Lattice QCD and QGP

Rachik Soualah
Dirk Krumbhorn

EMMI Seminar June 2008

Outline

? What is QCD -

? Why do we need Lattice QCD -

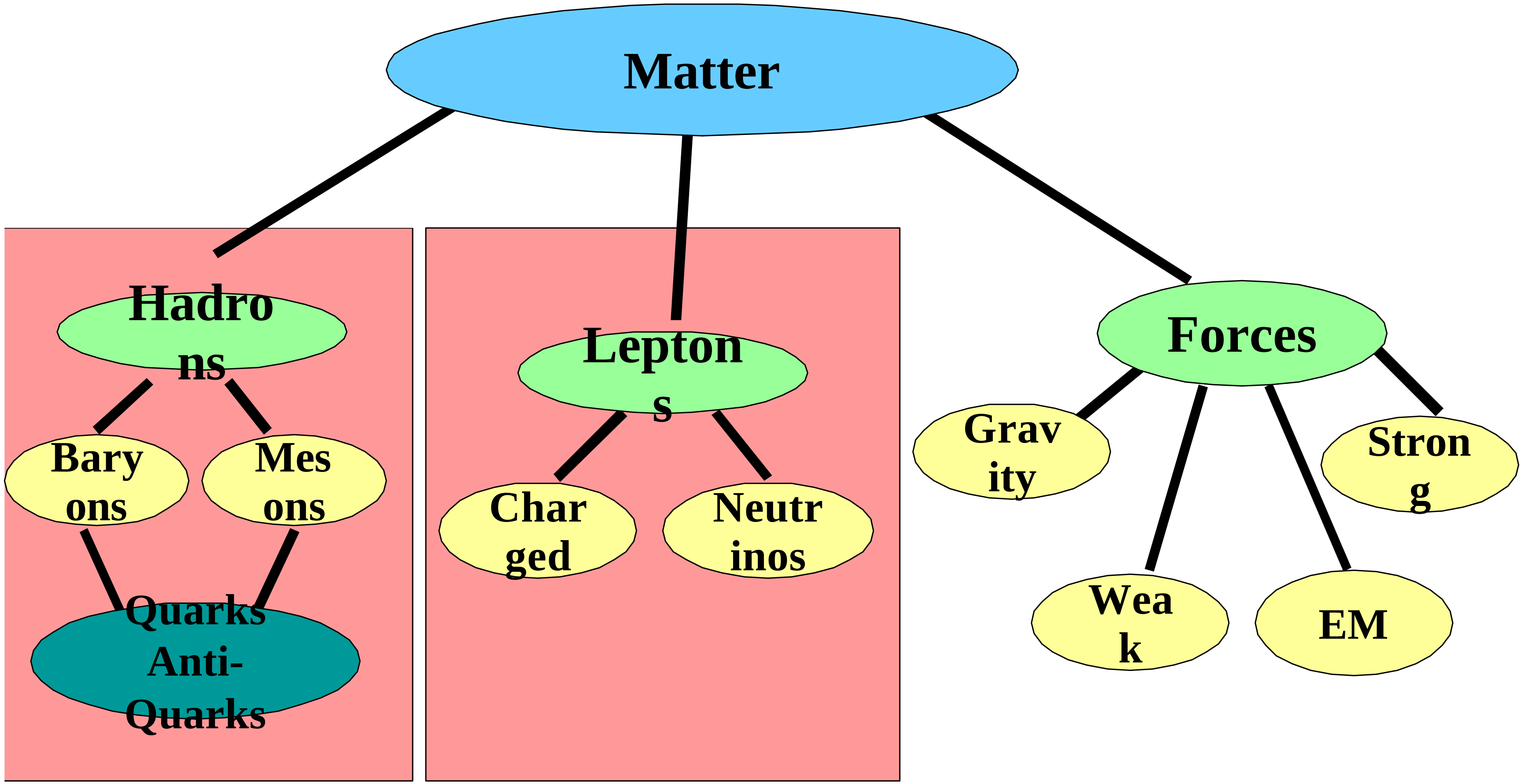
? What is Lattice QCD -

Lattice with Fermions -

nd part : Quark-Gluon Plasma2 -

? What is QCD

• Quantum Chromo Dynamics is the theory describing the interactions of quarks and gluons, the building blocks of the nucleons (proton + neutron).



? What is QCD

• Quantum Chromo Dynamics is the theory describing the interactions of quarks and gluons, the building blocks of the nucleons (proton + neutron).

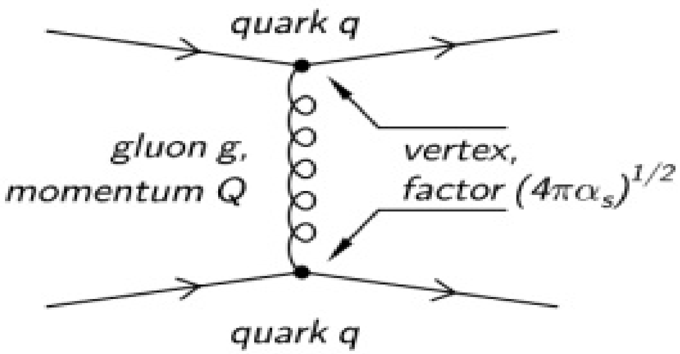
• QCD is an example of quantum field theory, where physical quantities are represented by functions ('fields') of space-time.

• QCD is 'gauge theory' whose essential ingredients are associated with the group SU (3).

q q q

➡ quarks carry a color charge q q q

➡ quarks interact among themselves via the exchange of the color filed quanta "gluons"



? What is QCD

Major Problem: Quarks are never observed as free particles, they turn in hadrons "confinement" (qq, qqq or $\bar{q}\bar{q}\bar{q}$).

→ All known hadron states are colors singlets ("white")

• The quantities one wants to calculate in quantum theory are: probabilities and expectation values of a certain combinations (functionals) of fields.

• From these numbers one can subsequently extract information about particles mass, life time decay amplitudes …etc

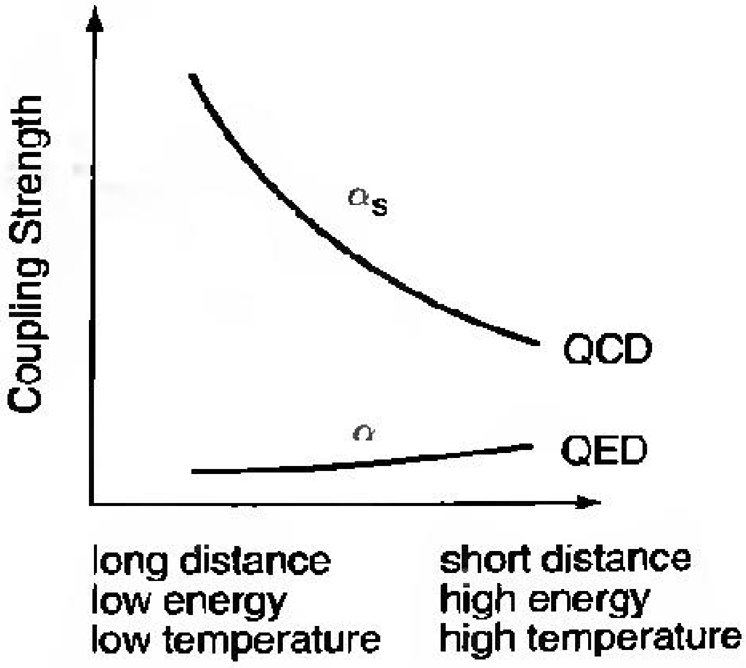
• The quarks and Gluons fields are controlled by the QCD Lagrangian:

$$\begin{aligned}\mathcal{L}_{\mathrm{QCD}} &= \bar{\psi}_i \left(i\gamma^\mu (D_\mu)_{ij} - m\,\delta_{ij}\right)\psi_j - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a \\ &= \bar{\psi}_i(i\gamma^\mu\partial_\mu - m)\psi_i - gG^a_\mu\bar{\psi}_i\gamma^\mu T^a_{ij}\psi_j - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a ,\end{aligned}$$

? Why we need Lattice QCD

- At high energies a_s is small. In this regime we treat quarks as free particles and observables can be calculated using perturbation theory.

- At low energies a_s is high. Perturbation theory fails we need a different approach.



? Why we need Lattice QCD

• Since the equilibrium phases and the phase transitions involve quarks and gluons over large distance scale they need to be studied within the framework of non-perturbative QCD.

• Lattice QCD is non-perturbative treatment of quantum chromodynamics on discrete lattice of space-time coordinates.

➡ So, what is Lattice simulation/calculation ?

- Formulation: Path integral method

- Basic quantities: to evaluate the path integral

(numerical treatment of QCD) ➡ (discretization of space-time)

- Lattice recipe

Birth of the Lattice QCD

PHYSICAL REVIEW D VOLUME 10, NUMBER 8 15 OCTOBER 1974

Nobel Laureate 1982

Confinement of quarks*

Kenneth G. Wilson
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850
(Received 12 June 1974)

A mechanism for total confinement of quarks, similar to that of Schwinger, is defined which requires the existence of Abelian or non-Abelian gauge fields. It is shown how to quantize a gauge field theory on a discrete lattice in Euclidean space-time, preserving exact gauge invariance and treating the gauge fields as angular variables (which makes a gauge-fixing term unnecessary). The lattice gauge theory has a computable strong-coupling limit; in this limit the binding mechanism applies and there are no free quarks. There is unfortunately no Lorentz (or Euclidean) invariance in the strong-coupling limit. The strong-coupling expansion involves sums over all quark paths and sums over all surfaces (on the lattice) joining quark paths. This structure is reminiscent of relativistic string models of hadrons.

I. INTRODUCTION

The success of the quark-constituent picture both for resonances and for deep-inelastic electron and neutrino processes makes it difficult to believe quarks do not exist. The problem is that quarks have not been seen. This suggests that quarks, for some reason, cannot appear as separate particles in a final state. A number of speculations have been offered as to how this might happen.[1]

Independently of the quark problem, Schwinger observed many years ago[2] that the vector mesons of a gauge theory can have a nonzero mass if vacuum polarization totally screens the charges in a gauge theory. Schwinger illustrated this result particles over short times and short distances. The polarization effects which prevent the appearance of electrons in the final state take place on a longer time scale (longer than $1/m_\gamma$, where m_γ is the photon mass).

A new mechanism which keeps quarks bound will be proposed in this paper. The mechanism applies to gauge theories only. The mechanism will be illustrated using the strong-coupling limit of a gauge theory in four-dimensional space-time. However, the model discussed here has a built-in ultraviolet cutoff, and in the strong-coupling limit all particle masses (including the gauge field masses) are much larger than the cutoff; in consequence the theory is far from covariant.

The confinement mechanism proposed here is

“ The Lattice spacing “

? What is Lattice QCD

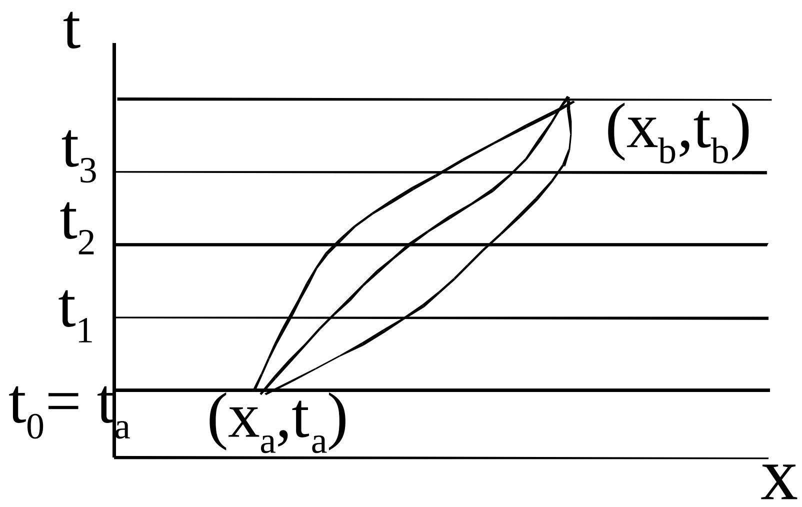
Formulation: **Path integral method**

- A single particle take the path $(x_a\ ,\ t_a)$ ➞ $(x_b\ ,\ t_b)$
- The action :

$$S_M = \int_{t_a}^{t_b} dt L\left(x(t), \dot{x}(t)\right)$$

- The partition function Z :

$$Z = \sum_{x_a} \left\langle x_a \left| e^{-bH} \right| x_a \right\rangle \quad , \quad b = 1/kT$$



- The quantum amplitude :

$$\left\langle x_b \left| e^{-iH(t_b - t_a)} \right| x_a \right\rangle = \sum_{\text{all paths}} e^{iS_M}$$

where

$$\sum_{\text{all paths}} e^{iS_M} = \int \prod_{i=1}^{n_t - 1} dx_i e^{iS_M}$$

substitute t = -it ➔ $$Z = \int \prod_{i=1}^{n_t} dx_i e^{-S_E}$$ Euclidean action

(we will obtain the partition function for gluons and quarks)

The basics calculation

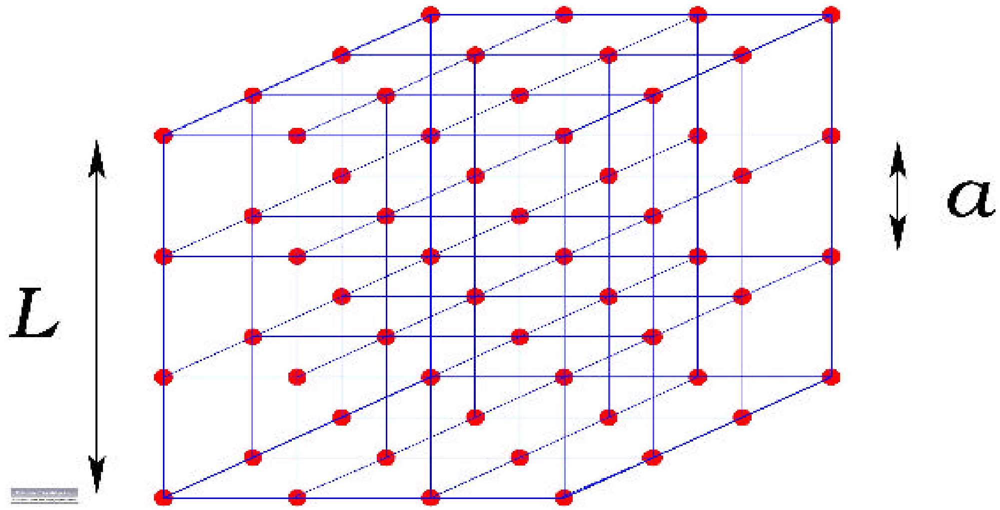
→ evaluate path integral

$$\langle \mathcal{O} \rangle = \int \mathcal{D}U \, \mathcal{O}(U) \, P_{\mathrm{eq}}(U)$$

where : $P_{\mathrm{eq}}(U)$ is the path integral distribution

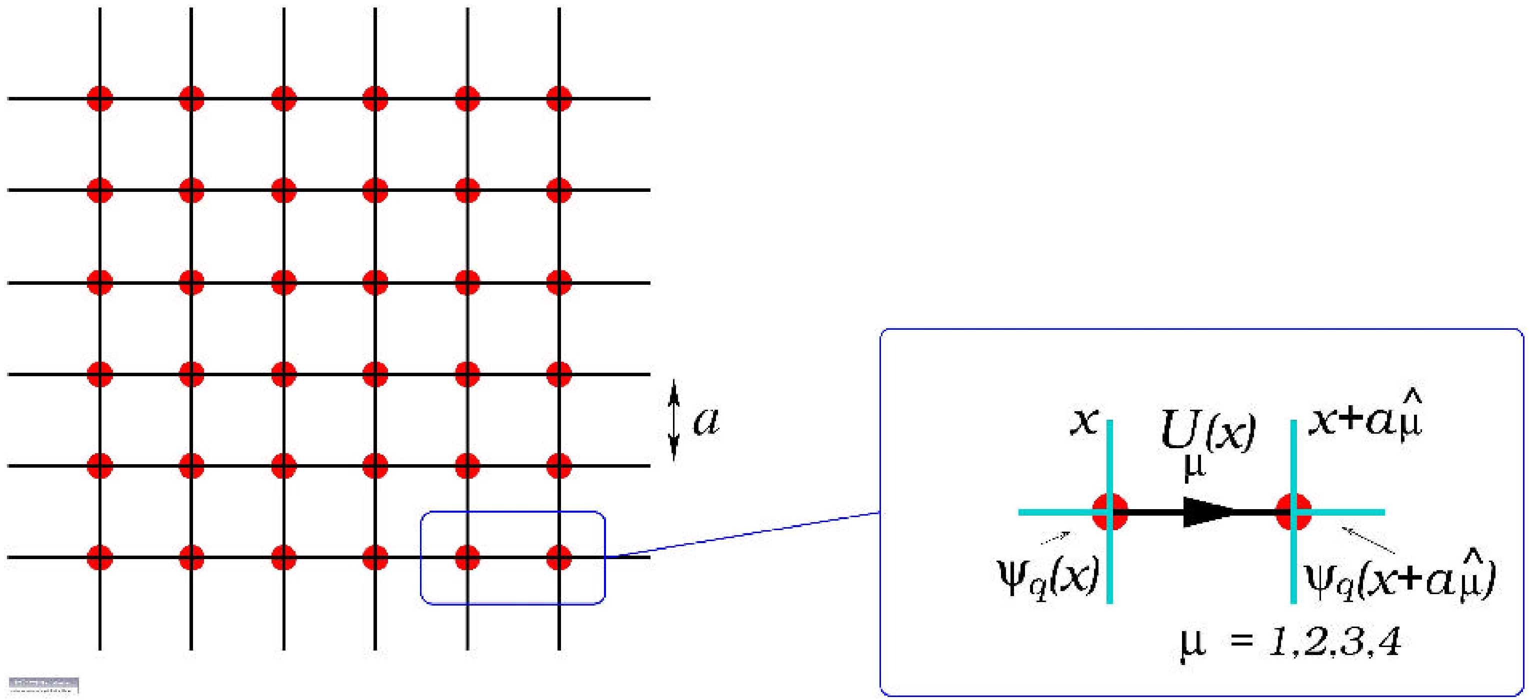
$$P_{\mathrm{eq}} = \frac{1}{\mathcal{Z}} e^{-S(U)}$$, S(U) is the theory action.

• Discretization of space-time is achieved introducing an Euclidean space-time lattice with spacing *a and volume* $L^3.T$



Lattice recipe

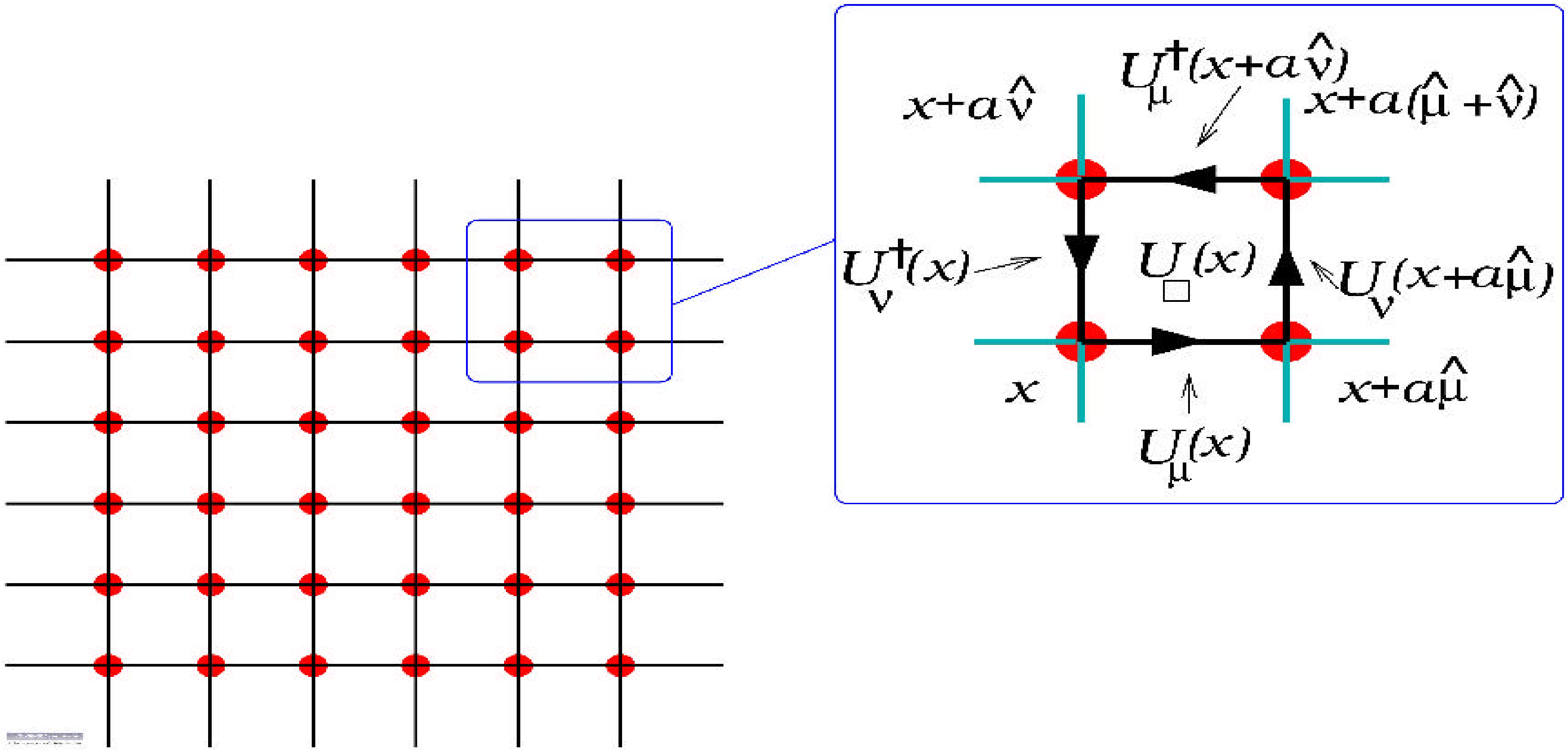
• *The quark and antiquark* fields *? (x), ?̄ (x)* leave in the lattice sites *x.*



• *Gauge fields* are represented by the ink variable $U_\mu(x)$ which are group elements in the SU(N) associated with line path connecting nearest neighbor pairs of lattices sites.

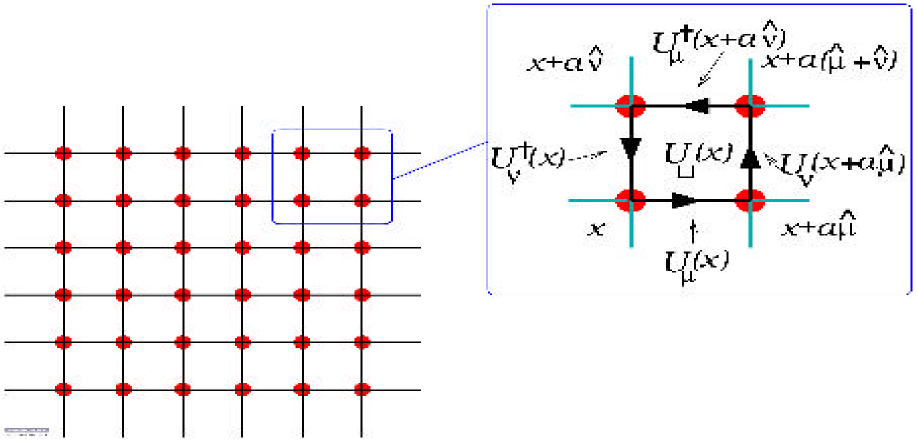
Lattice recipe

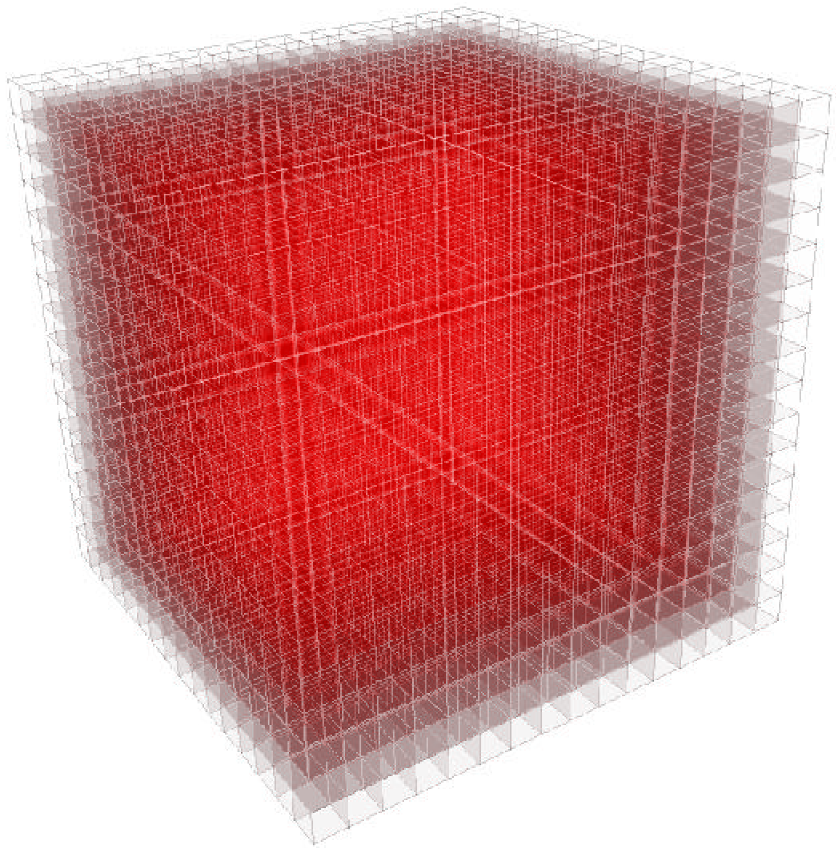
- *The quark and antiquark* fields *? (x), ?̄ (x)* leave in the lattice sites *x.*



Lattice recipe

• *The quark and antiquark* fields *? (x), ?̄ (x)* leave in the lattice sites *x.*





Lattice recipe

- Latticize :

$$\begin{cases} \mathcal{D}U \to \prod_{x,\mu} dU_{x,\mu} \\ S \to S_{\mathrm{latt}} \end{cases}$$

So the observable becomes:

$$\langle \mathcal{O}_{\mathrm{latt}} \rangle_a = \int \prod_{x,\mu} dU_{x,\mu}\ \mathcal{O}_{\mathrm{latt}}(U_{x,\mu})\ P^{\mathrm{latt}}_{\mathrm{eq}}(U_{x,\mu})$$

In order to evaluate <*O*> we have to set the continuum extrapolation : a → 0

However the lattice still infinite. By taking finite volume:

$$\langle \mathcal{O}_{\mathrm{latt}} \rangle_{a,V} = \int \prod_{x,\mu}^{V} dU_{x,\mu}\ \mathcal{O}_{\mathrm{latt}}(U_{x,\mu})\ P^{\mathrm{latt}}_{\mathrm{eq}}(U_{x,\mu})$$

→ finite volume effects

Lattice recipe

- $\langle \mathcal{O} \rangle_{a,V}$ still with high-dimensional integrals.
- Monte Carlo methods:

  - generate a gauge configurations U^i .

$$\int \prod_{x,\mu}^{V} dU_{x,\mu}\, \mathcal{O}_{\rm latt}(U_{x,\mu})\, P_{\rm eq}^{\rm latt}(U_{x,\mu}) \to \sum_{\{U^i\}} \mathcal{O}_{\rm latt}(U^i_{x,\mu})\, P_{\rm eq}^{\rm latt}(U^i_{x,\mu})$$

→ the statistical error decreases as $\frac{1}{\sqrt{N_U}}$

→ The generation of configurations ensembles $\{U^i\}$ is numerically costly and needs supercomputers.

Lattice recipe

- **The Analysis :**

  - Evaluation of the path integrals.
  - Take the appropriate limits.
  - Quantify the errors.

- **The errors :** the errors in this procedure have different origins.

→ **Systematic errors :**

the lattice discretization from the lattice spacing *a.*

the number of point in the lattice universe (finite volume)

→ **Statistical errors :** - the finite number of configurations used to compute

- the value of path integrals.

To control and quantify these errors: - simulate in big enough volume.

- reduce disretization error.
- different methods gives the same answer

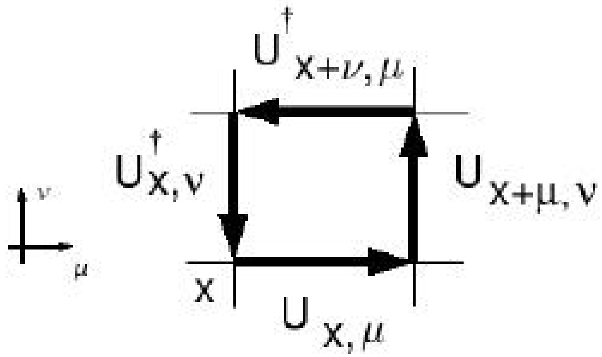
The Wilson gauge action

- We find the physics in:
  - The construction of the lattice gauge action.
  - The construction of the observables
- The plaquette is

$$\begin{aligned} U_{\mu\nu}(x) &= U_{x,\mu} U_{x+\hat{\mu},\nu} U_{x+\hat{\mu}+\hat{\nu},-\mu} U_{x+\hat{\nu},-\nu} \\ &= U_{x,\mu} U_{x+\hat{\mu},\nu} U^{\dagger}_{x+\hat{\nu},\mu} U^{\dagger}_{x,\nu}(x) \end{aligned}$$

where $U_{x,\mu}$ are unitary.

→ The trace of the products of gauge fields over closest paths is the lattice Wilson loop.



$$S^{\text{latt}} = \beta \sum_{x} \sum_{\mu \neq \nu} \frac{1}{2N_c} \left(\mathrm{Tr} U_{\mu\nu}(x) - U^{\dagger}_{\mu\nu}(x) \right)$$

where ß is the lattice version of the coupling.

The Wilson gauge action

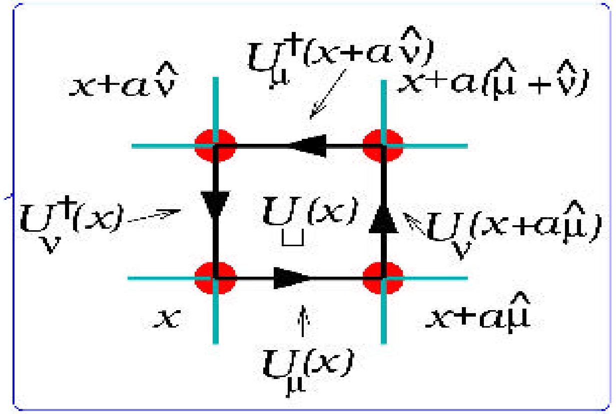
- **The gluon action :**

Using the smallest closed loop which connect the sites *n* and *n+μ*

$$U_\mu(n) = \exp\left(igaA_\mu(n)\right)$$

Plaquette:

$$U_{\mu\nu}(n) = U_\nu^\dagger(n) U_\mu^\dagger(n+\hat{\nu}) U_\nu(n+\hat{\mu}) U_\mu(n)$$



The action:

$$S_g = \frac{2N_c}{g^2} \sum_P \left(1 - \frac{1}{N_c} \text{Re tr } U_{\mu\nu}(n)\right)$$

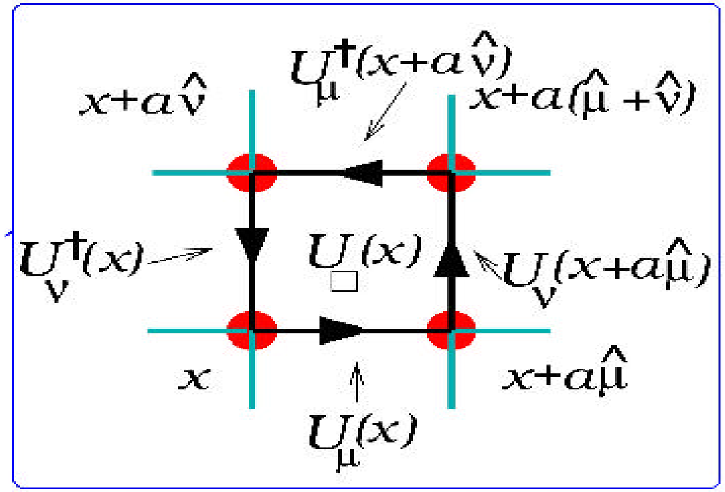
ß the lattice version of the coupling

The lattice with Fermions

- **The fermion action :**

Following the same steps with the gluon case,

$$S_W = a^4 \sum_n \left[m\bar{q}(n)q(n) - \frac{1}{2a}\sum_\mu \bar{q}(n+\hat{\mu})\Gamma_\mu U_\mu(n)q(n) \right.$$
$$\left. -\frac{r}{2a}\sum_\mu \left(\bar{q}(n+\hat{\mu})U_\mu(n)q(n) - \bar{q}(n)q(n)\right)\right]$$
$$\equiv a^4 \sum_{n',n} \bar{q}(n')\left(m\delta_{n',n} + D_W(n',n;r)\right)q(n),$$



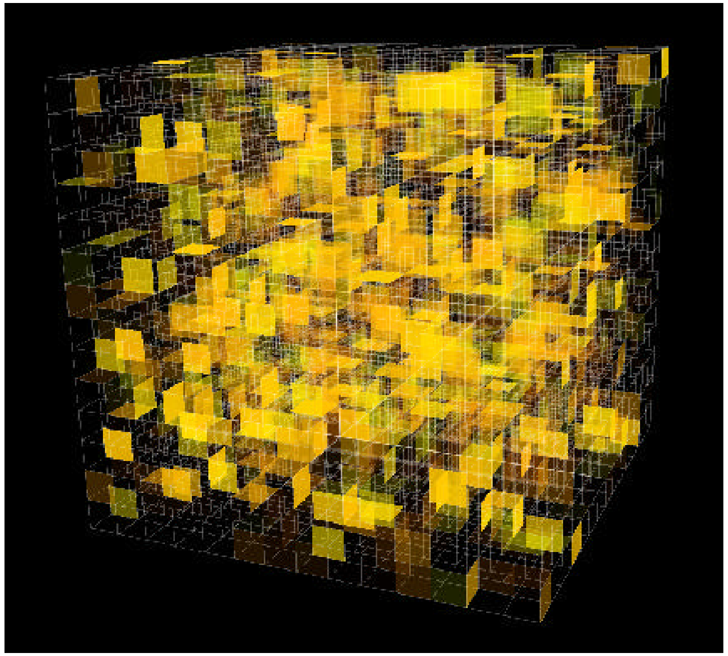
where D_w is the Wilson's Dirac operator

Lattice QCD at finite T

-Using hyper-cubic lattice with lattice spacing *a*

$$\longrightarrow \quad T^{-1} = N_t a, \quad V = (N_s a)^3$$

N_s : # of spatial sites and **N_t** : # of temporal sites



the U_μ of quark field satisfy the boundary conditions :

$$U_\mu(n_4 + N_t, \boldsymbol{n}) = U_\mu(n_4, \boldsymbol{n}),$$

$$\psi(n_4 + N_t, \boldsymbol{n}) = -\psi(n_4, \boldsymbol{n}).$$

→ in the thermodynamics limit $V^{?}$ >> T ⬌ $N_s >> N_t$

Lattice QCD at finite T

Two different ways to vary T:

→ change N_t and keep a and N_s fixed.

→ change a with N_s and N_t fixed.

↳ changing a is like changing the lattice coupling $g(a)$

small g(a) ⟶ ***small a*** ⟶ ***large T***

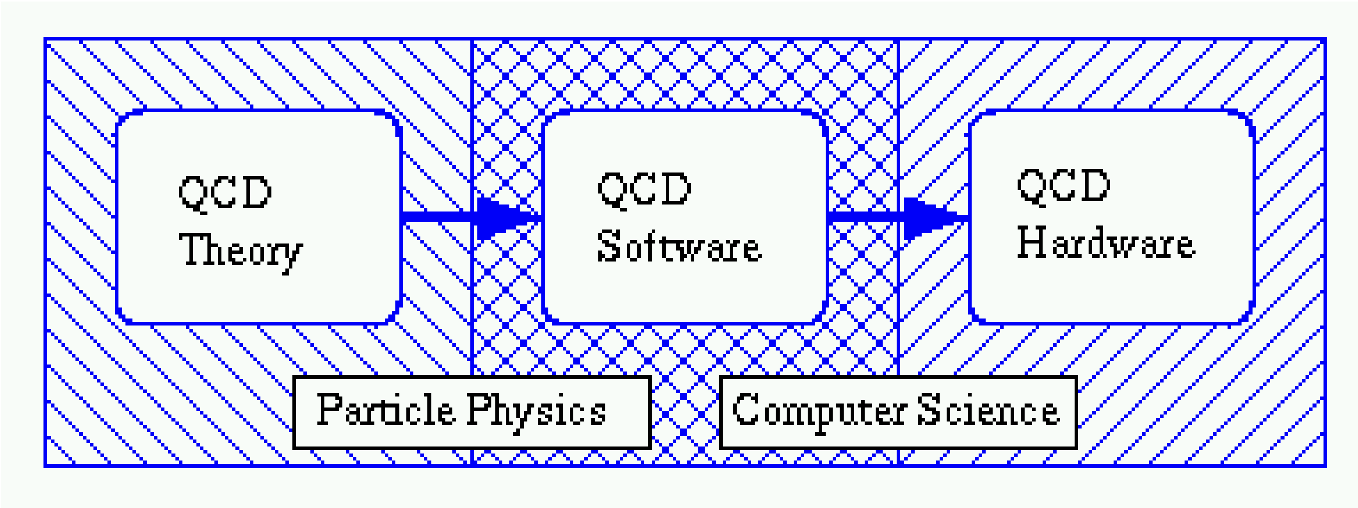
large g(a) ⟶ ***large a*** ⟶ ***small T***

By from the leading order of the coupling expansion a :

$$T \simeq \frac{\Lambda_{\mathrm{LAT}}}{N_t} \exp\left(\frac{1}{2\beta_0 g^2(a)}\right)$$

$?_{LAT}$ is the lattice scale parameter to be determined from the experimental input

The Lattice Computing



The Lattice Computing

• Lattice computing requires large amount

(Teraflops) of computer time.

• Improvements in algorithms and a steady increase

in the capabilities of computers

• In current lattice QCD problems, the representation of the lattice requires around 10 GB of memory and calculation rates of around 1 Tflops (10^12 floating-point operations per second) to make reasonable progress(a typical desktop PC is capable of about 1 Gflops.

• Need for parallel Computing (new computer architectures for lattice) where the tasks that don't depend on each other can be done simultaneously now … to the QGP part.