Quark-Gluon-Plasma and Cold Atomic Physics - SS 2008

Thermodynamics of relativistic gases and the QGP

Florian Beutler

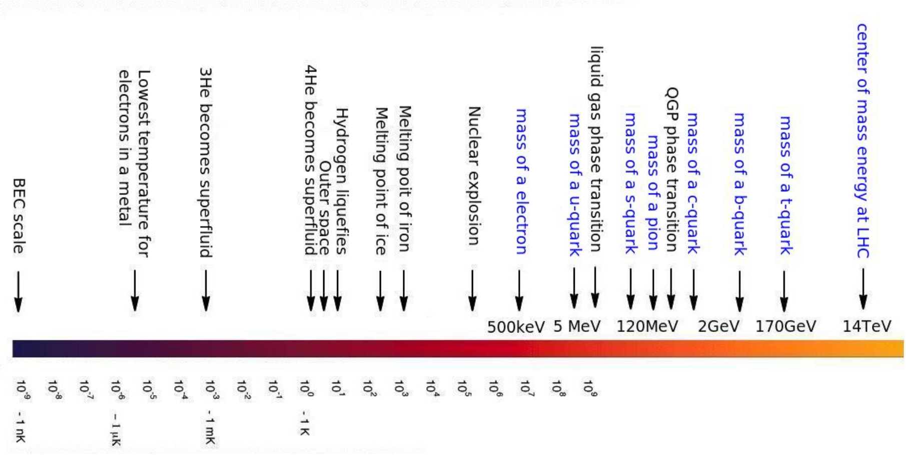
University of Heidelberg

14.04.2008

Outline

1. What is a quark gluon plasma?
   - The fundamentals of QCD
   - The MIT Bag model
   - Colour screening
2. Ideal relativistic gas
   - Hadronic gas
   - Gas of quarks and gluons
3. Construction of the phase diagram
   - Gibbs condition, estimation of T_c at $\mu = 0$
   - The $\mu \neq 0$ case
   - The energy density at the transition point
4. Is a pion gas a good approximation?
5. Conclusion

Energy scale



$$[\text{Kelvin}]/11605 = [\text{eV}]$$

What is a quark-gluon plasma?

Where we are?

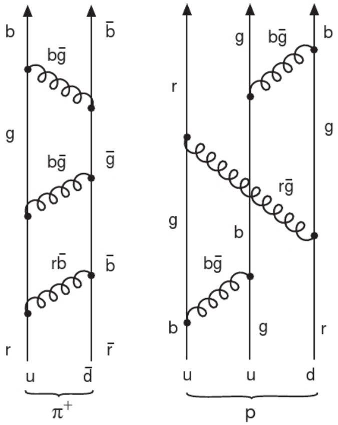
$\Rightarrow$ QCD = The theory of strong interactions.

In a QCD system at very high temperature and/or very high pressure the quarks and gluons are expected to become quasi-free. This deconfined dense state of matter is called a Quark Gluon Plasma (QGP).

But:

- How is it possible to get free quarks?
- What are the necessary conditions? $T_c = ?$, $n_c = ?$

The fundamentals of QCD



Postulate of the QCD:

All free particles are colourless!

- We will not find a free quark or a free gluon in nature.
- In QCD we have three different colours: red, green and blue.
- White objects are mesons and baryons $\rightarrow$ hadrons

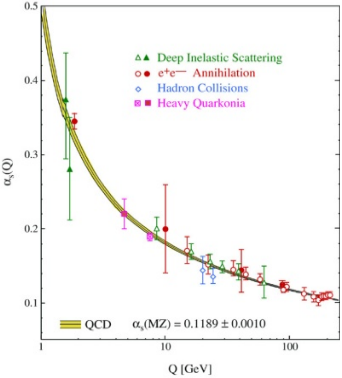
$$|\pi^+\rangle = \begin{cases} |u_r \bar{d}_{\bar{r}}\rangle \\ |u_g \bar{d}_{\bar{g}}\rangle \\ |u_b \bar{d}_{\bar{b}}\rangle \end{cases} \qquad |p\rangle = \begin{cases} |u_r u_g d_b\rangle \\ |u_g u_r d_b\rangle \\ |u_r u_b d_g\rangle \\ \vdots \end{cases}$$

The fundamentals of QCD

The QCD coupling constant:

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f)\ln(Q^2/\Lambda^2_{\rm QCD})}$$

- Strength of the coupling depends on the momenta exchange.
- Situation in QED and QCD is absolutely different
  - gluon loops
  - The QCD only calculable for $\alpha_s < 1$, means high energies
- At high momenta (small distances) the coupling constant becomes weaker → asymptotic freedom.



The MIT Bag model

Basic assumptions of the MIT Bag model:

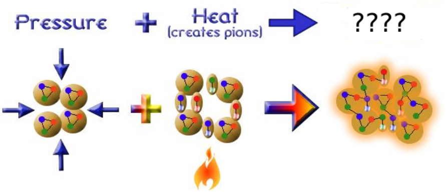
- Two different vacua, a confined and a deconfined vacuum.
- Quarks are caged in the volume of a hadron ("Bag") $\rightarrow$ confinement.
- Within this volume the quarks are free $\rightarrow$ asymptotic freedom.

This leads to the following consequences:

- To make room for a deconfined vacuum bubble of volume V in the confined vacuum, an energy $E = BV$ is necessary.
- To stabilize the bubble, the internal vapor pressure $p(T)$ must be equal to the external pressure B.

B is the Bag constant. The volume which is occupied by a hadron depends on B.

What happens at high temperature/density?

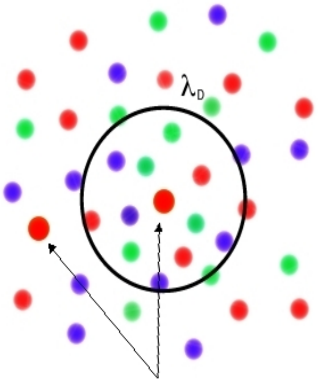


- Hadrons have intrinsic size $r_h \simeq 1fm$, need $V_h \simeq \frac{4\pi}{3} r_h^3$ to exist
  $\Rightarrow$Limiting density of hadronic matter: $n_c = 1/V_h$
  [Pomeranchuk 1951]
- Increasing temperature produces more and more particles (mainly pions). This leads to a limiting temperature: T_c = 150-200MeV

$\Rightarrow$ What lies beyond n_c, T_c?

Colour screening

- The charge of one particle is screened by the surrounding charges.
- (Debye) screening radius (λ_D): The distance at which the charge is reduced by 1/e.
- Originally defined for electromagnetic plasma, later extended to plasma of colour charges.

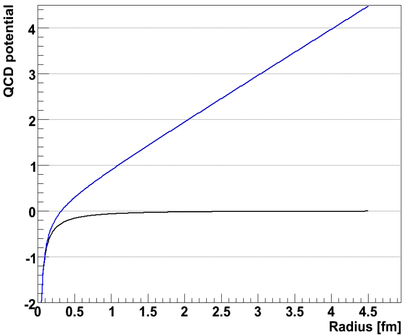


Colour screening

Normal QCD potential: $$V(r) = -\frac{\alpha(r)}{r} + \sigma r$$

Colour screened QCD potential: $$V(r) = -\frac{\alpha(r)}{r} e^{-\frac{r}{\lambda_D}}$$

- At $T \approx T_c$, a screening factor is introduced and the linear term in the potential vanishes.
- The screening length decreases with increasing temperature (and/or density) $\lambda_D = \lambda_D(T, n)$.
- Deconfinement sets in (we have a QGP).

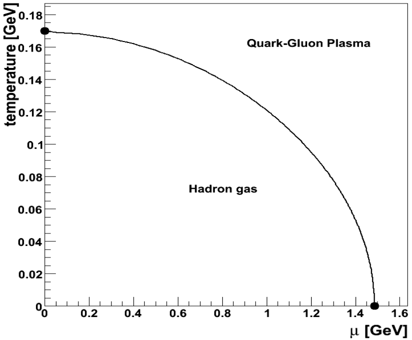


Phase transition

The Gibbs condition at a phase transition <u>of first order</u> is:

$$p_{\text{phase1}} = p_{\text{phase2}} \qquad \mu_{\text{phase1}} = \mu_{\text{phase2}}$$

In the case of a transition from hadron gas to QGP this will give:



Ideal relativistic gas

What can we do to simplify the problem?

⇒ For high energies: Approximation as a quantum ideal gas (neglecting interactions). Then we can apply the relativistic Equation of State (EoS)

$$\epsilon(T, \mu) = 3p$$

The hadron gas and the QGP states are described by a grand canonical partition function:

$$\ln Z(z, V, T) = \frac{gV}{(2\pi)^3} \int d^3p \ln(1 \pm e^{-\beta E} z)$$

g = d.o.f.
β = 1/T
E = $\sqrt{p^2 + m^2}$
z = $e^{\beta\mu}$

and the energy density is

$$\epsilon(T, \mu) = \frac{E}{V} = -\frac{1}{V}\frac{\partial}{\partial\beta} \ln Z(z, V, T)\bigg|_{z=const.} = \frac{g}{(2\pi)^3} \int \frac{E\ d^3p}{e^{(E-\mu)/T} \pm 1}$$

Bosons (pions and gluons)

The boson energy density:

$$\epsilon = \frac{g}{(2\pi)^3}\int\frac{E\,d^3p}{\exp((E-\mu)/T)-1} = \frac{4\pi g}{(2\pi)^3}\int\frac{p^3\,dp}{\exp(p/T)-1}$$

$$\epsilon = \frac{g}{(2\pi)^3}\int\frac{E\,d^3p}{\exp((E-\mu)/T)-1} = g\frac{3}{\pi^2}T^4\zeta(4) = g\frac{\pi^2}{30}T^4$$

with $\zeta(4) = \frac{\pi^4}{90}$.
And the number density

$$n = \frac{g}{(2\pi)^3}\int\frac{d^3p}{\exp((E-\mu)/T)-1}\bigg|_{\substack{m=0\\ \mu=0}} = \frac{1.2g}{\pi^2}T^3$$

Fermions (quarks and nucleons)

Fermion energy density (μ not generally $= 0$)

$$\epsilon = \frac{g}{(2\pi)^3} \int \frac{E\ d^3p}{\exp((E-\mu)/T)+1} = \frac{4\pi g}{(2\pi)^3} \int \frac{p^3\ dp}{\exp((p-\mu)/T)+1}$$

and we get for particle and antiparticle ($\epsilon(\mu) = \bar{\epsilon}(-\mu)$)

$$\epsilon + \bar{\epsilon} = g\left(\frac{7\pi^2}{120}T^4 + \frac{\mu^2}{4}T^2 + \frac{\mu^4}{8\pi^2}\right)$$

and the number density is

$$n = \frac{g}{(2\pi)^3} \int \frac{d^3p}{\exp((E-\mu)/T)+1} = \frac{4\pi g}{(2\pi)^3} \int \frac{p^2\ dp}{\exp((p-\mu)/T)+1}$$

what yields for particle and antiparticle

$$n - \bar{n} = g\left(\frac{\mu}{6}T^2 + \frac{\mu^3}{6\pi^2}\right) \rightarrow n = \bar{n} \Rightarrow \mu = 0$$

Hadronic gas ($T \neq 0$, $\mu = 0$ case)

The pressure of the hadronic phase is, at low temperature, dominated by the pressure of the lightest hadron. Therefore we approximate our hadron gas as a massless pion gas and neglect all other particles.

If we treat the hadron gas as an ideal pion gas

$$p_\pi(T) = g_\pi \frac{\pi^2}{90} T^4$$

$$\epsilon_\pi(T) = 3p_\pi(T) = g_\pi \frac{\pi^2}{30} T^4$$

with $g_\pi = 3$ and accordingly the entropy density

$$s = \frac{1}{T}(\epsilon + p) = 4g_\pi \frac{\pi^2}{90} T^3$$

Gas of quarks and gluons - The MIT Bag model

The pressure difference between the both vacua in the MIT Bag model is negative

$$\Delta p = p_{deconf} - p_{conf} = -B < 0$$

and causes the confinement.

Thus the energy density and pressure in the QCD vacuum should be modified according to the rule

$$\begin{aligned} p'_{QGP} &\rightarrow p_{QGP} = p'_{QGP} - B \\ \epsilon'_{QGP} &\rightarrow \epsilon_{QGP} = \epsilon(p_{QGP}) + B \end{aligned}$$

But B = B(T,n) and B is not well known (but we will fix B in the further treatment).

The vacuum structure keeps the coloured particles bound and confined. To be able to move colour charges within a region of space, one needs to "melt" the confining structure.

Gas of quarks and gluons (T $\neq$ 0, $\mu = 0$ case)

Using again the approximation of a ideal gas we get for the QGP consisting of just gluons and quarks.

$$\begin{aligned} p'_{QGP} &= 2_{q,\overline{q}} g_q \frac{7\pi^2}{720} T^4 + g_g \frac{\pi^2}{90} T^4 \\ &= (\frac{7}{8} 2_{q,\overline{q}} \cdot g_q + g_g) \frac{\pi^2}{90} T^4 \end{aligned}$$

and

$$\epsilon'_{QGP} = 3p'_{QGP} = (\frac{7}{8} 2_{q,\overline{q}} \cdot g_q + g_g) \frac{\pi^2}{30} T^4$$

Account for the MIT Bag model

$$\begin{aligned} p_{QGP} &= p'_{QGP} - B \\ \epsilon_{QGP} &= 3(p_{QGP} + B) + B = 3p_{QGP} + 4B \end{aligned}$$

Degrees of freedom for a QGP

The effective number of partonic degrees of freedom in a QGP state

$$g = \frac{7}{8} 2_{q,\bar{q}} \cdot g_q + g_g$$

where g_q and g_g are the d.o.f. of, respectively, the quark and gluon states

$$\begin{aligned} g_q &= 2_{spin} \cdot 3_{colour} \cdot n_f = 6n_f \\ g_g &= 2_{spin} \cdot 8_{colour} = 16 \end{aligned}$$

which yields the value g = 37(95/2) for an $n_f = 2(3)$ flavour QGP.

Rollback -Quarks-

Quarks are:

- Fermions with spin $= \pm\frac{1}{2}$.
- Carry colour charge, weak charge, electric charge and mass.

name	flavour	symbol	electric charge	mass[MeV]
Up		u	$+\frac{2}{3}$	1.5-3.0
Down		d	$-\frac{1}{3}$	3-7
Strange	S = -1	s	$-\frac{1}{3}$	95±25
Charm	C = +1	c	$+\frac{2}{3}$	1250±90
Bottom	B = -1	b	$-\frac{1}{3}$	4200±70
Top	T = +1	t	$+\frac{2}{3}$	170900±1800

Rollback -Gluons-

Gluons are:

- Exchange particle of the strong interaction
- Massless
- Boson (Spin = 1)
- Carry colour charge and anticharge

We have 8 independent possibilities (SU(3) has 8 generators ⇒ 8 gluons):

$$|r\bar{g}\rangle,\ |r\bar{b}\rangle,\ |g\bar{r}\rangle,\ |g\bar{b}\rangle,\ |b\bar{r}\rangle,\ |b\bar{g}\rangle,$$

and

$$\frac{1}{\sqrt{6}}(|r\bar{r}\rangle + |g\bar{g}\rangle - 2|b\bar{b}\rangle)$$
$$\frac{1}{\sqrt{2}}(|r\bar{r}\rangle - |g\bar{g}\rangle)$$

Degrees of freedom for a QGP

The effective number of partonic degrees of freedom in a QGP state

$$g = \frac{7}{8} 2_{q,\bar{q}} \cdot g_q + g_g$$

where g_q and g_g are the d.o.f. of, respectively, the quark and gluon states

$$\begin{aligned} g_q &= 2_{spin} \cdot 3_{colour} \cdot n_f = 6n_f \\ g_g &= 2_{spin} \cdot 8_{colour} = 16 \end{aligned}$$

which yields the value g = 37(95/2) for an $n_f = 2(3)$ flavour QGP.

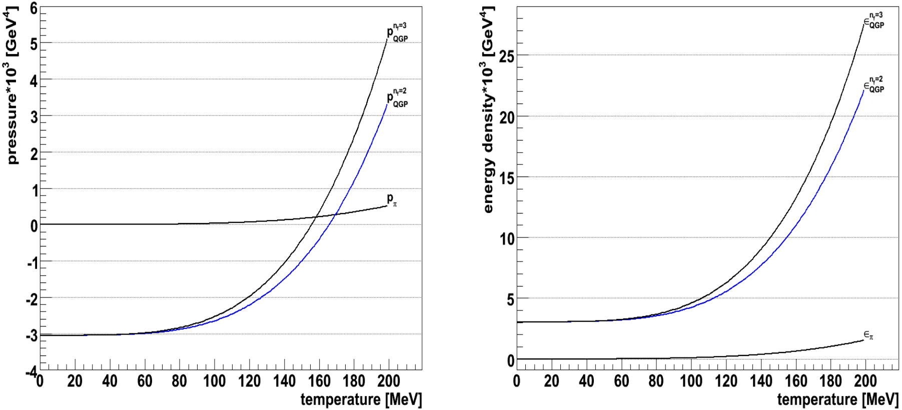


Figure: (a) Determination of the phase boundary in the T-μ plane. (b) Energy density for a pion gas and a QGP.

Estimation of T_c

$$p_{QGP}(T_c) = p_\pi(T_c) \quad \text{and} \quad T_{QGP} = T_\pi = T_c$$

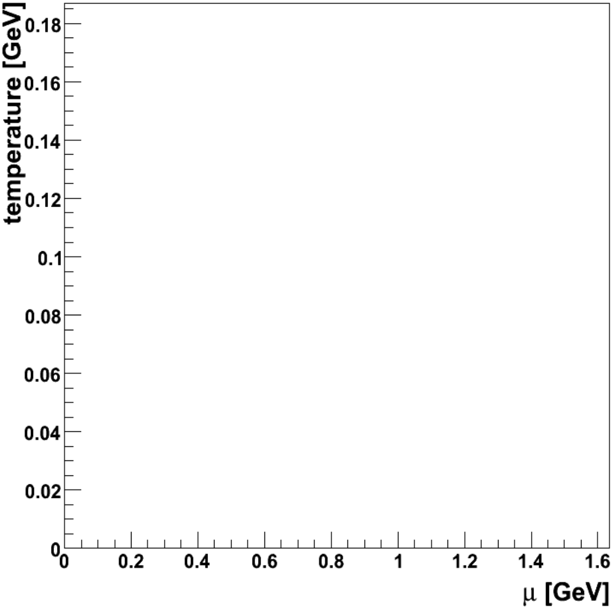
what yields

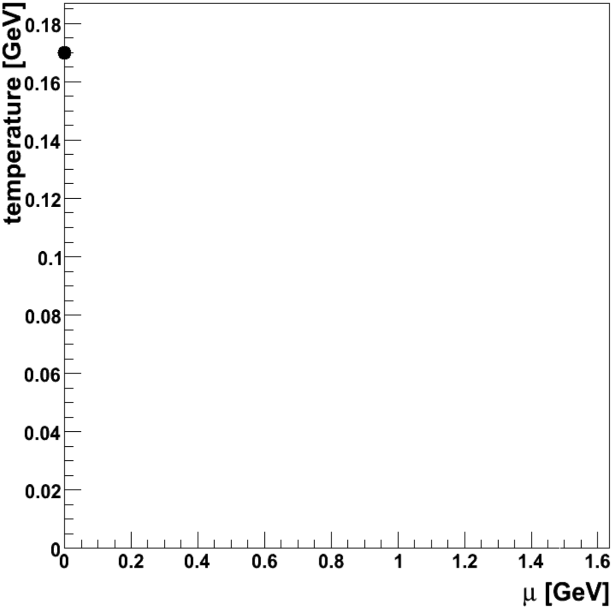
$$g_\pi \frac{\pi^2}{90} T_c^4 = g_{QGP} \frac{\pi^2}{90} T_c^4 - B$$

$$\Leftrightarrow \quad B = \Delta g \frac{\pi^2}{90} T_c^4$$

$$\Leftrightarrow \quad T_c = \sqrt[4]{\frac{90B}{\Delta g \pi^2}}$$

taking $\mathrm{B}^{1/4} = 0.235\mathrm{GeV}$ we get $T_c \simeq 169\mathrm{MeV}$.


temperature [GeV]
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
μ [GeV]


temperature [GeV]
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
μ [GeV]

The T = 0, $\mu \neq 0$ case

At T = 0 (no pions) and $\mu \neq 0$ we can approximate our gas as a gas of nucleons (lightest baryons)
(mass is not neglectable)

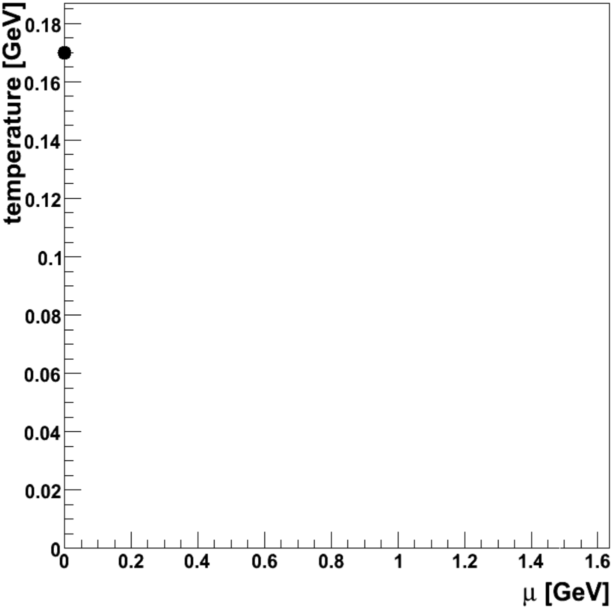
$$p_{nuc} + \overline{p}_{nuc} = \frac{g_{nuc} 4\pi}{(2\pi)^3} \int \frac{Ep^2 dp}{e^{(E-\mu)/T} + 1} + \frac{g_{nuc} 4\pi}{(2\pi)^3} \int \frac{Ep^2 dp}{e^{(E+\mu)/T} + 1}$$

with $g_{nuc} = 4$

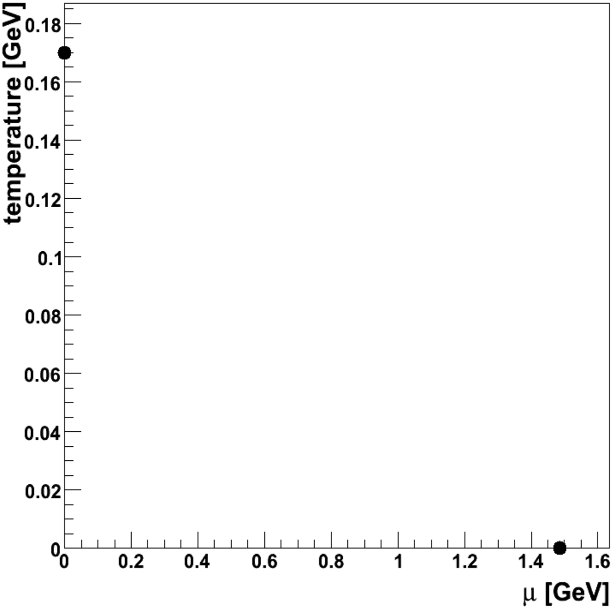
$$p_{QGP} = \frac{g_q \mu_q^4}{24\pi^2} - B$$

with $\mu = 3\mu_q$ and $g_q = 12$ (only quarks, no gluons)

$$p_{nuc} + \overline{p}_{nuc} = p_{QGP} \overset{\text{n.s.}}{\Longrightarrow} \underline{\mu = 1485\,MeV}$$



temperature [GeV]
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
μ [GeV]



temperature [GeV]
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
μ [GeV]

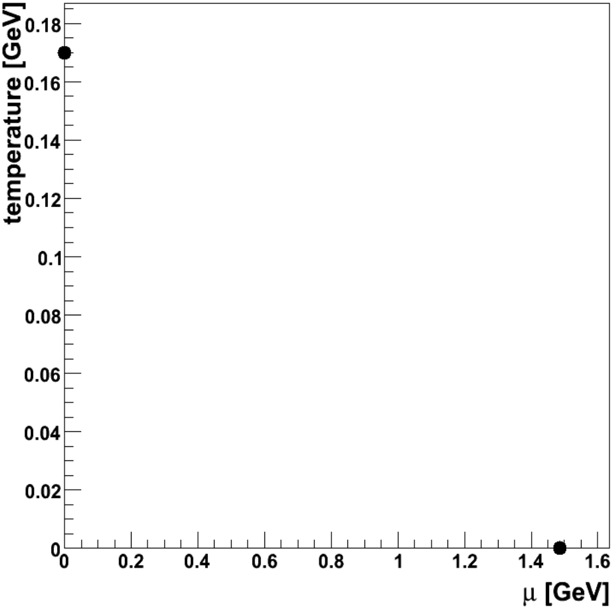
Constructing the phase diagram

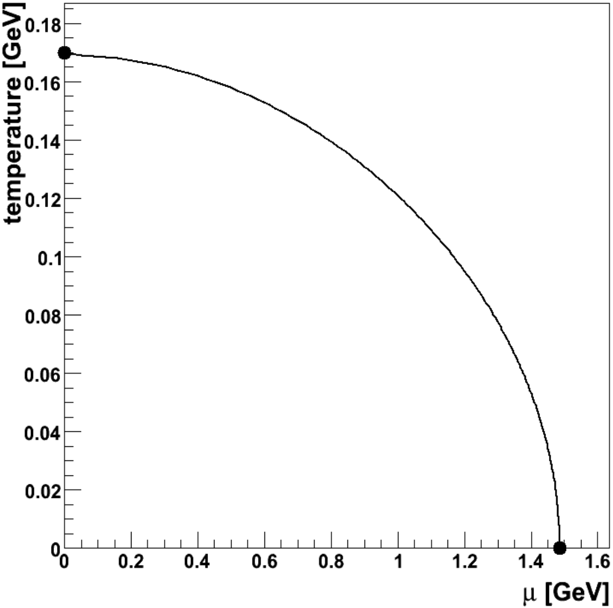
The T $\neq$ 0 and $\mu \neq 0$ case:

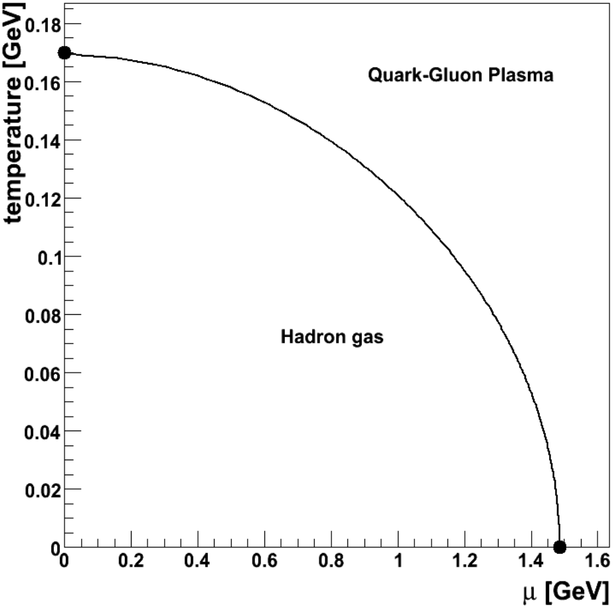
$$g_\pi \frac{\pi^2}{90} T^4 + \sum_{p+\overline{p}} \frac{g_{\text{nuc}} 4\pi}{(2\pi)^3} \int \frac{E p^2 dp}{e^{(E \pm \mu)/T} + 1} = \frac{g_{q\overline{q}}}{3} \left(\frac{7\pi^2}{120} T^4 + \frac{\mu_q^2}{4} T^2 + \frac{\mu_q^4}{8\pi^2} \right) + g_g \frac{\pi^2}{90} T^4 - B$$

with

$$\begin{aligned} g_{q\overline{q}} &= 6 n_f = 12 \\ g_g &= 16 \\ g_\pi &= 3 \\ g_{\text{nuc}} &= 4 \end{aligned}$$


temperature [GeV]
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
μ [GeV]


temperature [GeV]
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
μ [GeV]

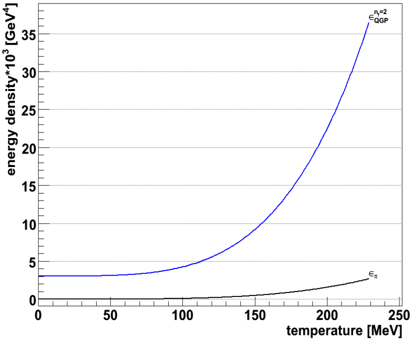


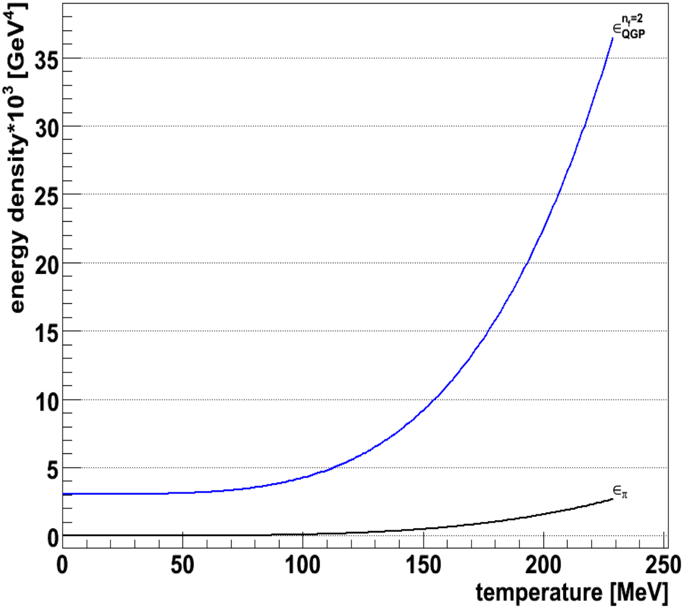
temperature [GeV]
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
μ [GeV]
Quark-Gluon Plasma
Hadron gas

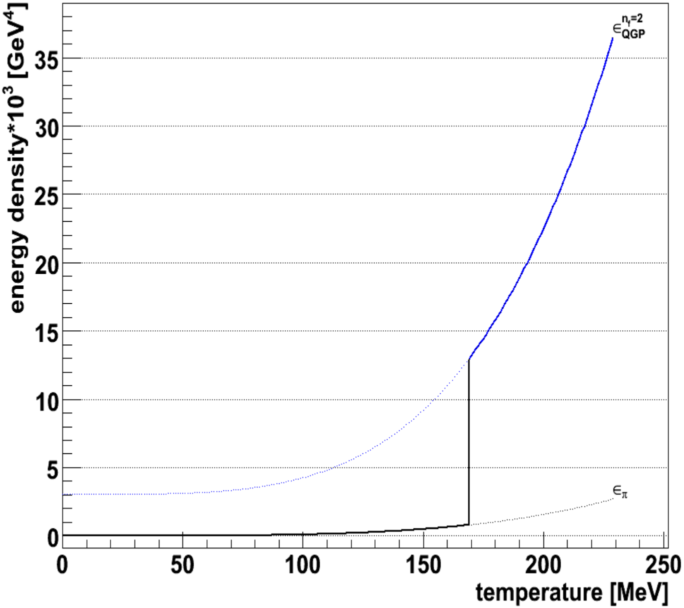
The energy density at the transition point

The latent heat is (again the $\mu = 0$ case):

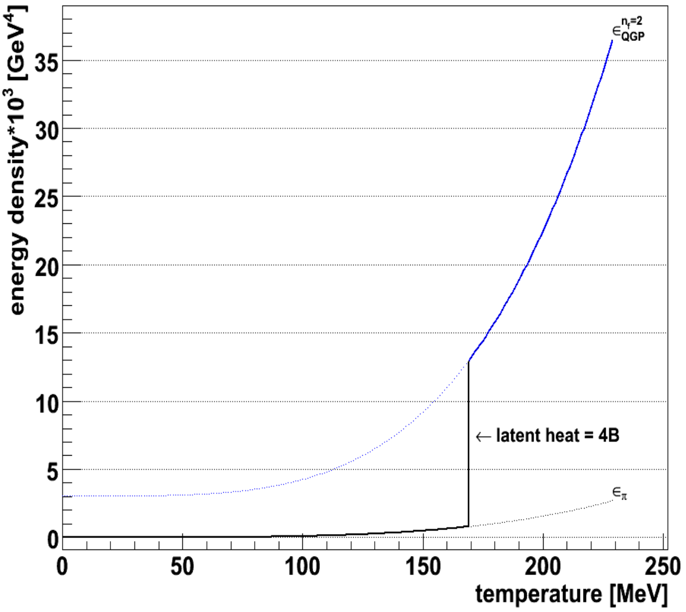
$$Q = \epsilon_{QGP} - \epsilon_\pi = (g_{QGP}\frac{\pi^2}{30}T^4 + B) - g_\pi \frac{\pi^2}{30}T^4 = 4B$$




energy density*10^3 [GeV^4]
0
5
10
15
20
25
30
35
0
50
100
150
200
250
temperature [MeV]
$\epsilon_{QGP}^{n_f=2}$
ϵ_{π}



energy density*10^3 [GeV^4]
35
30
25
20
15
10
5
0
0
50
100
150
200
250
temperature [MeV]
ϵ^{n_f=2}_{QGP}
ϵ_π



energy density*10^3 [GeV^4]
35
30
25
20
15
10
5
0
0
50
100
150
200
250
temperature [MeV]
ε_QGP^(n_f=2)
← latent heat = 4B
ε_π

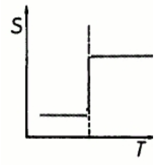
The energy density at the transition point

What happens with the energy density at the phase transition?

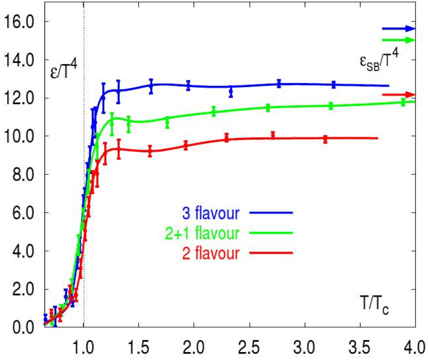
$$\frac{\epsilon_{QGP}(T_c)}{\epsilon_\pi(T_c)} = \frac{(\frac{7}{8}2_{q\bar{q}} \cdot g_q + g_g)\frac{\pi^2}{30}T^4 + B}{g_\pi \frac{\pi^2}{30}T^4} = \dots = \frac{4\frac{g_{QGP}}{g_\pi} - 1}{3} \approx 16$$

$\Rightarrow$ Step in the energy density at the transition point. Transition from hadronic to partonic degrees of freedom.

The latent heat Q = 4B is an increase of energy with fixed temperature and causes therefore a step in the entropy density.



$$\frac{s_{QGP}(T_c)}{s_\pi(T_c)} = \frac{\beta(\epsilon_{QGP} + B + p_{QGP} - B)}{\beta(\epsilon_\pi + p_{pi})} = \dots = \frac{g_{QGP}}{g_\pi} \approx 12$$



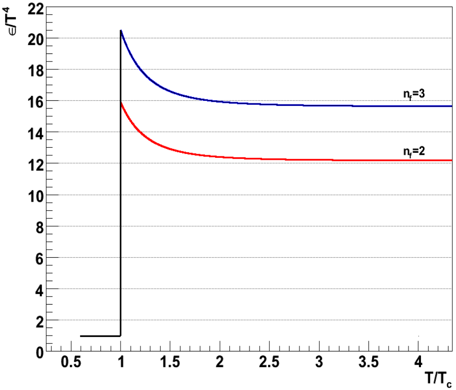


Figure: Energy density ϵ as a function of temperature (left) from lattice calculations and (right) with relativistic gas approximation.

left: [F. Karsch et al. hep-lat/0109017]

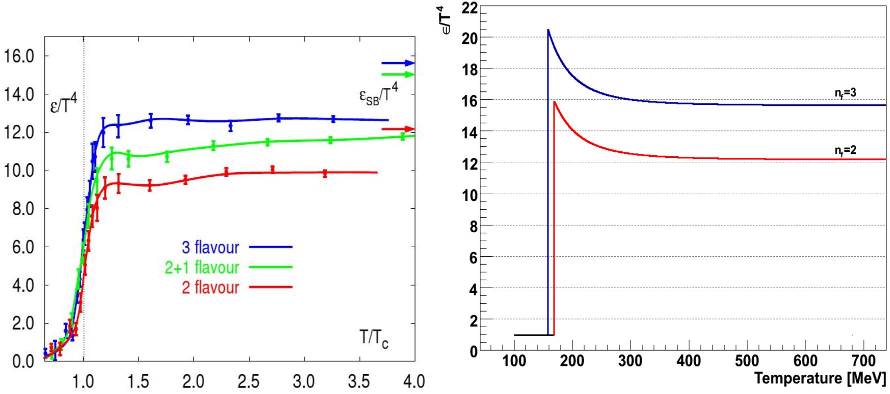


Figure: Energy density ϵ as a function of temperature (left) from lattice calculations and (right) with relativistic gas approximation.

left: [F. Karsch et al. hep-lat/0109017]

Is a pion gas a good approximation?

What is the reason for the difference between lattice QCD calculations and our results?

- Negligence of interactions.
- The hadron gas approximation as a pion gas is questionable for high temperatures (is lattice QCD better?).

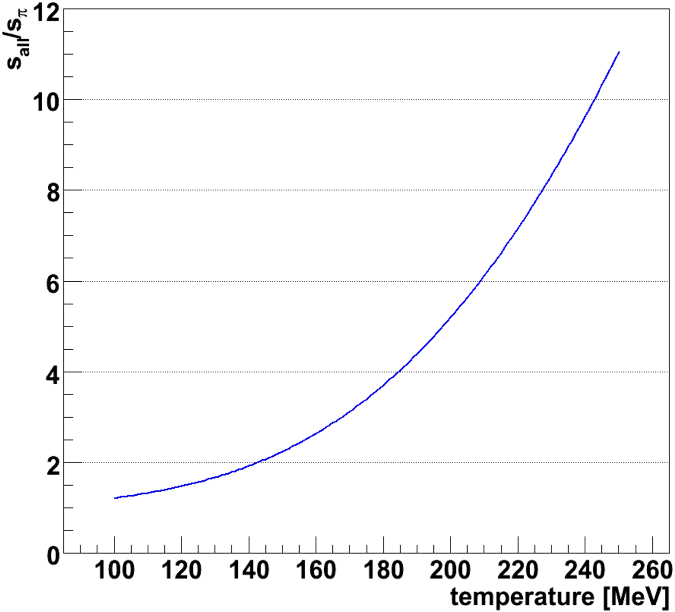
Check with all particles (the most common 330 particle types)

$$\frac{s_{all}}{s_\pi} = \frac{\beta(\epsilon_{all} + p_{all})}{\beta(\epsilon_\pi + p_\pi)} = \frac{g_{all}}{g_\pi}$$

with

$$s_{all}/T = \sum_i \epsilon_i + \sum_i p_i$$

If pion gas is a good approximation, we would expect that this ratio is 1...


s_{all}/s_{π}
0
2
4
6
8
10
12
100
120
140
160
180
200
220
240
260
temperature [MeV]

Conclusion

- There is a transition in strong interaction thermodynamics at which:
  - Deconfinement sets in (we get a QGP).
  - Latent heat increases the energy density ($\epsilon_c \approx 1.7 GeV/fm^3$).
  - The transition temperature is $T_c \simeq$ 150-180MeV.
- To reach this state of matter (QGP), high energies are needed (Big Bang, neutron stars, heavy ion collisions...)
- In a first approximation it is possible to use the ideal gas equation to estimate the critical parameters (neglecting interactions). Further approximations are
  - Hadron gas as massless pion gas.
  - QGP as a massless gas of u, d (and s) quarks and gluons.
- But this approximations give errors of about 20%.
- Lattice QCD calculations are the best calculations available, up to now.

Please keep in mind

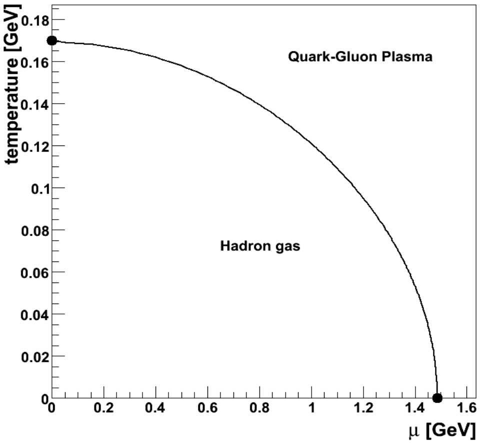


Figure: The nuclear matter phase diagram