

(Lattice Results For) Quark-Gluon Plasma

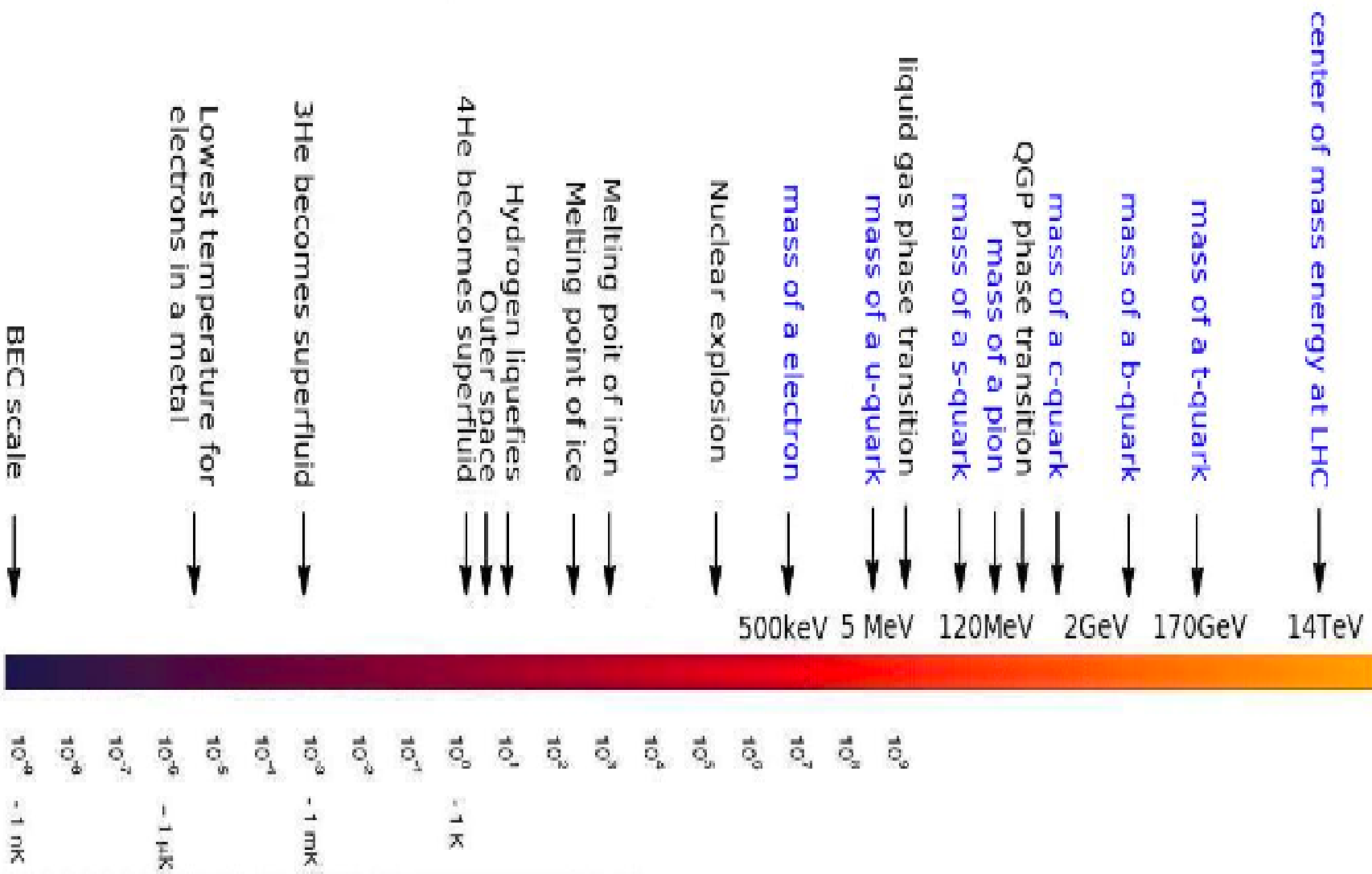
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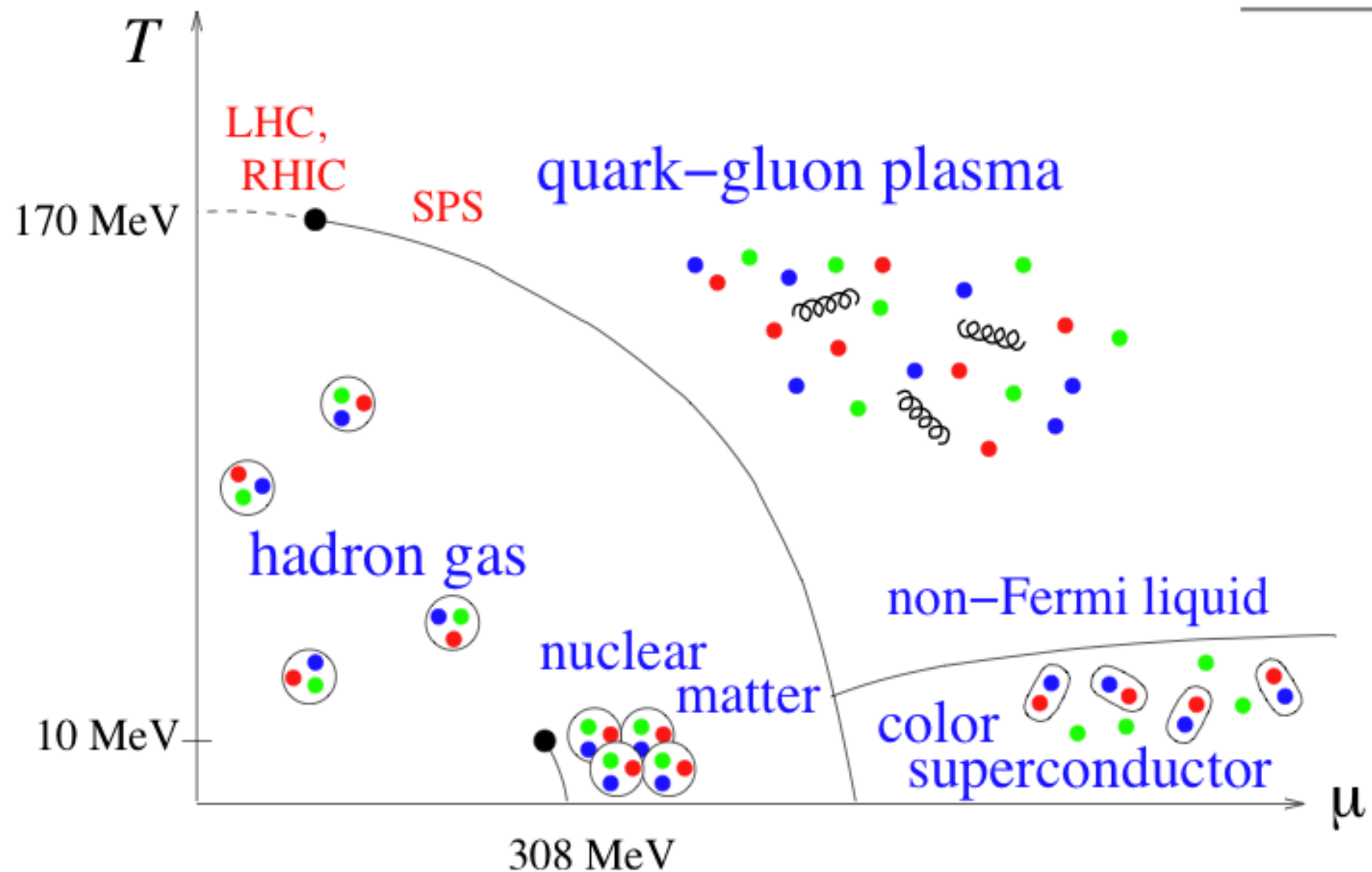
09.06.08

Outline

- 1 introduction
- 2 equation of state
 - pressure
 - energy density
- 3 phase transition
 - sensitive observables
 - $\mu \neq 0$
 - (strange quark) mass sensitivity
- 4 summary



phase diagram of strongly interacting matter



thermodynamics

- partition function

$$Z(\vec{K}) = \int [dU] F(U) e^{-S_E(U)}$$

U gauge field; $F(U)$ quark contribution; $S_E = S_G + S_F$

- physical observables:

continuum ($a \rightarrow 0$) and thermodynamic ($V \rightarrow \infty$) limit

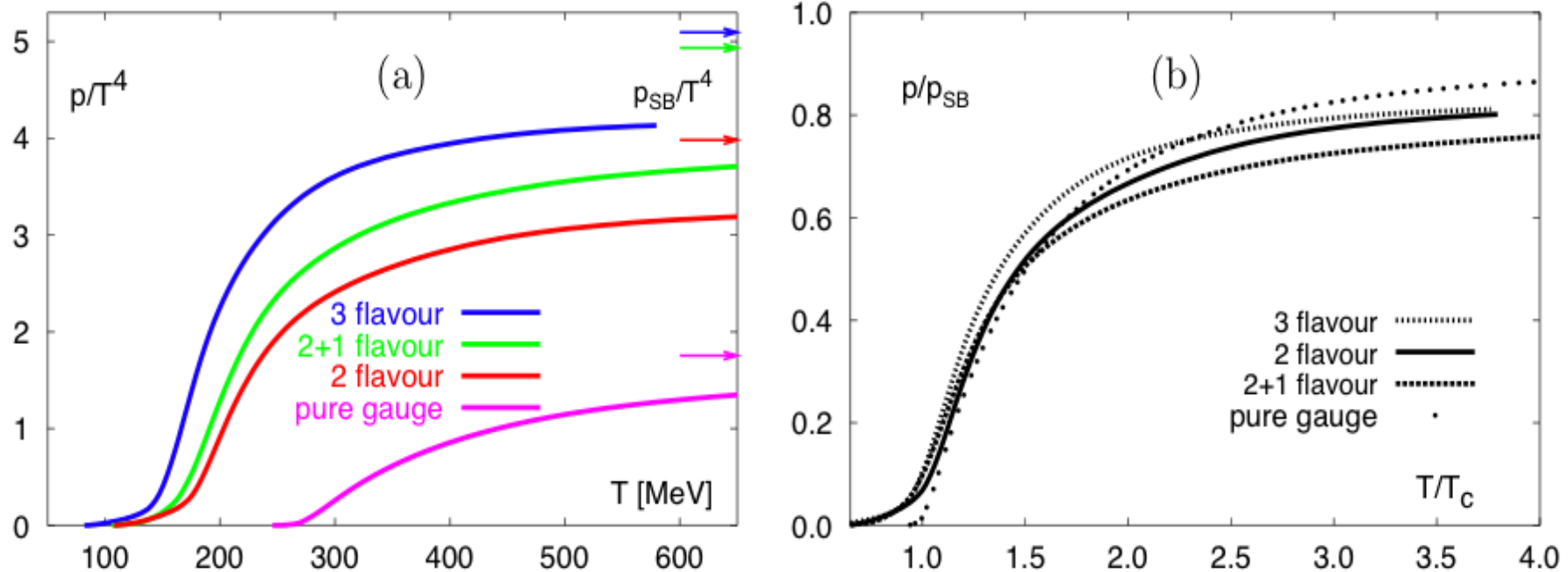
- pressure

$$p(\vec{K}) = \frac{T}{V} \ln Z(\vec{K})$$

- energy density

$$\frac{\epsilon - 3p}{T^4} = T \frac{d}{dT} \left(\frac{p}{T^4} \right)_{\text{fixed } \mu/T}$$

- in the following: $\mu = 0$

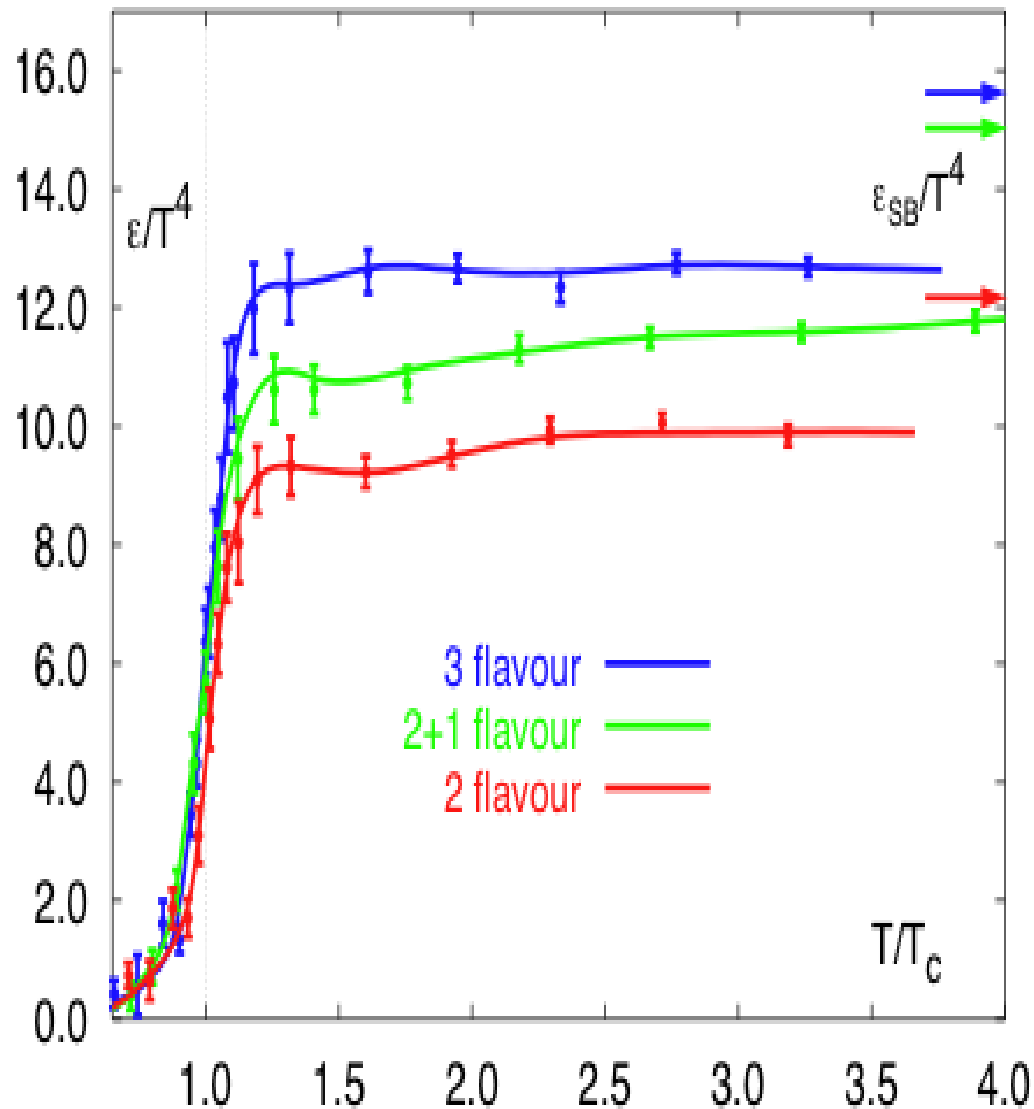


- left

- pressure in QCD with: $n_f = 0, 2$ and 3 light quarks
- and 2 light and a heavier quark
- pure gauge: continuum extrapolation
- arrows: ideal gas pressure
- p and T_c dependent on degrees of freedom

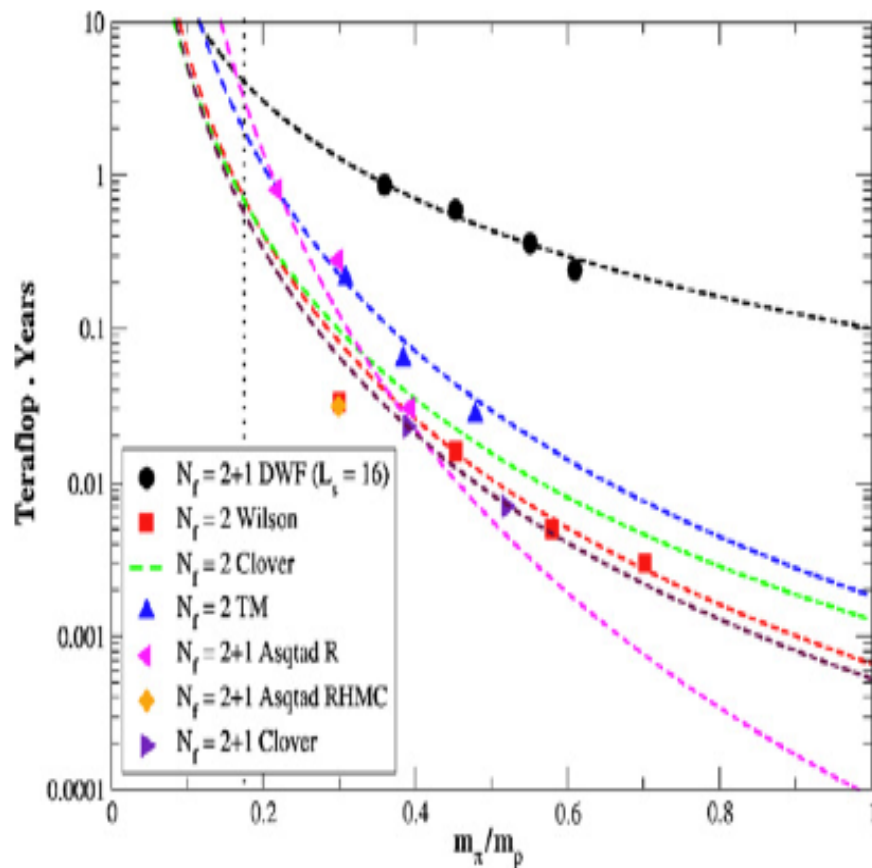
- right:

- almost flavour independent



- $N_\sigma = 16; N_\tau = 4$
- **2,3**: $m_l/T = 0.4$
- **+1**: $m_h/T = 1$
- StefanBoltzmann not reached
- $\epsilon_c \approx 6T_c^4;$
 $T_c \approx 175\text{MeV}$
- $\epsilon_c \approx 0.7\text{GeV}/\text{fm}^3$

berlin wall plot



cost to generate 1000 independent gauge configurations

on $24^3 \times 40$ lattice with $a = 0.08 \text{ fm}$

- 10^{12} **F**loating point **O**perations **P**er **S**econd
- P4, 3.2 GHz ≈ 7.2 GigaFlops
- $\pi, \rho: |ud\rangle; S_\pi = 0, S_\rho = 1$
- $m_{\text{tot}} = m_{q_1} + m_{q_2} + \Delta m_{ss}$
- physical values:
 $m_\pi = 140 \text{ MeV}$
 $m_\rho = 775 \text{ MeV}$
- increase due to “sign problem”

QCD: two phase transitions

1 restoration of chiral (helicity) symmetry ($m_q \rightarrow 0$)

$$SU_{L+R}(N_f) \rightarrow SU_L(N_f) \times SU_R(N_f)$$

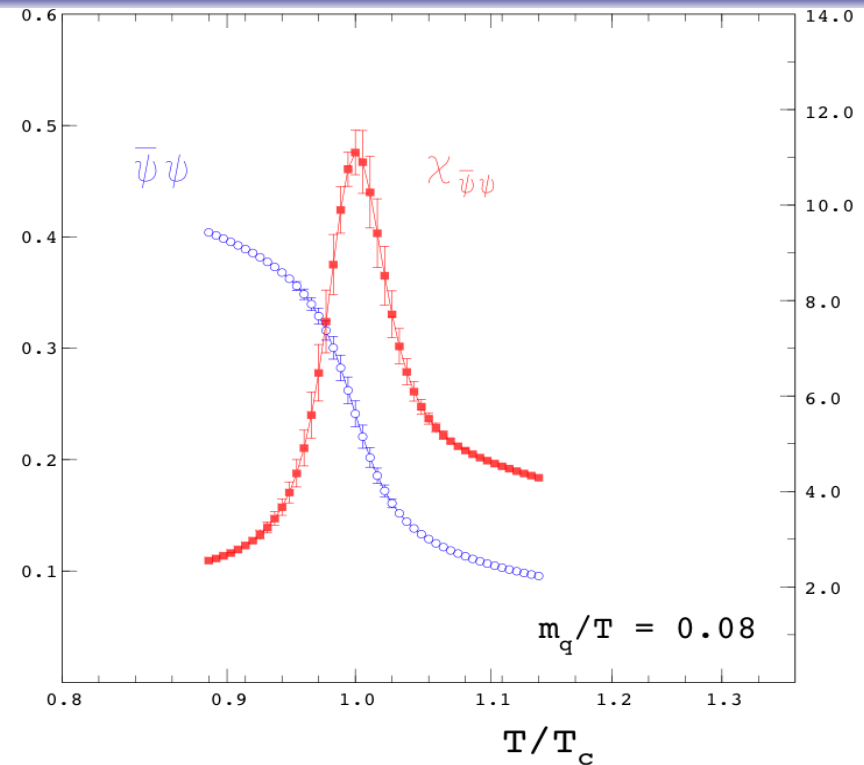
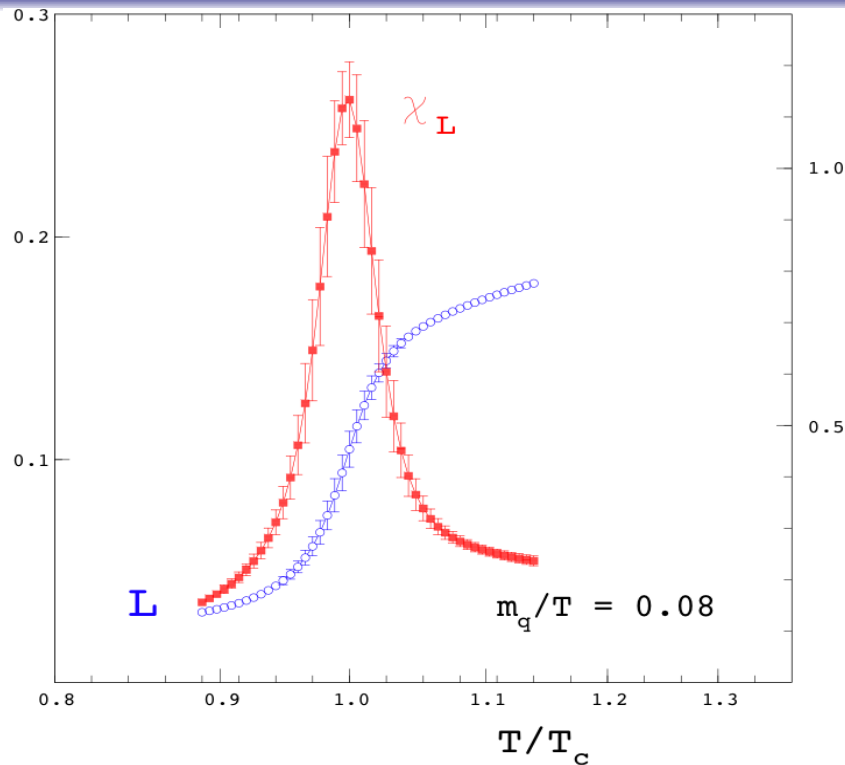
order parameter (chiral condensate) :

- $\langle \bar{\psi}_f \psi_f \rangle = \langle \bar{q} q \rangle = \frac{T}{V} \frac{\partial}{\partial m_f} \ln Z(T, V, \mu)$
- bare quark mass (QGP): $m_{u,d} \approx 5 \text{ MeV}$ satisfies chiral limit well (hadron: $m_{\text{Proton}} \approx 1 \text{ GeV}$)

2 de/confinement

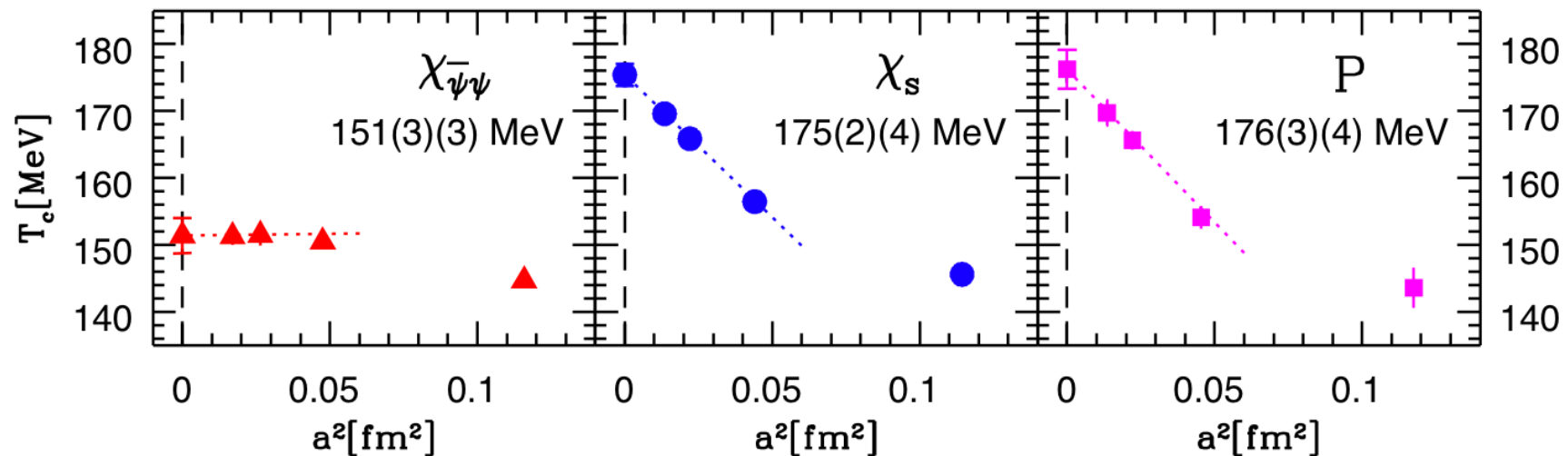
order parameter (Polyakovloop, $m_q \rightarrow \infty$):

- $\langle L \rangle = \frac{1}{V} \langle \sum_{\vec{x}} \text{Tr} \mathcal{P} L(\vec{x}) \rangle$
 $L(\vec{x}) = \mathcal{P} e^{-\int_0^{1/T} dx_0 A_0(x_0, \vec{x})}$
 (closed line Integral over gluon field representing static quark source)
- symmetry explicitly broken in presence of light dynamical quarks, still:
- free energy of a static quark placed in thermal heat bath:
 $F_q(T) = -T \ln(\langle L \rangle)$



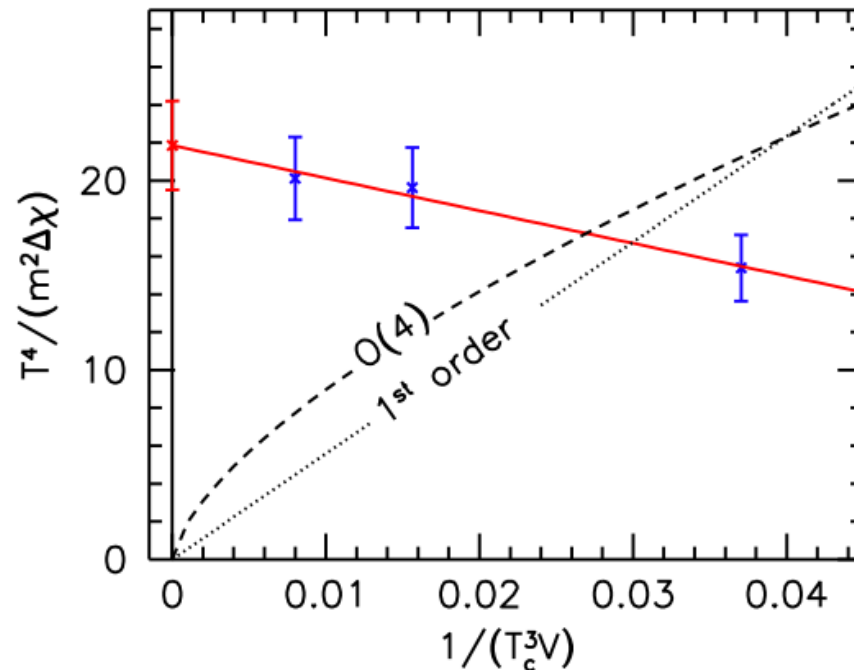
- (left) Polyakov loop
 - $\langle L \rangle = 0 \leftrightarrow$ confined phase, $T < T_c$
 - $\langle L \rangle > 0 \leftrightarrow$ deconfined phase, $T > T_c$
- (right) chiral condensate
 - $\langle \bar{q}q \rangle > 0 \leftrightarrow$ symmetry broken, $T < T_c$
 - $\langle \bar{q}q \rangle = 0 \leftrightarrow$ symmetric phase, $T > T_c$
- and their temperature derivatives ($\chi_{L, \bar{\psi}\psi}$)
- both transitions almost at same temperature

critical temperature T_c



- continuum *extrapolation* from $N_\tau = 6, 8, 10$; $a^2 \propto \frac{1}{N_\tau^2}$
 - (left) chiral condensate susceptibility
 - (middle) strange quark number susceptibility: $\chi_s = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial \mu_s^2}$
 - (right) normalized Polyakov loop
- T_c depends **a lot** on the variable
- and the group: Fodor/Katz (above) vs Karsch

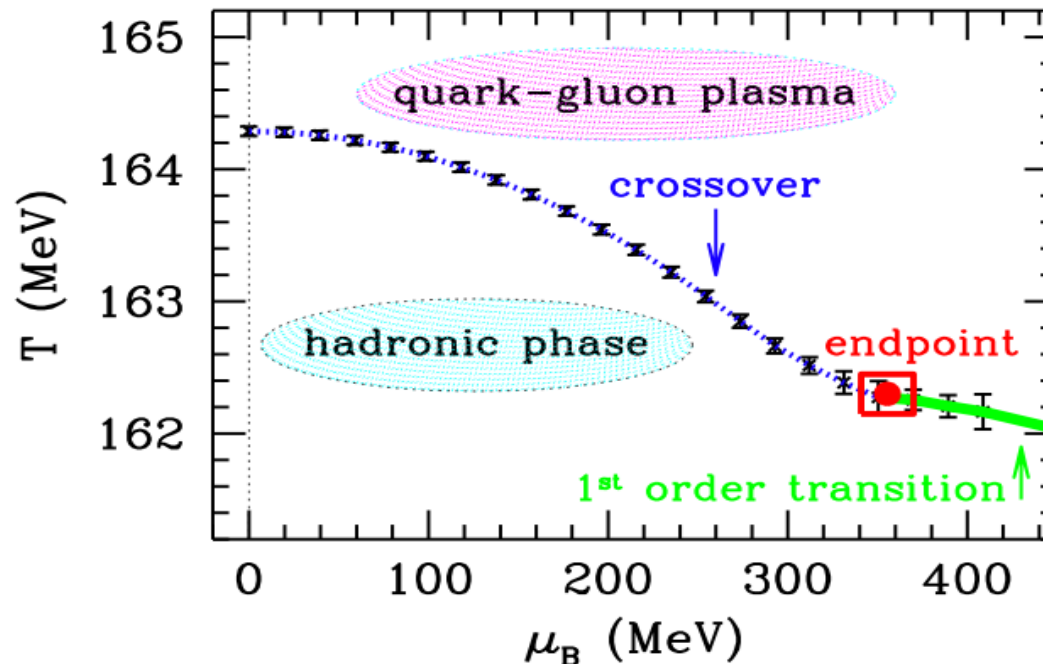
order of phase transition



- $\Delta\chi$: peak of chiral susceptibility: $\chi(T) - \chi(0)$; $m = m_{u,d}$
- 22(2) is 11σ away from zero
- no phase-transition-like volume dependence
- ⇒ crossover
- no real T_c , but pseudo-critical

$$\mu \neq 0$$

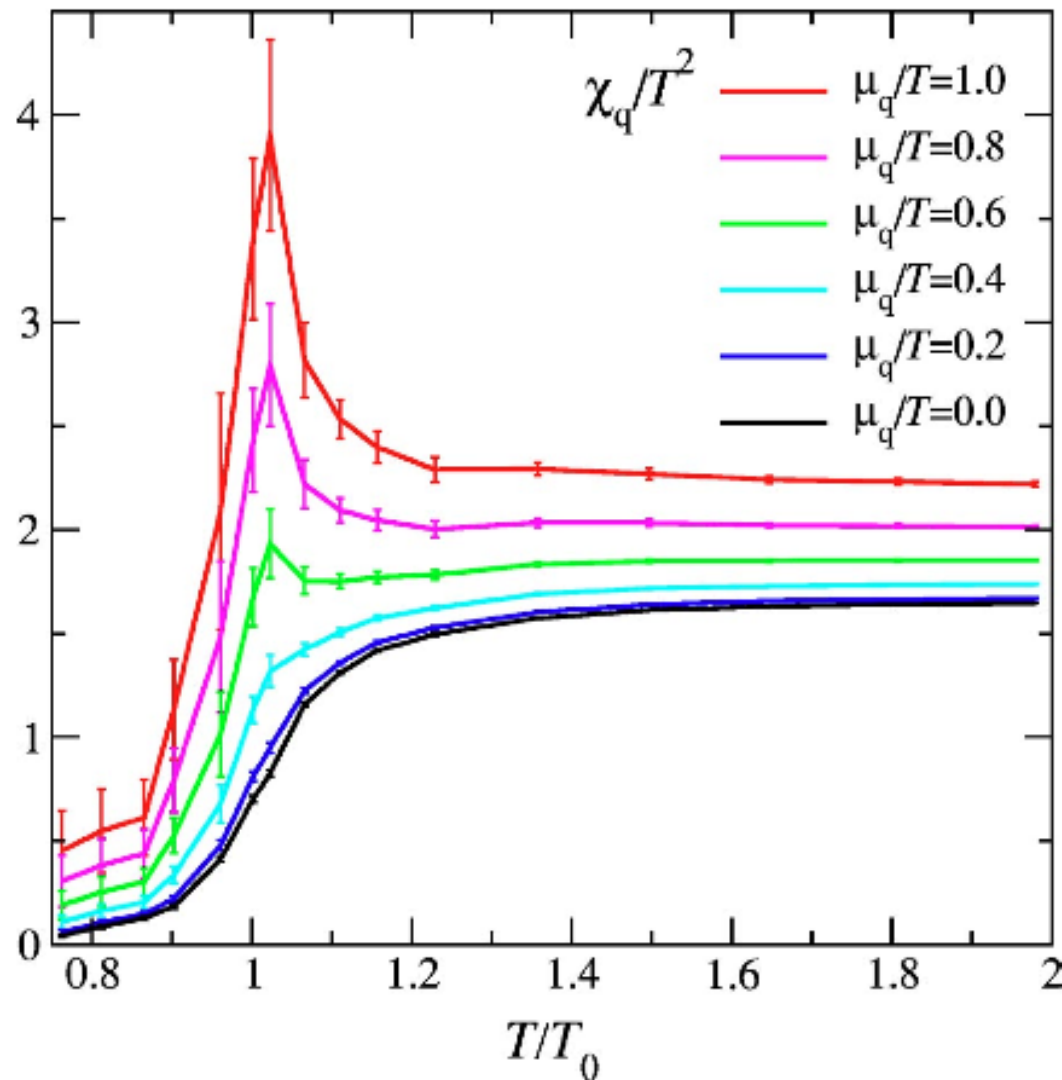
T_c and critical end point



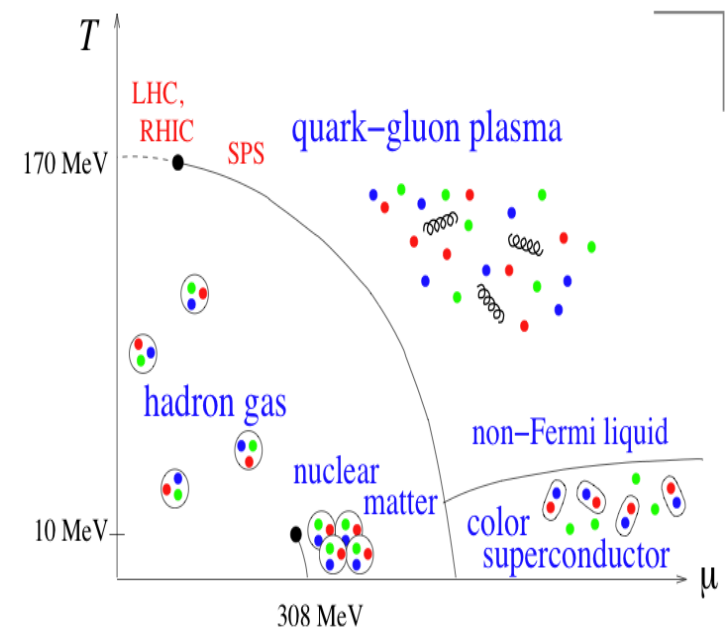
- methods:
 - extrapolation from numerical simulations at $\mu = 0$ to $\mu > 0$
 - simulations with imaginary μ analytically continued to real μ
- $\mu = 0$: $T_c = 164 \pm 2 \text{ MeV}$
- $T_E = 162 \pm 2 \text{ MeV}$, $\mu_E = 360 \pm 40 \text{ MeV}$

$\mu \neq 0$

baryon density

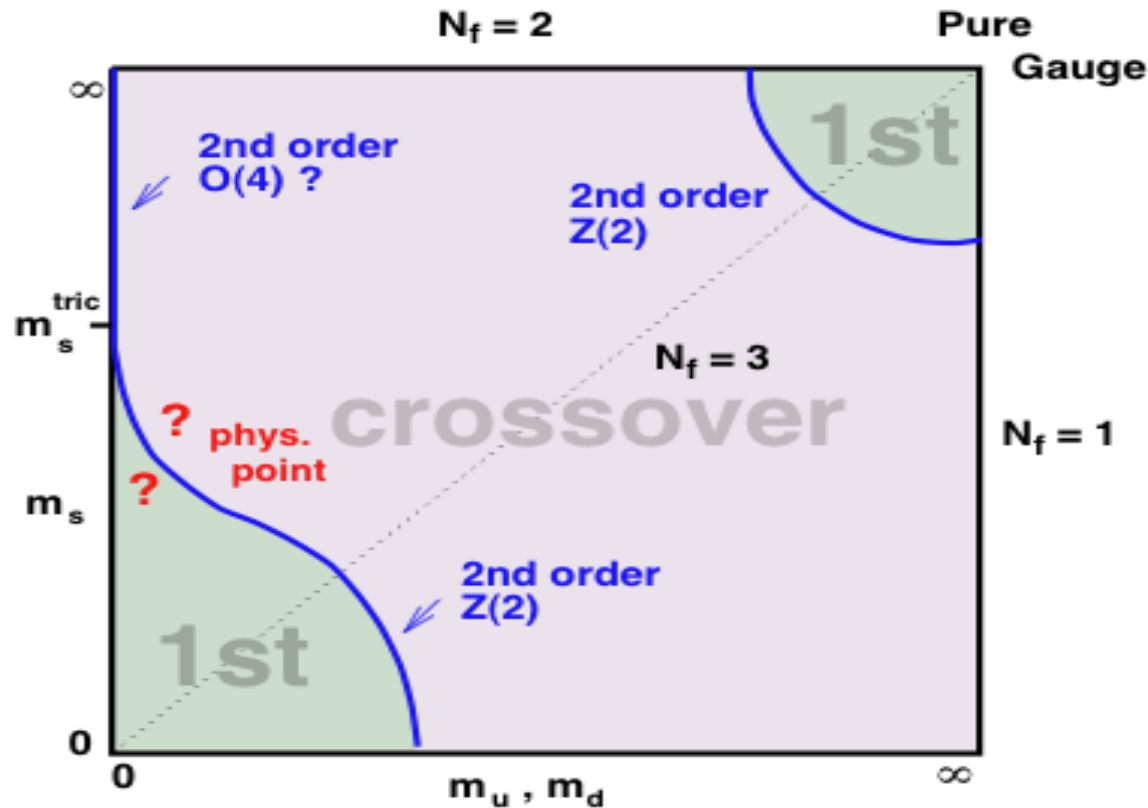


- $\frac{\chi_q}{T^2} = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial \mu^2}$
- crossover $\rightarrow$ 2nd order transition at:
- $\mu_b = 3\mu_q \gtrsim 500 \text{ MeV}$



(strange quark) mass sensitivity

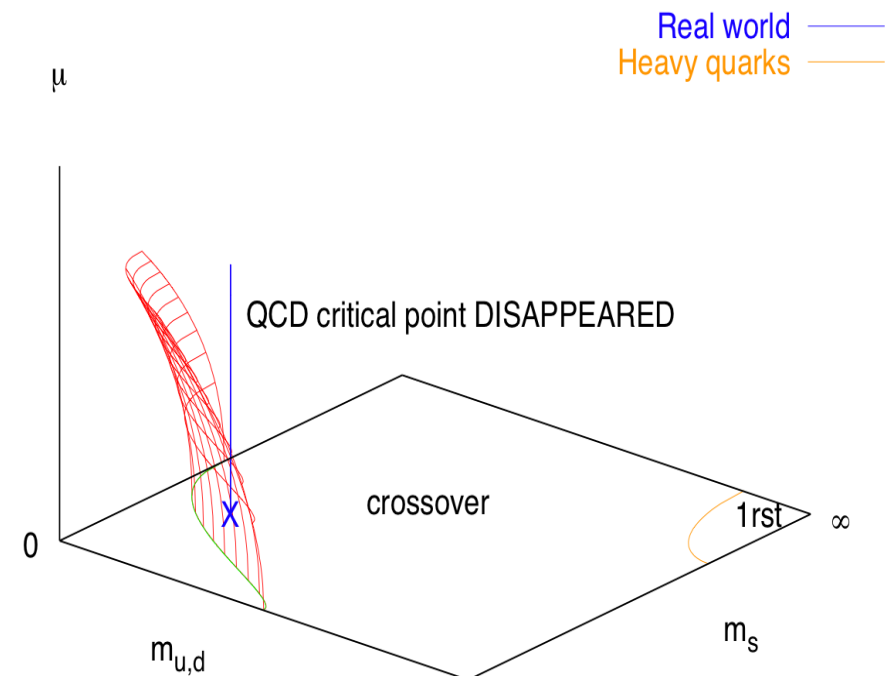
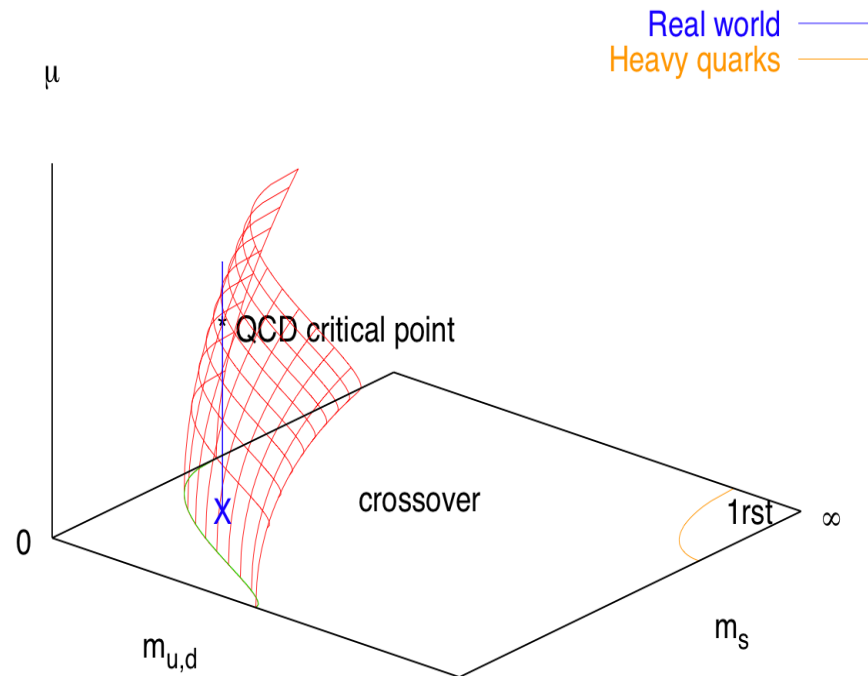
$$\mu = 0$$



- phase diagram dependence on number of flavors and quark masses

(strange quark) mass sensitivity

$$\mu \neq 0$$



- (left)

- common expectation
- curvature positive $\leftrightarrow$ 1st order region expands
- “standard” phase diagram

- (right)

- curvature negative $\leftrightarrow$ 1st order region shrinks
- only crossover (favored by calculations !)

summary

- QCD results only on lattice quite good
- improved constantly
- critical temperatures for $\mu = 0$ vary
($T_c = 151 \pm 5 \rightarrow 192 \pm 8$)
- dependencies/ cpu cost: $m_q, \mu, a, (N_\tau, N_\sigma)$
physical light quark masses not yet reached
- order of phase transition depends on μ
- critical end point (second order transition): $T_E = 162 \text{ MeV}$
 $\mu_b = 3\mu_q = 360 \text{ MeV}$

literature

- sorry for not mentioning all the authors
- pictures:
 - (p. 6 & 7): Lattice Results on QCD Thermodynamics
F.Karsch; 06/2001; hep-ph/0103314
 - (p. 8): Bulk and spectral observables in lattice QCD
Hatsuda; 2007; g7_8_s14
 - (p. 10): Karsch
 - (p. 11 & 12): The order of the quantum chromo dynamics transition predicted by the standard model of particle physics
Fodor, Katz; 10 Nov 2006; arXiv:hep-lat/0611014
 - (p 13): Critical point of QCD at finite T and μ , lattice results for physical quark masses
Fodor, Katz; 8 Feb 2004; arXiv:hep-lat/0402006 v1
 - (p 14): Equation of state for two flavor QCD at nonzero chemical potential
Karsch, Learmann; 2003; phys rev 014507
 - (p 15): The chiral critical line of $N_f = 2 + 1$ QCD at zero and non-zero baryon density
Forcrand, Philipsen; Jan 2007; arXiv:hep-lat/0607017
 - the others: literature on the seminar web page
- special recommendation
 - Thermodynamics and in-medium hadron properties from lattice QCD
Karsch, Learmann; 2003; arXiv:hep-lat/0305025
 - Bulk and Spectral Observables in Lattice QCD
Hatsuda; Feb 2007; arXiv:hep-lat/0702293v1