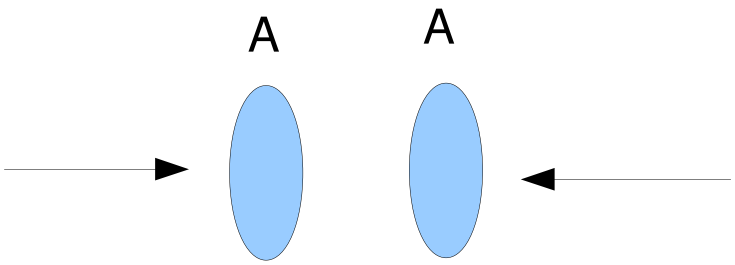
Relativistic Hydrodynamic and the Bjorken Model

emmi Seminar 16 Jun 08

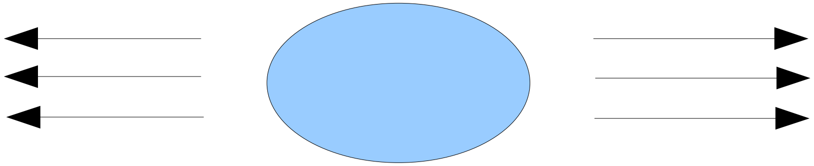
Stefan Zimmer

The Experiment

- the setup



- the question: How does this matter evolve in time



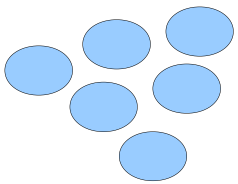
- the aim: what can we conclude for the properties (equation of state, temperature,density, lifetime)

Outline

- relativistic hydrodynamics
  - speed of sound
- equation of state
- signatures of hydrodynamic expansion
  - Bjorken model
  - elliptic flow
- characterization of the QGP

relativistic hydrodynamics

- differential Equation for nuclear matter
- assuming LOCAL THERMODYNAMIC EQUILIBRIUM (the mean free path of particles is much smaller than any other size)



  - pressure and temperature varying so slowly in the neighborhood that thermodynamic equilibrium can be assumed
  - Isotropy
- all thermodynamic quantities associated with the fluid element are defined in the fluid rest frame

baryon number conservation

- mass conservation in non relativistic hydrodynamic

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$

- in relativistic collisions baryon number conservation; if there is enough energy a particle and antiparticle can be produced
- baryon density in the lab frame Lorenz contracted
- relativistic hydrodynamic

$$\partial_\mu (n u^\mu) = 0$$

equation of state

- high kinetic energy -> Maxwell Boltzmann statistic

$$n = \frac{1}{V}\sum_{\vec{p}} e^{(-E_{\vec{p}}+\mu)/T}$$

$$\epsilon = \frac{1}{V}\sum_{\vec{p}} E_{\vec{p}}\, e^{(-E_{\vec{p}}+\mu)/T}$$

$$P = \frac{1}{V}\sum_{\vec{p}} p_x v_x e^{(-E_{\vec{p}}+\mu)/T}$$

- baryonless fluid -> $\mu = 0$

$$n = \frac{g}{\pi^2\hbar^3}T^3 \qquad s = 4n$$

$$\epsilon = 3P = 3nT$$

energy and momentum conservation

- local thermodynamic equilibrium
  -> isotropy

$$T_{(0)} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

- energy-momentum tensor in the fluid rest frame

- first order Lorenz transformation into
  the moving frame

$$\Lambda = \begin{pmatrix} 1 & v_x & v_y & v_z \\ v_x & 1 & 0 & 0 \\ v_y & 0 & 1 & 0 \\ v_z & 0 & 0 & 1 \end{pmatrix}$$

$$T = \begin{pmatrix} \epsilon & (\epsilon+P)v_x & (\epsilon+P)v_y & (\epsilon+P)v_z \\ (\epsilon+P)v_x & P & 0 & 0 \\ (\epsilon+P)v_y & 0 & P & 0 \\ (\epsilon+P)v_z & 0 & 0 & P \end{pmatrix}$$

- energy and momentum conservation law for non-viscous liquids

$$\partial_\mu T^{\mu\nu} = 0$$

equations of heavy ion collisions

- equation of state (n: particle density; g = 40)

$$n = \frac{g}{\pi^2 \hbar^3} T^3 \qquad\qquad s = 4n$$

$$\epsilon = 3P = 3nT$$

- baryon conservation (n: baryon density)

$$\partial_\mu (n u^\mu) = 0$$

- energy – momentum conservation

$$\partial_\mu T^{\mu\nu} = 0 \qquad T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu - P g^{\mu\nu}$$

The speed of sound

- velocity of sound: $$c_s = \left(\frac{\partial P}{\partial \epsilon}\right)^{1/2}$$
- wave equation $\partial_\mu T^{\mu\nu} = 0$

$$\epsilon(t,x,y,z) = \epsilon_0 + \delta\epsilon(t,x,y,z)$$
$$P(t,x,y,z) = P_0 + \delta P(t,x,y,z)$$

sound wave

$$\frac{\partial(\delta\epsilon)}{\partial t} + (\epsilon_0 + P_0)\vec{\nabla}\cdot\vec{v} = 0$$

energy conservation

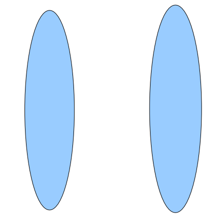
$$(\epsilon_0 + P_0)\frac{\partial \vec{v}}{\partial t} + \vec{\nabla}\delta P = \vec{0}$$

Newtons 2nd law

$$\frac{\partial^2(\delta\epsilon)}{\partial t^2} - c_s^2 \Delta(\delta\epsilon) = 0$$

wave equation

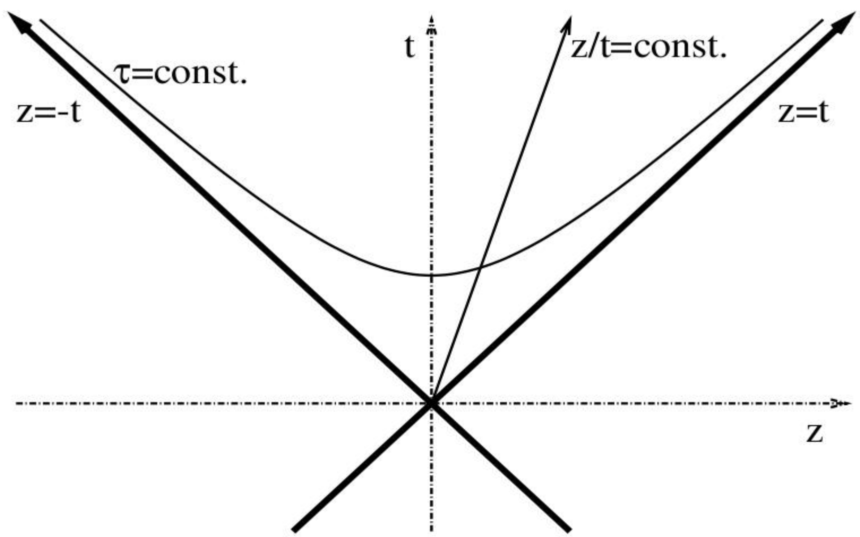
Signatures of hydrodynamic expansion



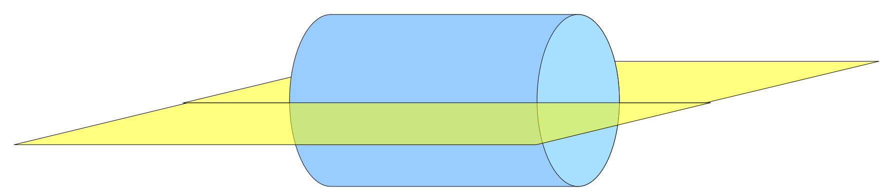
- Initial condition
  - at RHIC Lorenz contraction by a Lorenz factor 100
    (at RHIC energy of a nucleon-nucleon collision 100 Gev)
  - two nuclei pass through each other in 0.15 fm/c
  - factor 100 smaller than any other characteristic size
    (size of a nucleon)
  - QGP – particles are produced isotropically in the collision on a very short timescale
  - transverse components of the fluid velocity averages to zero

Hint 1: longitudinal spectra

- rapidity coordinates



$$t = \tau \cosh \eta_s$$
$$z = \tau \sinh \eta_s$$
$$v_z = \tanh Y.$$



- behavior under Lorenzboost

τ -> τ proper time

η_s -> η_s + const space rapidity

Y -> Y + const

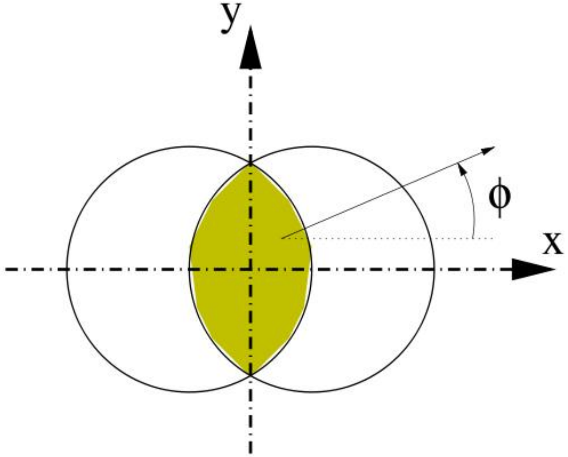
Models

- Bjorken Model
  assumption : all particles get their velocity at the initial collision.
  No acceleration later on.

$$v_z = z/t \qquad\qquad Y = \eta_s$$

- Gaussian density profile in the overlap region $s = 4n$
  remember the equation of state

$$s(x, y, \eta_s) \propto \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{\eta_s^2}{2\sigma_\eta^2}\right)$$



Longitudinal acceleration

- Energy momentum conservation

$$\partial_\mu T^{\mu\nu} = 0 \quad \frac{\partial}{\partial t}((\epsilon + P)v_z) + \frac{\partial}{\partial z}P = 0$$

- calculate acceleration at $z = 0$ Bjorken assumption $v_z = 0$

isentropic process ds = dN =0

$$\frac{\partial v_z}{\partial t} = -\frac{1}{\epsilon + P}\frac{\partial P}{\partial z} = -c_s^2 \frac{\partial \ln s}{\partial z}$$

- transform to rapidly coordinates $$\frac{\partial Y}{\partial \tau} = -\frac{c_s^2}{\tau}\frac{\partial \ln s}{\partial \eta_s}$$
- equation valid for all z (transform via Lorenzboost)
- necessary assumption for the Bjorken assumption

$$(\partial s/\partial \eta_s)_\tau = 0$$

rapidity spectrum

- necessary condition for the Bjorken assumption:

$$(\partial s/\partial \eta_s)_\tau = 0$$

- initial condition

$$s(x, y, \eta_s) \propto \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{\eta_s^2}{2\sigma_\eta^2}\right)$$

- flat rapidity spectra of outgoing particle (assuming the rapidity of the outgoing particles is equal to the fluid rapidity)
- In reality the Bjorken assumption does not hold for the whole expansion, therefore no total flat rapidity spectra.

rapidity spectrum from RHIC compared to RDM (Rolf Kuiper and Georg Wolschin, Heidelberg)

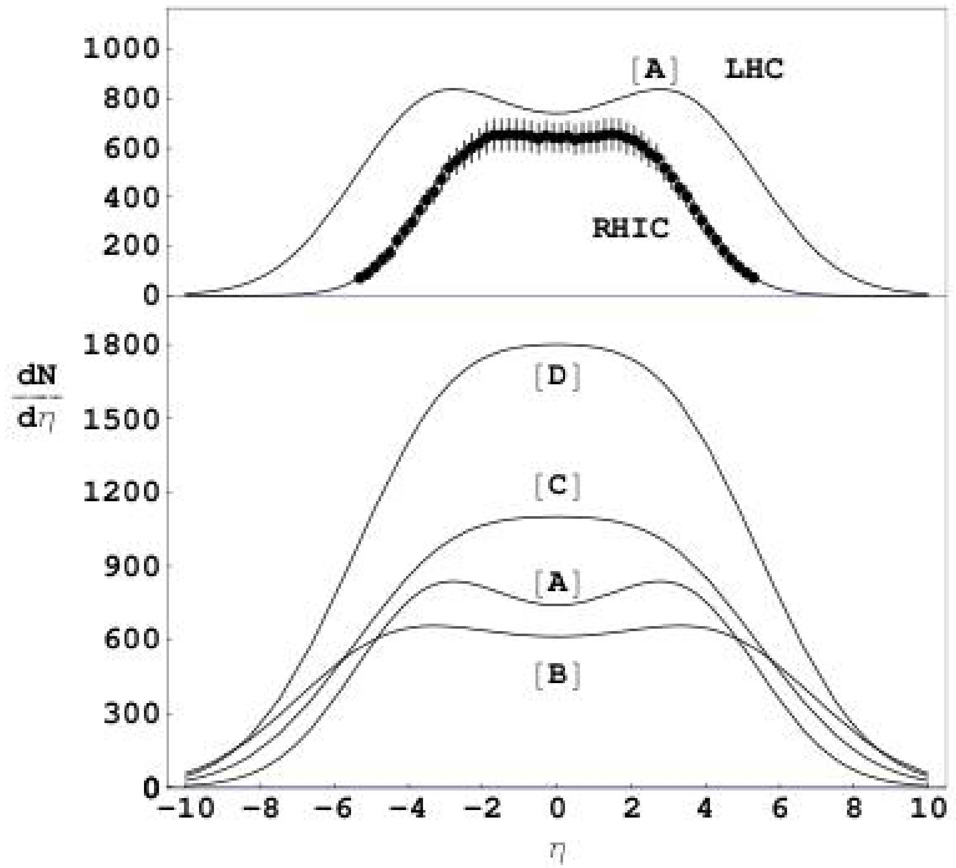
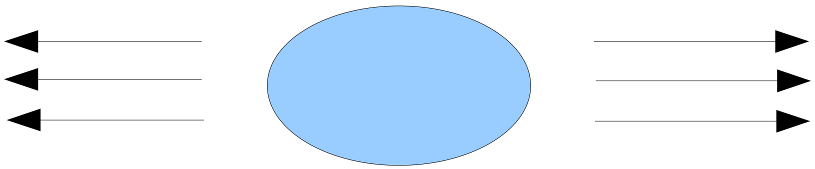


FIG. 2: Produced charged hadrons for central Au + Au collisions at RHIC in the RDM compared with 200 A GeV PHOBOS data [3], and diffusion-model extrapolation to Pb + Pb at LHC energies. See [11] for curves [A] to [D] at LHC energies.

HINT 2: total energy

- QCD predicts much higher (factor 3) initial energy than the measured final energy.
- The QGP cools down while expanding in z direction (longitudinal cooling)



longitudinal cooling

- relativistic hydrodynamic

$$\partial_\mu T^{\mu\nu} = 0$$

- evaluate at z = 0; generalization possible by transforming to rapidity coordinates

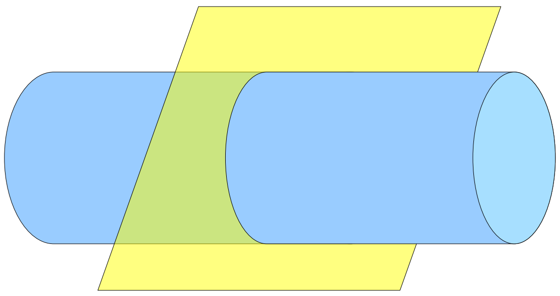
$$\frac{\partial \epsilon}{\partial t} + \frac{\epsilon + P}{t} = 0.$$

$$d(\epsilon t) = -P dt$$

- the comoving energy decreases. This relies on local thermodynamic equilibrium

HINT 3: transverse spectra

- initially there is no transverse velocity
  if there are effects based on transverse velocity this is a clear signal for hydrodynamic behavior



- determined by the Boltzmann factor

$$\frac{dN}{d^3x d^3p} = \frac{2S+1}{(2\pi\hbar)^3} \exp\left(-\frac{E^*}{T}\right)$$

$$E^* = p^\mu u_\mu = m_t u^0 - p_t u$$

- use only particles that are faster than the fluid, as Boltzmann factor probe.

HINT 3 a: radial flow

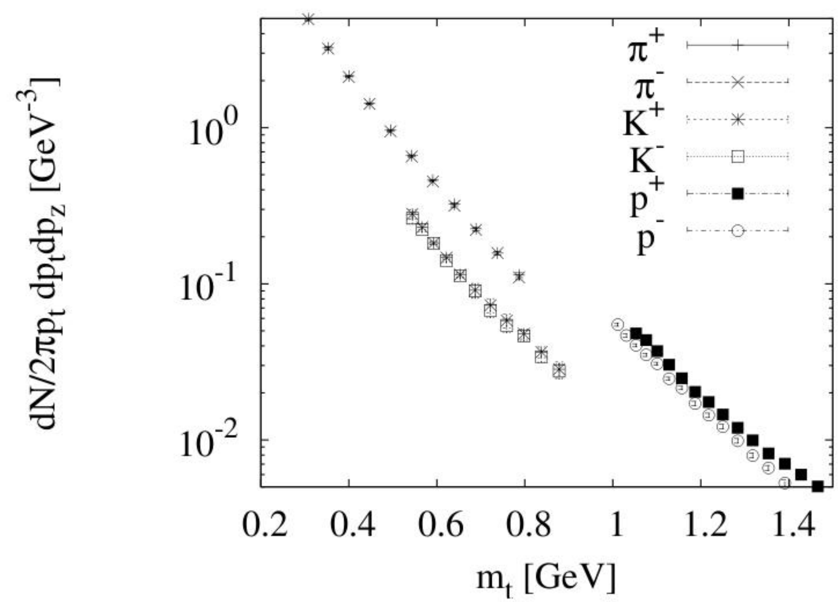
- central collisions
- rewriting the Boltzmann factor using rotation symmetry in the transverse plane

$$\frac{dN}{2\pi p_t dp_t dp_z} \propto \exp\left(\frac{-m_t u_0 + p_t u}{T}\right)$$

- slope is determined by T (kinetic temperature)

$$T_f \simeq 100 \text{ MeV}$$

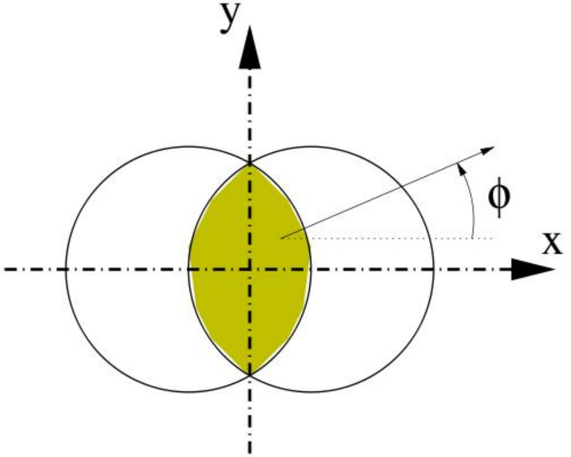
- spectrum for p -p



HINT 3 b: ecliptic flow

- non central collisions
- the elliptic flow combines the Boltzmann factor and the hydrodynamic description of the QGP.
  One of the best criteria to check the hydrodynamic description

$$\frac{dN}{d\phi} \propto 1 + 2v_2 \cos 2\phi$$



elliptic flow in theory

- assuming initial Gaussian entropy density and constant velocity of sound. $s = 4n$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\frac{\partial v_x}{\partial t} = -\frac{1}{\epsilon + P}\frac{\partial P}{\partial x} = -c_s^2 \frac{\partial \ln s}{\partial x}$$

$$v_x = \frac{c_s^2 x}{\sigma_x^2} t, \qquad v_y = \frac{c_s^2 y}{\sigma_y^2} t.$$

- in a non – central collision $\sigma_x < \sigma_y$
- more particles go in x direction than in y direction

elliptic flow in the experiment

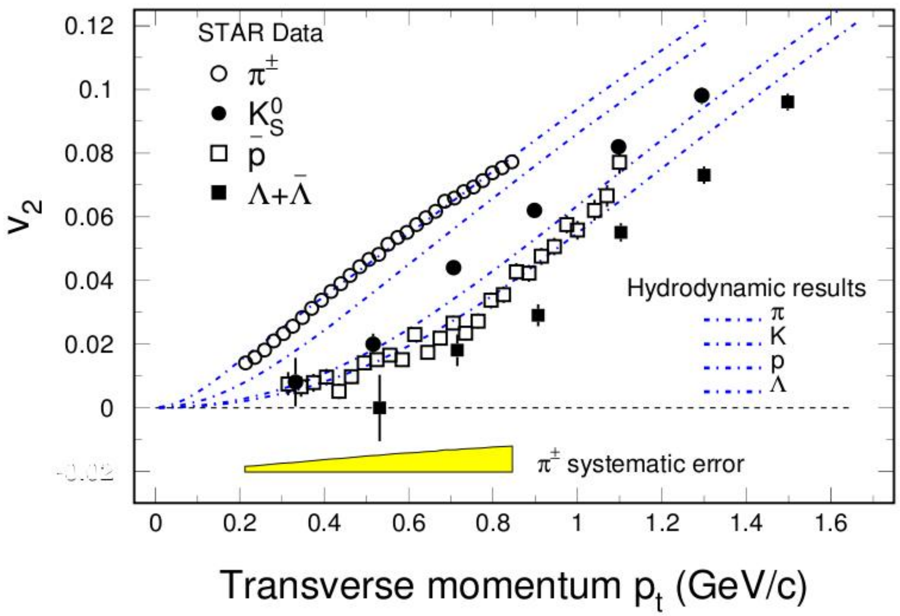
- starting from the Boltzmann factor

$$\frac{dN}{p_t dp_t dp_z d\phi} \propto \exp\left(\frac{-m_t u_0(\phi) + p_t u(\phi)}{T}\right)$$

- fluid velocity in the x, y plane $u(\phi) = u + 2\alpha \cos 2\phi$
- expand to α and compare to

$$\frac{dN}{d\phi} \propto 1 + 2v_2 \cos 2\phi$$

$$v_2 = \frac{\alpha}{T}\left(p_t - v m_t\right)$$



Characterization of the QGP

particle ratios

- integrate the Boltzmann factor over momentum

$$\frac{dN}{2\pi p_t dp_t dp_z} \propto \exp\left(\frac{-m_t u_0 + p_t u}{T}\right)$$

- compare the ratio of two particle species; the Temperature is the only free parameter
- chemical freeze out temperature

$$T_c \simeq 170 \; MeV$$

chemical vs. kinetic freeze out temperature

- below chemical freeze out temperature no inelastic collisions; no chemical equilibrium; but still elastic collisions to maintain the Boltzmann distribution the kinetic equilibrium

$$T_c \simeq 170 \ MeV$$

- kinetic freeze out temperature; the kinetic equilibrium is broken

$$T_f \simeq 100 \text{ MeV}$$

Density of the QGP

- equation of state

$$n = \frac{g}{\pi^2 \hbar^3} T^3$$
$$\epsilon = 3P = 3nT$$

- Lattice QCD

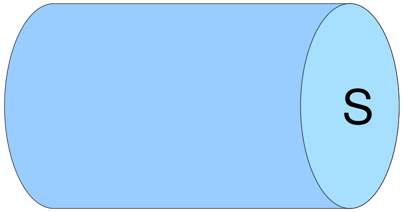
$$T_c \approx 192 \text{ MeV}$$

- minimum particle density and energy density

$$n \simeq 3.75 \,\text{fm}^{-3}$$

Measure the lifetime of the QGP

- assuming particle conservation



$$n(t) = \frac{1}{S}\frac{dN}{dz} = \frac{1}{St}\frac{dN}{dv_z}$$

- from the experiment

$$dN_{\rm ch}/d\theta \simeq 700$$

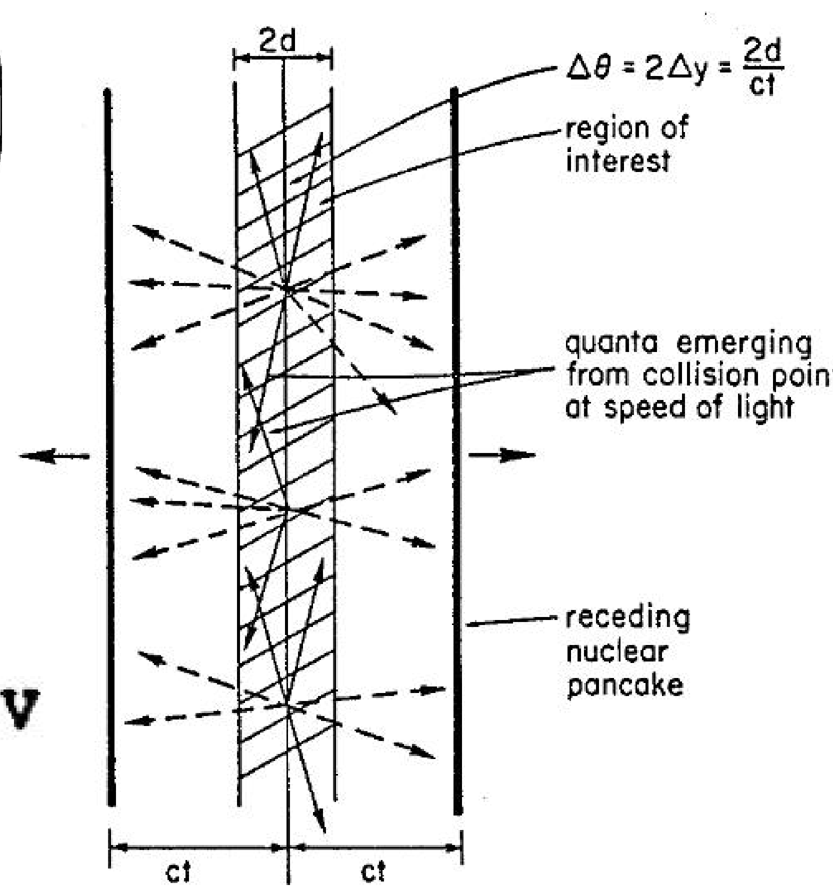
- Lattice QCD requires minimum density $n \simeq 3.75\,{\rm fm}^{-3}$
- The QGP exists for $t < 3.5\,{\rm fm/c}$

measure the energy density

- Bjorkens formula; neglecting transversal cooling

$$E = N\frac{d\langle E\rangle}{dy}\,\Delta y = N\cdot\frac{d\langle E\rangle}{dy}\cdot\frac{1}{2}\left(\frac{2d}{t}\right)$$

$$\epsilon \simeq \frac{N}{A}\cdot\frac{d\langle E\rangle}{dy}\cdot\frac{1}{2t}$$



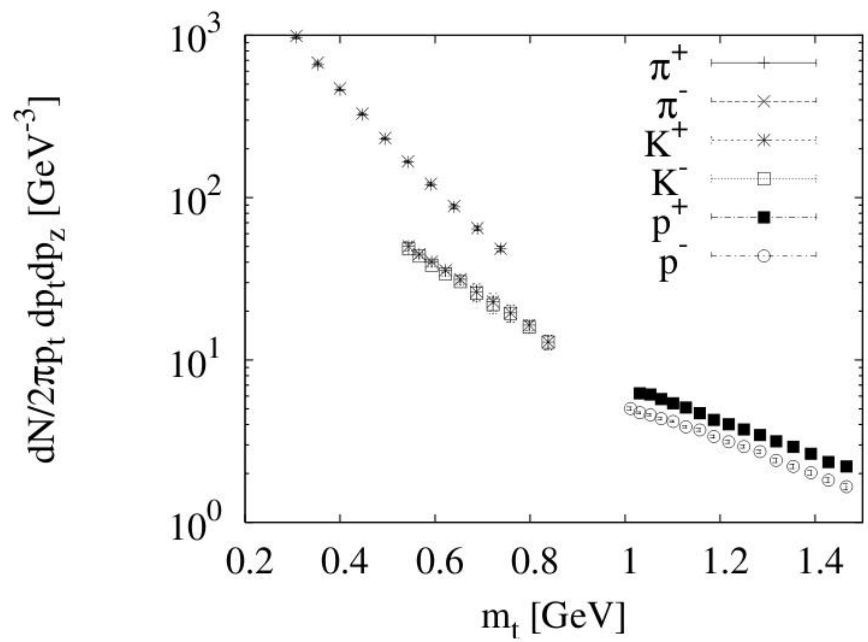
- at SPS

$$\frac{d\langle E\rangle}{dy} \sim 3 \times 0.4 \times 1.5 \simeq 1.8\ \mathrm{GeV}$$

$$\epsilon_0 \simeq 1 - 10\ \mathrm{GeV}/_{f}^{3}$$

Outlook

- reality is not inviscid
- statistical model can somehow also describe p-p and e-e collisions, but most likely due to other effects
- next week A-A



Summary

- Heavy Ion collisions can be described using Boltzmann statistic and hydrodynamic methods
- most significant signature for hydrodynamic is the anisotropy of the transverse spectra encoded in the v_2 parameter
- we could calculate lifetime, temperature and density of the QGP
- current research in viscid hydrodynamics

Literature

- Jean-Yves Ollitrault Relativistic hydrodynamics for heavy-ion collisions (review-article)
  Spires arXiv: 0708.2433v2 [nucl-th] 7 Jan 2008
- Bjorken 1983 Highly Relativistic Nucleus-Nucleus Collisions: The Central Rapidity Region