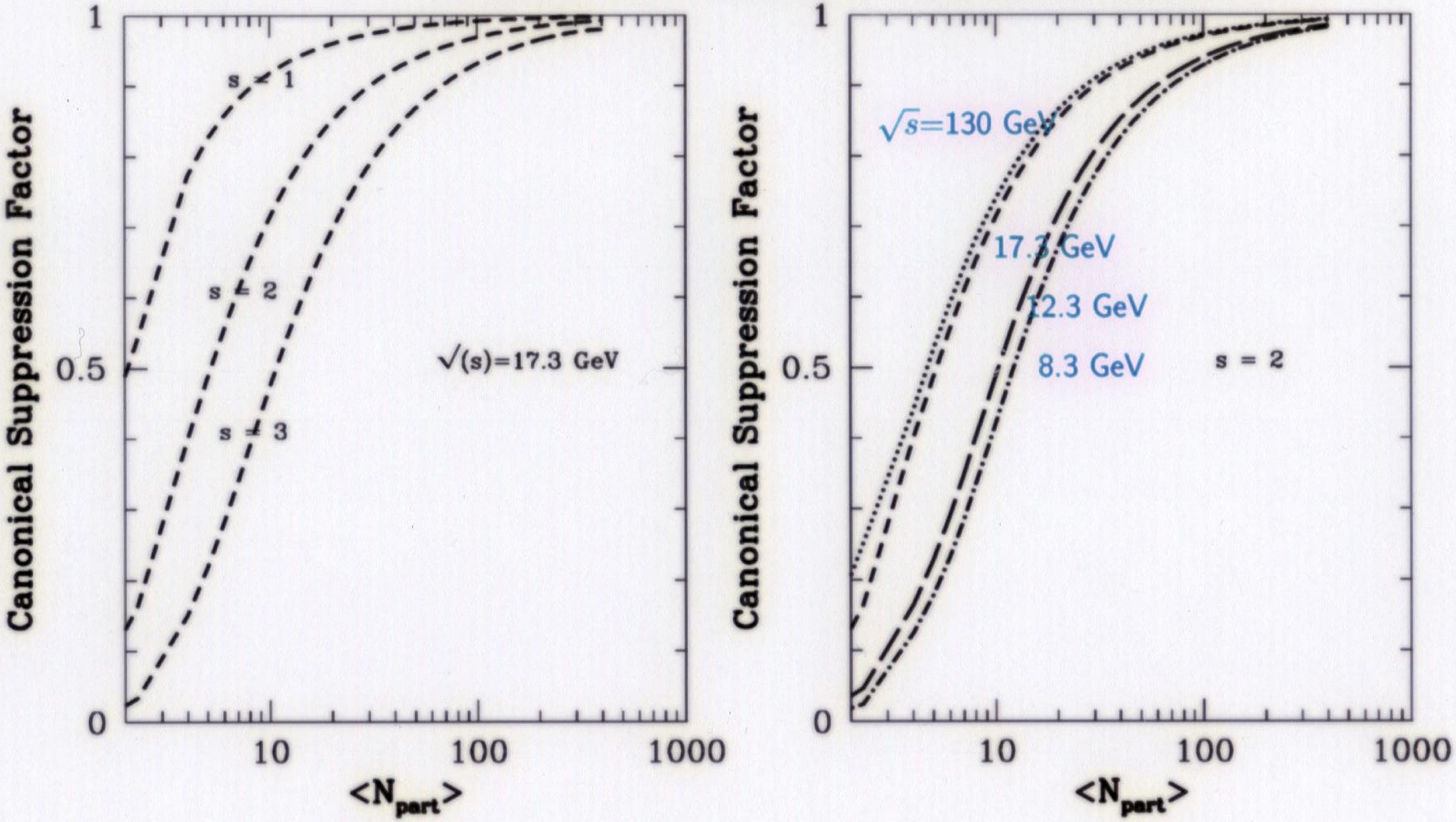
$$\lim_{x\to 0} \frac{I_n(x)}{I_0(x)} \to (x/2)^n \qquad \propto V^n$$

---

[1] first seen for charge conservation in pion production Horn/Silver 1970 and similar Shuryak 1972

Canonical Suppression Factor

Tounsi and Redlich, hep-ph/0211159



for $N_{part} \geq 60$ Grand Canonical ok to better 10%

Grand Canonical Ensemble

$$\ln Z_i = \frac{V g_i}{2\pi^2} \int_0^\infty \pm p^2 dp \ln[1 \pm \exp[-(E_i - \mu_i)/T]]$$

$$n_i = N/V = -\frac{T}{V}\frac{\partial \ln Z_i}{\partial \mu} = \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2\,\mathrm{d}p}{\exp[(E_i - \mu_i)/T] \pm 1}$$

$$\mu_i = \mu_B B_i + \mu_S S_i + \mu_{I_3} I_i^3$$

for every conserved quantum number there is a chemical potential μ
but can use conservation laws to constrain:

- Baryon number: $V \sum_i n_i B_i = Z + N$ $\rightarrow V$
- Strangeness: $V \sum_i n_i S_i = 0$ $\rightarrow \mu_S$
- Charge: $V \sum_i n_i I_i^3 = \frac{Z - N}{2}$ $\rightarrow \mu_{I_3}$

This leaves only μ_b and T as free parameter when 4π considered

for rapidity slice fix volume e.g. by $\mathrm{dN}_{ch}/\mathrm{dy}$

- finite volume correction: a la Balian-Bloch

$$f = 1 - \frac{3\pi}{4pR} + \frac{1}{(pR)^2}$$

- interactions: van der Waals type via excluded volume correction

thermodynamically consistent form Rischke/Gorenstein/Stöcker/Greiner 1990

$$p^{excl.}(T,\mu) = p^{id.gas}(T,\hat{\mu}); \qquad \text{with } \hat{\mu} = \mu - v_{eigen}\, p^{excl.}(T,\mu)$$

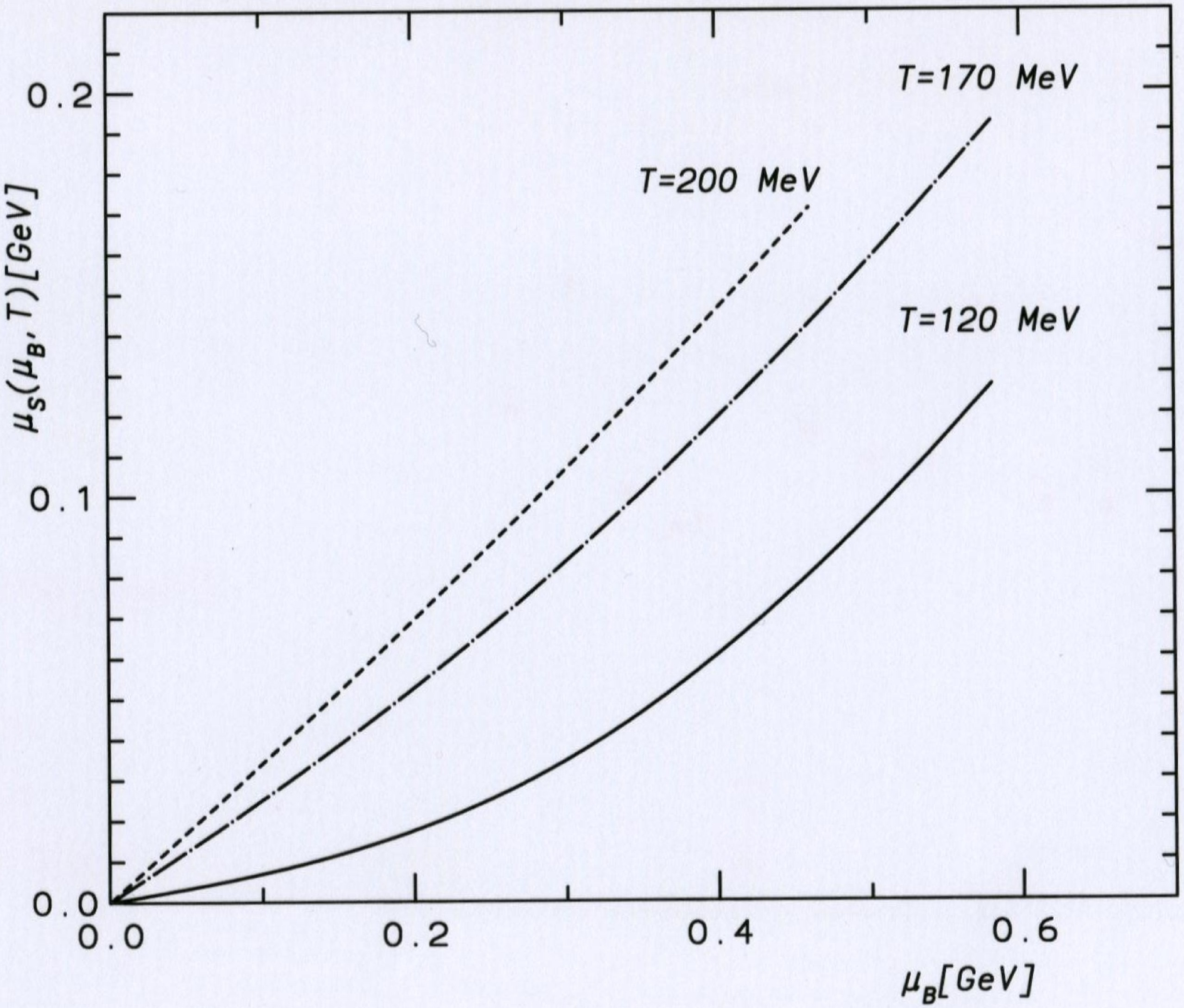
use r=0.3 fm (hard core of nucleon-nucleon interaction)

- resonance width: e.g. Weinhold, Friman, Nörenberg, 1996

$$\ln Z_R = N \frac{V d_R}{2\pi^2} T \exp[\mu/T] \int_{s_{min}}^{s_{max}} ds\, s\, K_2(\sqrt{s}/T)\, \frac{1}{\pi} \frac{m_R \Gamma_R}{(s-m_R^2)^2 + m_R^2 \Gamma_R^2}$$

not important for large enough T

Dependence of μ_S on T and μ_B



CERN SPS Data and Thermal Model

P. Braun-Munzinger, I. Heppe, J.Stachel, Phys.Lett.**B465** (1999) 15 + reanalysis in 2003 with more data

free parameters:
$T = 0.170 \pm 0.005$ GeV
$\mu_b = 0.255 \pm 0.010$ GeV

fixed by conservation laws:
$\mu_s = 0.074$ GeV from $\Delta S=0$
$\mu_{I_3}=0.005$ GeV from $\Delta Q=0$

reduced χ^2 (excluding ϕ and d)

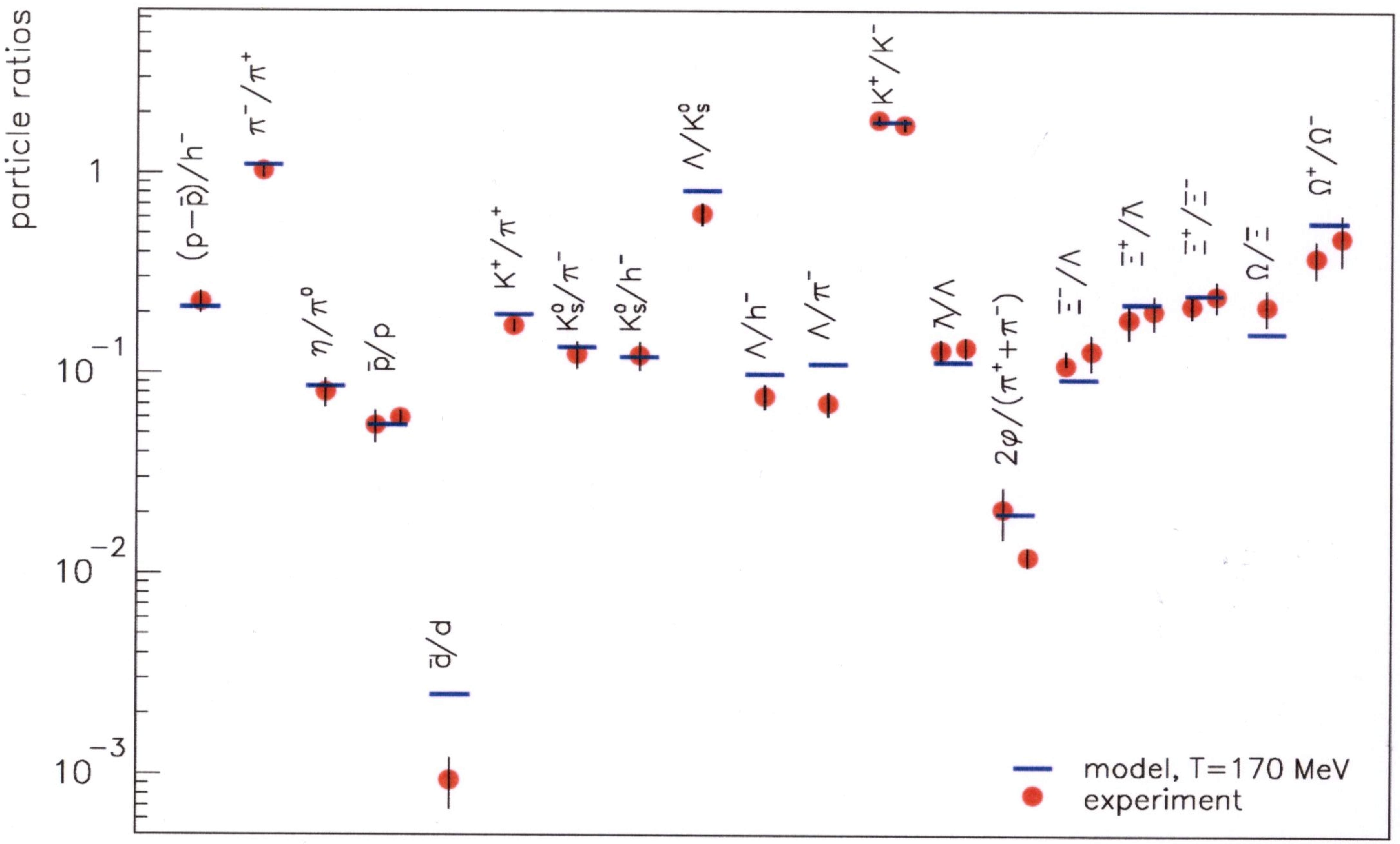
2.0

largest contribution:

weak feeding of π

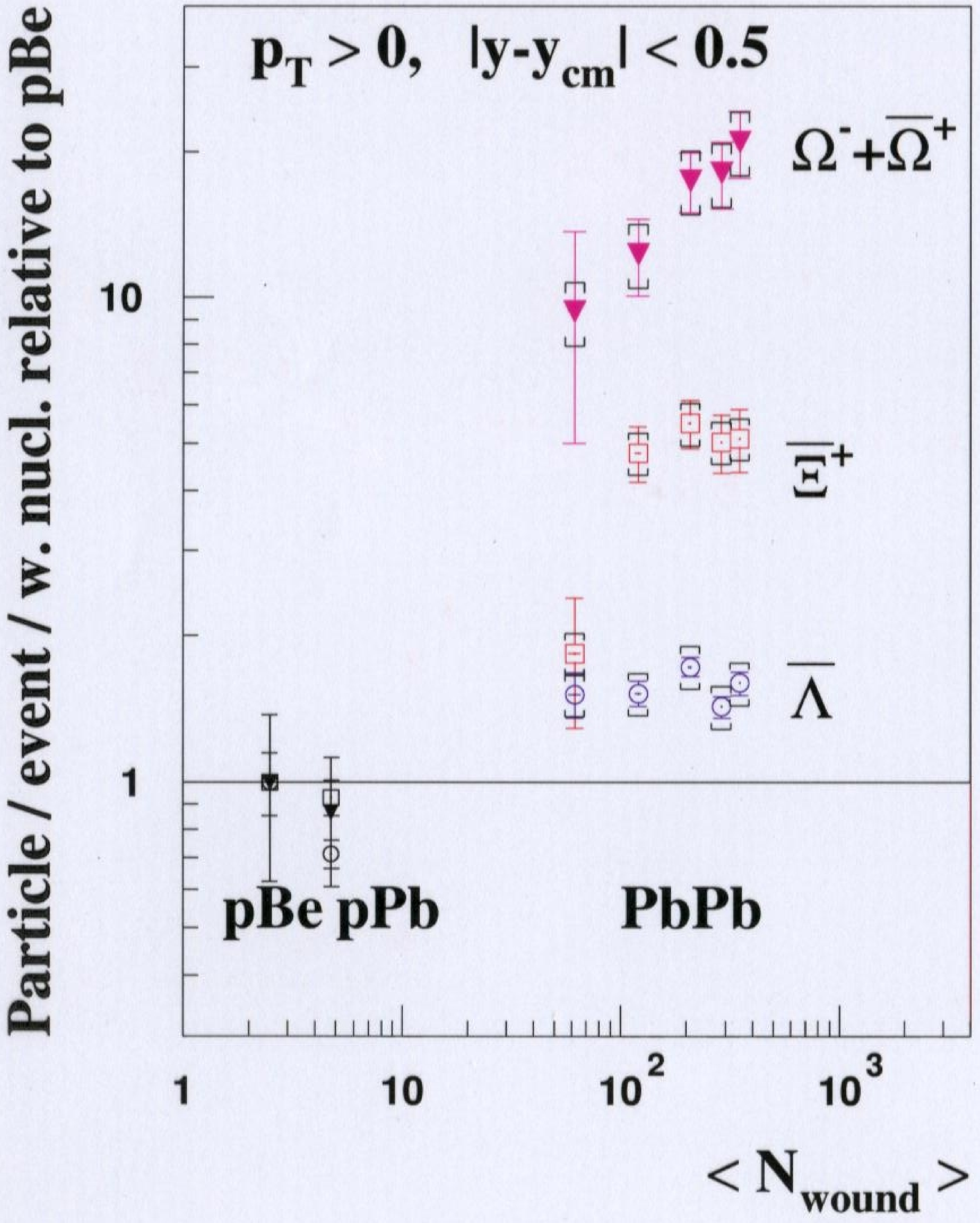
affects Λ/π and Λ/h^-

central 158 A GeV/c Pb + Pb collisions

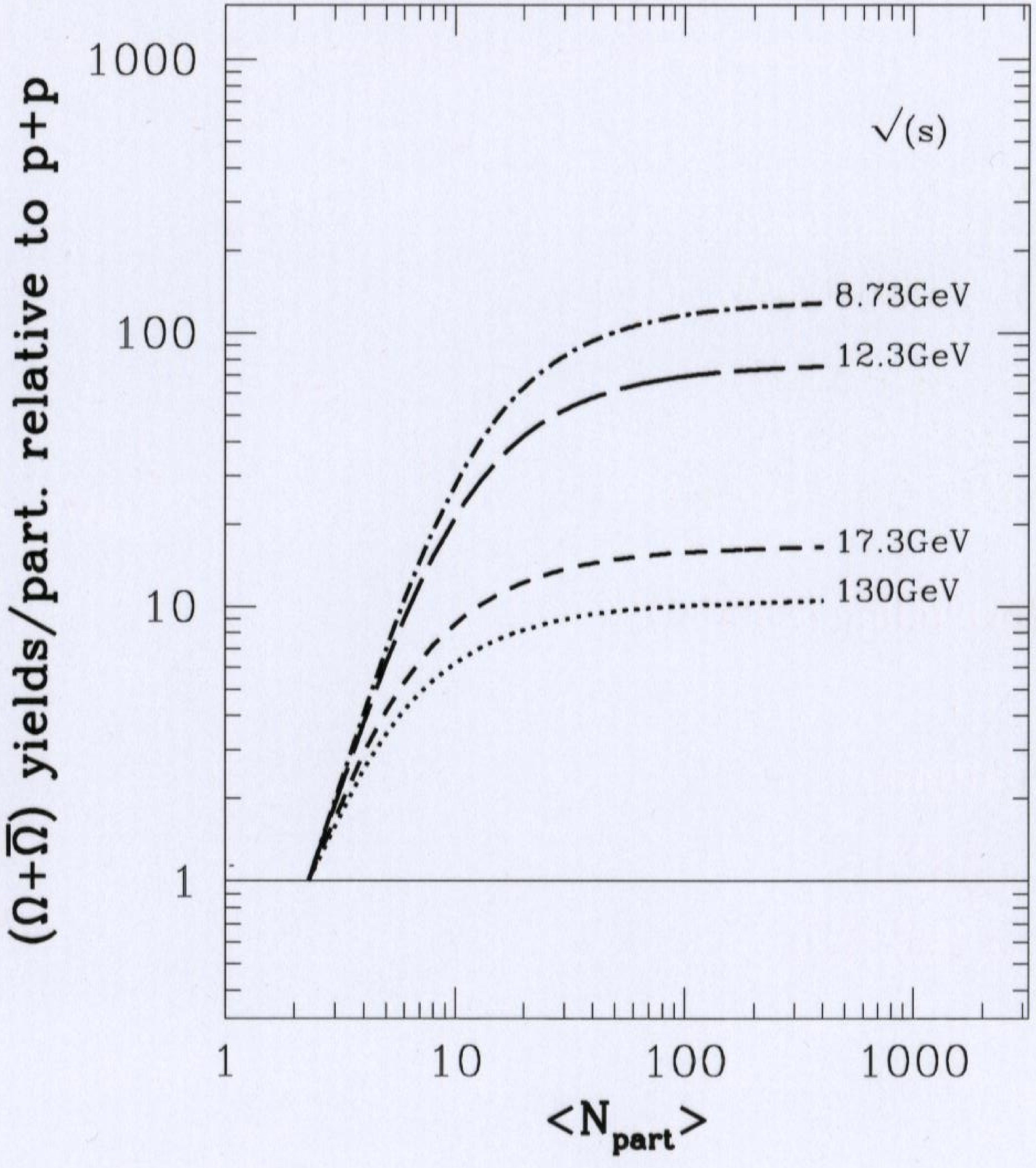


Strangeness Enhancement in 158 A GeV/c Pb + Pb Collisions

NA57



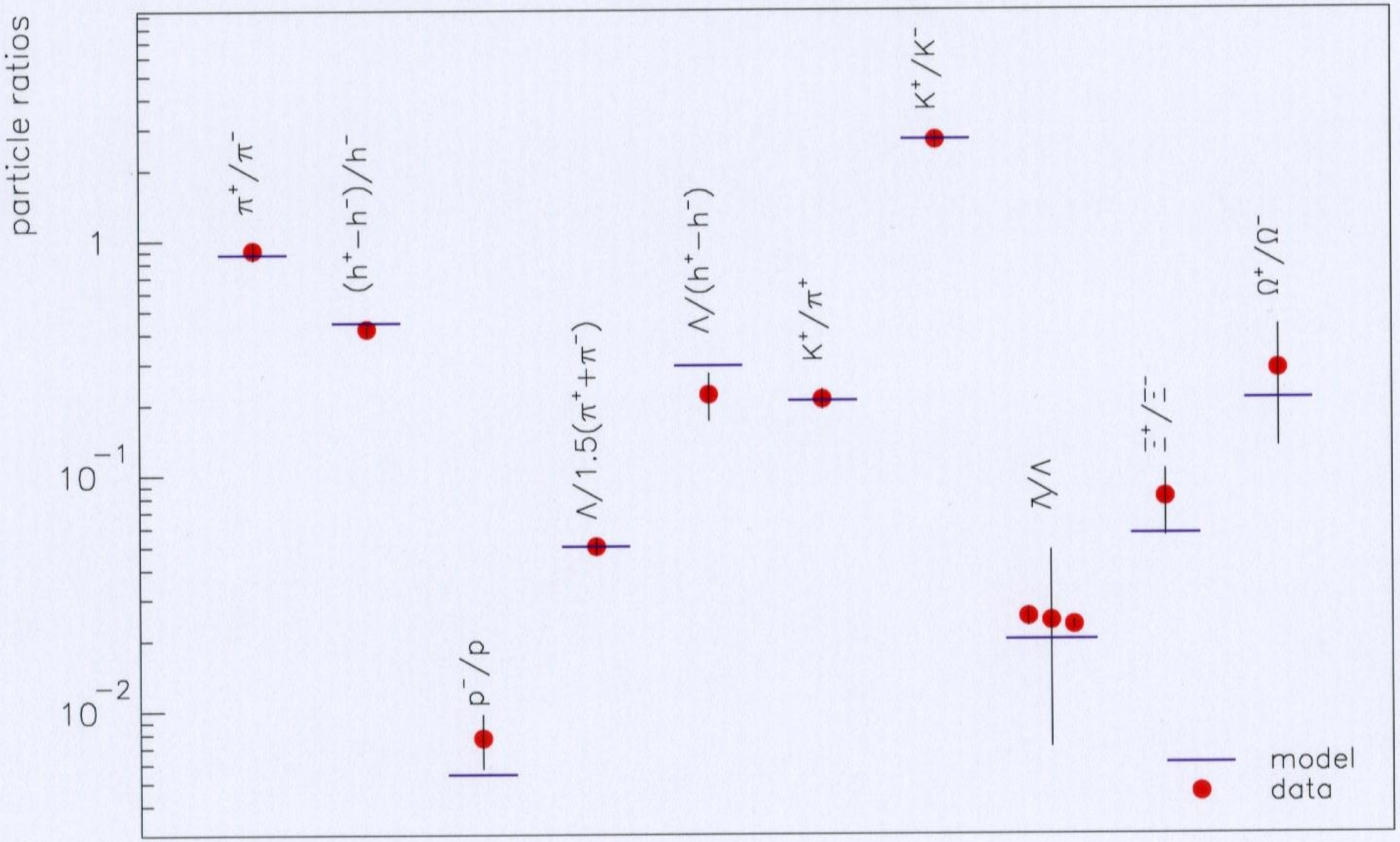
Tounsi/Redlich



Hadron Yields at SPS and Thermal Model

P. Braun-Munzinger, D. Magestro, J. Stachel, Dec. 02

central 40 A GeV/c Pb + Pb collisions - thermal model parameters: T $=$ 148 MeV, $\mu_b = 400$ MeV

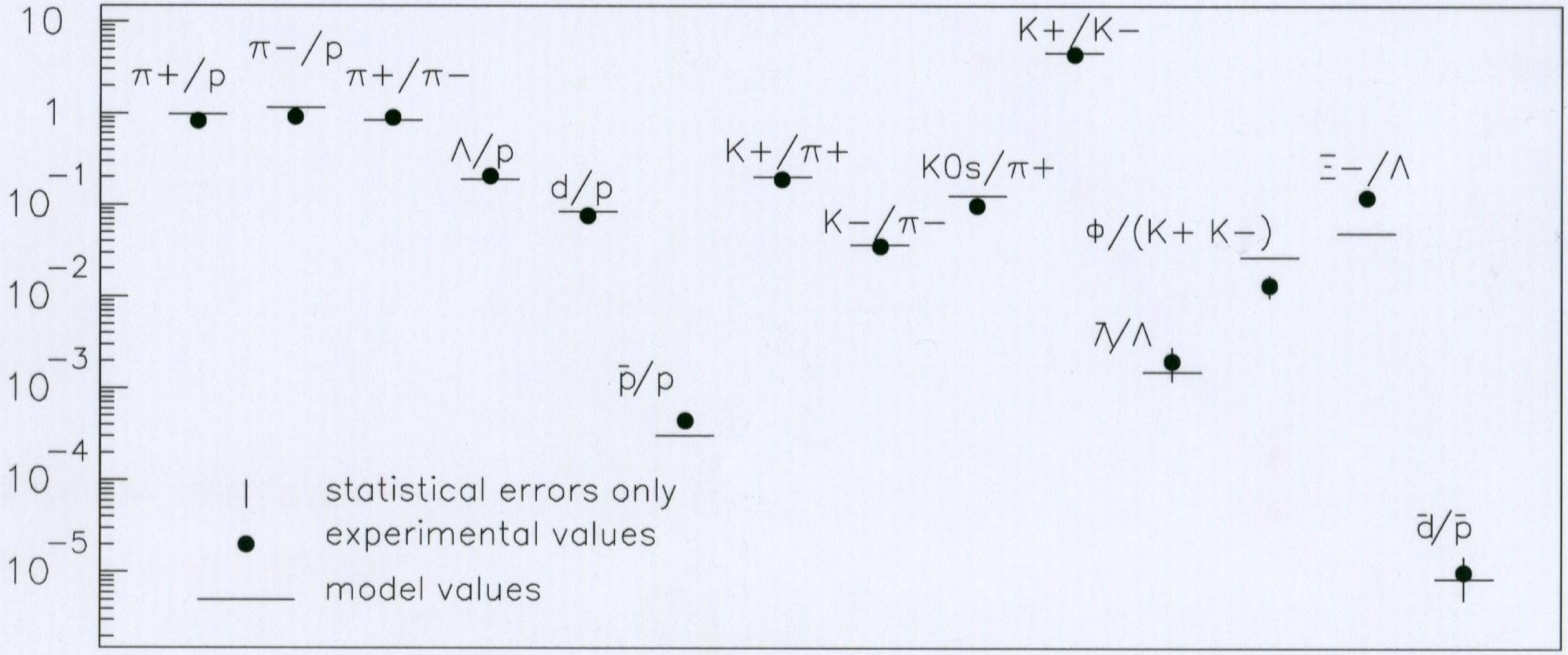


reduced $\chi^2 = 1.1$

Hadron Yields at AGS and Thermal Model

P. Braun-Munzinger, I. Heppe, J. Stachel, Phys. Lett. **B465** (1999) 5
and I. Heppe, Diploma thesis, U. Heidelberg 1998

central 14.6 A GeV/c Si + Au collisions
thermal model parameters: T = 125 MeV, μ_b = 540 MeV



D. Prorok: also E_T and N_{ch} described by same parameters – consistency check

yields for 11.5 A GeV/c Au + Au are very similar

Yields of Light Nuclei at AGS and Thermal Model

Addition of every nucleon → penalty factor R_p=48
but data are at very low p_t
p_t int. with A-dependent slope → R_p =26

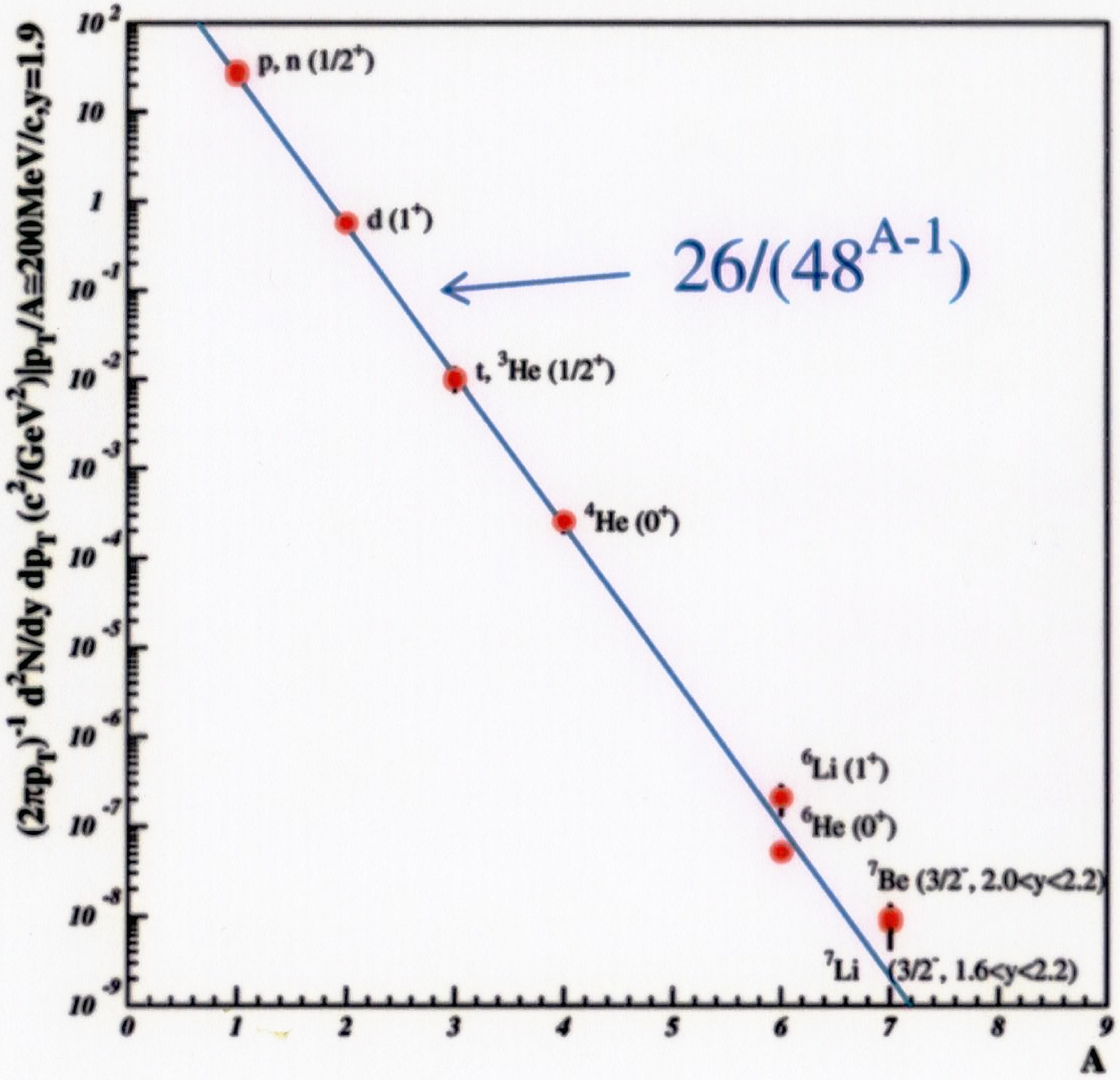
Grand Canonical Ensemble:
$R_p \approx \exp[(m_n \pm \mu_b)/T]$
for T=125 MeV and $\mu_b = 540$ MeV
→ R_p= 23 good agreement!

also good for antideuterons
data: R_p=2±1 10^5 GC: R_p=1.3 10^5

P.Braun-Munzinger, J.Stachel
J.Phys.**G28**(2002)1971

Note: AGS may be special here since
chemical and thermal freeze-out coincide

E864 Collaboration, Phys. Rev. **C61** (2000) 064908



$26/(48^{A-1})$

The Perennial Question of Use of γ_s

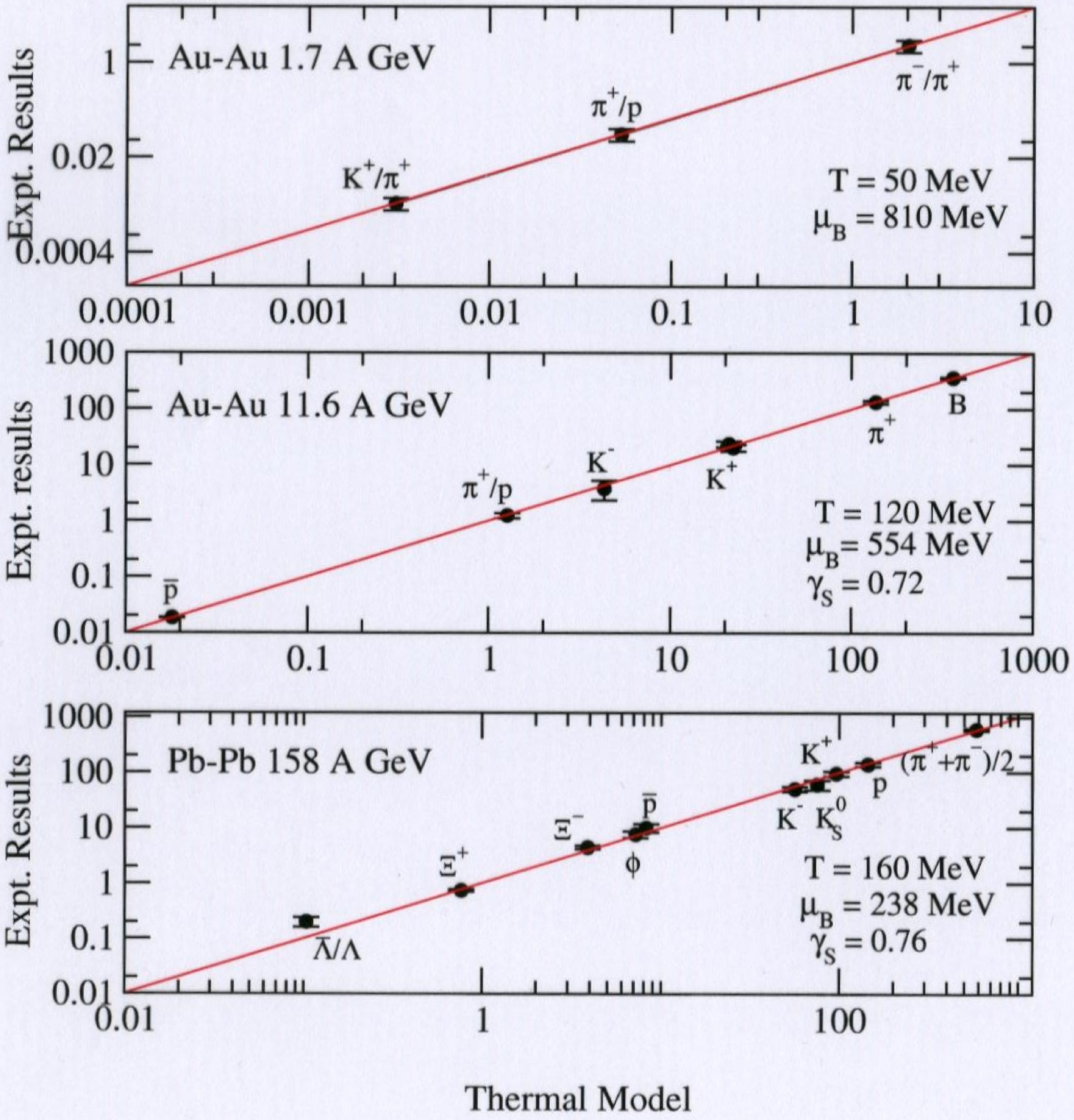
$$n_i(S) = \gamma_s^S \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2\,\mathrm{d}p}{\exp[(E_i - \mu_i)/T] \pm 1}$$

- Looks similar to canonical suppression but in that case dependence on V_0 and N_{part} fixed.
- γ_s as free fit parameter for every system and centrality has no thermodynamic justification, rather a measure of lack of equilibration
- but if really no thermalization, need dynamical treatment, doubtful that this could be replaced by factor γ_s

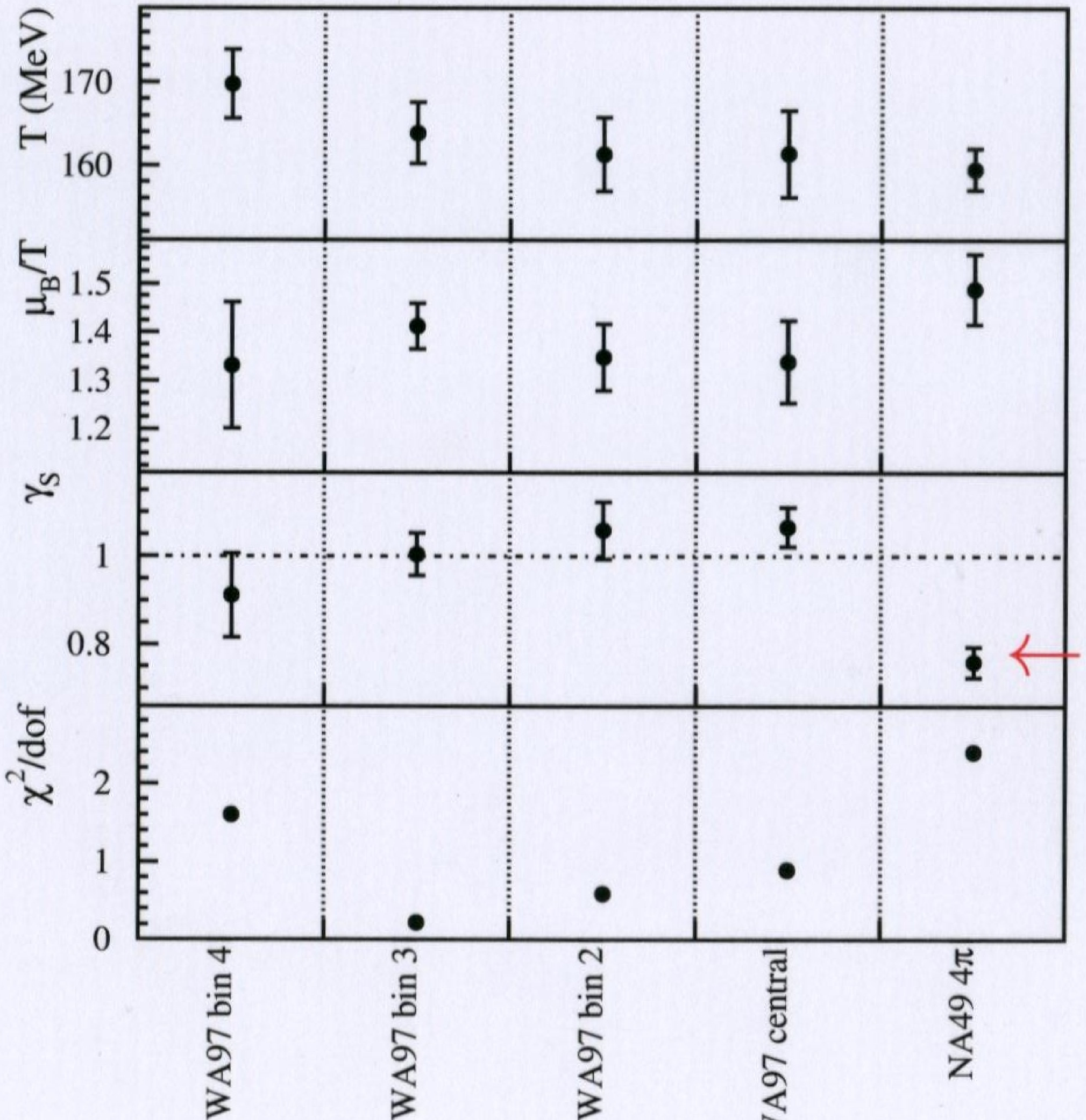
Strangeness Undersaturation?

central Au + Au and Pb + Pb collisions, from SIS to SPS

F. Becattini, J. Cleymans, A. Keranen, E. Suhonen, K. Redlich, hep-ph/0011322



F. Becattini, J.Phys. G28 (2002) 1553



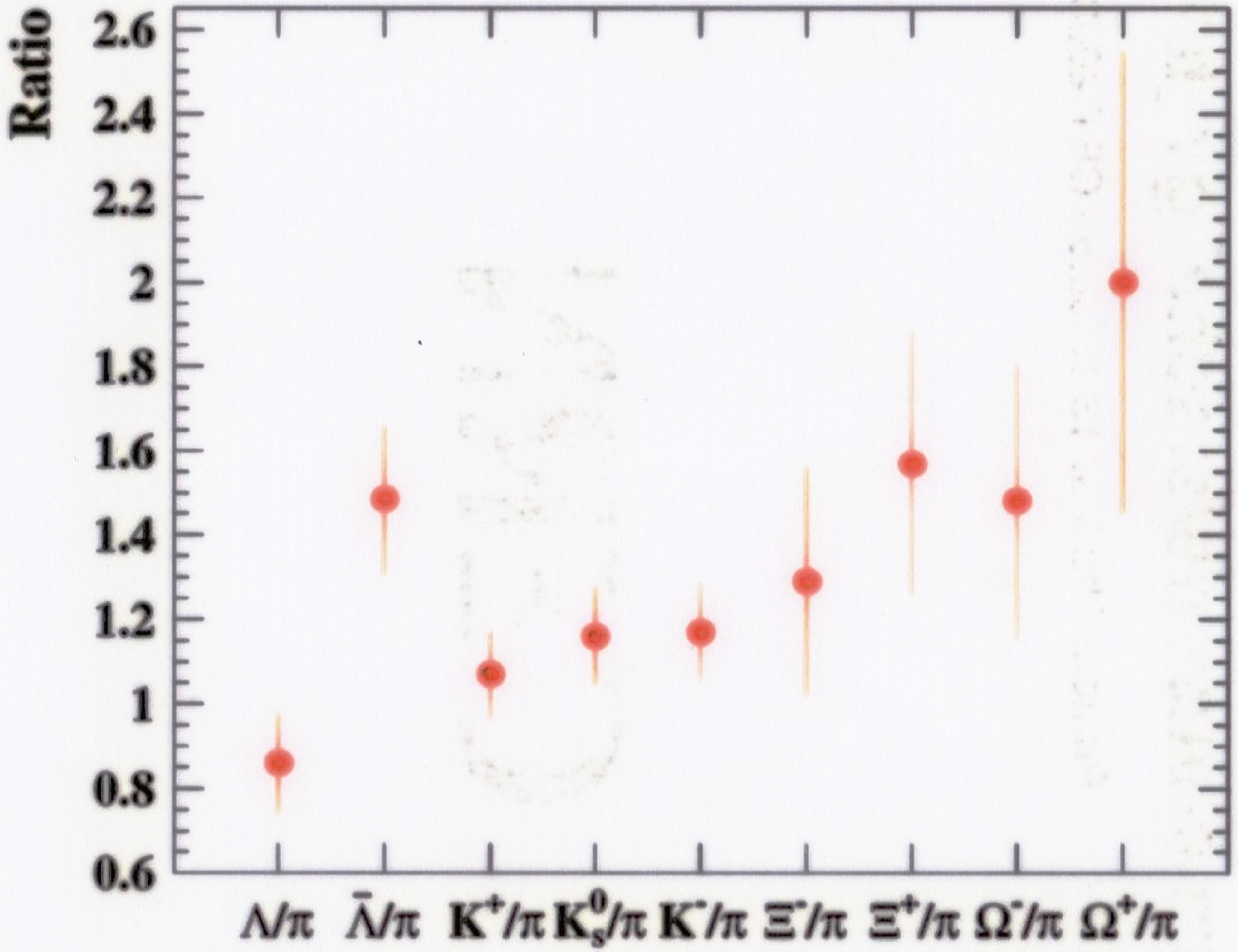
← ?

Difference Mid-rapidity vs. 4π Data

all data NA49, all measured or scaled to top 10 % centrality

158 A GeV/c Pb + Pb collisions

y=0 / 4π



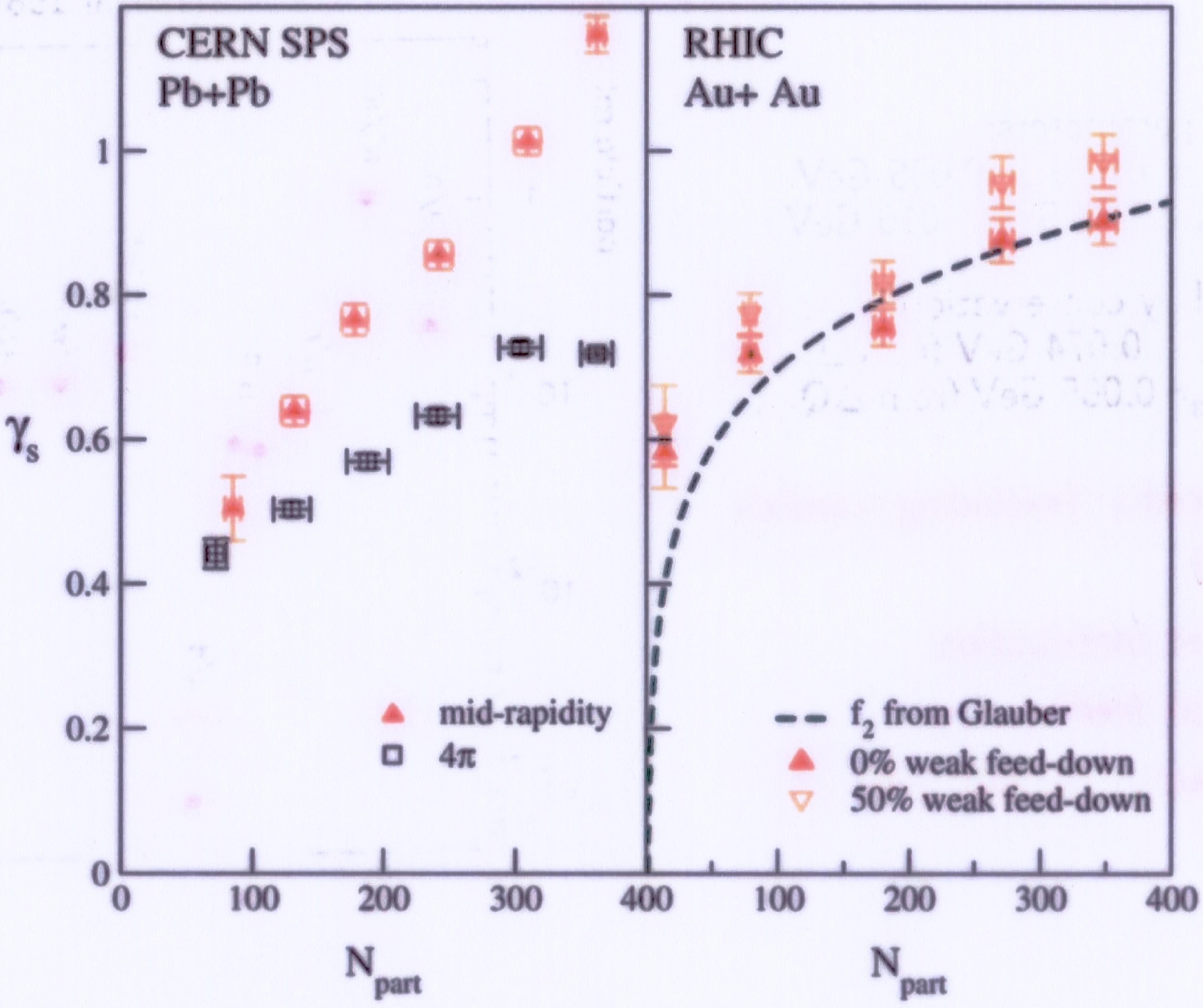
Fit of mid-rapidity data does not need strangeness suppression

Centrality Dependence of Strangeness Saturation

Cleymans, Kämpfer, Steinberg, Wheaton, hep-ph/0212335

Fit μ_B and γ_S to π, K, p yields

f_2 fraction of N_{part} with multiple collisions



Central collisions reach strangeness saturation at mid-rapidity

constant ϕ/π from STAR does not support $\gamma_S \leq 1$ for more peripheral

centrality dependence of R (NA57) not consistent w. above γ_s

Lack of Strangeness Saturation for Light Systems and Peripheral Coll.

centrality dependence of PbPb data and of CC and SiSi C. Höhne, NA49, QM02

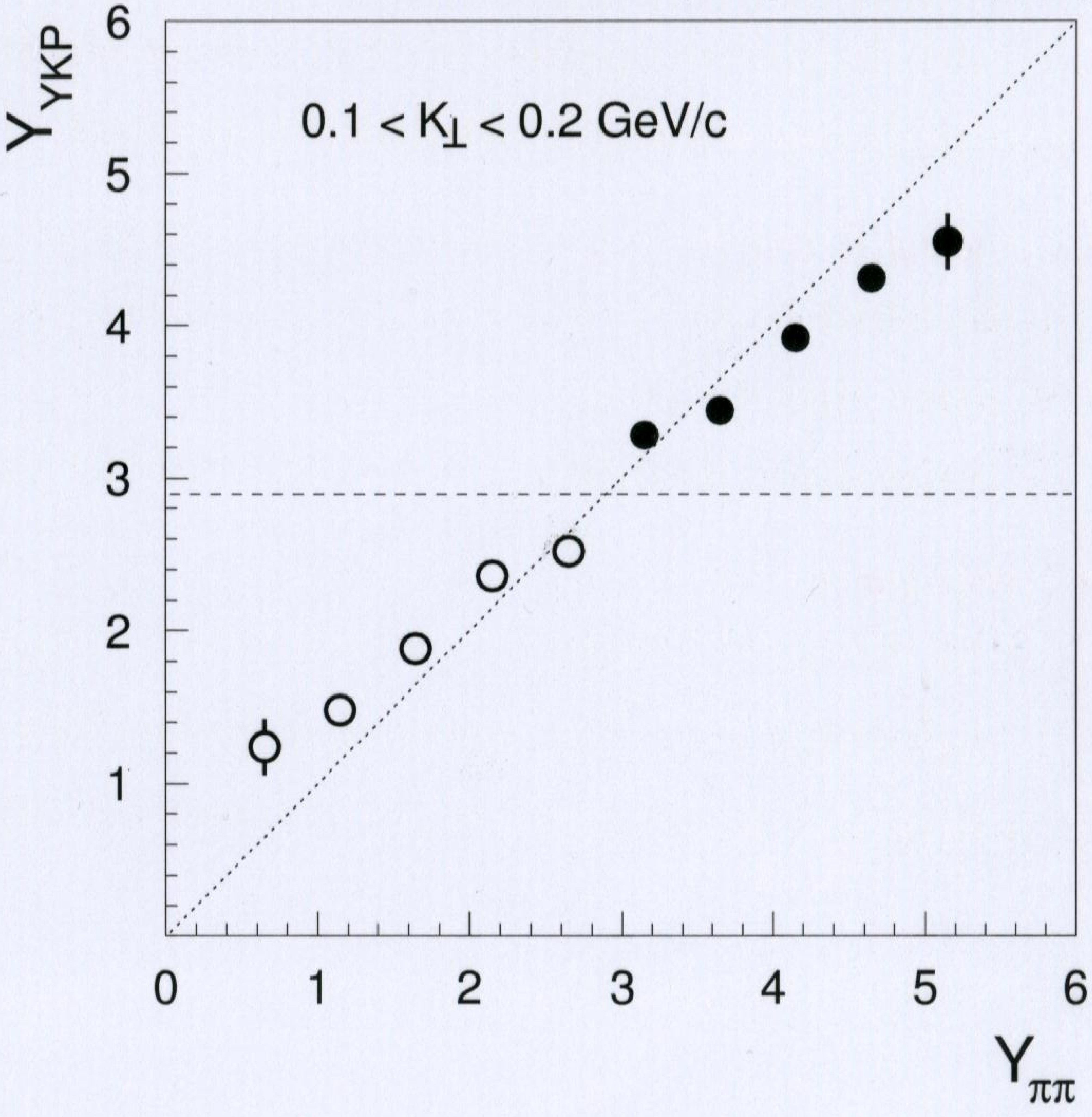
- no simple N_{part} scaling as for e.g. canonical suppression

- scales rather with number of collisions or fraction of nucleons having undergone multiple collisions

Choice of Rapidity Window

- Bjorken expansion: boost invariant
  $\rightarrow$ any rapidity window ok
- AGS and lower: no separation of central and fragmentation regions
  $\rightarrow$ choose 4 π
- RHIC: rapidity plateau $\Delta y \approx \pm 2$
  $\rightarrow$ within this window any cut should be okay
- Full energy SPS: evidence that some transparency sets in, distinction central region - fragmentation region becomes meaningful
  $\rightarrow$ assess width of plateau from HBT and get $\Delta y \approx \pm 1$

NA49: source rapidity vs pair rapidity

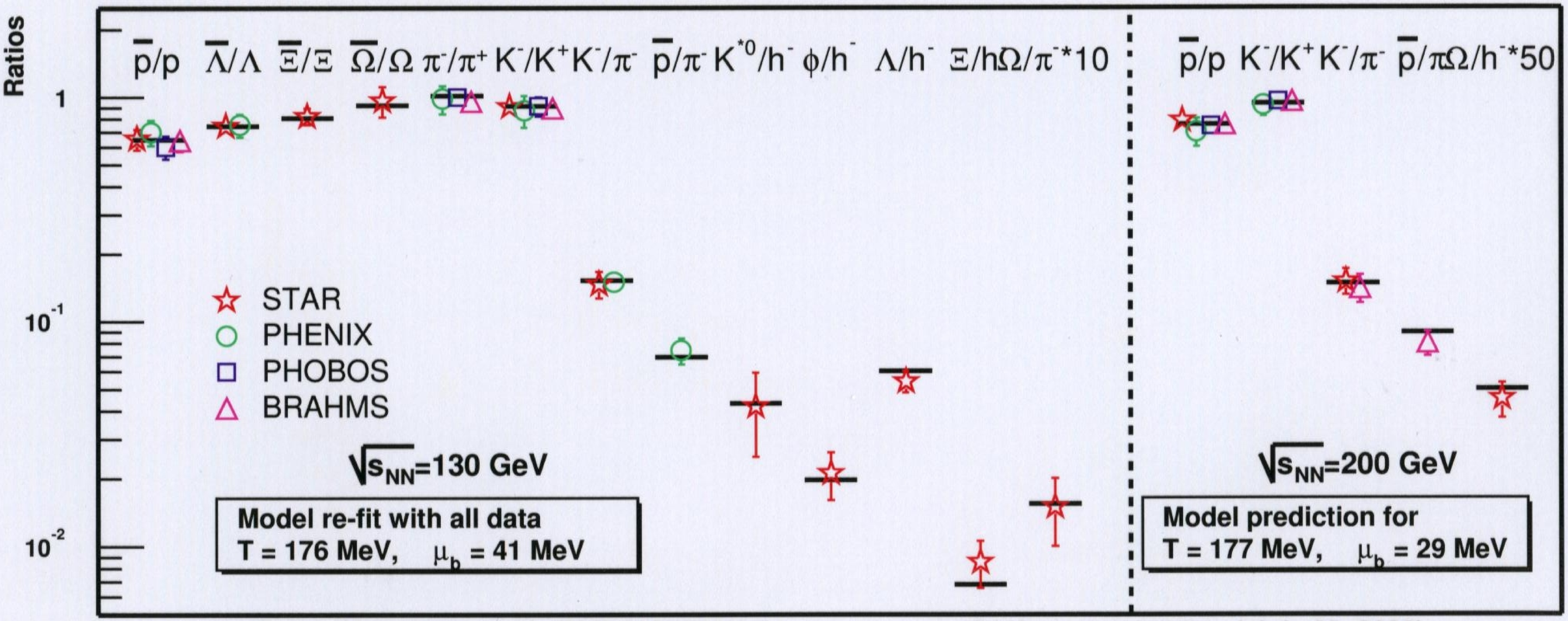


RHIC Data and Thermal Model

P.Braun-Munzinger, D. Magestro, K. Redlich, J.Stachel, Phys. Lett. B518 (2001) 41

central Au + Au collisions, data from all experiments combined

$\chi_r^2 = 0.8$ $\chi_r^2 = 1.1$

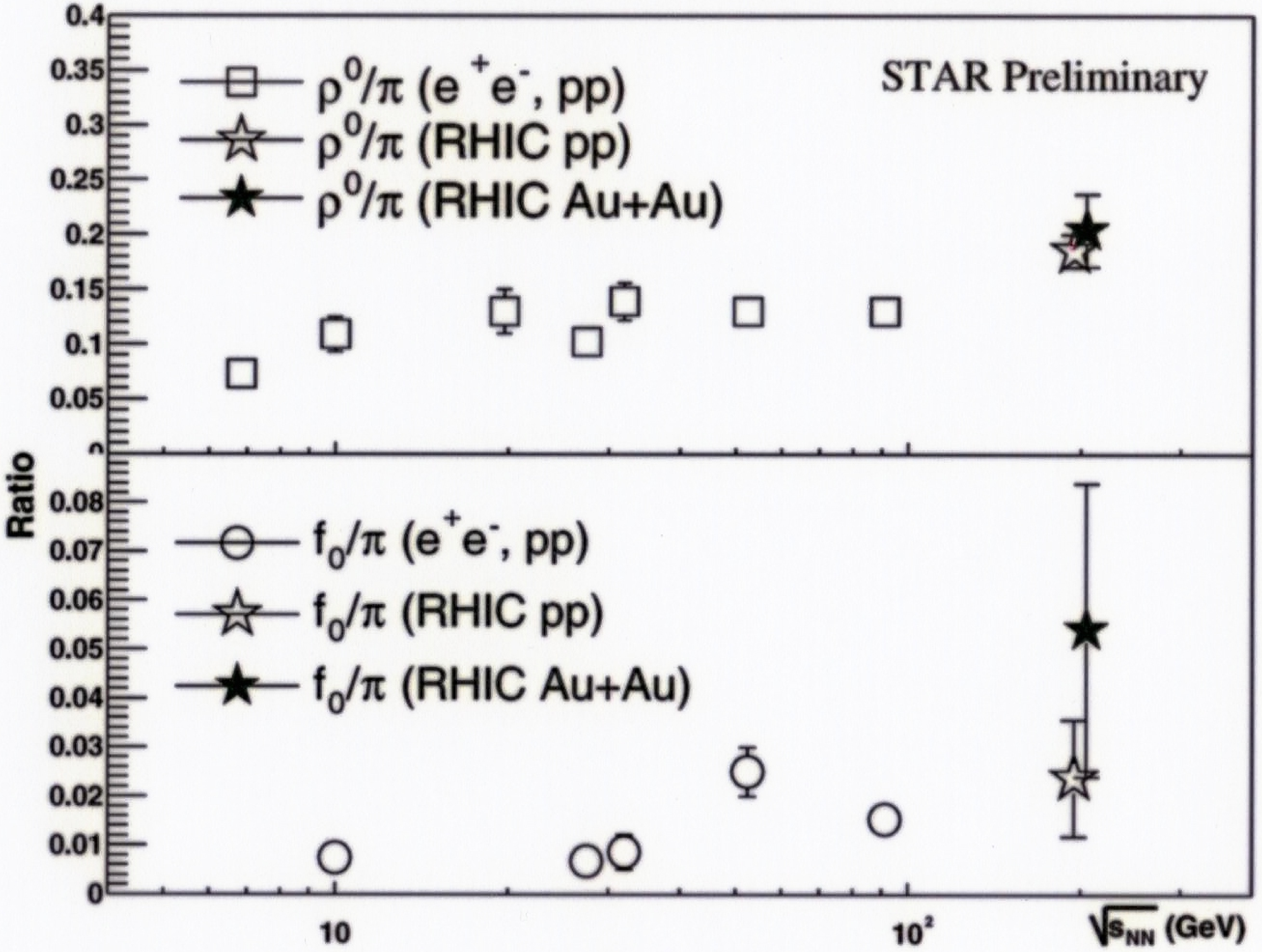


Braun-Munzinger et al., PLB 518 (2001) 41 D. Magestro (updated July 22, 2002)

fit result confirmed by Becattini and
Xu / Kaneta

ρ^0 and f_0 yield at RHIC

P. Fachini, QM2002, nucl-ex/0211001



statistical model T=177 MeV: $\rho^0/\pi^- = 0.11$ and T=120 MeV: 4 10^{-4}

even with growth of μ_π to nearly m_π and $\Delta m(\rho) \approx 60$ MeV difficult!

Mass changes close to T_c ?

repeat fit of RHIC data with several assumptions

- change all masses by constant factor:
  similar fit quality if variation $\lesssim 20\%$
  (see also Michalec, Florkowski, Broniowski
  nucl-th/0103029)

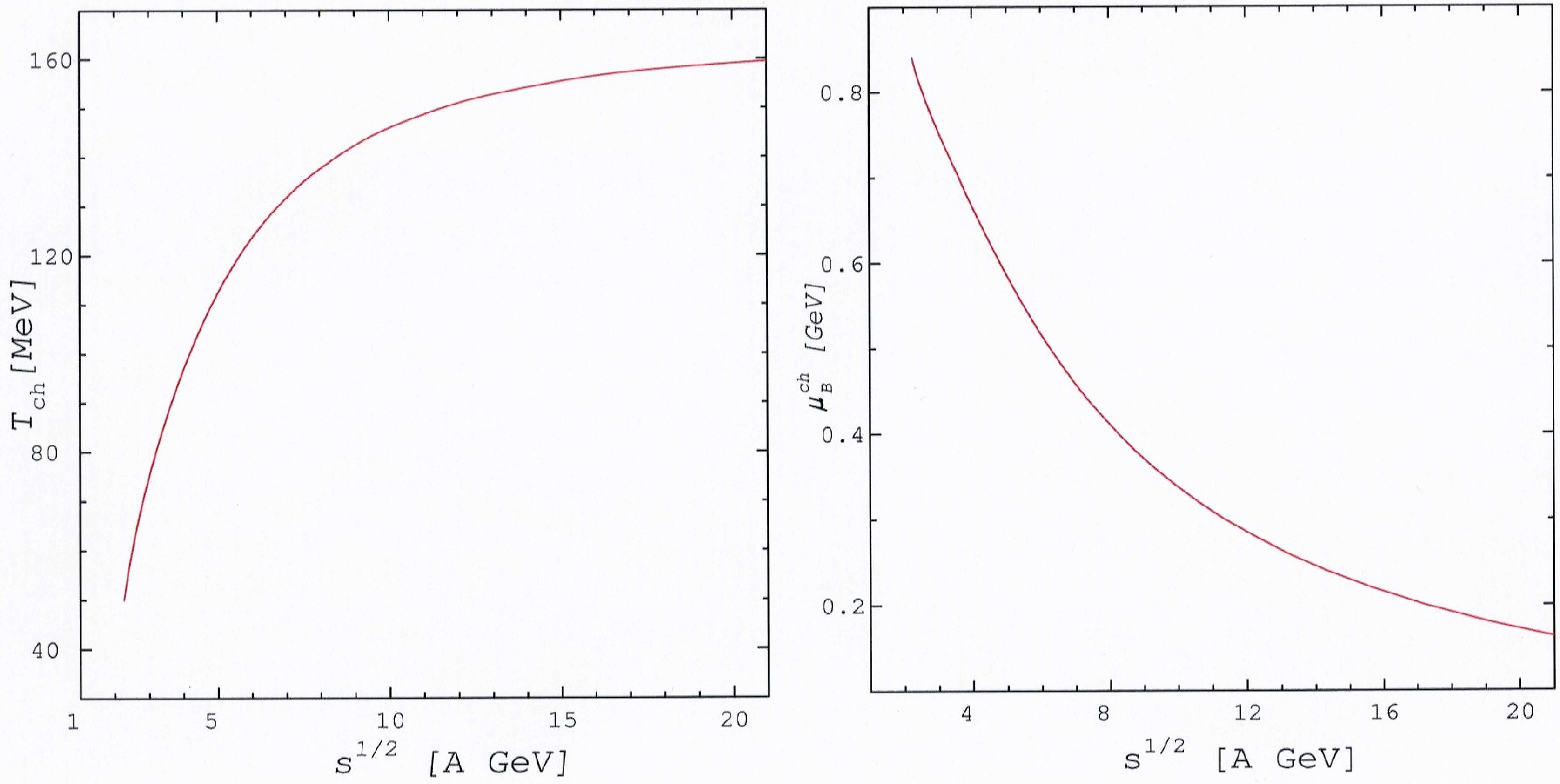
- reduce m_ϕ by 5% : 3σ discrepancy w. data

- reduce $m_{K^{0*}}$ by 10% : 2.5σ „

no room for very significant changes

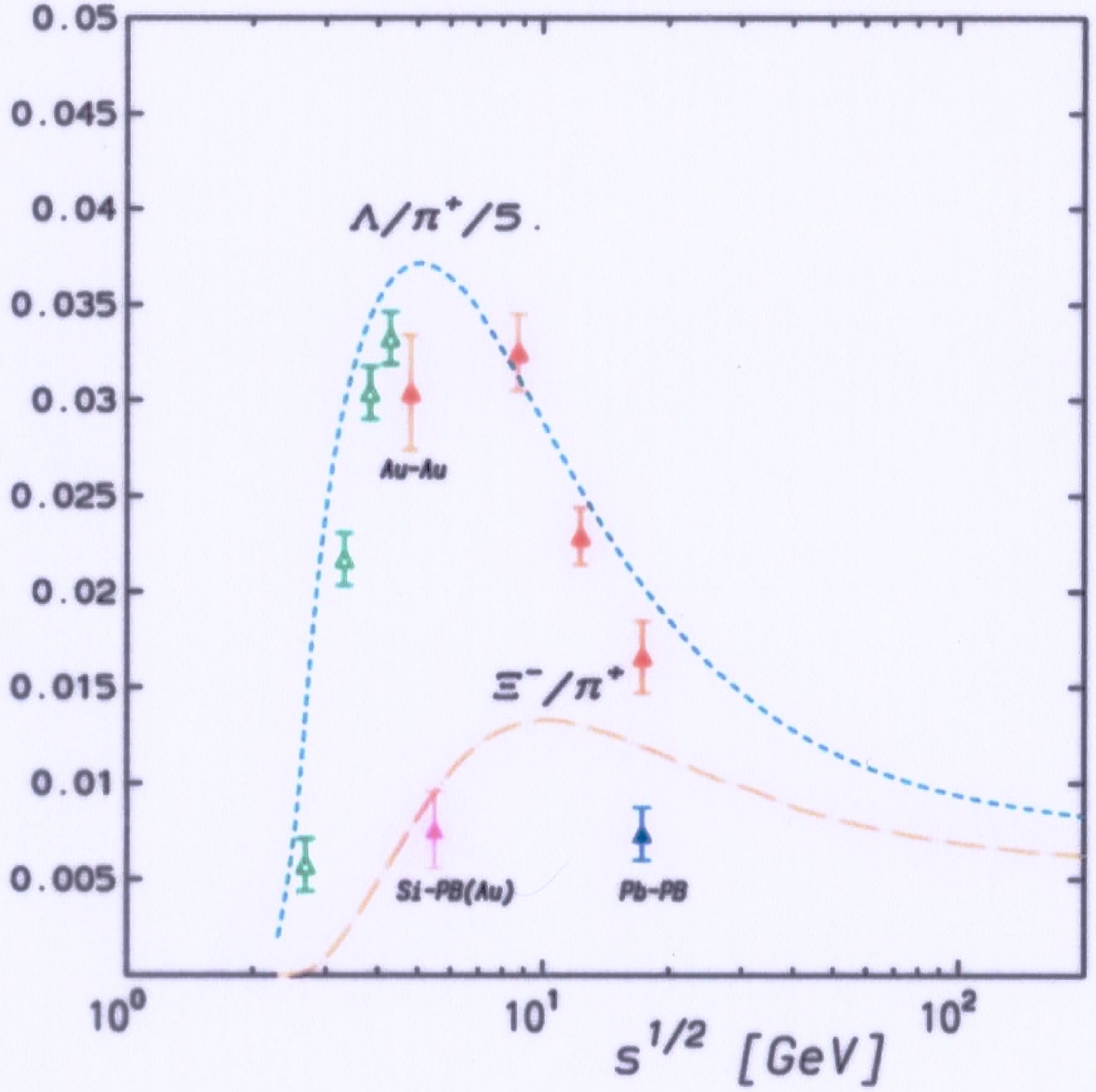
Evolution of Thermal Parameters with $\sqrt{s}$

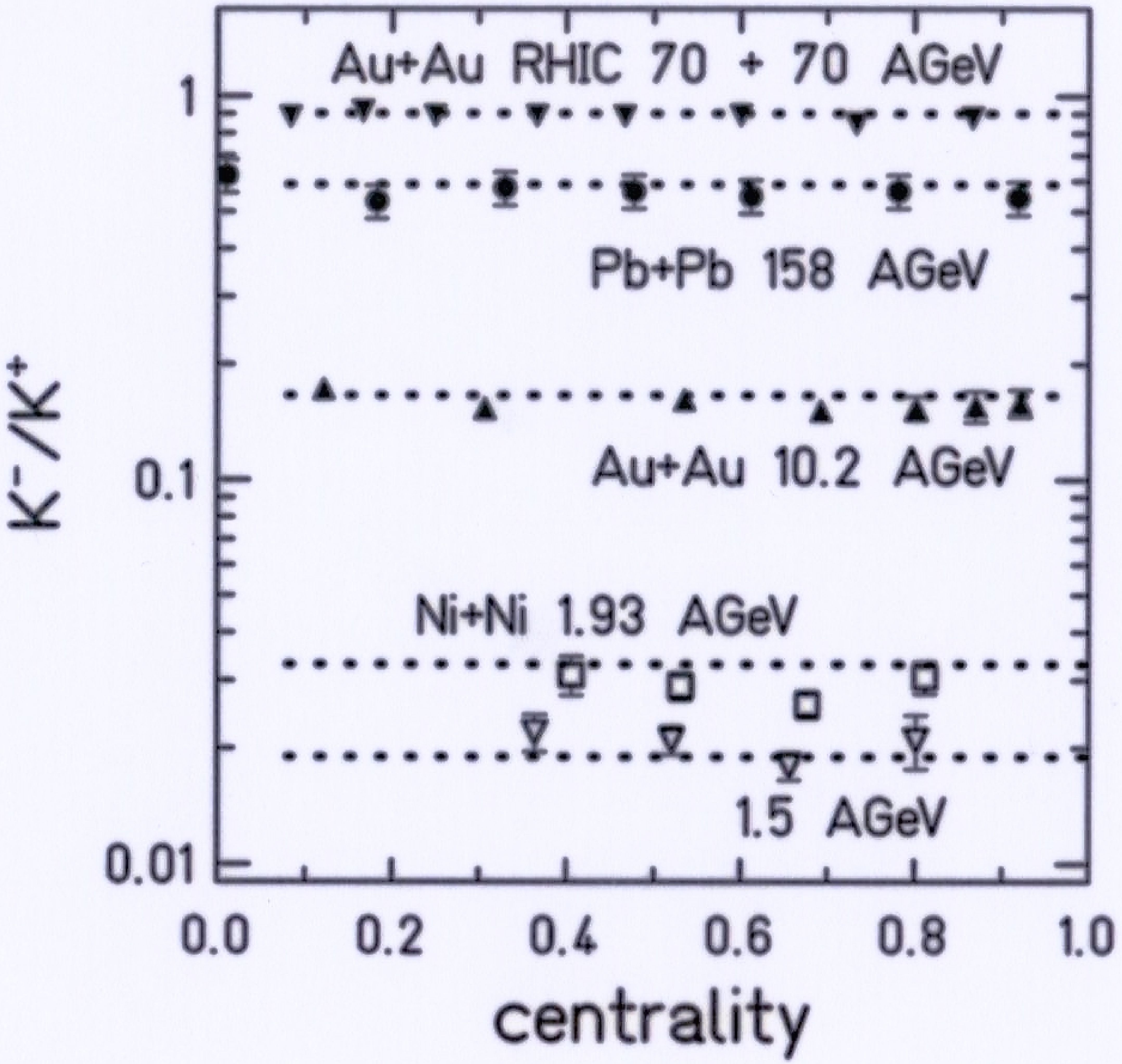
Cleymans/Redlich PR C60 (1999) 054908



Evolution of Thermal Parameters with $\sqrt{s}$

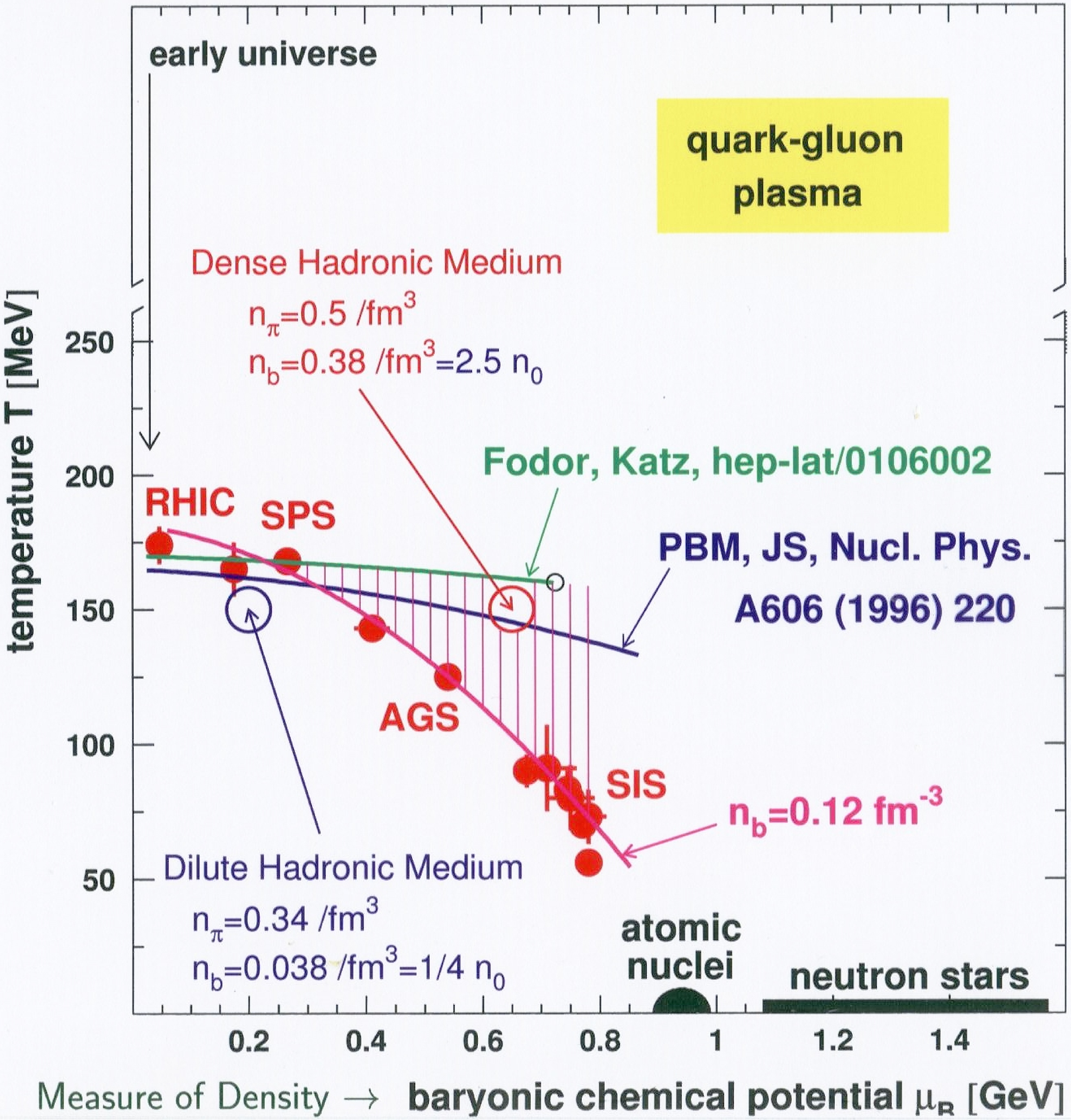
Braun-Munzinger, Cleymans, Oeschler, Redlich Nucl. Phys. A697 (2002) 902





Phase Diagram of Nuclear Matter

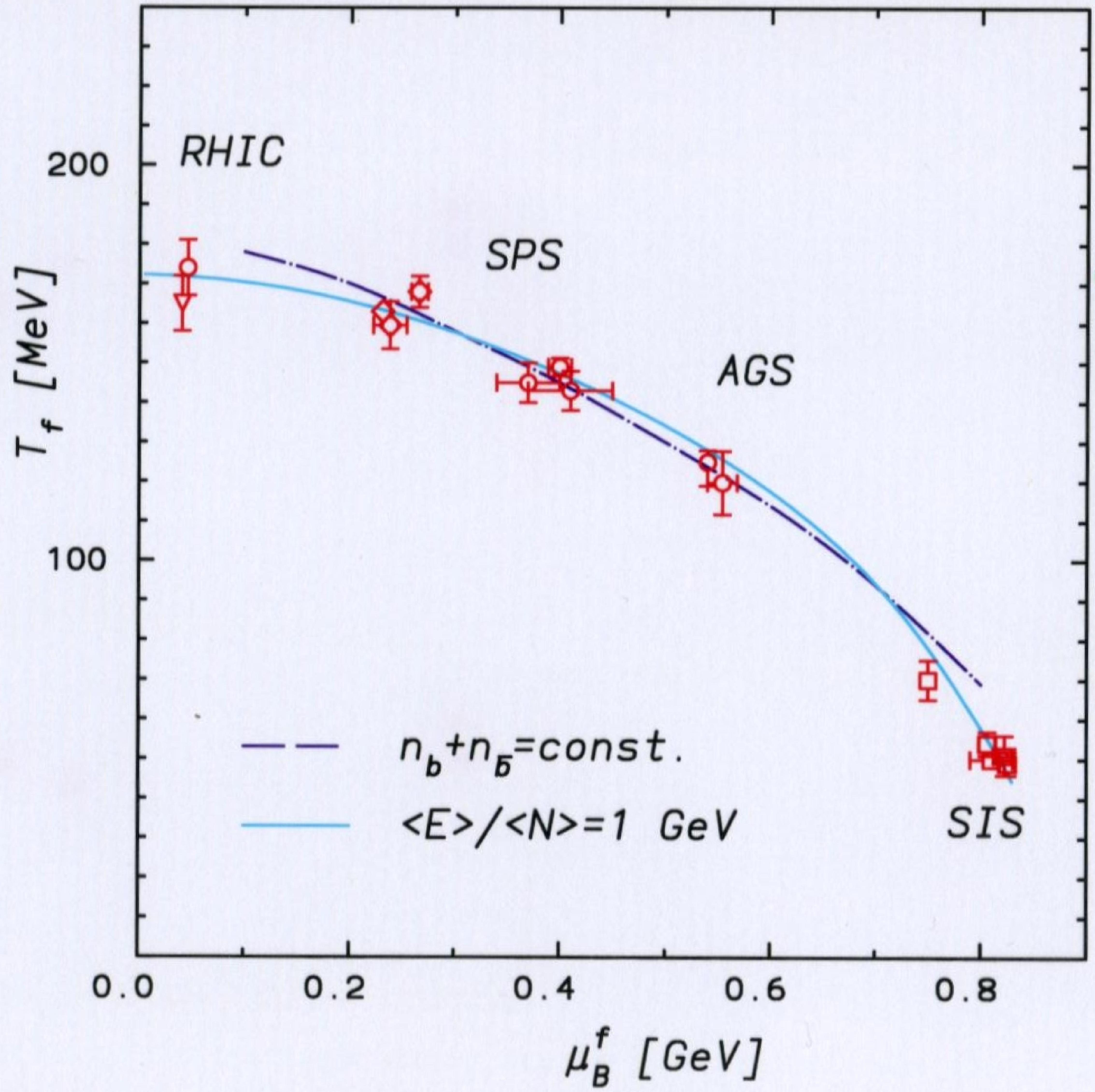
1. hadron yields equilibrated
2. for full SPS energy and above: hadron yields frozen at phase boundary
3. how is equilibrium achieved? at SPS and RHIC not with hadronic cross sections
   → QGP much more efficient equilibrator



What Characterizes Chemical Freeze-out Curve?

Cleymans/Redlich PRL 81 (1998) 5284: Constant energy per hadron E/h = 1 GeV

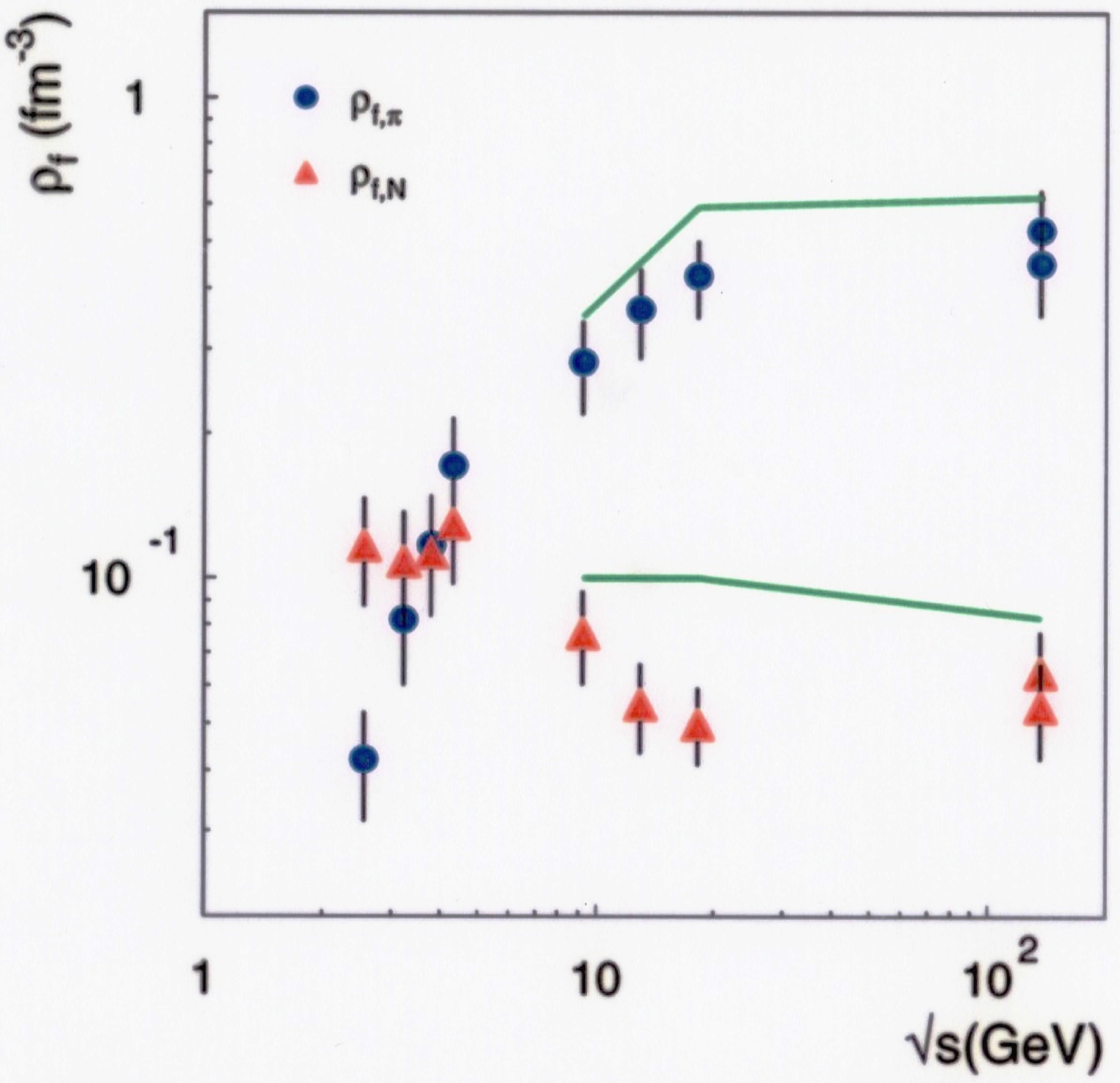
Braun-Munzinger/Stachel JP G28 (2002) 1971: Constant total baryon density $n_B = 0.12\ \mathrm{fm}^{-3}$



- constant total baryon dens. could indicate $b\bar{b}$ and $m\bar{b}$ cross section relevant for chemical freeze-out
- for full SPS energy and above freeze-out points below -·- suggestive that actually phase boundary met

Freeze-Out Density from Pion HBT

CERES PRL 90 (2003) 023001



HBT gives density at thermal freeze-out

← pion density at chemical freeze-out

← nucleon density at chemical freeze-out

Volume appears to only grow 30 % between chemical and thermal freeze-out!

Hadronic Phase after Chemical Freeze-out

- Densities from HBT as compared to chemical freeze-out:
  indicate growth $\Delta R_s \approx 15$ % (at most 40 %)
  at average expansion velocity 0.5c this takes 2 fm/c (max 4 fm/c)
- Role of annihilation for $\bar{\text{p}}$/p and $\bar{\text{d}}$/d; in UrQMD factor 2 change due to annihila
  in data no room for that

  → no indication for a longlived hadronic phase
- During hadronization # dof increases by factor 3.5
  volume has to grow accordingly

  → more time at T$_c$ than in hadronic phase

CERES enhancement understood in this context?

Excluded Volume to take into account interactions (van der Waals-type)

- different choices in literature
- a thermodynamically consistent correction proposed by Gorenstein, Rischke & Co

  $p^{excl}(T,\mu) = p^{igas}(T,\hat{\mu})$

  with $\hat{\mu} = \mu - V_{eigen}\, p^{excl}(T,\mu)$

  recursive ... find $\hat{\mu}$ and then compute any variable (T,μ) like ε, ρ, s ...

- choice of V_{eigen}? relevant distance is where interaction becomes repulsive
  for nn at 0.3 fm
  mesons?
  our choice $r = 0.3$ fm for all

➡ note: as long as r is same for all ratios practically not affected by excluded volume correction but absolute densities are!
caveat for users of freeze-out geometry for e.g. lepton pairs, photons, ...
for full energy SPS densities reduced by factor 2 - 2.5

fugacities (strangeness suppression γ_s) in grandcanonical

- theoretical limit: this is the canonical strangeness suppression factor

- further (Redlich, ...) F_{cs} depends explicitly on volume over which quantum number conserv. is enforced

for $x_1 << 1$ $$\frac{I_1(x_1)}{I_0(x_1)} \simeq \frac{1}{2} x_1 \propto V_0 \leftarrow \frac{K}{\pi}$$

$$V_0^3 \leftarrow \frac{\Omega}{\pi}$$

- for pp and e^+e^- $V_0 = 7\ \text{fm}^3$ volume of hadron
  reason for strangeness supp.

heavy ion coll. (Redlich, ...) $V = V_0 \cdot \frac{N_{part}}{2}$

- Au+Au at SIS $1400\ \text{fm}^3$

HI for higher energies: V exceeds this value and looses its meaning; strangeness percolates over large volume

suggestive → (QGP)

φ from NA50

N. Willis, proc. QM99 NPA in print

for central PbPb

$\varphi / \rho + \omega$ = 1.10 ± 0.08 stat ± 0.20 syst

$T_{\varphi} = 219 \pm 5$ MeV $T_{\rho,\omega} = 222 \pm 6$ MeV

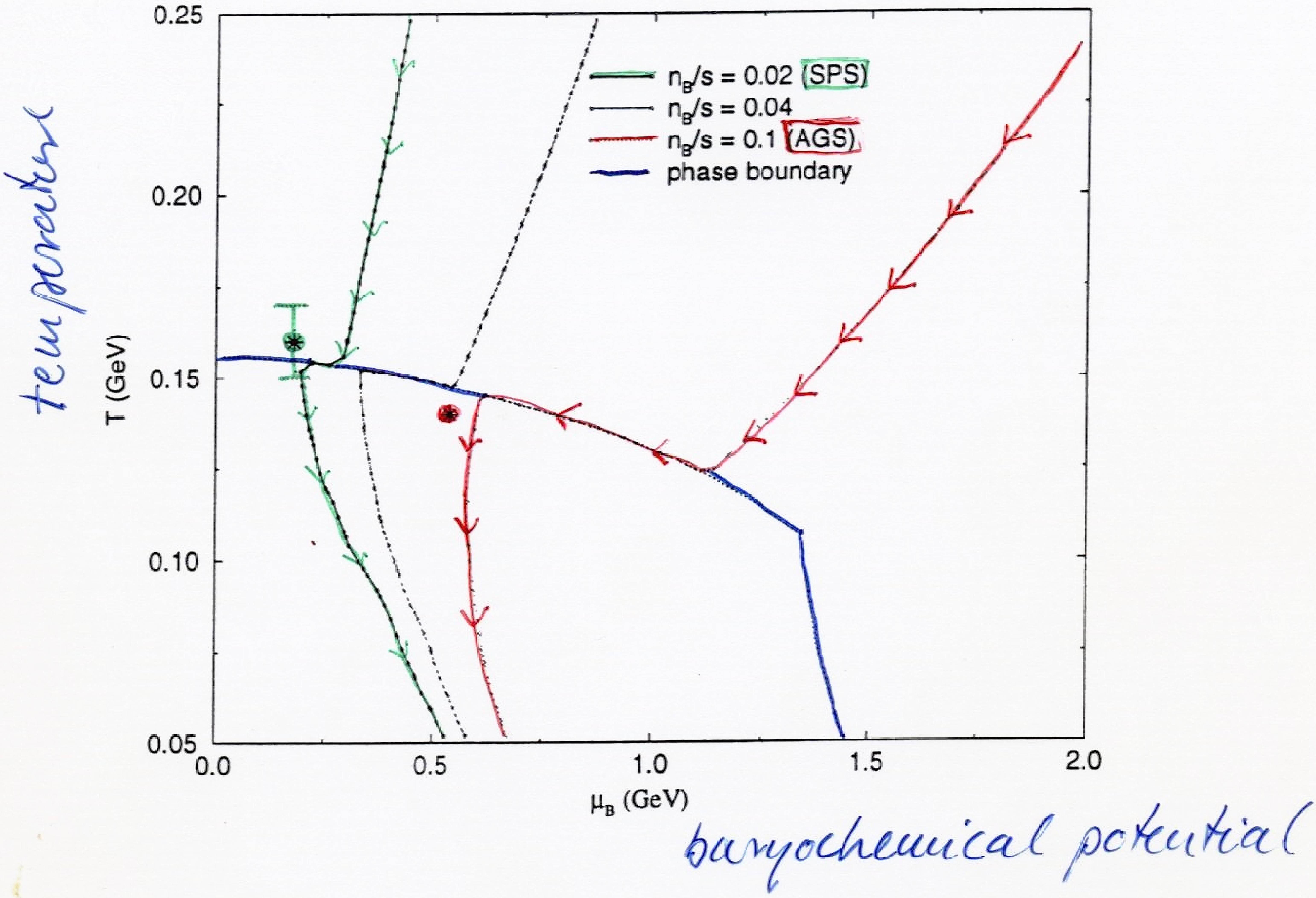
Thermal Model

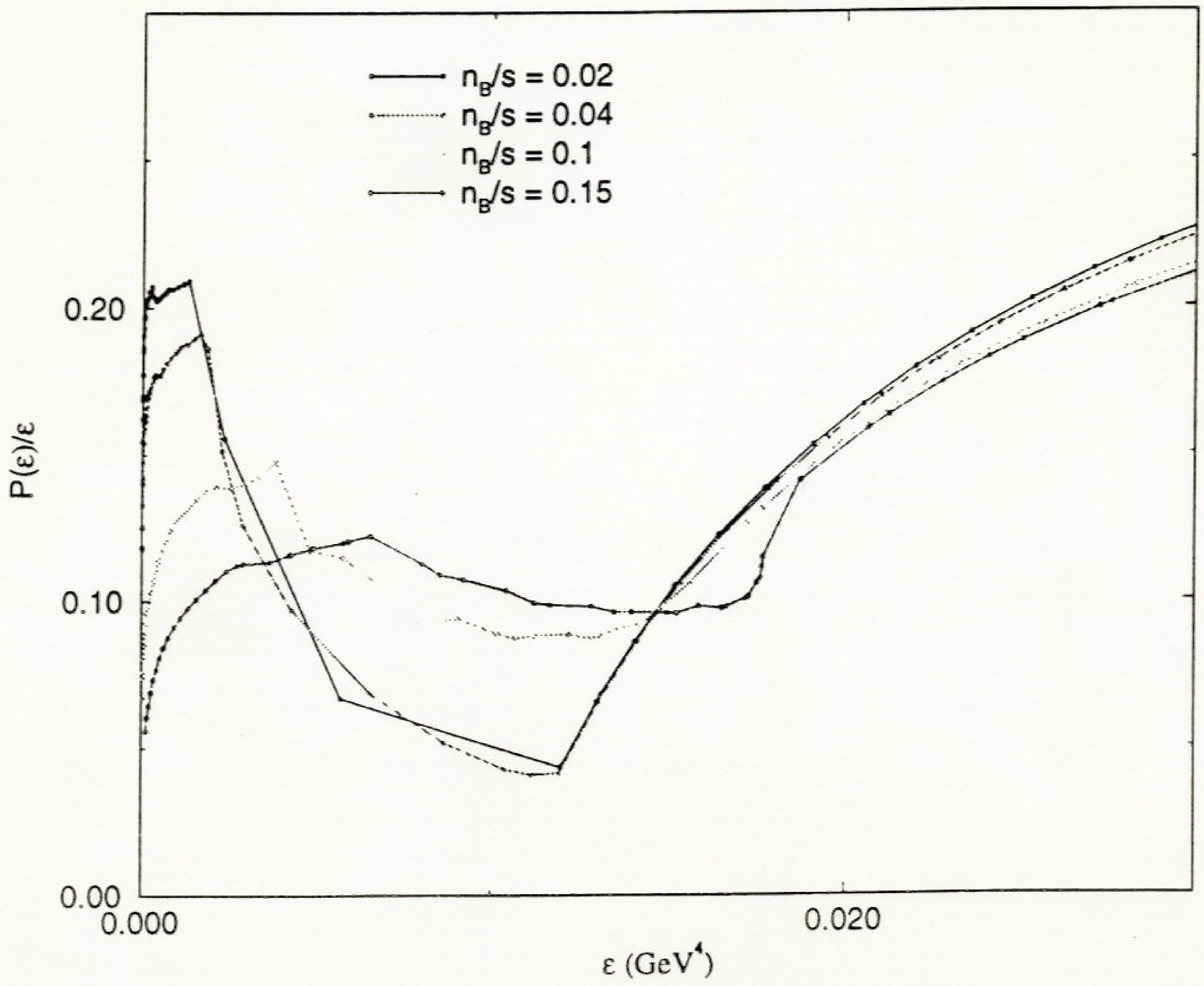
	yield	$B_{\mu\mu}$	$y \cdot B_{\mu\mu}$
ρ_0	0.0182	$4.60(28) \cdot 10^{-5}$	$8.37 \cdot 10^{-7}$
ω	0.0136	$7.07^{*}(19) \cdot 10^{-5}$	$9.62 \cdot 10^{-7}$
φ	0.00345	$2.50(40) \cdot 10^{-4}$	$8.63 \cdot 10^{-7}$

$$\left.\right\} \frac{\varphi}{\rho + \omega} = 0.48 \pm 0.08$$

*ee

Phase Diagram and Hydrodynamic Evolution (E.V. Shuryak, Hirschegg 1997)





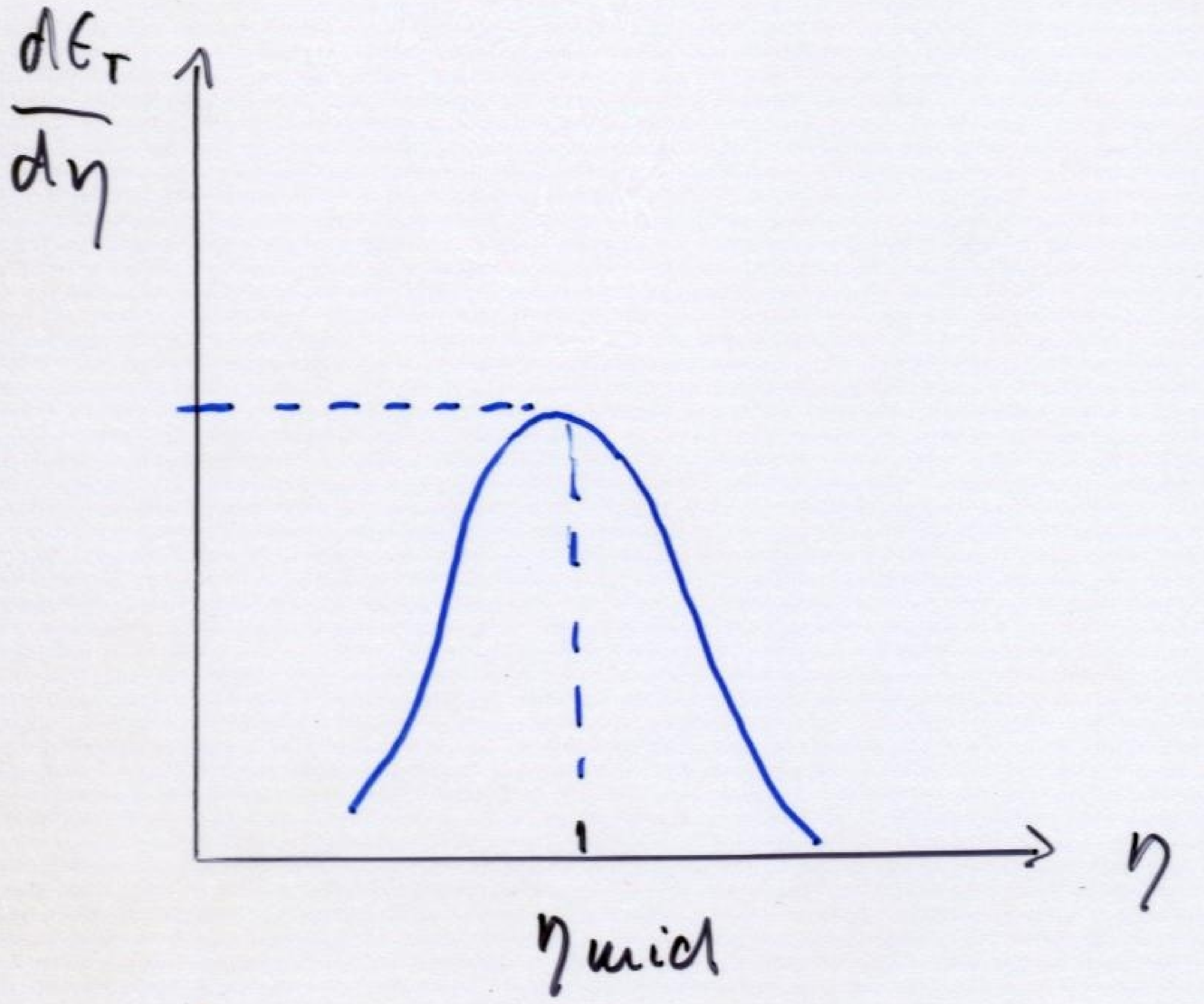
estimate of initial energy density

Bjorken 1982

$$\varepsilon_0 = \frac{dE_T}{d\eta}\frac{1}{A_\perp}\frac{d\eta}{dz} \simeq \frac{dE_T}{d\eta}\frac{1}{\pi R_A^2}\frac{1}{\tau_0}$$

$\tau_0 \simeq 1\,\mathrm{fm}$

$\frac{1}{\pi R_A^2} \simeq \frac{1}{\pi(1.2\,\mathrm{fm}\,A^{1/3})^2} = 150\,\mathrm{fm}^2$ for $A = 200$

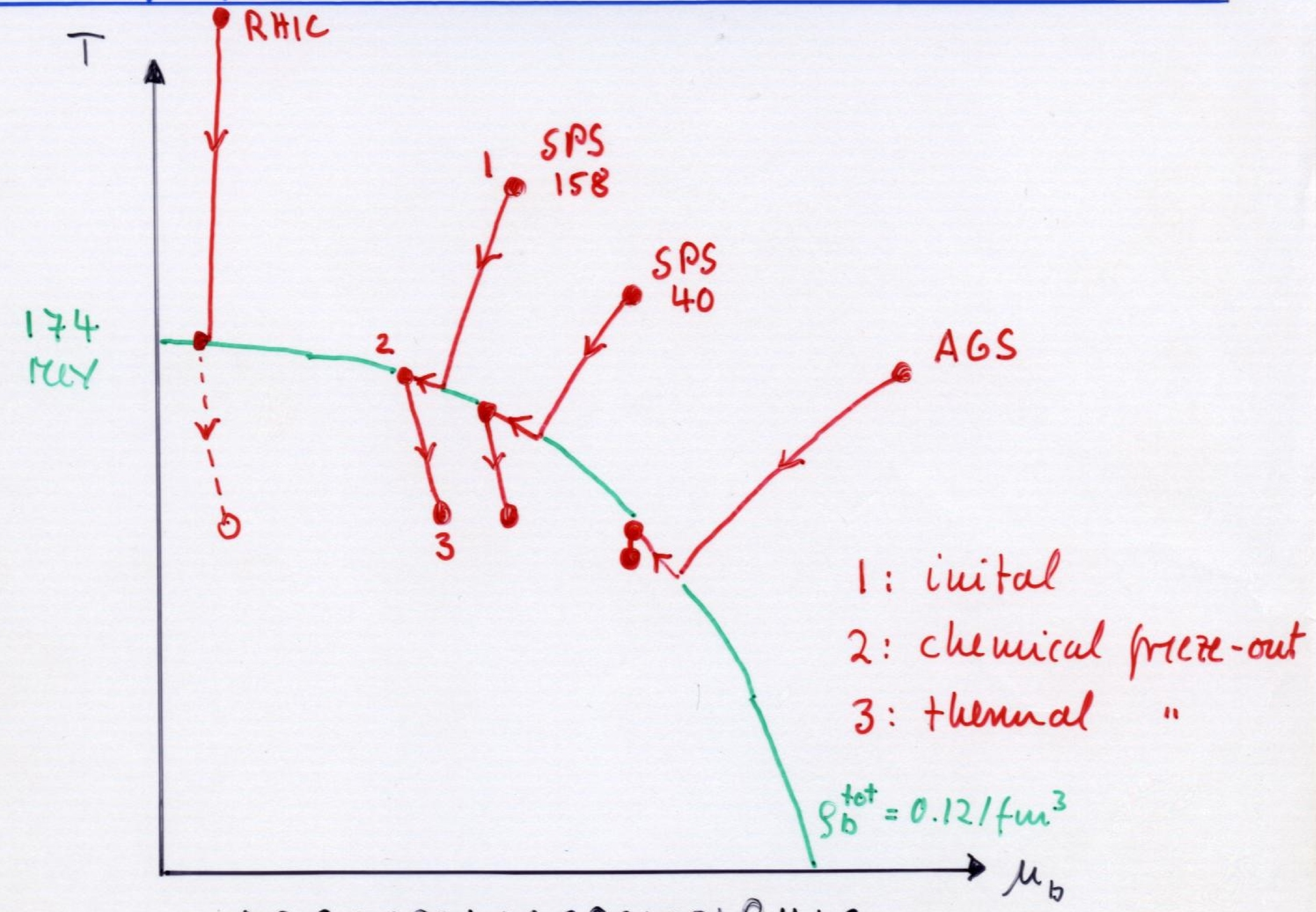


similarly initial baryon density ρ_0

$$\rho_0 = \frac{dN_b}{d\eta}\frac{1}{A_\perp}\frac{d\eta}{dz}$$

	AGS	SPS40	SPS158	RHIC
$dE_T/d\eta\,\|_{max}$	230	–	380	600
$2\,dN_p/d\eta$	150	114	76	
ε_0	1.5	–	2.5	$\tau = 0.2 \div 0.33\,\mathrm{fm}$ > 4 (10–20) GeV/fm³
ρ_0	1.0	0.75	0.50	1/fm³
ρ_0/ρ_{nuc}	≃ 7	5	3.3	

Temperature and Density at various stages of fireball evolution



	AGS	SPS40	SPS158	RHIC	
initial ρ_0	1.0	0.75	0.50		} dE_T/dy
ε_0	1.5		2.5	10-20	dN_p/dy
chem T/μ_b	125/540	148/400	170/255	177/41	} stat. model
nucl. dens.⊛	.126	.10	.10	.08	↔ hadron yields
pion dens	.17	.35	.59	.62	
thermal					
nucl. dens	.106	.077(5)	.063(5)	.060(9)	} HBT 2π-corr
pion dens	.15	.28(3)	.43(3)	.49(10)	
T	120	120	120	120?	
V_{therm}/V_{chem}	.86	← ≃ 1.3 →			
	≃ equal 3 & 2 coincide				

3d expansion R: 5.5 → 6.0 fm

at v/c ≃ 0.25 2 fm/c

(known from hydro) → 0.5 1 fm/c

⊛ note:
total baryon
density const
≃ 0.12/fm³

T	$\varepsilon_{lat}\left(\frac{MeV}{fm^3}\right)$	$\frac{\varepsilon}{T^4}$?	ε_{HG}
105.7	0		20.9
117.3	4.06		39.3
131.3	17.0		80.3
147.9	85.9		173.3
167.1	413.9		368.5
175.6	730.4		489.0
189.3	1471		719.3
214.3	2945		–

P. Chung: E895 Ξ^- analysis, neural network, & Λ

$\Xi/\Lambda+\Sigma^0$

4 0.007 | centrality ↓ 6 GeV very good agreement w. statistical model

4 0.01

6

8 0.014

H. Huang: K^+ and K^- comparison SiAu AuAu also does not scale w. Npart

argues $\Sigma^0 \approx 0.35\Lambda$, but then more Ξ than Σ ?? unlikely

thermal model [illegible] check: Ξ^- to Σ^0 Λ

0.0016 (0.0009 p)

0.0029 (0.0017 p)

0.013 (0.0025 pmin)

R_{AA} $\Lambda+\bar{\Lambda}$ goes close to 1 drops for $p_t \gtrsim 4$ GeV

K^0_S 0.7