## 8.4 Bag Models

Despite extensive searches in the late 60's (continuing to the present day), no free quarks were found. It was therefore clear, even before QCD, that one needed some form of confinement mechanism. The most obvious was to make the quarks extremely massive, but have them bound in an extremely deep potential so that the observed hadronic states could still have reasonable masses. A simple model of this kind was constructed by Bogoliubov [Bog 67]. He set the quark masses to $m$, with $m \to \infty$, but placed them in a spherical volume of radius $R$, within which they felt an attractive scalar field of strength $m$. This extremely simple model led to an accurate prediction for the ratio of the mass of the Roper resonance to that of the nucleon, to a simple explanation of the deviation of $g_A$ from the naive $SU(6)$ result of $\frac{5}{3}$ and many other interesting results.

However, the real renaissance of interest in such models came only after it was realized that QCD is asymptotically free, at short distances, as well as (probably) confining at large distances. The MIT bag model [Ch 74, DeG 75] was constructed in such a way as to build these properties into a simple phenomenological model. Space was divided into two regions, the interior of the bag in which the quarks had very small (current) masses and felt only weak forces and the exterior in which the quarks were not allowed to propagate and which had a different (lower) vacuum energy. It was soon realized that as far as the motion of the quarks was concerned, at least for a static, spherical cavity, the wave functions in the MIT bag were identical to those in the model of Bogoliubov in the limit where the parameter $m \to \infty$. Bearing this in mind, we begin our discussion of bag models with a review of the model of Bogoliubov which is mathematically very simple.

### 8.4.1 The Model of Bogoliubov

Consider the Dirac equation for a particle of mass $m$ moving in a spherical cavity of radius $R$ within which it feels a constant scalar potential $-V_s$:

$$[\vec{\alpha} \cdot \vec{p} + \beta(m - V_s)]\, \psi = E\psi. \tag{8.71}$$

The operators representing the total angular momentum, $\vec{j} = \vec{l} + \vec{\sigma}/2$, and also $K = \beta(\vec{\sigma}\cdot\vec{l}+1)$ commute with this Hamiltonian and may therefore be used to classify the eigenstates of Eq. (8.71). Taking the eigenvalues of $\vec{j}^2, j_z$ and $K$ to be $j(j+1), \mu$ and $-\kappa$, respectively, one can show that $\kappa$ is equal to $\pm(j+\frac{1}{2})$ and it is therefore sufficient to label $\psi$ by $\kappa$ and $\mu$, $\psi^\mu_\kappa$. The $4\times 4$ matrix operator $K$ obviously has the $2\times 2$ matrices $\pm(\vec{l}\cdot\vec{\sigma}+1) \equiv \pm k$ down the diagonal. Taking the eigenfunctions of $k$ and $j_z$ to be $\chi^\mu_\kappa$ we may therefore write:

$$\psi^\mu_\kappa = \begin{pmatrix} g(r)\chi^\mu_\kappa \\ i\,f(r)\chi^\mu_{-\kappa} \end{pmatrix}. \tag{8.72}$$

Using the operator identity

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} - i\frac{\hat{r}}{r}\times\vec{l}, \tag{8.73}$$

we find

$$\vec{\alpha}\cdot\vec{p} = -i\vec{\alpha}\cdot\hat{r}\frac{\partial}{\partial r} + \frac{i}{r}\vec{\alpha}\cdot\hat{r}(\beta K - 1). \tag{8.74}$$

It is then simple to write down the two coupled differential equations for $g(r)$ and $f(r)$:

$$\begin{aligned} (E + V_s - m)g &= -\left(\frac{df}{dr} + \frac{f}{r}\right) + \kappa\frac{f}{r}, \\ (E - V_s + m)f &= \left(\frac{dg}{dr} + \frac{g}{r}\right) + \kappa\frac{g}{r}. \end{aligned} \tag{8.75}$$

If we now take $V_s = m$ for $r < R$ and $V_s = 0$ for $r > R$ it is easy to solve these coupled equations. For $\kappa = -1$ (in which case the upper component is in s-wave) we find:

$$g(r) = \begin{cases} A\dfrac{\sin Er}{r} & r < R, \\ A\dfrac{\sin Er}{r}e^{-\sqrt{m^2-E^2}(r-R)} & r > R, \end{cases} \tag{8.76}$$

where the eigenenergy is obtained by matching the first derivatives at $r = R$.

The particular case of interest here is the limit where $m \to \infty$, so that the quarks behave as though they were free (with zero effective mass) inside $R$, but have infinite mass outside – and are therefore confined. In that limit, matching the derivative inside and outside gives the condition

$$j_0(ER) = j_1(ER), \tag{8.77}$$

where $j_0$ and $j_1$ are the spherical Bessel functions

$$j_0(x) = \frac{\sin x}{x}; \qquad j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x}. \tag{8.78}$$

Equation (8.77) has the solutions $ER = 2.04, 5.40, \ldots$, so that if we label the eigenenergies by the principal quantum number, $n$, and $\kappa$ we have $E_{n,\kappa} = \omega_{n,\kappa}/R$ with, for example, $\omega_{n,-1} = 2.04, 5.40, \ldots$. Using the property

$$\chi^\mu_{-\kappa} = -\vec{\sigma}\cdot\hat{r}\chi^\mu_\kappa, \tag{8.79}$$

we can now write down the wave functions for the confined quark in this simple problem

$$\psi^{\mu}_{n,-1} = \frac{N_{n,-1}}{\sqrt{4\pi}} \begin{bmatrix} j_0(\omega_{n,-1}\frac{r}{R}) \\ i\vec{\sigma}\cdot\hat{r}j_1(\omega_{n,-1}\frac{r}{R}) \end{bmatrix} \chi^{\mu}_{-1}, \tag{8.80}$$

for $r < R$. (It is, of course, zero for $r > R$.) In the present case, the upper component is in an s-state and the spin-angle function, $\chi^{\mu}_{-1}$, is just a Pauli spinor. The normalization constant is given by:

$$N_{n,-1} = \frac{\omega^3_{n,-1}}{2R^3(\omega_{n,-1}-1)\sin^2(\omega_{n,-1})}. \tag{8.81}$$

**The first excited state**

The immediate success of this simple model, which was observed by Bogoliubov, is that the first excited state of the nucleon (with positive parity) has two quarks in the 1s-state and one in the 2s-state. The ratio of the mass of this excited state to that of the nucleon is therefore $(5.40 + 2 \times 2.04)/(3 \times 2.04) = 1.55$, which is in remarkable agreement with the ratio of the mass of the Roper resonance to that of the nucleon, namely 1.56. Of course, this close agreement cannot now be regarded as anything more than a happy coincidence as there are many subtleties ignored in this extremely simple model. Nevertheless, it is an indication that the model is not grossly incorrect and certainly served to encourage early work.

**The axial coupling constant**

Within the non-relativistic $SU(6)$ quark model we have already seen that the axial coupling constant of the nucleon is predicted to be $\frac{5}{3}$, in disagreement with the experimental value of 1.26. In a relativistic treatment the axial current involves $\bar{\psi}\gamma^{\mu}\gamma_5\frac{\lambda_a}{2}\psi$ and for the case of massless, confined quarks one can easily see that, in the Bogoliubov model, the non-relativistic result is just multiplied by a factor

$$\frac{\int_0^R drr^2[j_0^2 - \frac{1}{3}j_1^2]}{\int_0^R drr^2[j_0^2 + j_1^2]} = 1 - \frac{1}{3}\left(\frac{2\omega_{1,-1}-3}{\omega_{1,-1}-1}\right) = 0.65. \tag{8.82}$$

This reduces $g_A$ from $\frac{5}{3}$ to 1.09 – somewhat below the experimental value but much closer to it.

## 8.4.2 The MIT Bag Model

Although, from the modern point of view, the model of Bogoliubov was quite attractive, building in asymptotic freedom and confinement in a simple way, it was not very useful for spectroscopy. In particular, the radius of the spherical cavity was put in by hand, rather than being determined dynamically. Indeed, minimizing the energy of the hadron would lead us to favour $R \to \infty$. In addition, one would prefer to formulate a model of hadron structure covariantly, even if, in practice, one needed to make approximations to find a solution.

The MIT bag model satisfies all of these criteria, while retaining an elegant simplicity. In its most general form, for massless quarks, the model involves a three-dimensional volume $V$ – the "bag" – with surface $S$. If $\theta_V$ is one inside the volume and zero outside the Lagrangian density for the MIT bag is

$$\mathcal{L} = \left(i\bar{\psi}\gamma^\mu\partial_\mu\psi - B\right)\theta_V - \frac{1}{2}\bar{\psi}\psi\delta_S, \tag{8.83}$$

where $\delta_S$ is a $\delta$ function at the bag surface. The additional constant, $B$, is supposed to be the extra energy per unit volume required to create a region of space where the vacuum is perturbative – as opposed to the region outside the bag, where there are no free quarks and the vacuum is non-perturbative.

Requiring that the action associated with $\mathcal{L}$ be stationary with respect to variations in the field and the bag surface, $S$, leads to three equations. The first is the free Dirac equation for a massless quark inside the bag

$$i\gamma^\mu\partial_\mu\psi = 0. \tag{8.84}$$

The other two equations are boundary conditions for $\psi$ on $S$, a linear boundary condition (l.b.c.):

$$i\gamma^\mu\eta_\mu\psi(x) = \psi(x), \qquad x \in \mathrm{S}, \tag{8.85}$$

and a non-linear boundary condition (n.l.b.c.):

$$B = -\frac{1}{2}\eta_\mu\partial^\mu\left(\bar{\psi}\psi(x)\right), \qquad x \in \mathrm{S}, \tag{8.86}$$

where $\eta^\mu$ is a normal to the surface (with $\eta^2 = -1$).

The l.b.c. can be used to show that the component of the quark current, $j^\mu = \bar{\psi}\gamma^\mu\psi$, normal to the bag surface is zero:

$$\eta_\mu\bar{\psi}\gamma^\mu\psi(x) = 0, \qquad x \in \mathrm{S}. \tag{8.87}$$

The proof is easy. Taking the Hermitian conjugate of Eq. (8.85) and multiplying to the right by $\gamma^0$ we find

$$\bar{\psi} = -i\bar{\psi}\gamma\cdot\eta. \tag{8.88}$$

Hence, we may write

$$\begin{aligned} i\eta_\mu j^\mu &= \left(\bar{\psi}i\gamma\cdot\eta\right)\psi = -\bar{\psi}\psi \\ &= \bar{\psi}\left(i\gamma\cdot\eta\psi\right) = +\bar{\psi}\psi \\ &= 0. \end{aligned} \tag{8.89}$$

As there is no component of the current normal to the surface the quarks are confined to the interior of the bag. In the case of a static, spherical bag, $\eta^\mu = (0, \hat{r})$, and substituting

Eq. (8.80), which for arbitrary $\omega_{n,-1}$ is the solution of the free Dirac equation (in s-wave), into Eq. (8.85), the l.b.c., we find (at $r = R$):

$$j_0(\omega_{n,-1}) = j_1(\omega_{n,-1}). \tag{8.90}$$

This is exactly the same eigenvalue condition as we found in the model of Bogoliubov.

On the other hand, the n.l.b.c., Eq. (8.86), is new. It involves a relationship between the normal derivative of $\bar{\psi}\psi$ at the surface of the bag and $B$. This has a simple interpretation as a stability condition because, if we consider the energy-momentum tensor for the model

$$T^{\mu\nu} = \left(\frac{i}{2}\bar{\psi}(\overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu)\psi + Bg^{\mu\nu}\right)\theta_V, \tag{8.91}$$

we find, using the relation

$$\delta_S = -\eta \cdot \partial\theta_V, \tag{8.92}$$

that the n.l.b.c. ensures that $\partial_\mu T^{\mu\nu} = 0$ on the surface (it is guaranteed by the Dirac equation elsewhere). Thus $B$ is essential to ensure energy momentum conservation. From the energy momentum tensor we identify $-\frac{1}{2}\eta \cdot \partial(\bar{\psi}\psi)$ as the normal component of the pressure exerted by the gas of free Dirac particles inside the bag. The n.l.b.c. ensures that this is exactly balanced at the surface by the difference in the energy density inside and outside the bag – and hence that the system is stable.

Using the energy momentum tensor we can construct the energy of the bag as

$$P^0 = \int d^3x\, T^{00}. \tag{8.93}$$

Specializing again to the simplest case of a static spherical bag of radius $R$, with three quarks in the 1s-state, this yields

$$P^0 = 3\,\frac{\omega_{1-1}}{R} + \frac{4\pi}{3}\,BR^3, \tag{8.94}$$

and the n.l.b.c. requires

$$\frac{\partial P^0}{\partial R} = 0. \tag{8.95}$$

Thus the internal energy of the quarks determines the radius of the cavity if $B$ is taken to be a universal constant – independent of the hadron being considered. One can now discuss excited states with confidence.

**Static spherical cavity**

The MIT bag model, as formulated above, is a very complicated problem in quantum field theory. It has only been solved in one space and one time dimension [Jaf 81]. For practical purposes it has mainly been used in a much simpler form where the cavity is taken to be a static, spherical cavity of radius $R$, fixed by satisfying the n.l.b.c. on average over the surface.

(One can only satisfy the n.l.b.c. exactly over a spherical surface for states with quantum numbers $s_{\frac{1}{2}}$ and $p_{\frac{1}{2}}$.) As we have seen this is the model of Bogoliubov, but with the radius determined dynamically.

The MIT bag model is constructed to model QCD and one must therefore also include the glue required to ensure local gauge invariance. Of course, the bag itself is supposed to be generated by the non-perturbative interactions between gluons so there is some danger of double counting. For this reason, while the gluons are included in the Lagrangian density describing the bag model, they are typically treated only in low order perturbation theory. (An exception to the rule is the treatment of glueball or hybrid states, but those are not of direct concern to us in the context of the structure of the nucleon.) Taking the volume to be $V$ and the surface $S$ in this case, the Lagrangian density is

$$L = \left( i\bar{\psi}\gamma^\mu D_\mu \psi - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a - B \right) \theta_V - \frac{1}{2} \bar{\psi}\psi\delta_S. \tag{8.96}$$

In Eq. (8.96) $D^\mu = \partial^\mu - igA^\mu$ (with $A^\mu = \frac{\lambda_a}{2} A^\mu_a$), the usual covariant derivative in QCD.

If we choose $\eta^\mu$ to be the outward normal for a static spherical bag

$$\eta^\mu = (0, \hat{r}), \tag{8.97}$$

minimizing the action associated with Eq. (8.96) leads to the quark and gluon field equations:

$$\left[ i\gamma^\mu \partial_\mu + g\gamma^\mu A_\mu \right] \psi = 0, \qquad r \leq R \tag{8.98}$$

$$\partial_\nu G^{\mu\nu}_a = g \left[ \bar{\psi}\gamma^\mu \frac{\lambda_a}{2} \psi + f_{abc} G^{\mu\nu}_b A_{c\nu} \right], \qquad r \leq R, \tag{8.99}$$

linear boundary conditions (l.b.c.) for the quarks and gluons:

$$-i\vec{\gamma} \cdot \hat{r}\psi = \psi, \qquad r = R, \tag{8.100}$$

$$\hat{r} \cdot \vec{E}_a = 0, \qquad \hat{r} \times \vec{B}_a = 0, \qquad r = R, \tag{8.101}$$

and finally a generalization of the n.l.b.c. to include the pressure of the confined gluons:

$$B = -\frac{1}{2} \frac{\partial}{\partial r} (\bar{\psi}\psi) - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a, \qquad r = R. \tag{8.102}$$

If we treat the effect of the gluons in low order perturbation theory, for the reasons explained earlier, Eqs. (8.98) and (8.100) are just the free Dirac equation and the linear boundary condition for which we obtained the s-wave solution above. The solutions with arbitrary angular momentum are (recall $\kappa = \pm(j + \frac{1}{2})$ )

$$\psi^\mu_{n,\kappa} = N_{n,\kappa} \begin{pmatrix} j_{-\kappa-1}(\omega_{n,\kappa} \frac{r}{R}) \\ i\,\vec{\sigma} \cdot \hat{r}\, j_{-\kappa}(\omega_{n,\kappa} \frac{r}{R}) \end{pmatrix} \chi^\mu_\kappa; \qquad \text{for} \quad \kappa < 0 \tag{8.103}$$

and

$$\psi^\mu_{n,\kappa} = N_{n,\kappa} \begin{pmatrix} i\,\vec{\sigma} \cdot \hat{r}\, j_\kappa(\omega_{n,\kappa} \frac{r}{R}) \\ j_{\kappa-1}(\omega_{n,\kappa} \frac{r}{R}) \end{pmatrix} \chi^\mu_\kappa, \qquad \text{for} \quad \kappa > 0. \tag{8.104}$$

In general $\chi^{\mu}_{\kappa}$ is given by:

$$\chi^{\mu}_{\kappa} = \sum_{m,\mu} C^{m\nu\mu}_{l\frac{1}{2}j}\, Y^{m}_{l}\, \chi^{\nu}_{\frac{1}{2}}. \tag{8.105}$$

As we have already seen the eigenvalues, $\omega_{n,\kappa}$, are determined by the l.b.c., Eq. (8.100), which implies

$$j_{-\kappa-1}(\omega_{n,\kappa}) = j_{-\kappa}(\omega_{n,\kappa}); \qquad j_{\kappa}(\omega_{n,\kappa}) = -j_{\kappa-1}(\omega_{n,\kappa}). \tag{8.106}$$

(Of course, the first and second cases in Eq. (8.106) correspond to $\kappa$ less than or greater than zero, respectively.) For example, for $ns_{1/2}$ one has $\omega_{n,-1} = 2.043,\ 5.396,\ 8.578, \ldots$ and for $np_{\frac{1}{2}}$, $\omega_{n,+1} = 3.812,\ 7.002, \ldots$, etc.

A final feature of the bag which needs some mention is that in practical applications it is common to introduce an additional correction to the bag energy of the form $-Z_0/R$. This was originally introduced by DeGrand et al.[DeG 75] as an approximation to the Casimir energy associated with gluon and $q\bar{q}$ fluctuations in an empty bag. In practice there does seem to be a need for such a term, with $Z_0 \sim 1.8$. It has the effect of raising the value of $B$ needed to fit the nucleon mass and hence making the nucleon bag radius somewhat smaller. However, its origin in the Casimir effect is rather unclear. We simply note that the center of mass correction, which will be discussed in more detail below, also tends to lower the total energy of the hadronic states in the bag by an amount proportional to $1/R$. From the practical point of view, the term $-Z_0/R$ may be viewed as including both the center of mass and Casimir corrections in a phenomenological way.

### Spectroscopy

The key additions to the model required in order that it can be applied to the lowest mass baryons and vector mesons are: (a) a non-zero mass for the strange quark, and (b) the spin-dependent energy shift arising from perturbative gluon exchange. Introducing a quark mass term into Eqs. (8.96) and (8.98) adds no significant complications to what we have already discussed. For example, for an s-state ($\kappa = -1$) we find

$$\psi_{n,-1} = N_{n,-1} \begin{pmatrix} \alpha_{+}\, j_0(\frac{xr}{R}) \\ \alpha_{-}\, i\,\vec{\sigma}\cdot\hat{r}\, j_1(\frac{xr}{R}) \end{pmatrix} \chi^{\mu}_{-1}, \tag{8.107}$$

where the quark energy is related to the spatial frequency, $x$, by

$$E = \frac{\Omega}{R}; \qquad \Omega = \sqrt{x^2 + (mR)^2}, \tag{8.108}$$

and $x$ is determined by the l.b.c.

$$\tan x = \frac{x}{1 - mR - \Omega}. \tag{8.109}$$

The coefficients $\alpha_{\pm}$ are $\sqrt{(E \pm m)/m}$ and the normalization constant is

$$N^2 = \frac{\Omega(\Omega - mR)}{R^3 j_0^2(x)\left[2\,\Omega(\Omega - 1) + mR\right]}. \tag{8.110}$$

By introducing $m_s \neq m_{u,d}$ one can fit the $SU(3)$ breaking in the hadron mass spectrum, just as in the non-relativistic quark model.

The treatment of gluon exchange is somewhat more idiosyncratic, as the total Coulomb energy is supposed to be zero because the physical hadrons are color singlets. Thus the only term retained in second order perturbation theory is the spin-dependent one gluon exchange between different quarks

$$\Delta E_g^M = -\frac{\alpha_s}{2} \sum_{i<j} \int d^3x\, \vec{B}_i^a \cdot \vec{B}_j^a. \tag{8.111}$$

Here $\vec{B}_i^a$ is the solution of the classical field equations for the gluon field strength inside the cavity, including the boundary conditions, generated by the color current of quark $i$. (Note that each $\vec{B}_i^a$ separately satisfies the l.b.c. $\hat{r} \times \vec{B}_i^a = 0$.) Equation (8.111) leads to

$$\Delta E_g^M = -3\,\frac{\alpha_s}{R} \sum_{i<j} (\vec{\sigma}_i \lambda_i^a) \cdot (\vec{\sigma}_j \lambda_j^a) M(m_i, m_j, R), \tag{8.112}$$

where $M$ can be obtained in closed form. Clearly Eq. (8.112) has exactly the same spin-flavor structure as the hyperfine interaction in the non-relativistic quark model and since $\alpha_s$ is treated as an adjustable parameter one can fit the $N - \Delta$ mass difference and the $\Sigma - \Lambda$ splitting – as was discussed in Section 8.2.6.

In summary, the energy of a bag, including all the effects we have discussed, is

$$E(R) = \sum_i \frac{\Omega_i}{R} + \frac{4\pi}{3} R^3 B + \Delta E_g^M - \frac{Z}{R} \tag{8.113}$$

and the radius is determined by the n.l.b.c., $\partial E(R)/\partial R = 0$. By adjusting $B, \alpha_s, Z$ and $m_s$ one can obtain a very good fit to the masses of the ground state baryon octet and decuplet as well as the vector mesons. $B$ is typically $(146\ \text{MeV})^{\frac{1}{4}}$, or roughly 58 MeV/fm$^3$, and the nucleon bag radius is about 1 fm. (The other parameters are $m_s = 279$ MeV and $Z_0 \sim 1.84$, while $\alpha_s \sim 2.2$ is even larger than in the non-relativistic quark model.)

One major difference between the non-relativistic oscillator model and the bag is that in the latter the spin and spatial wave functions are inextricably mixed. For the [56] representation of $SU(6)$, which contains the nucleon spin-$\frac{1}{2}$ octet and the delta spin-$\frac{3}{2}$ decuplet, this is not a complication as the wave function is a totally anti-symmetric product of flavor, color and spin wave functions:

$$|\Psi\rangle \sim SU(3)_{\text{flavor}} \times SU(3)_{\text{color}} \times SU(2)_{\text{"spin"}}. \tag{8.114}$$

Here $SU(2)_{\text{"spin"}}$ is the full Dirac wave function for the $1s_{1/2}$ state, but in any case the spatial wave function is totally symmetric. However, for excited states, for example, where one quark is in a $p_{\frac{1}{2}}$ state, the situation becomes much more complicated.

Whereas one can remove the center of mass motion exactly in the oscillator model, leaving two internal, spatial degrees of freedom, this is not possible in the bag. Equation (8.114) corresponds to three, independent spatial degrees of freedom in the fixed cavity. This is physically incorrect and a great deal of effort has gone into attempts to remove spurious center of mass motion leading to unphysical excitations. For example, whereas it is trivial to see that the [56] $1^-$ is purely a spurious excitation of the center of mass in the non-relativistic harmonic oscillator model, this is much less obvious in the bag model. In particular, one could imagine a genuine excitation in which the quarks oscillate with respect to the non-perturbative glue making up the bag. In order to address this one must go beyond the static bag approximation and allow at least small surface excitations of the bag. Finally, we note that, because the spin and orbital angular momentum are linked through the Dirac wave function, the natural coupling scheme for the bag is j-j coupling. On the other hand, the success of the non-relativistic harmonic oscillator model strongly suggests that L-S coupling is the better scheme within which to tackle the spectrum of excited states.

In summary, the discussion of excited states within the bag model is technically complicated because of the spurious center of mass motion and the fact that j-j coupling is most natural. The overall description of the baryon spectrum is certainly less impressive in the bag model than in the non-relativistic harmonic oscillator model, but it is not clear to what extent this is a consequence of the technical difficulties, rather than a failing of the model. Certainly the relativistic quark wave functions in the bag model have led to dramatic improvements in the predictions for some photoproduction amplitudes (e.g., the [70] $1^-$) in comparison with the non-relativistic models [He 83]. The new feature, which could be extremely important, is the possibility of genuinely new states, not present in the oscillator model, in which the non-perturbative glue and the quarks move relative to each other or in which the glue itself is excited. As an important example of the latter phenomenon, it has been suggested that the Roper resonance may involve a breathing mode ($0^+$) excitation of the bag itself [Gui 85]. These are all important topics for future research.

### Charge and current densities

The bag model Lagrangian density is clearly invariant under the phase transformations

$$\begin{aligned} \psi &\to \psi + i\varepsilon\psi, \\ \bar{\psi} &\to \bar{\psi} - i\varepsilon\bar{\psi}. \end{aligned} \tag{8.115}$$

and hence the vector current, $j^\mu = \bar{\psi}\gamma^\mu\psi\theta_V$ is conserved. Noting that the upper and lower components of the quark wave functions must be equal at the bag boundary and that the charge density is

$$\begin{aligned} j^0 &\equiv \rho = \bar{\psi}\gamma^0\psi \\ &\propto \left[ {j_0}^2\left(\frac{\omega r}{R}\right) + {j_1}^2\left(\frac{\omega r}{R}\right)\right], \end{aligned} \tag{8.116}$$

we can easily understand the forms shown in Fig. 8.10.

The calculation of the magnetic moment is a little more interesting because massless quarks have no Dirac moment. This is completely changed by the confinement as we now illustrate. Suppose that the bag is immersed in a constant magnetic field $\vec{B}$, with corresponding vector potential $\vec{A} = \frac{1}{2}\vec{B} \times \vec{r}$. Then the confined quark will have a magnetic moment given by

$$\vec{\mu} \cdot \vec{B} = \int d^3r\, \vec{j} \cdot \vec{A},$$

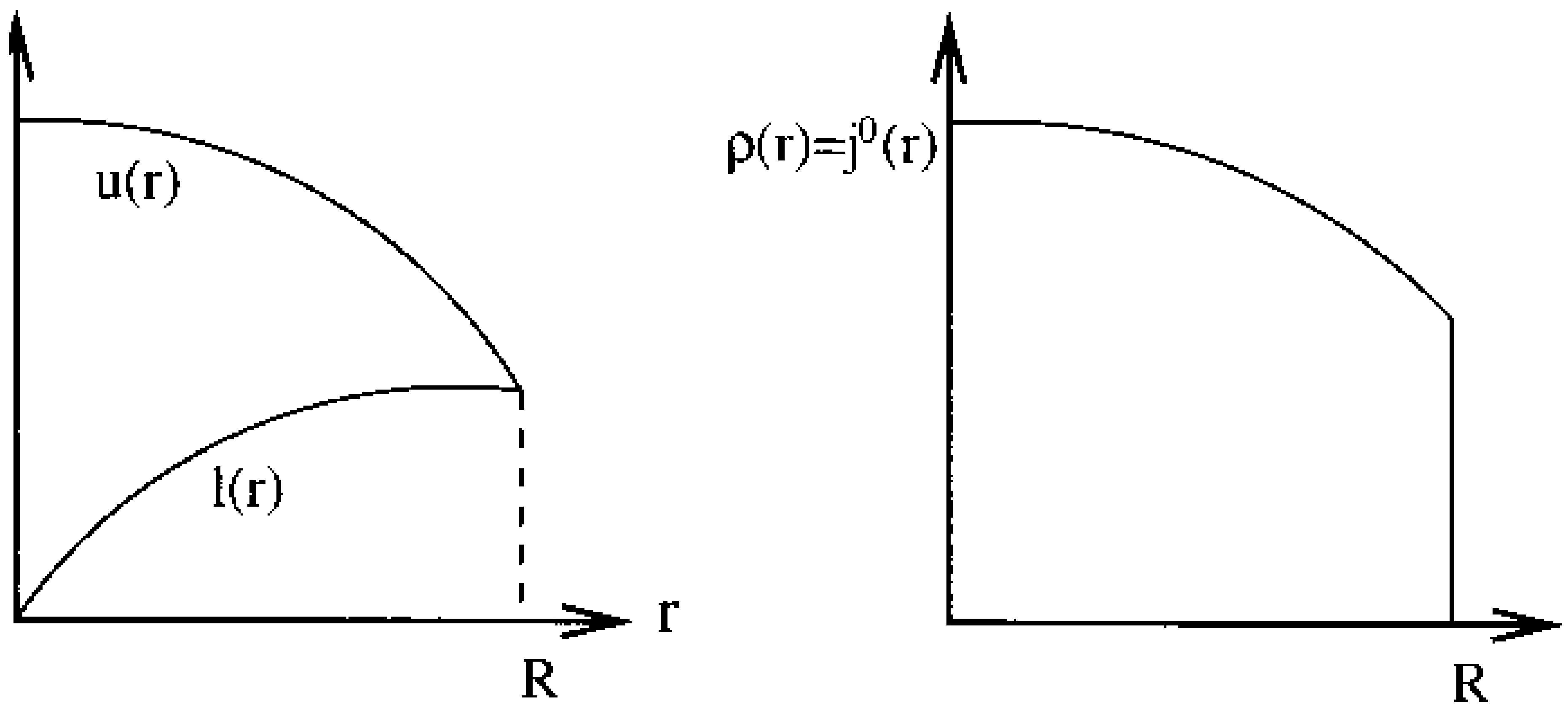


**Figure 8.10:** Illustration of the bag boundary condition that the upper and lower components of the Dirac wavefunction (u(r) and l(r), respectively) should be equal at $r = R$, together with the corresponding charge distribution.

where the integral is taken over the bag volume. Substituting for $\vec{j}$ the spatial piece of the conserved vector current this gives

$$\begin{aligned}\vec{\mu} \cdot \vec{B} &= \frac{e}{2} \int d^3r \, (\psi^\dagger \vec{\alpha} \psi) \cdot \left(\vec{B} \times \vec{r}\right) \\ &= \frac{e}{2} \int d^3r \, \vec{r} \times (\psi^\dagger \vec{\alpha} \psi) \cdot \vec{B},\end{aligned} \tag{8.118}$$

and hence

$$\begin{aligned}\vec{\mu} &= \frac{e}{2} \int d^3r \, \vec{r} \times \psi^\dagger \vec{\alpha} \psi \\ &= \mu_{\text{conf}} \vec{\sigma},\end{aligned} \tag{8.119}$$

where

$$\mu_{\text{conf}} = \frac{eR}{2\Omega} \cdot \left\{ \frac{4\Omega - 3}{6(\Omega - 1)} \right\}. \tag{8.120}$$

Thus the energy of the confined quark, $\Omega/R$, acts something like a constituent quark mass. We note that the famous relationship $\frac{\mu_p}{\mu_n} = -\frac{3}{2}$ is preserved here too.