Color superconductivity in dense quark matter

Florian Marhauser

Motivation

Superconductivity

„standard“ superconductivity

- loss of electric resistivity
- Expulsion of magnetic fields (Meißner effect)
- Type I and II
- BCS theory: condensation of Cooper pairs

$$\left\langle \psi(\vec{k},\uparrow)\ \psi(-\vec{k},\downarrow) \right\rangle$$

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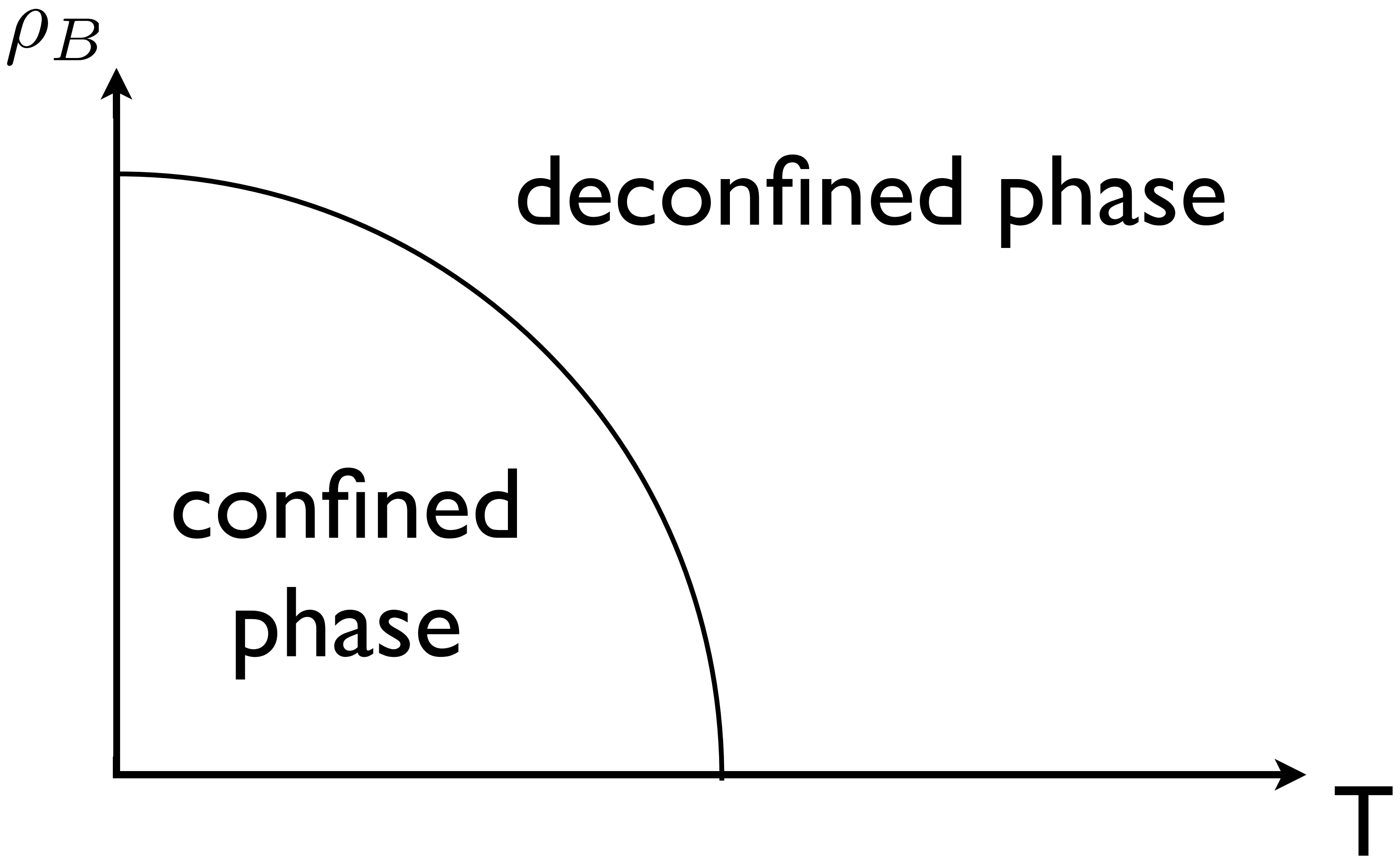
QCD is a gauge theory

➡color superconductivity

History of the QCD phase diagram

first conjectured phase diagram: (Cabbibo and Parisi 1975)



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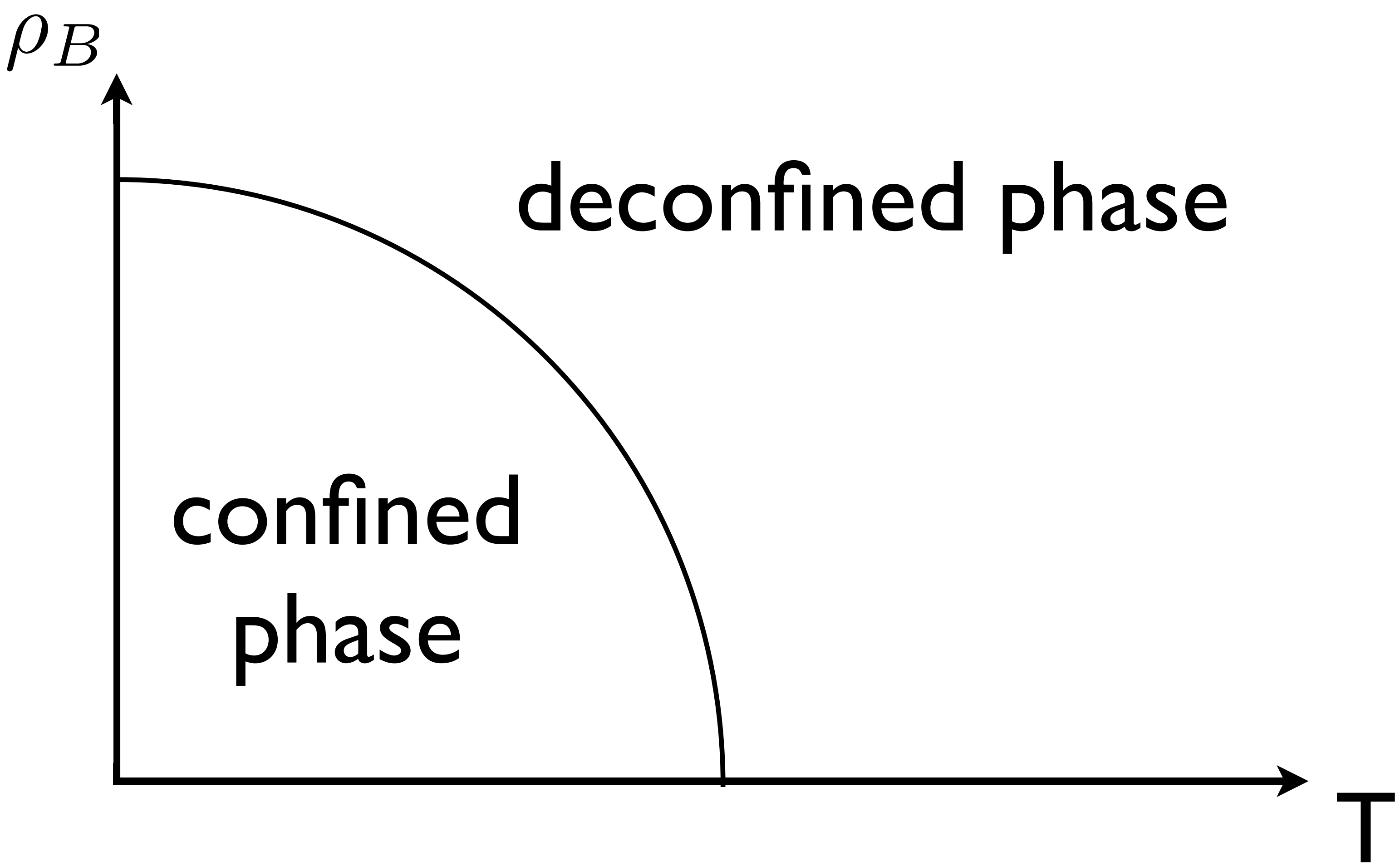

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Collins and Perry (1975):

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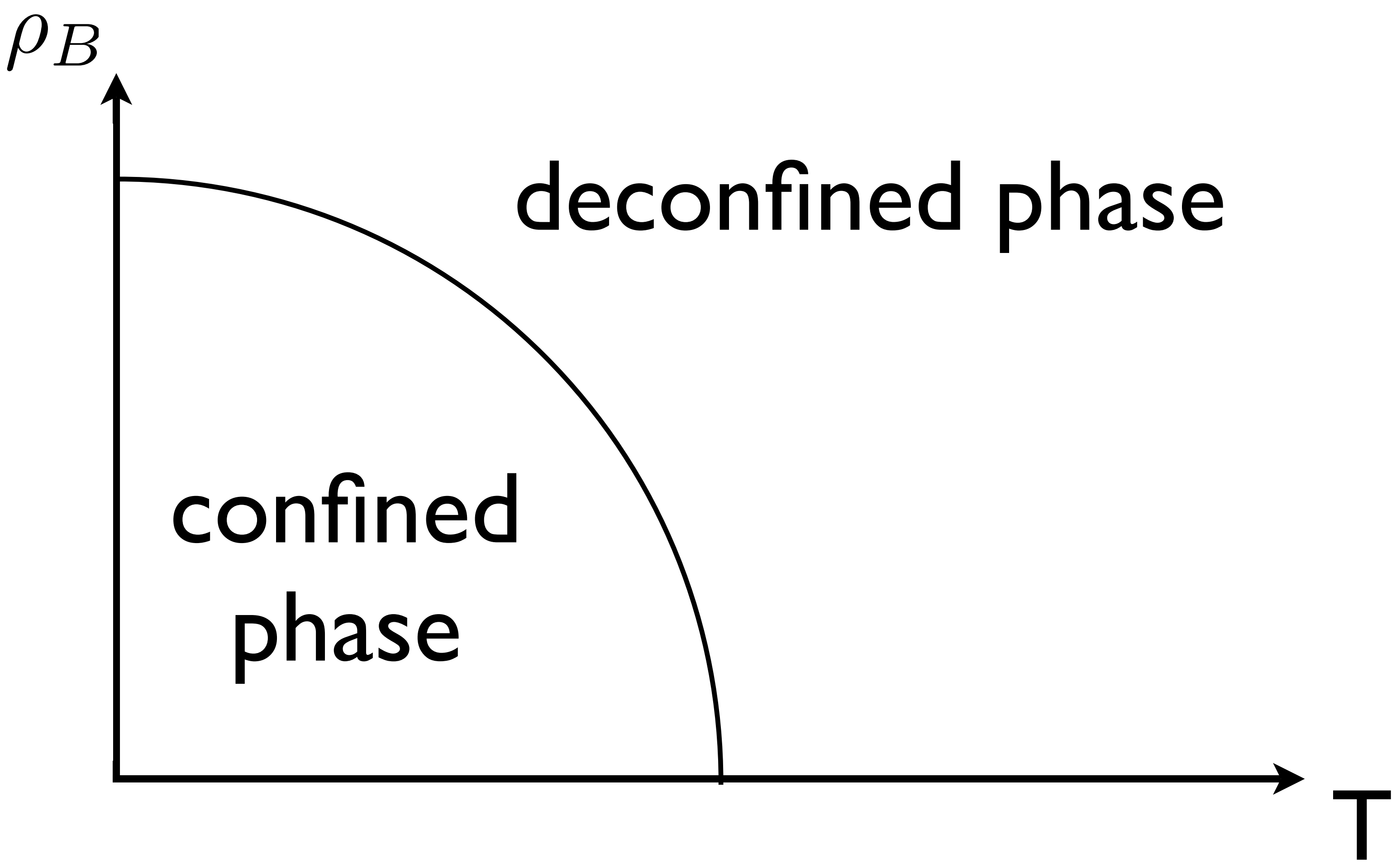

„the true phase diagram may actually be substantially more complex“

Collins and Perry (1975):

„Also we might expect superfluidity or superconductivity, since the interquark forces are attractive in some channels“

History of the QCD phase diagram

rediscovery of color superconductivity

(Alford, Rajagopal, Wilczek (1998); Rapp, Schäfer, Shuryak, Velkovsky (1998))

- larger diquark gaps $\Delta \sim 100$ MeV
- sizeable critical temperatures BCS : $T_c = 0.57\Delta(T = 0)$

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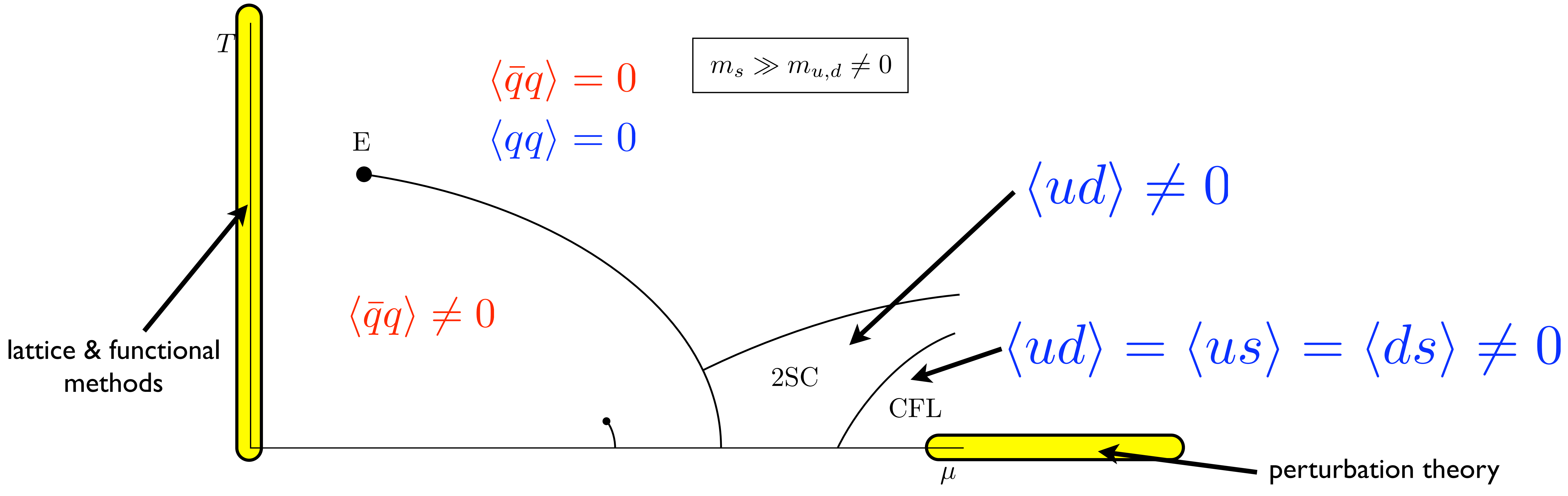
➡ schematic phase diagram (Rajagopal (1999))



Recent Developments

- imposing electric and color neutrality
  - ➡ no 2SC phase? (Alford and Rajagopal (2002))
- alternative condensation patterns
  - crystalline (LOFF) phases ➡ J. Klein
  - CFL + kaon condensates
  - gapless phases
  - ...

Introduction

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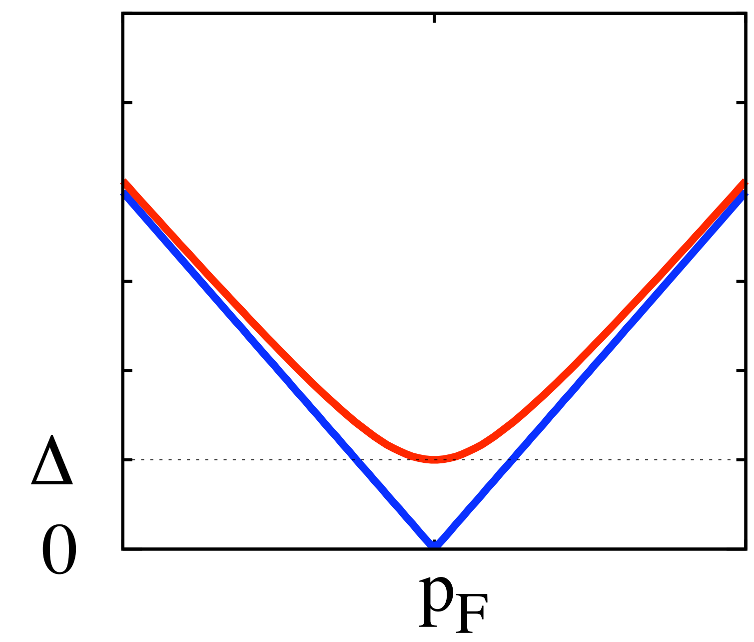
Fermi gas with attractive interaction

- Fermi surface
- condensation into Cooper pairs
- gaps in the excitation spectrum



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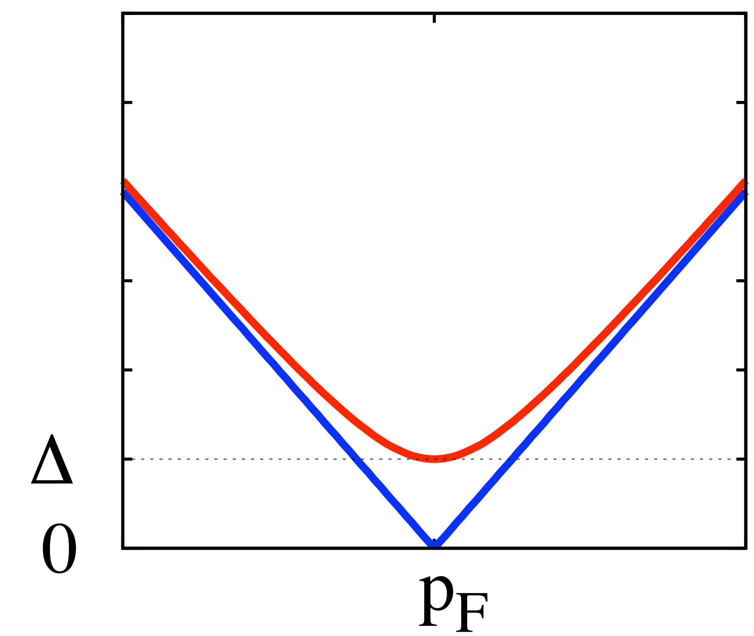

CSC realised in system free quarks (quark gluon plasma)

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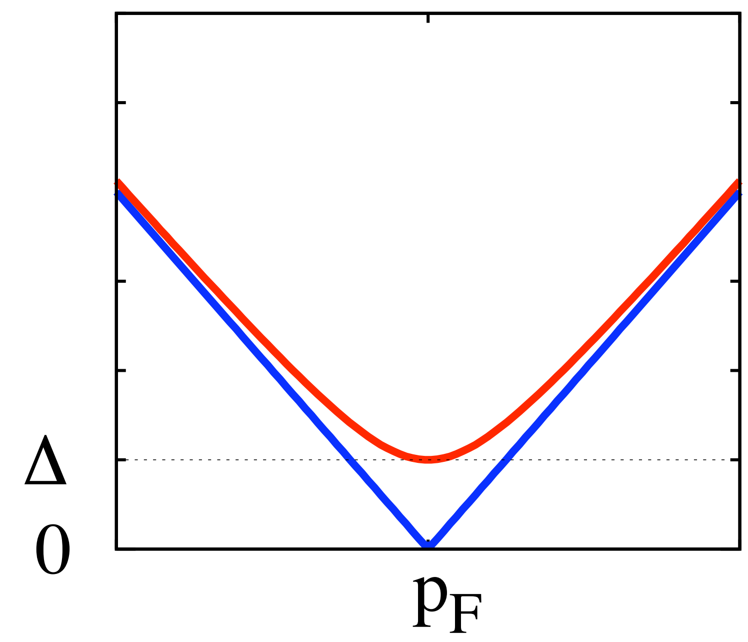

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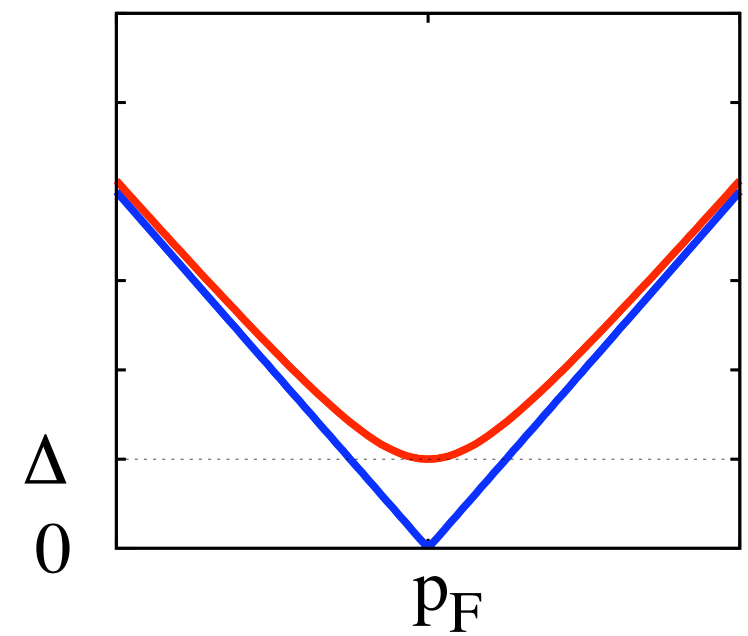

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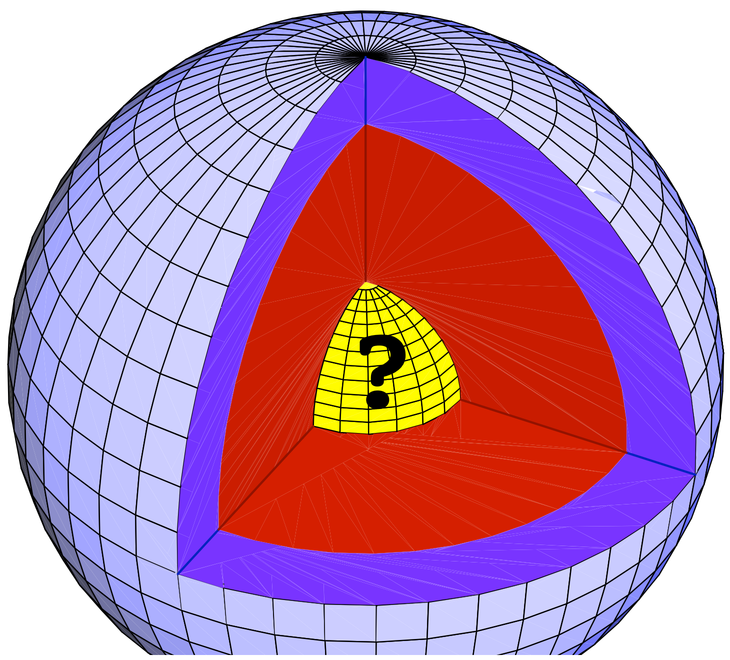

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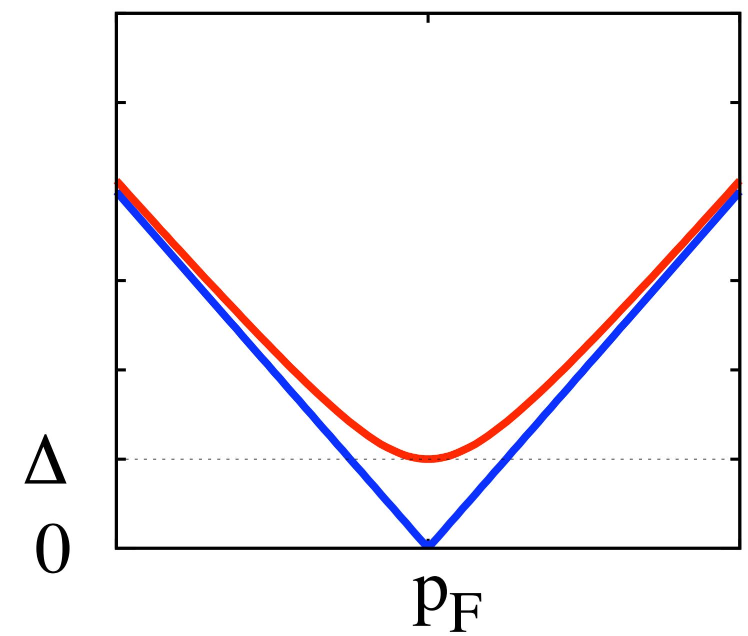

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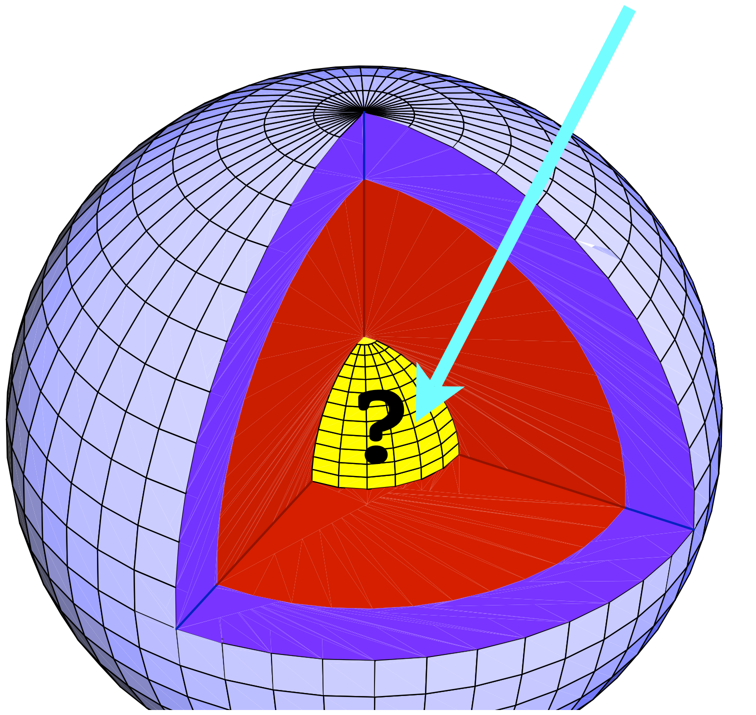

Pairing Patterns

only constraint on quark pairing pattern:

Pauli principle

in QCD 3 different types of degrees of freedom:

color	-	-	+	+
spin	-	+	-	+
flavor	-	+	+	-

\- antisymmetric

\+ symmetric

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➡ antisymmetric flavor channels

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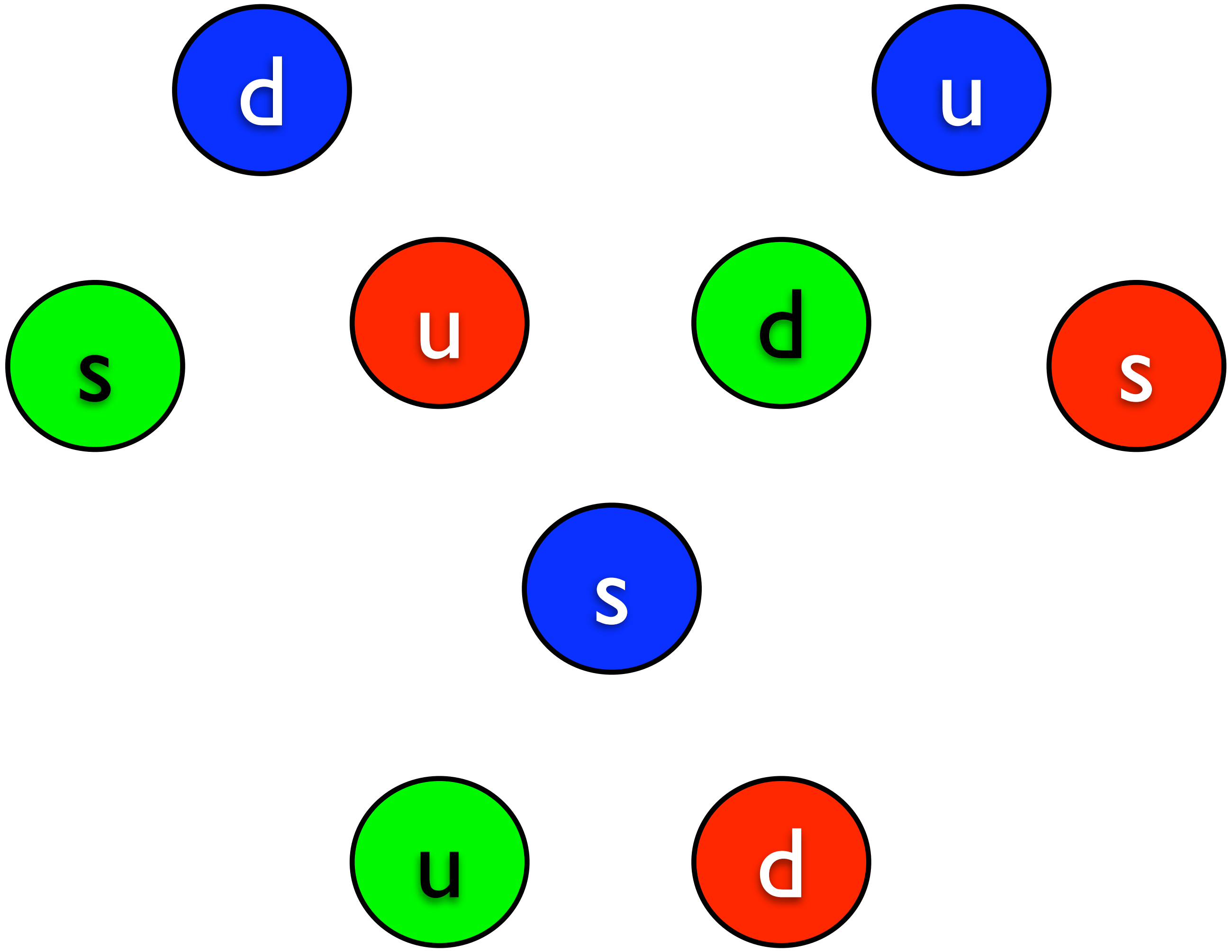

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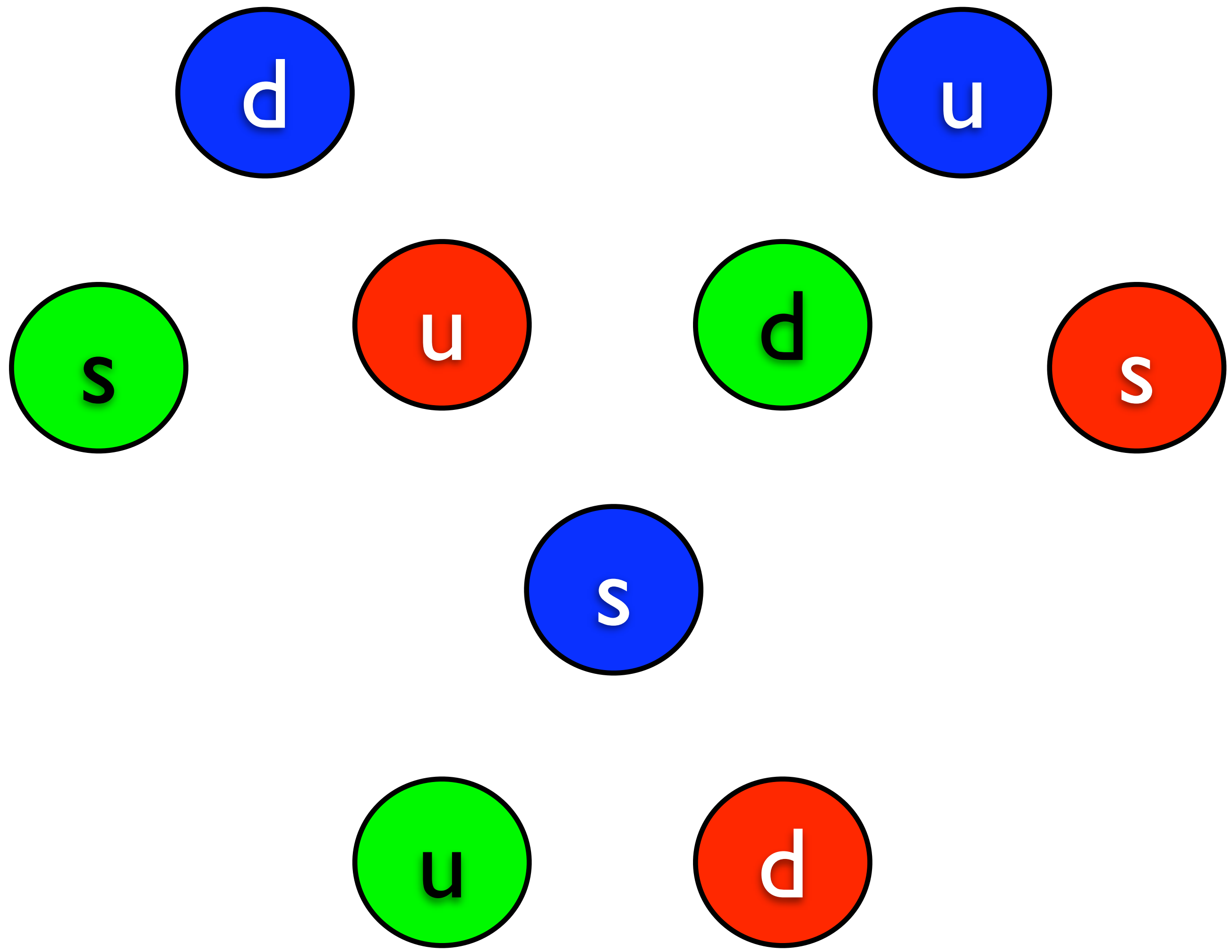
strange quark heavy: only u and d quarks



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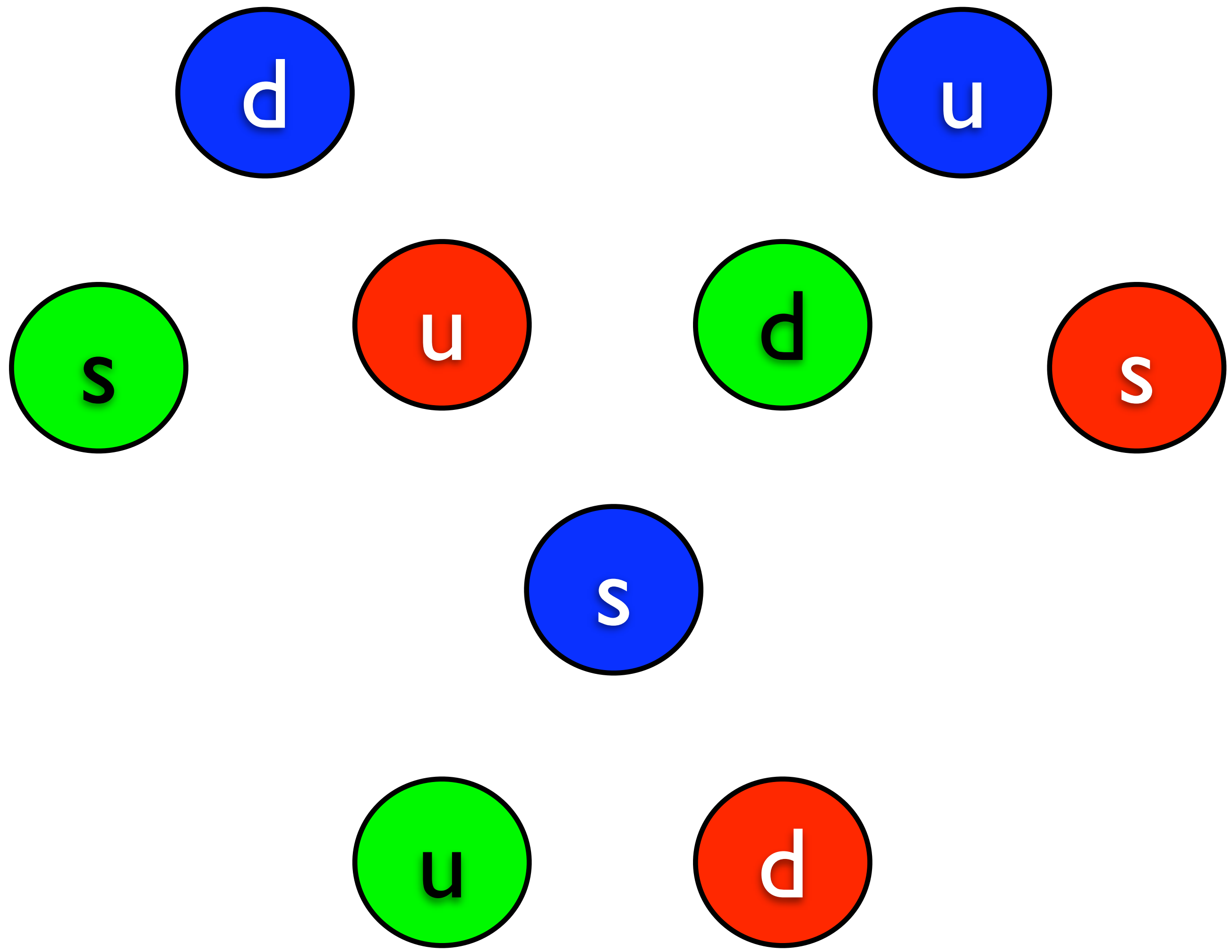
2SC: only up and down pair

2SC = 2 flavor color superconductor



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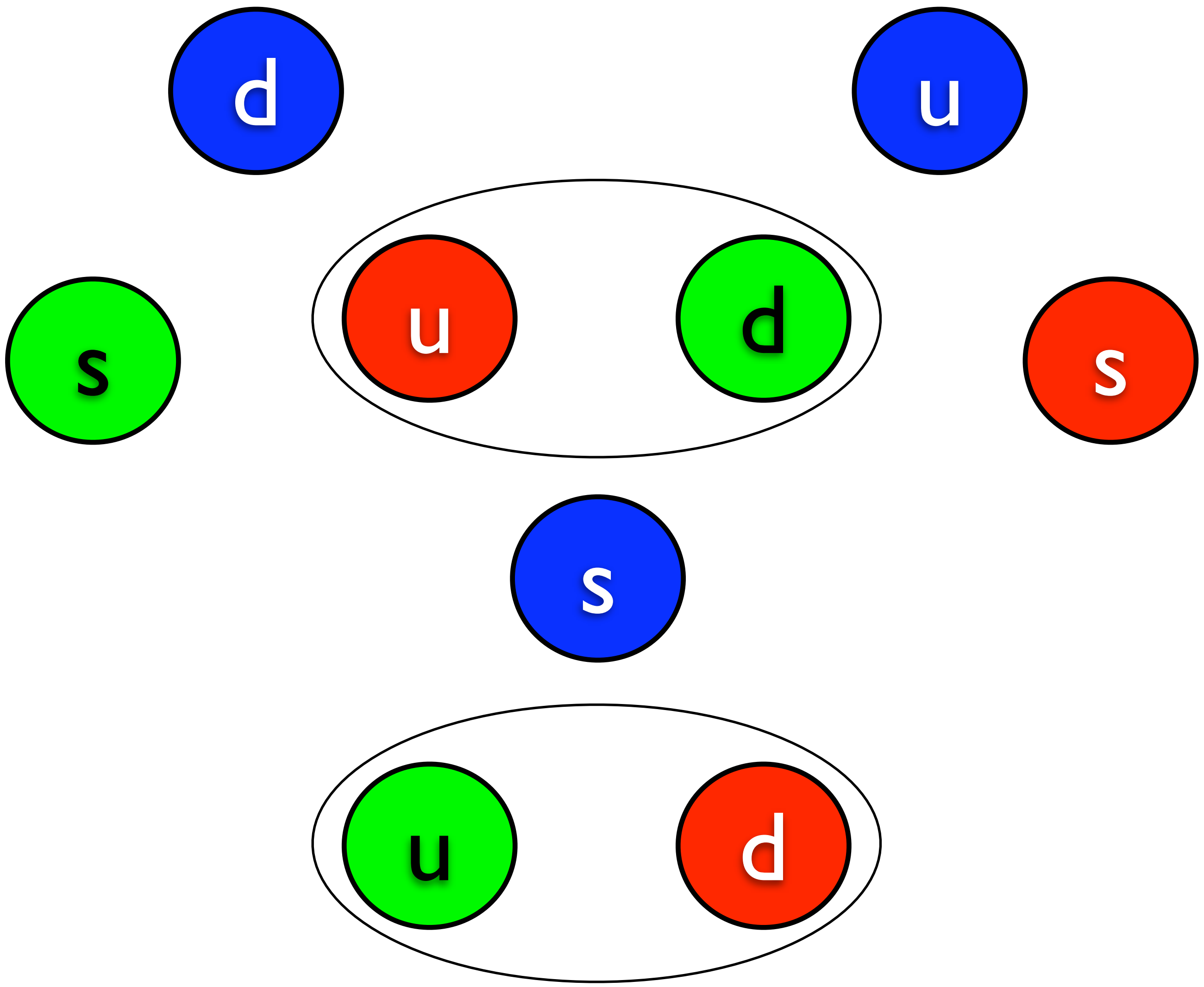

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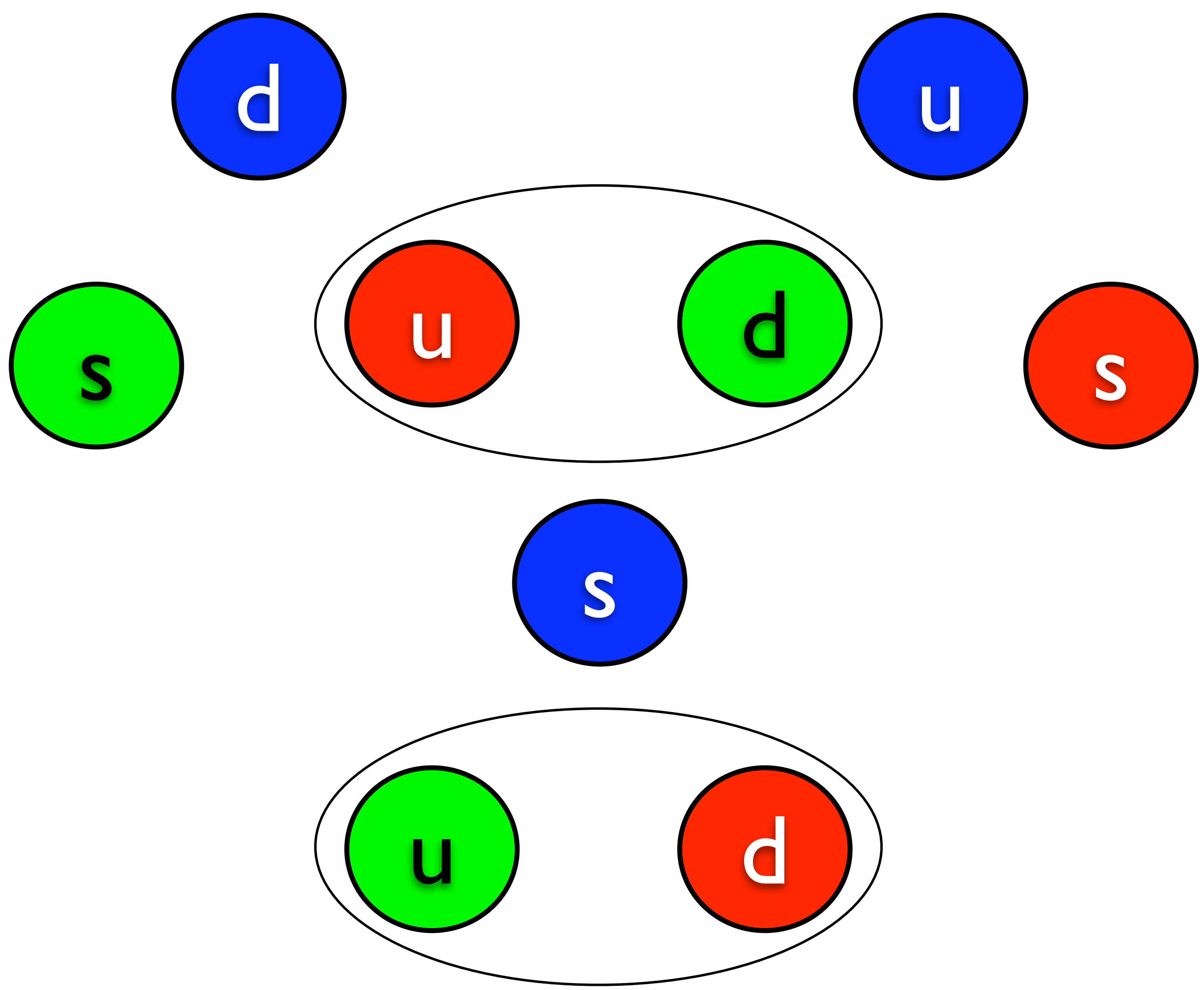

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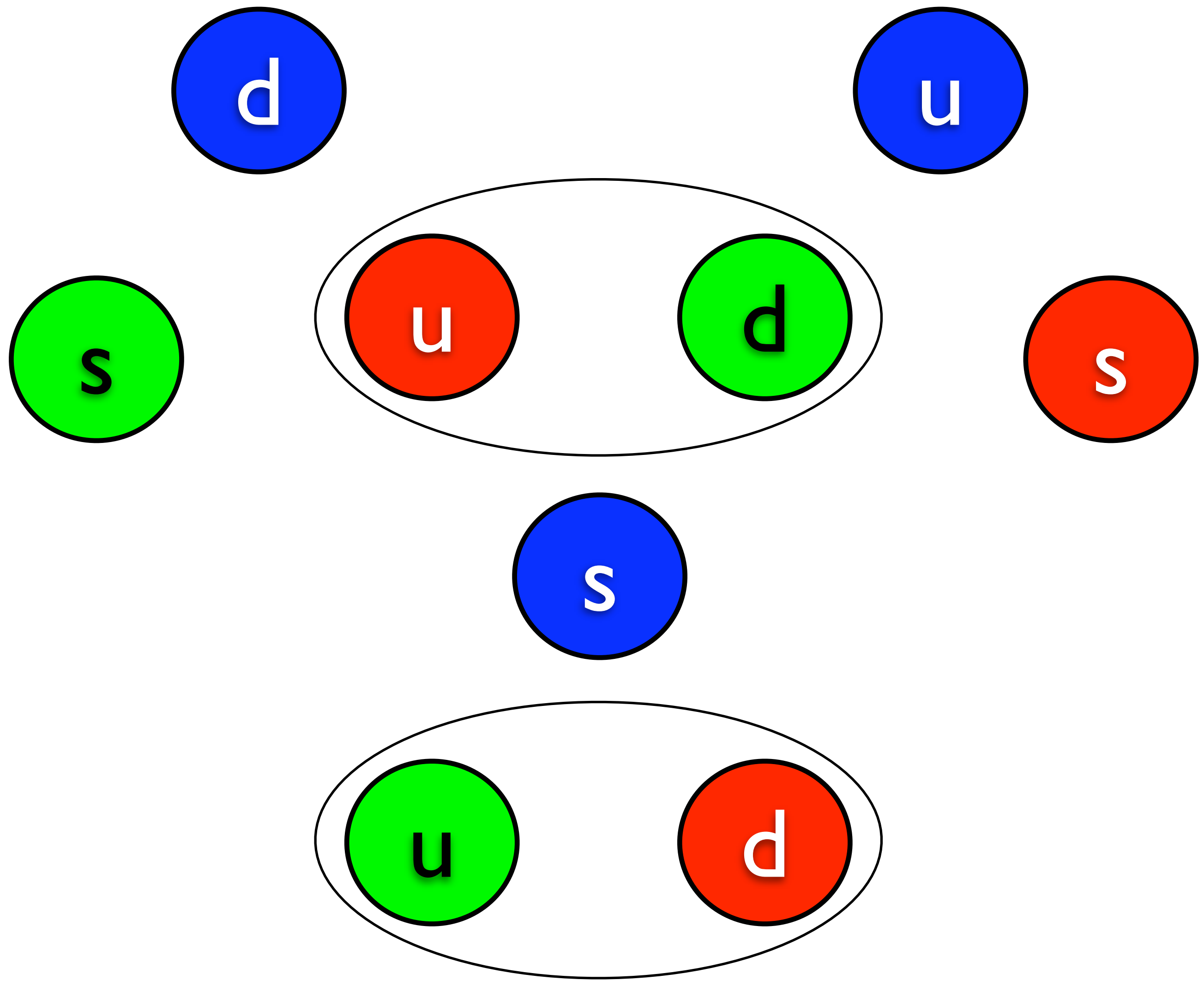

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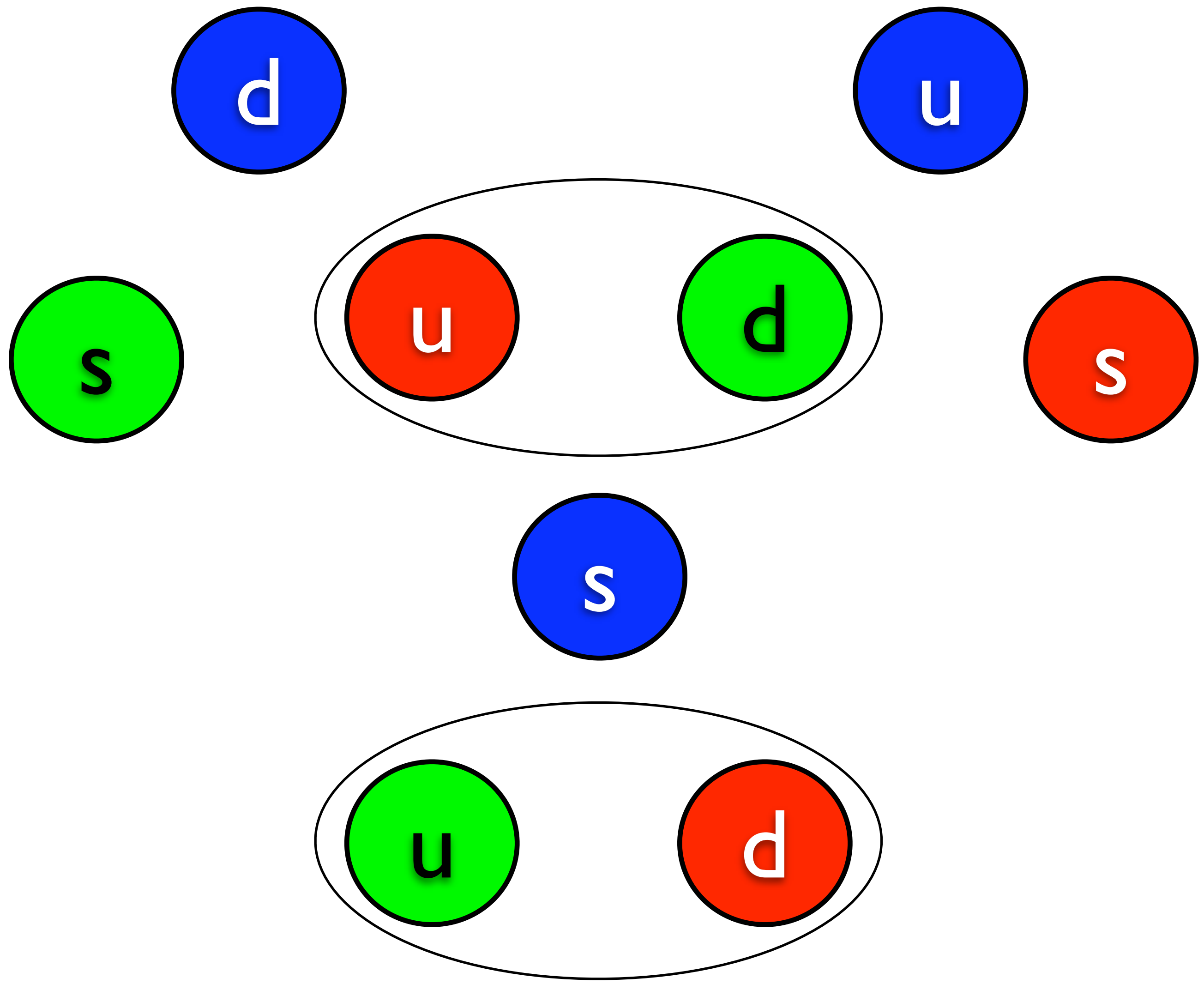

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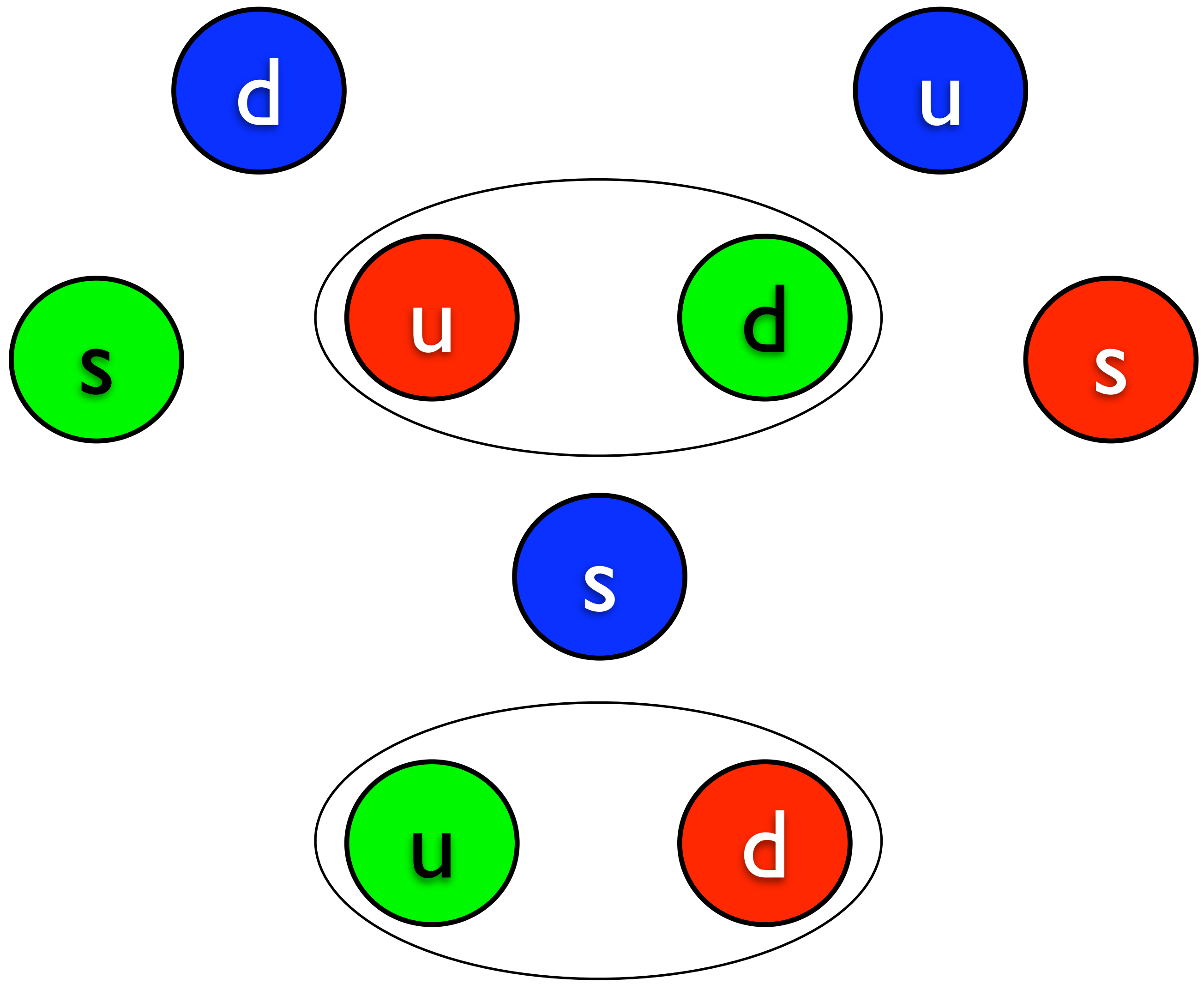

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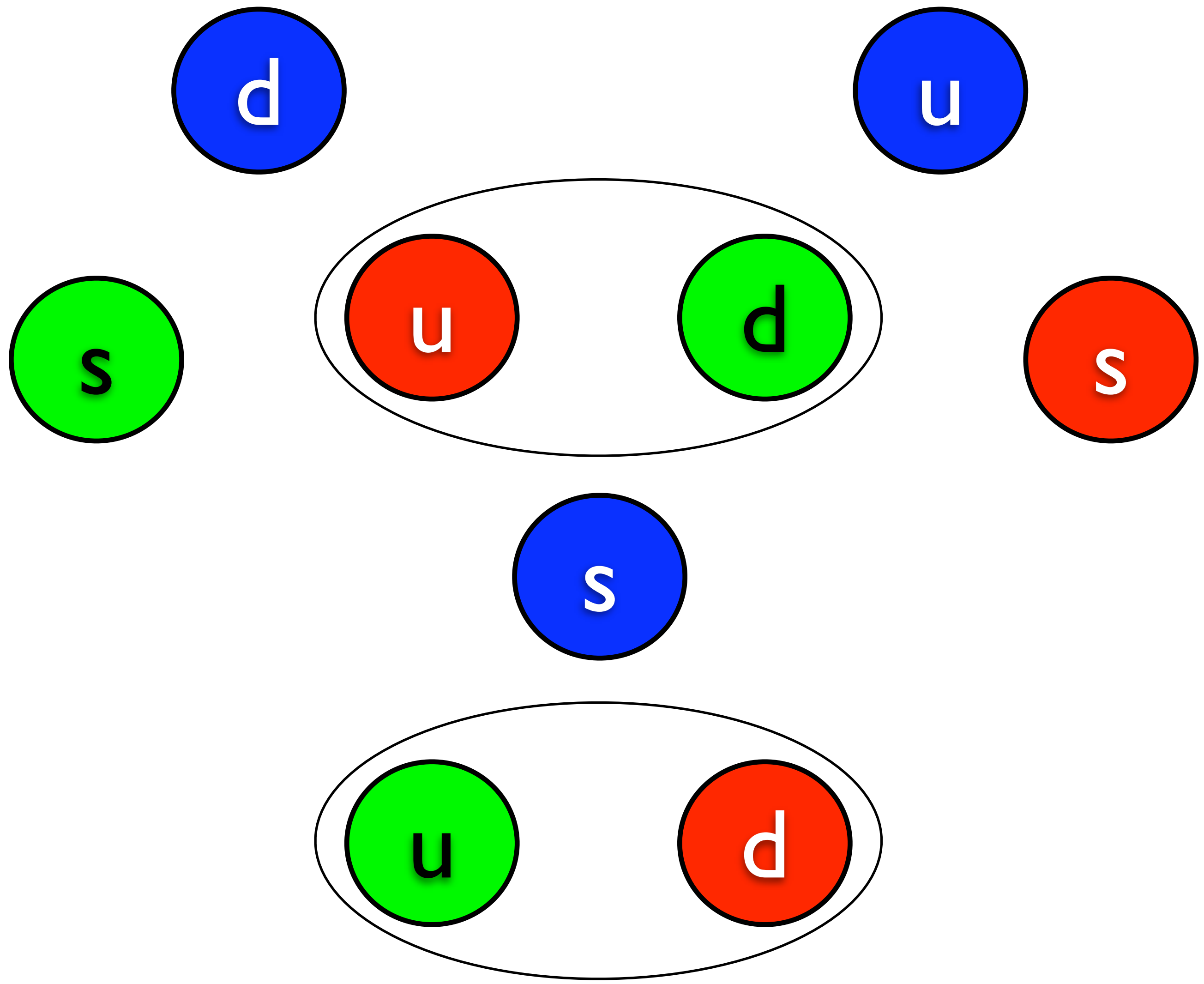

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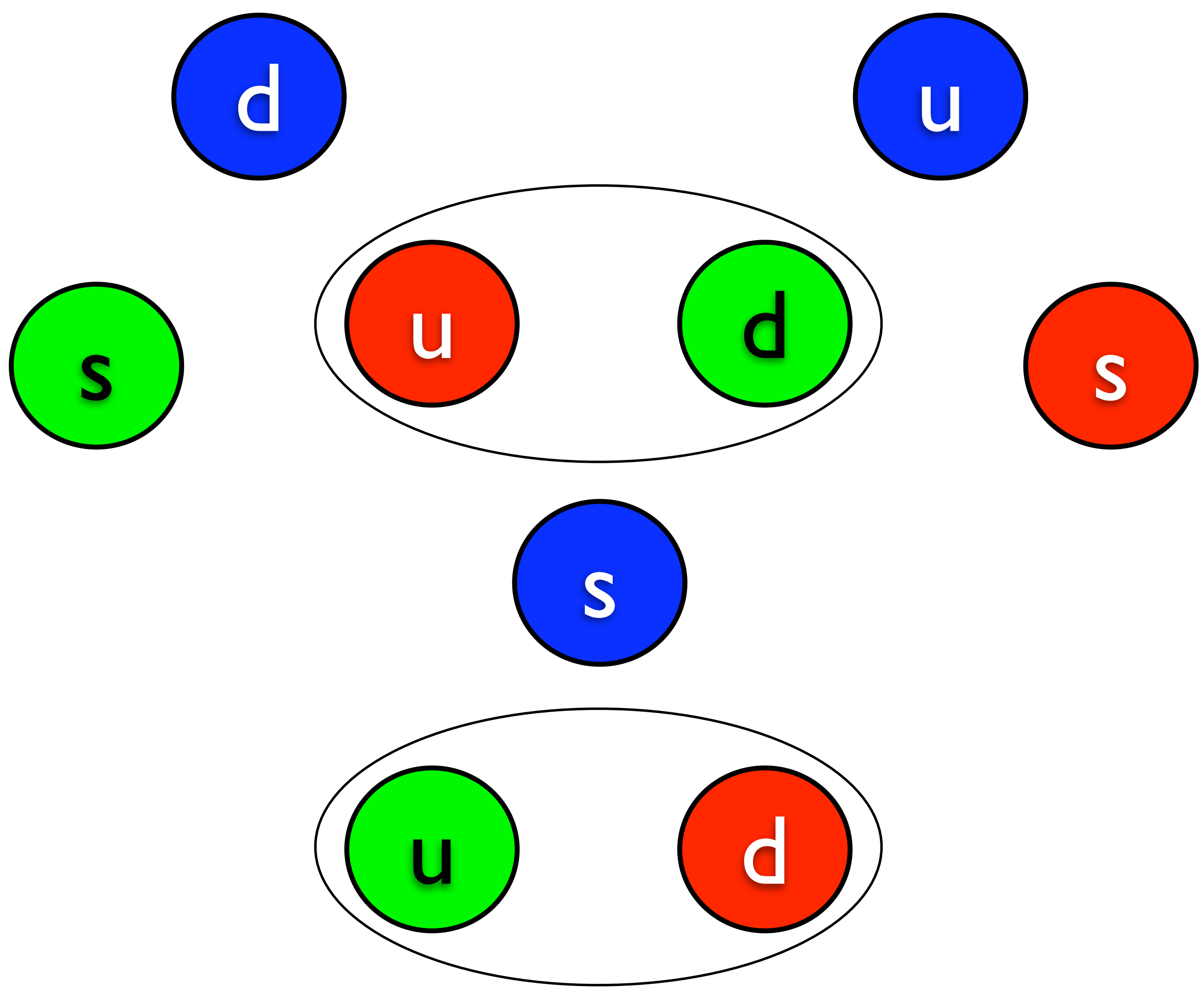
CFL: all flavors pair

CFL = color flavor locking



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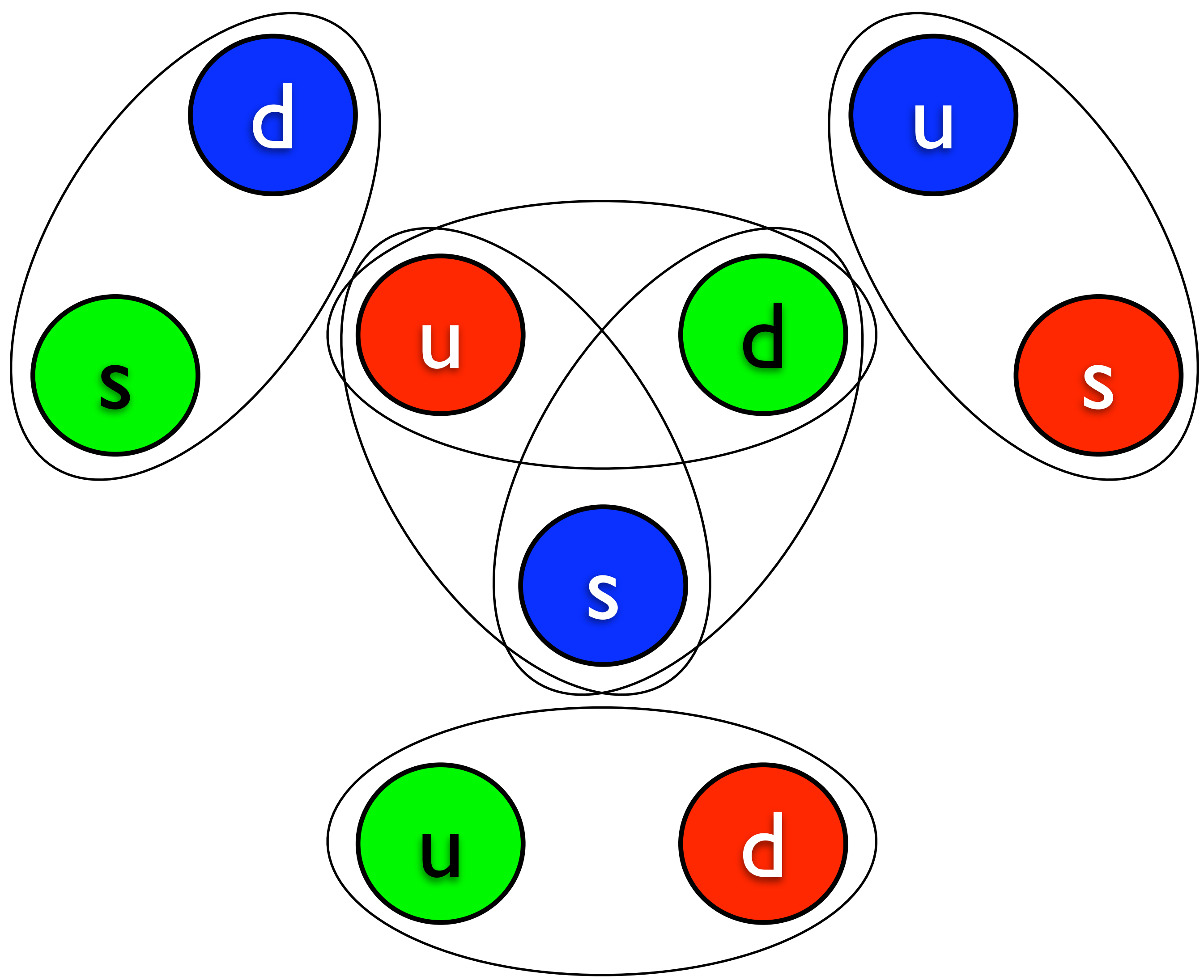

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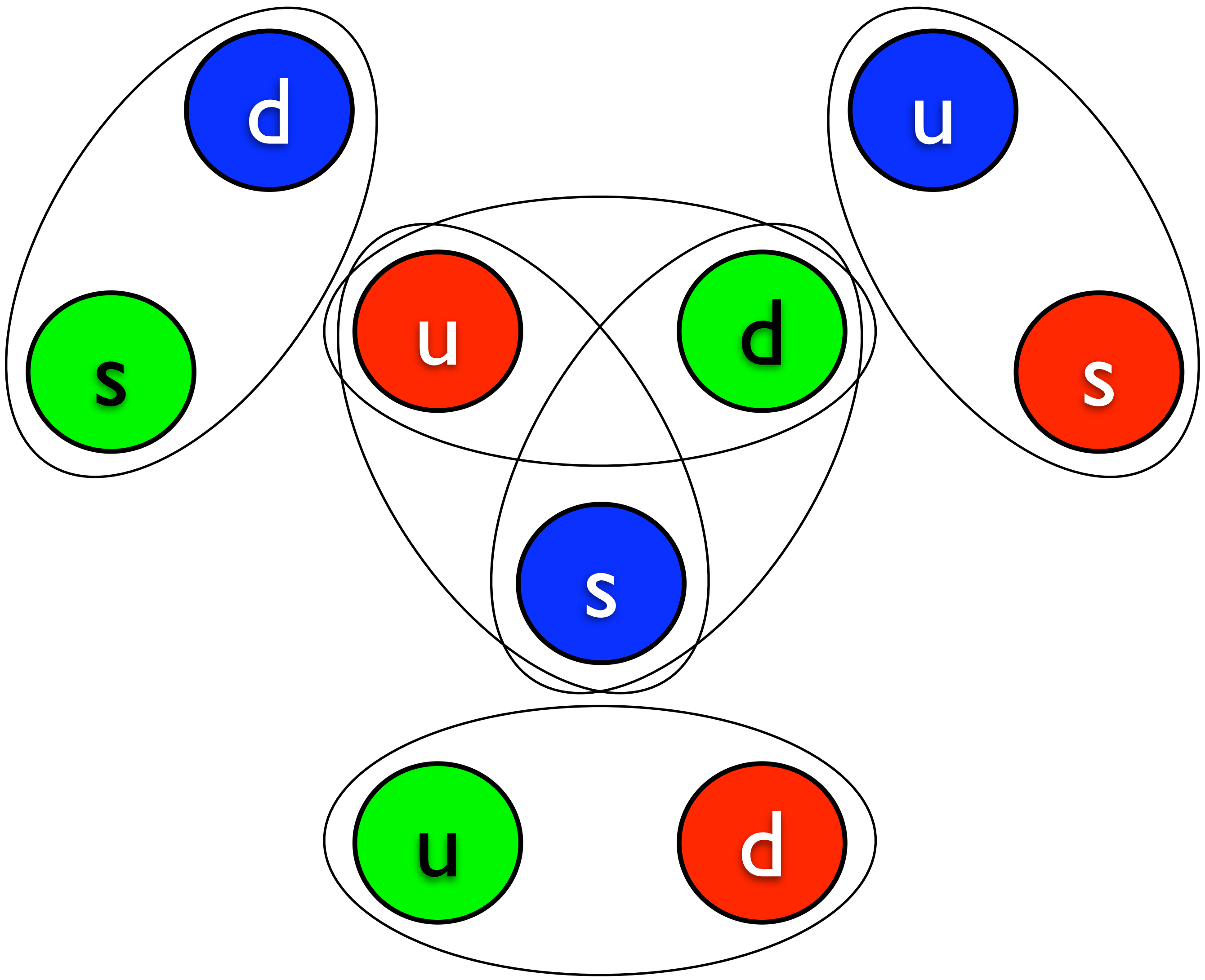
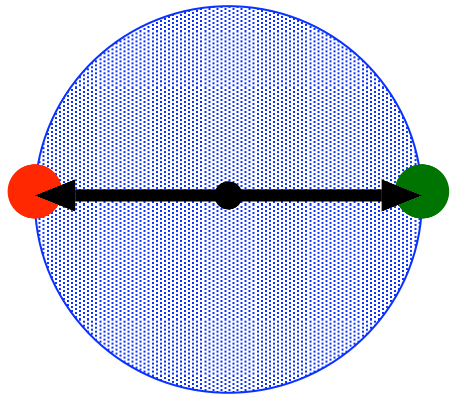


neutrality conditions can prevent pairing

Pairing Obstructions

BCS theory:

- opposite momenta
- momenta close to Fermi momentum



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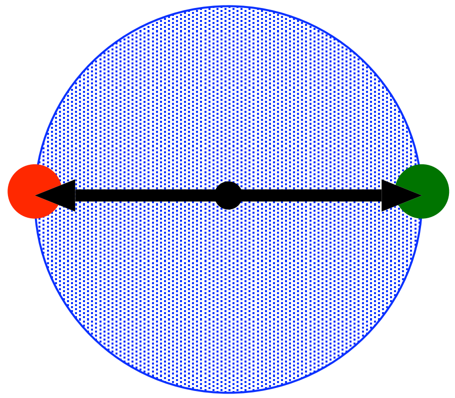
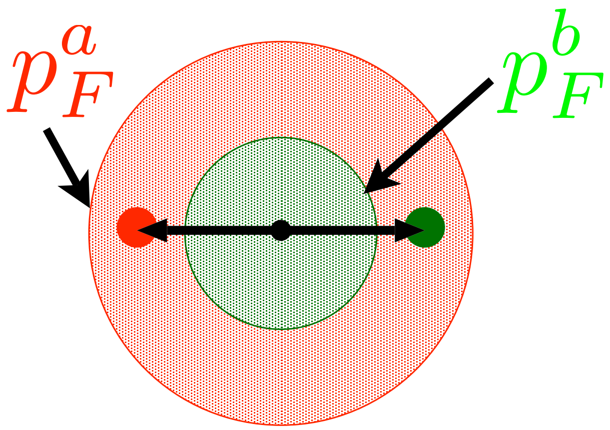


stressed pairing (e.g. different masses):

- Fermi momentum $p_F = \sqrt{\mu^2 - M^2}$
- pairing favored for $|p_F^a - p_F^b| \lesssim \sqrt{2}\Delta_{ab}$



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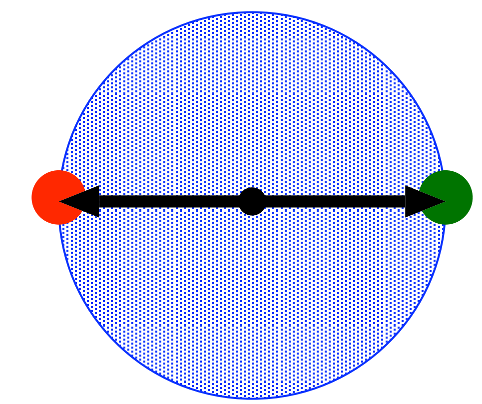

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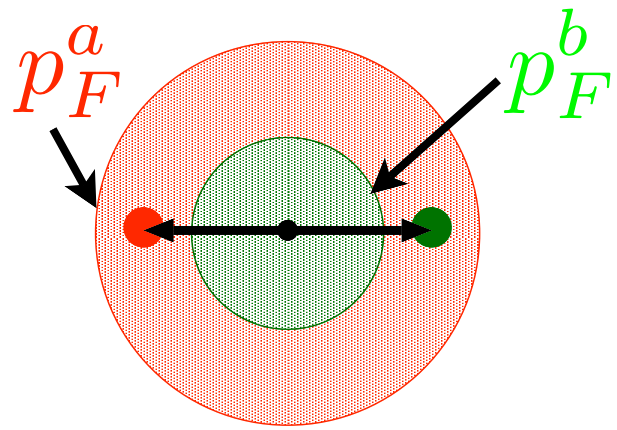

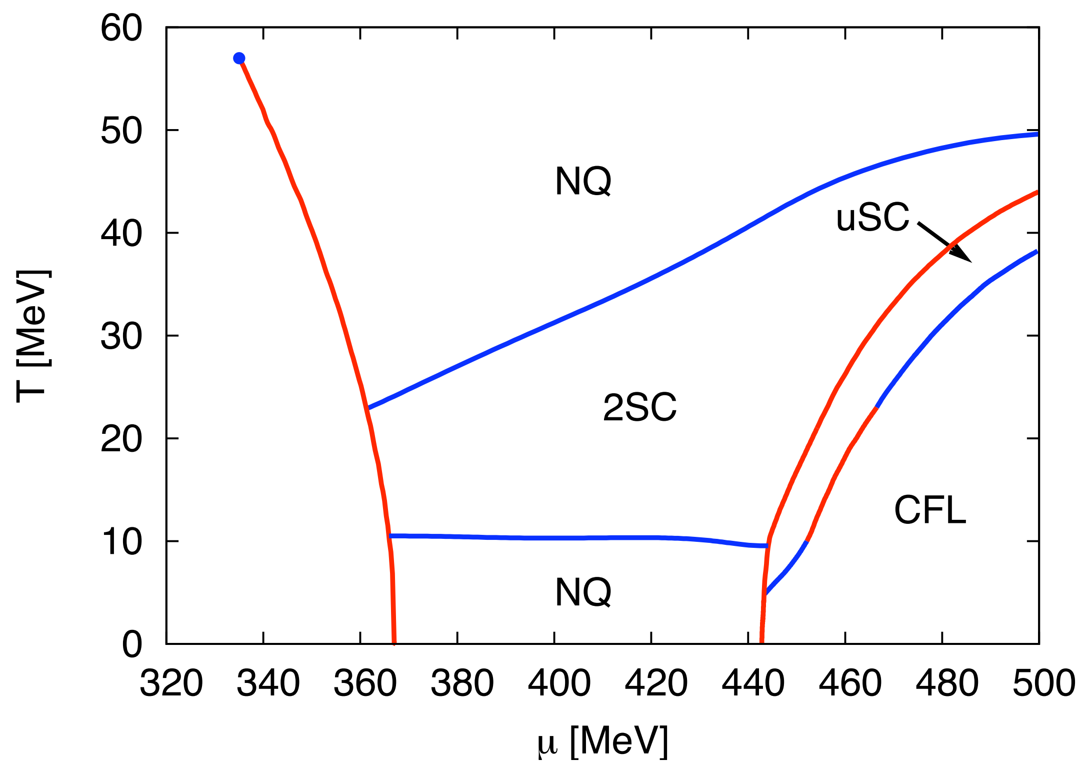

Rüster, Werth, Buballa, Shovkovy, '05

Neutrality

local color neutrality is energetically not costly

➡ focus only on electric neutrality: $2n_u - n_d - n_s \approx 0$

$$m_d = m_u = m_s \quad \Rightarrow \quad n_u = n_d = n_s$$ ➡ CFL

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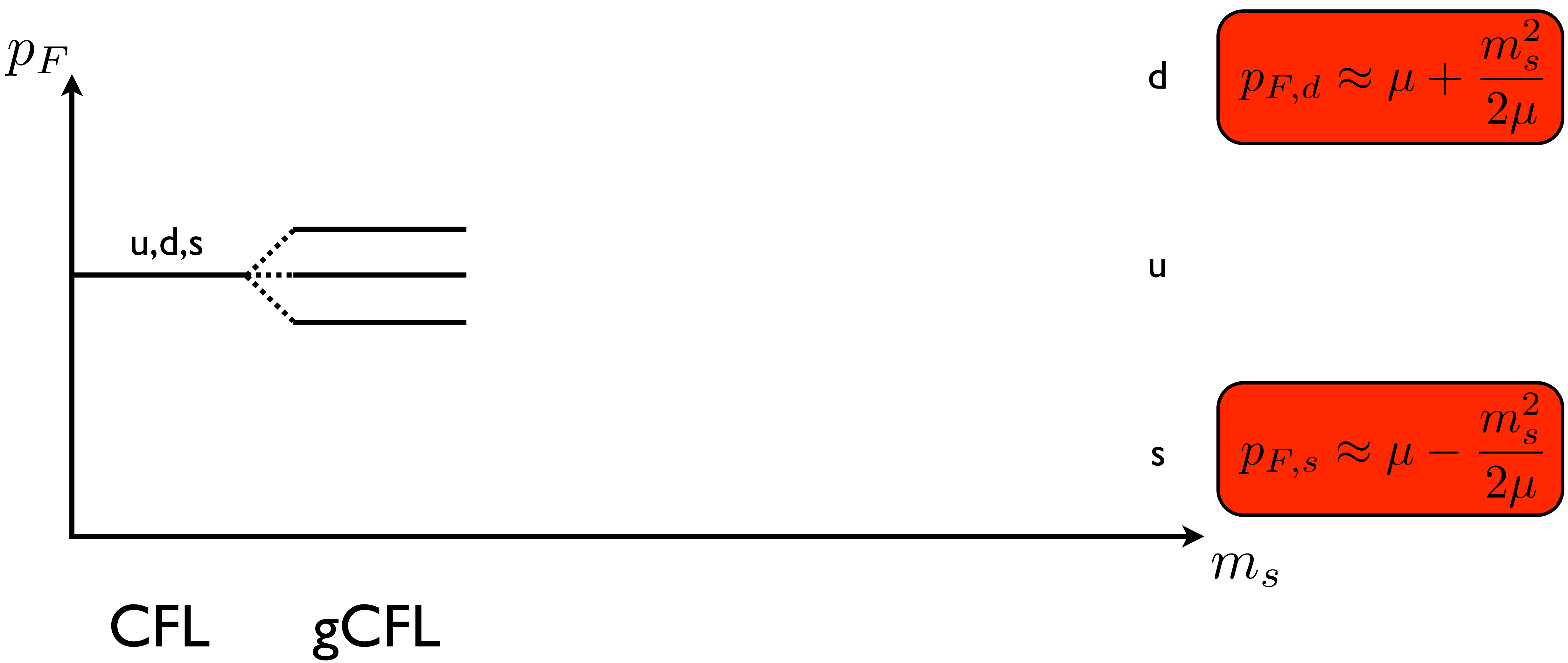

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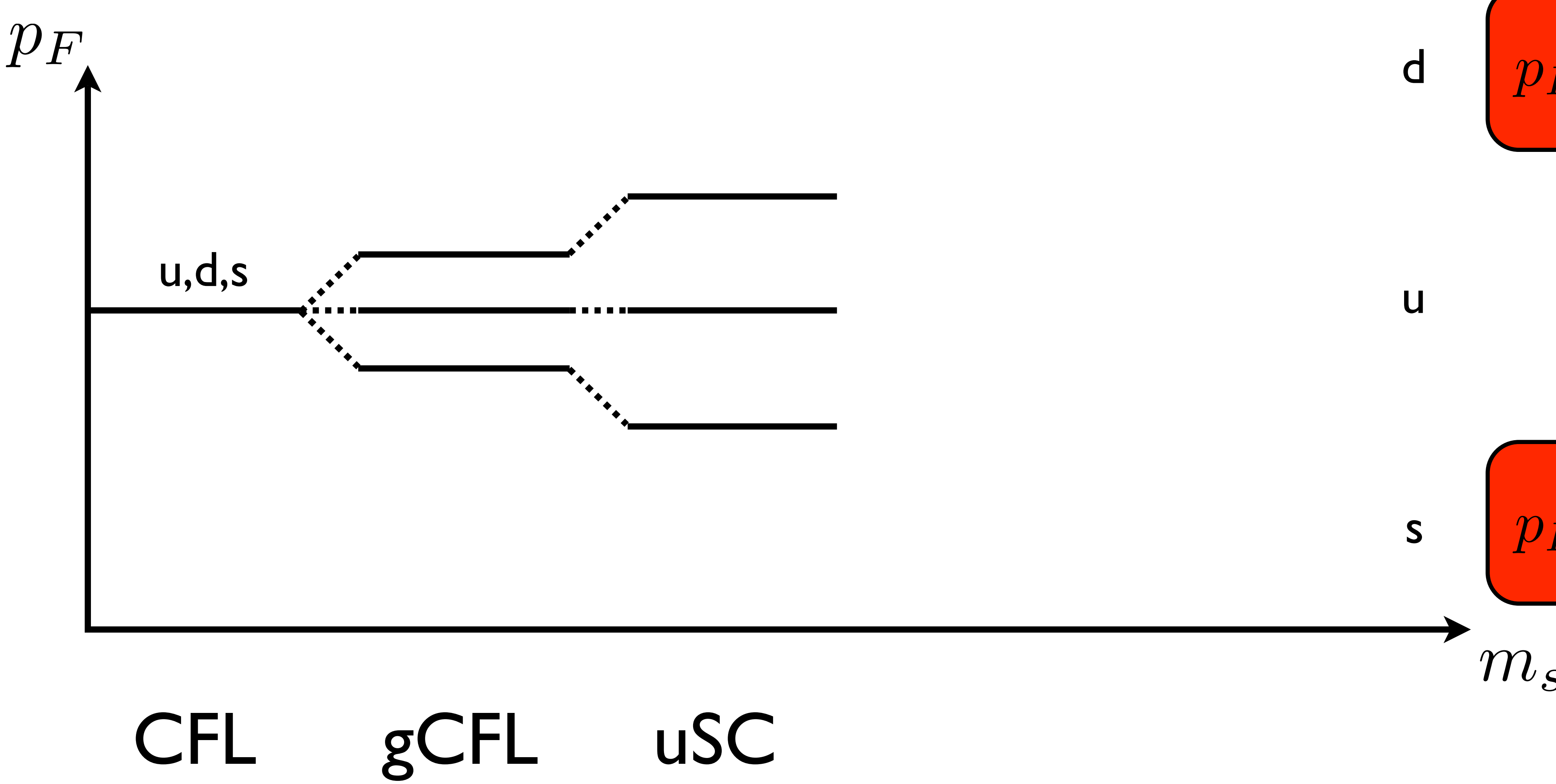

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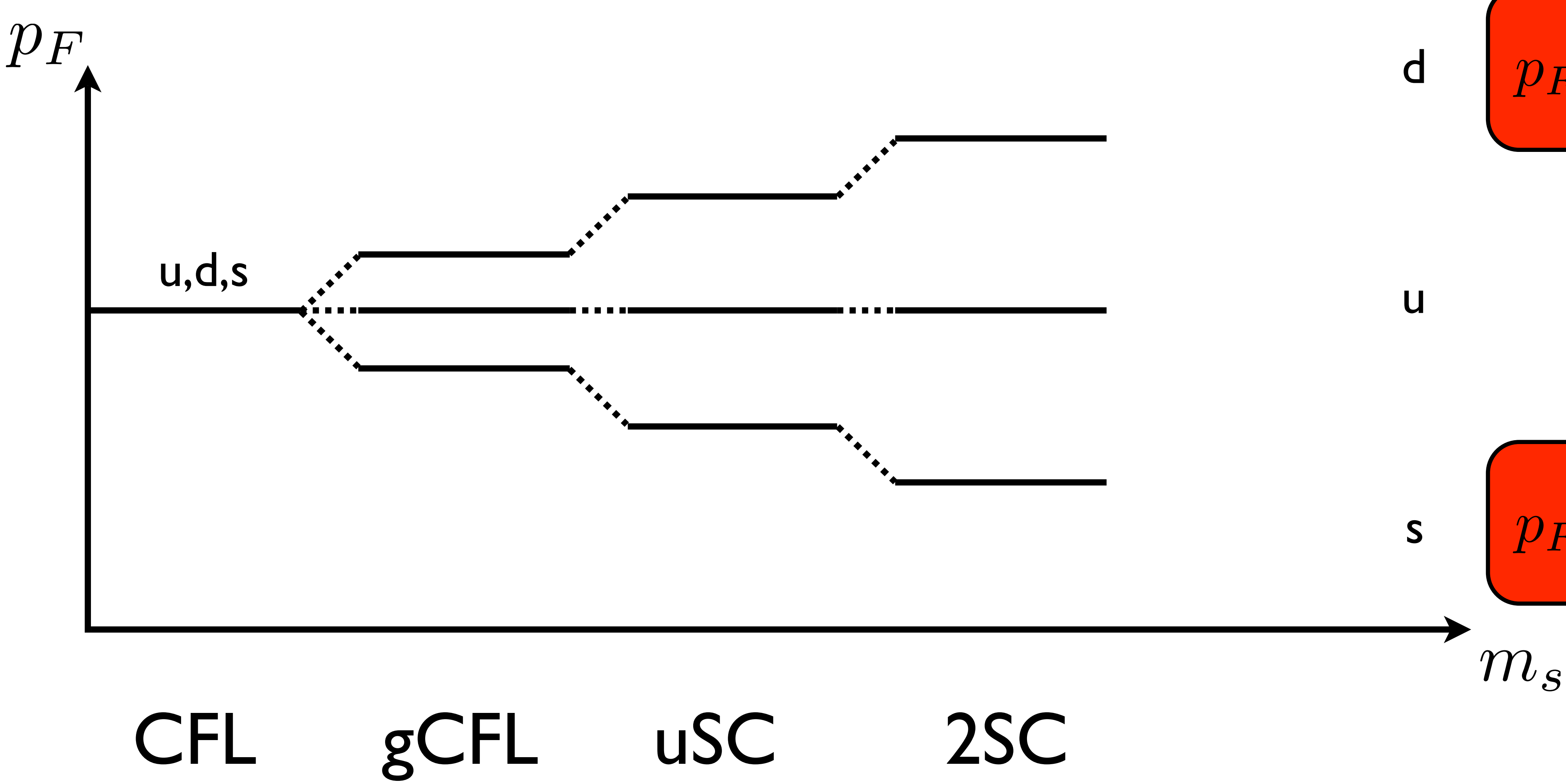

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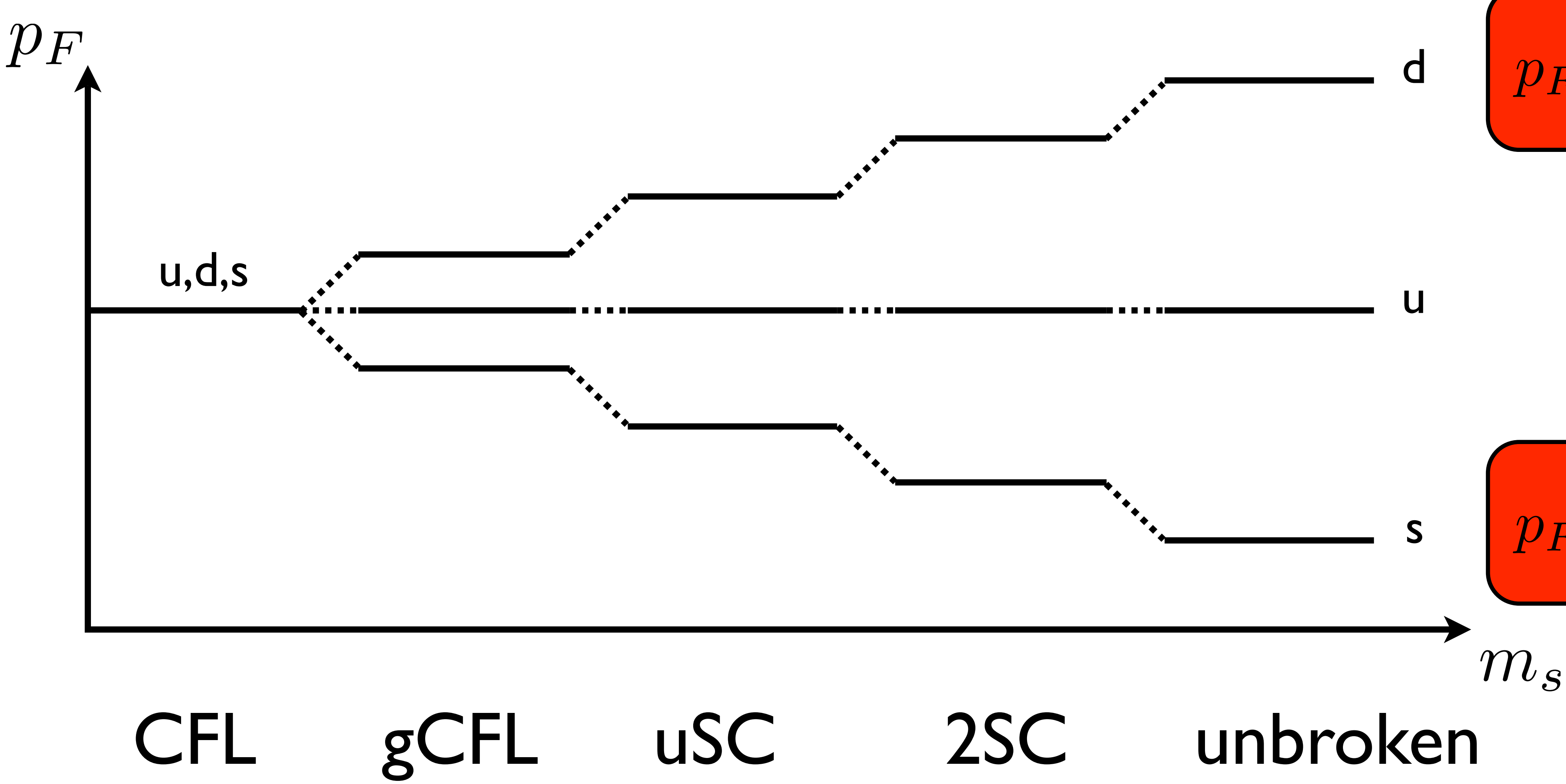

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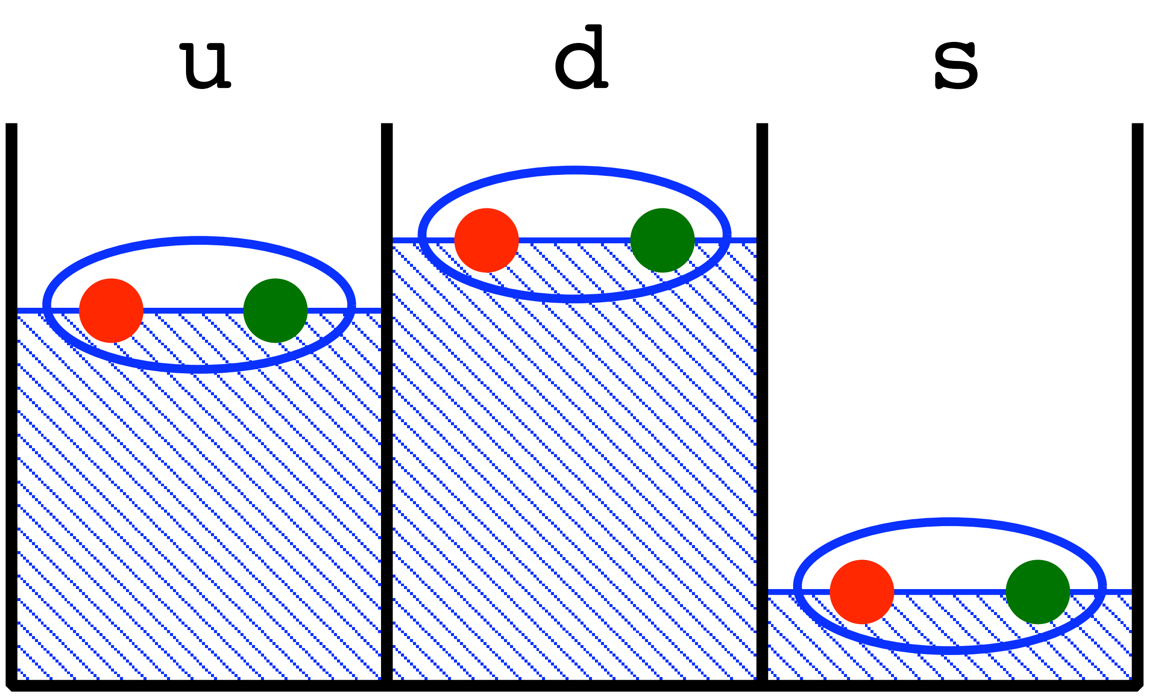
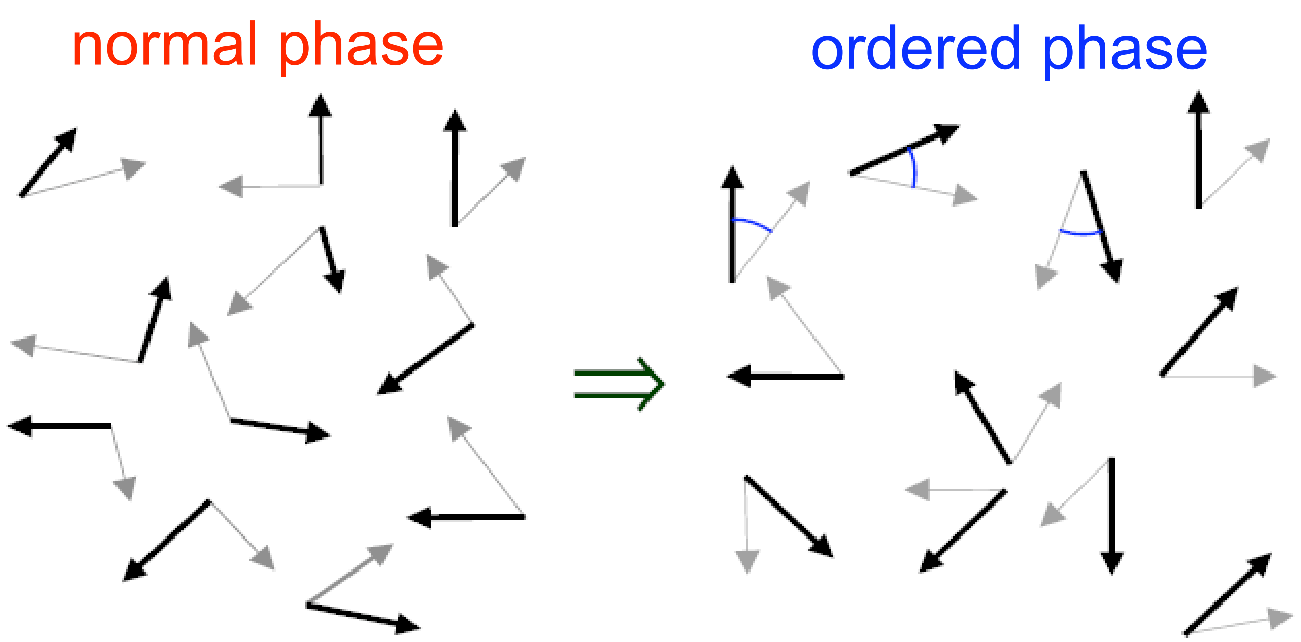
Further Phases

Spin-1 phases:

- flavors pair independently
- different Fermi energies do not prevent pairing
- usually energetically less favored
- e.g. „Color-spin-locking"
- similarity to condensed matter pairing (Helium-3)





A. Legget, Nobel Lecture

Realisation in Nature?

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to be clarified:

1) reach of critical density for phase transition NM to QM

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current state:

rather unlikely, need experimental signatures

thank you for
your attention

Goldstone Bosons

finite quark masses

➡modification of GB chemical potential

$$\mu_{\pi^+} = \mu_Q + \frac{m_d^2 - m_u^2}{2\mu_Q} \qquad \mu_+ = \mu_Q + \frac{m_s^2 - m_u^2}{2\mu_Q} \qquad \mu_0 = \frac{m_s^2 - m_d^2}{2\mu_Q}$$

- if $\mu_{GB} \geq \mu_{GB}$ Goldstone Bosons condense
- Goldstone Bosons contribute to the pressure
  - ➡treatment of GB can change phase structure in phase diagram
  - ➡ e.g. CFL + K favoured to 2SC, even if 2SC favoured to CFL