From BEC to BCS

Molecular BECs and Fermionic Condensates of Cooper Pairs

Preseminar Extreme Matter Institute EMMI

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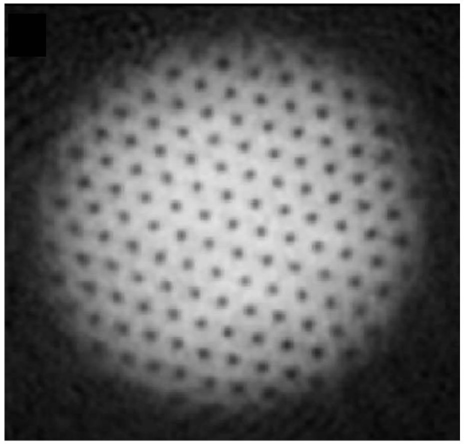
Motivation

Superfluidity, the frictionless flow, is still not fully understood. Although it has now been studied for almost 100 years.

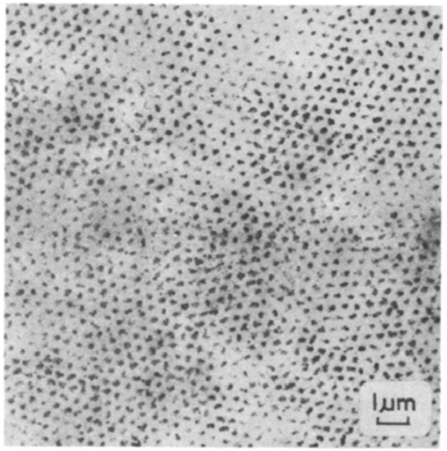
So far there are mainly two different theories describing superfluidity:

BEC Theory
(for bosons)



vortex lattice in sodium BEC [1]

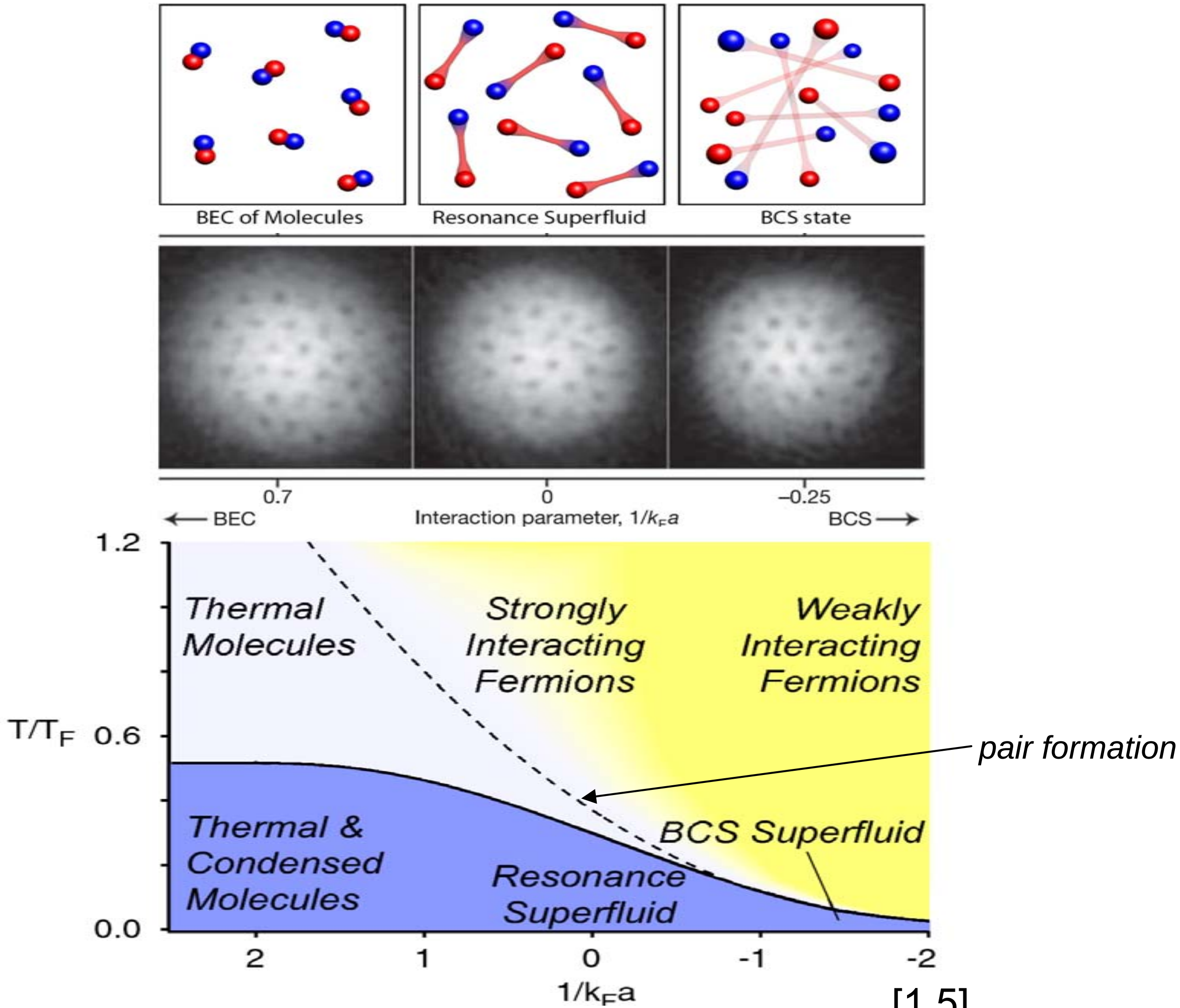
Bardeen Cooper Schrieffer (BCS) Theory
(for fermions)



vortex lattice in superconductor [2]

ultracold fermionic gases can establish a connection between these two descriptions

Phase Diagram



[1,5]

Outline

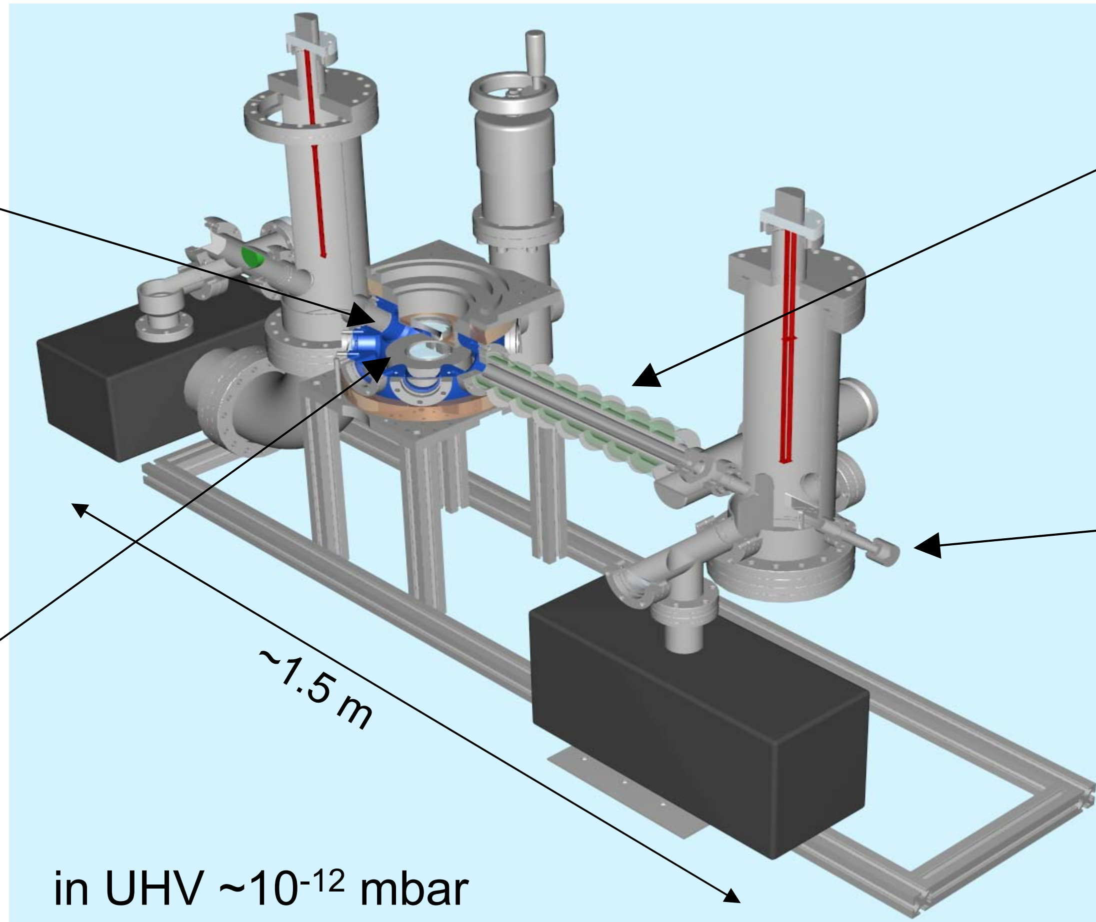
- Brief reminder (ultracold fermions, Feshbach resonances)
- Molecule formation and molecular BECs
- Superfluidity and vortices

- Bardeen Cooper Schrieffer (BCS) Theory
- BCS pairing
- Unitary regime
- Summary

Reminder: Experimental Setup

MOT
T~470μK
v~1 m/s
(two lowest hyper-fine states of ^{6}Li)

Zeeman Slower
v~50 m/s



fermionic ^{6}Li oven
(360 °C)
v~1000 m/s

optical dipole trap
evaporative cooling to
T~100 nK

Reminder: Experimental Setup

MOT
T~470µK
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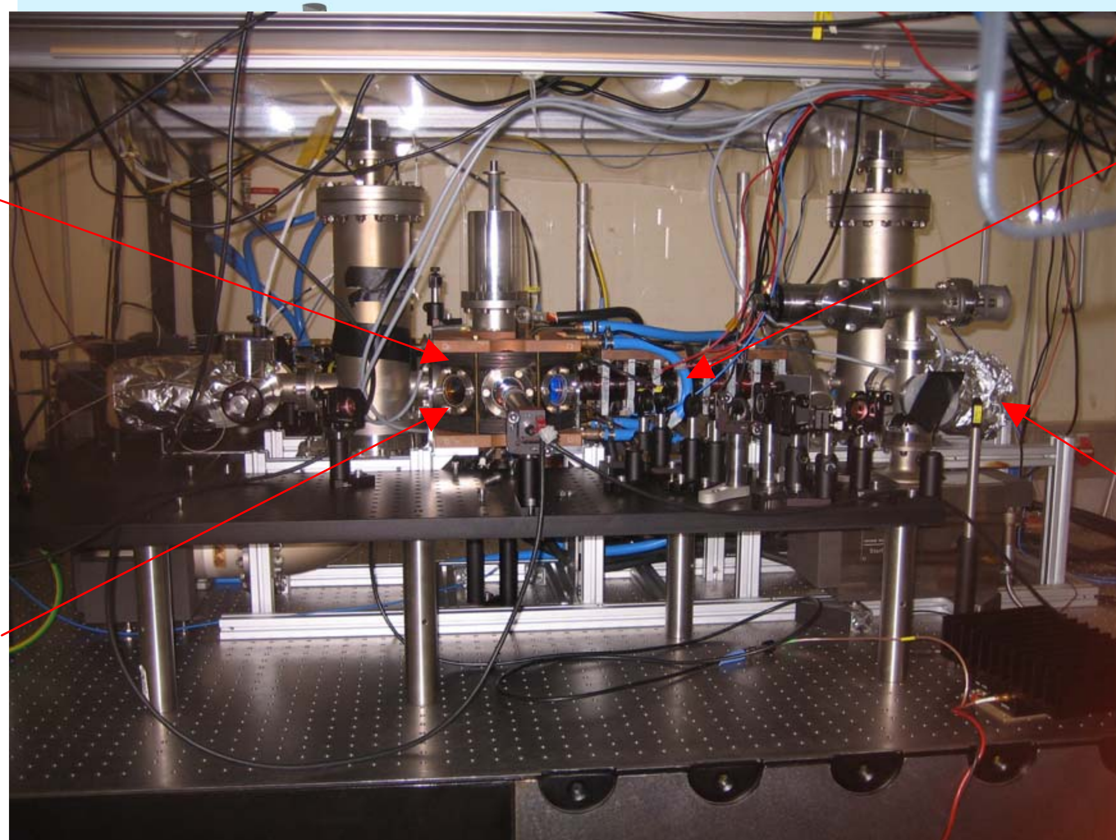


Zeeman Slower
v~50 m/s

fermionic ^{6}Li oven
(360 °C)
v~1000 m/s

in UHV ~10^{-12} mbar

Reminder: Ultracold Fermions

dilute gas: $n^{-1/3}$= d =interparticle spacing >> size of the interaction potential
→ description via effective δ-potential
for ultracold temperatures only s-wave scattering:
single parameter to describe interaction: the scattering length a

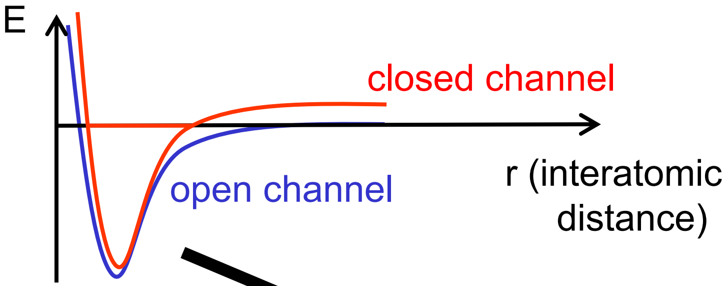
^{6}Li:
3 protons
3 neutron → fermion
3 electrons

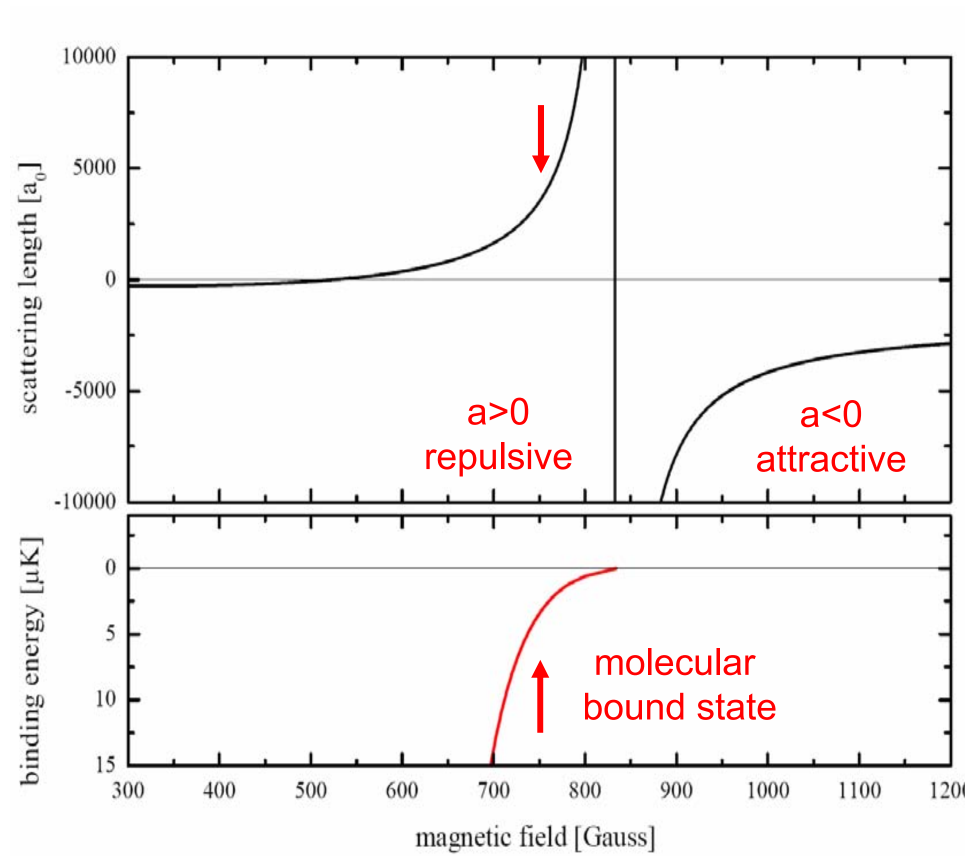
for ultracold temperatures **no s-wave scattering** (due to Pauli blocking) for identical fermions

→ use cooling agent (e.g. Na, ^{7}Li) or 2 species of same atom (two different hyperfine states)

strong interactions: a=scattering length >> interparticle spacing
possible to tune interactions via **Fesbach resonances**

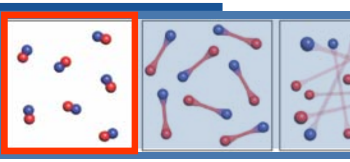
Reminder: Feshbach Resonances



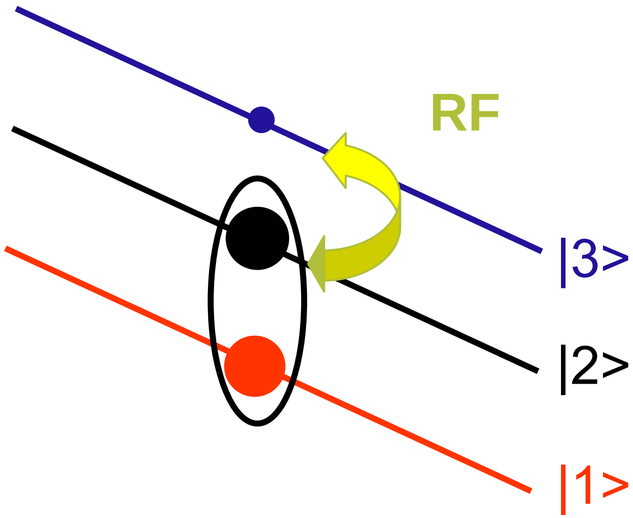


cool evaporatively at 750 G
($a \sim 3500\ a_0$, $E_b \sim k_B\ 3\ \mu K$)
→ if temperature of the sample
~ E_b/k_B formation of molecules
sets in

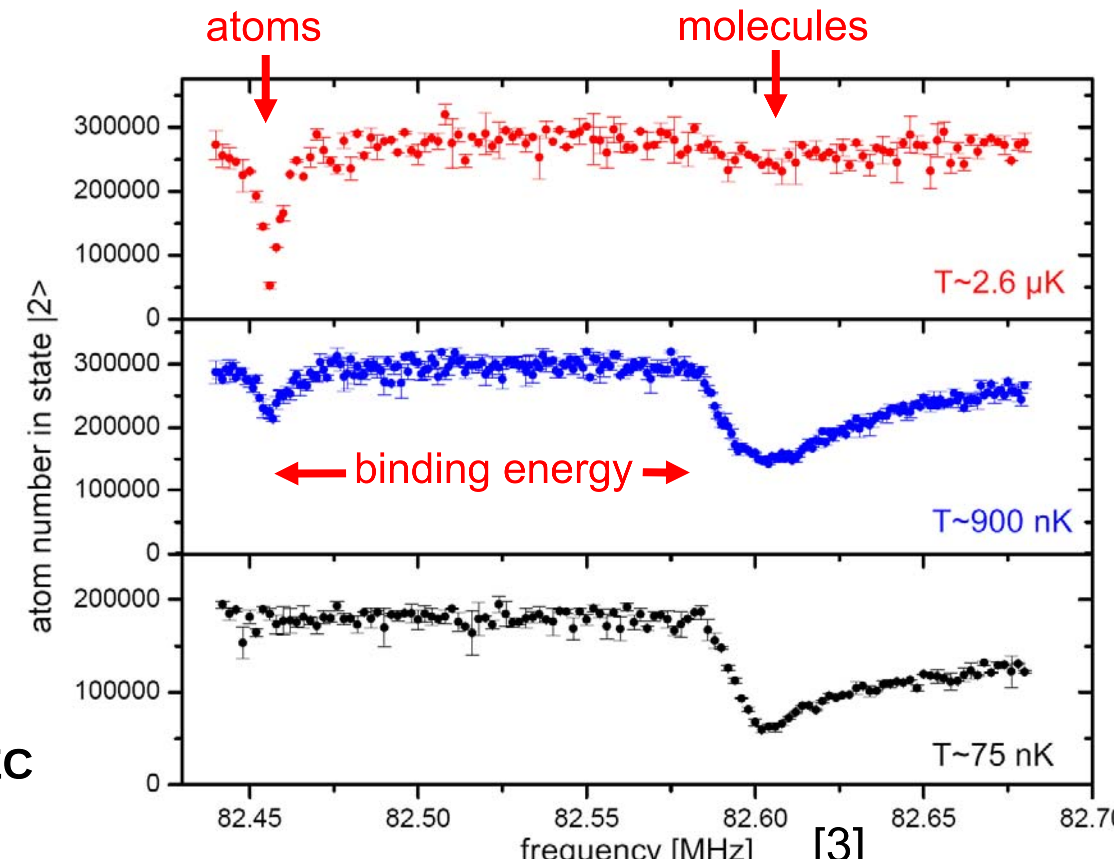
Molecule Formation



[5]



Our gas consists of atoms in states |1> and |2>.
Transition from |2> to |3> can be driven via RF fields.
Additional energy is needed to break up the pairs.

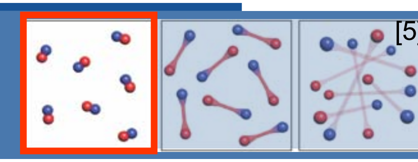


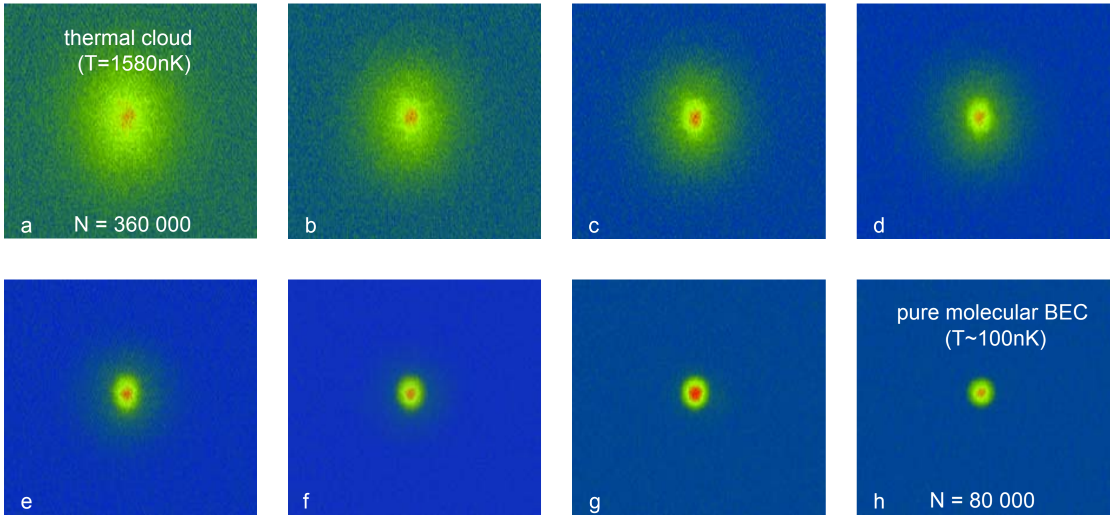
[3]

getting colder:
→ less atoms
→ more molecules

molecules are bosonic:
can condense into molecular BEC

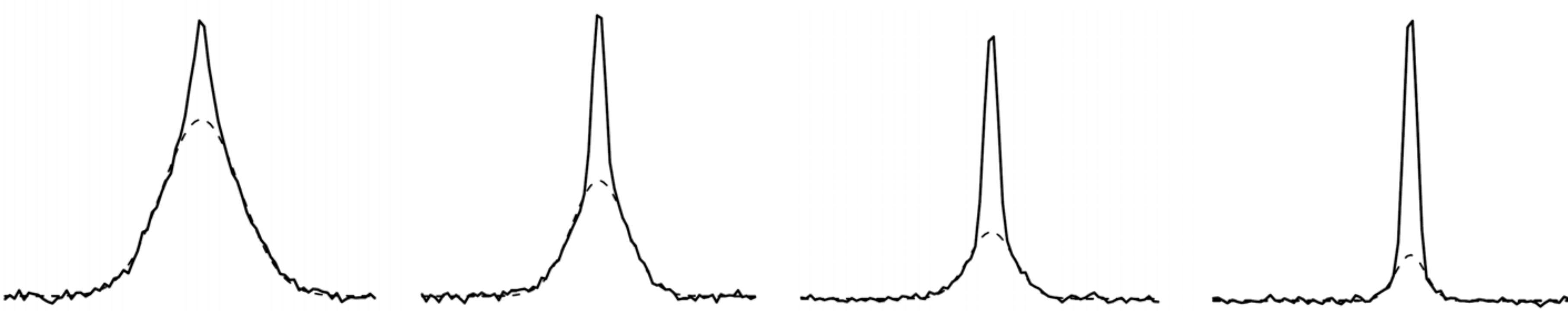
Molecular BEC (mBEC)





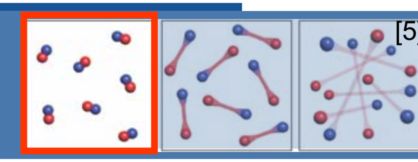
Images taken via absortion imaging after ~10ms time-of-flight.

The bimodal distribution is clearly visible:



gaussian wings + parabolic TF distribution [4]

Description of the mBEC


[5]

Gross-Pitaevskii equation (stationary):

$$\left(-\frac{\hbar^2}{2m}\Delta + V(r) + \frac{4\pi\hbar^2 a}{m} |\Psi(r)|^2 \right) \Psi(r) = \mu \Psi(r)$$

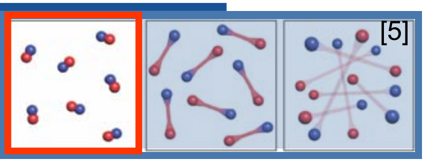
In the TF-limit (N a/a_{ho}>>1, means: interactions >> kinetic energy term) leads to inverted parabola for the mBEC in a harmonic potential:

$$|\Psi(r)|^2 = n(r) = \frac{m}{4\pi\hbar^2 a}(\mu - V(r))$$

Pairing mechanism:

Real two body bound state, which is associated with the Feshbach resonance, leads to weakly bound molecules .

Superfluidity



Landau criterion: If E_k is the dispersion relation of a medium, an object moving with velocity $v<v_c$ (where $v_c = \min_k \frac{E_k}{\hbar k}$) can not scatter from the medium [5,6].

BEC without interaction: $E_k \sim k^2$ thus $v_c = 0$

BEC with weak interaction: $v_c = c = \sqrt{\frac{\mu}{m}} > 0$ with c = speed of sound

→ for particles with $v<v_c$ frictionless superfluid

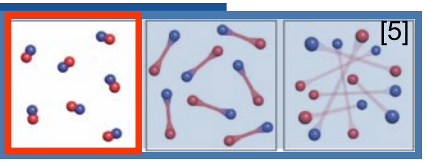
Modern definition: If there is a (off diagonal) long range order (→stiff phase over whole cloud) then the sample is superfluid.

Proofs of superfluidity:

1. **Collective excitation frequencies**: excite oscillation of the cloud, frequency changes when the sample is superfluid.
   Problem: collisional hydrodynamic gas has almost the same frequencies ($\sqrt{12/5}\,\omega = 1.549\,\omega$; $\sqrt{5/2}\,\omega = 1.581\,\omega$).

2. **Quantized vortices:** excite vortices in rotating superfluid, they build a vortex lattice.
   Problem: experimentally difficult to realize, but „smoking gun proof“ for superfluidity.

Vortices



Search for solutions of the GPE with cylindrical symmetry, for example (see [6]):

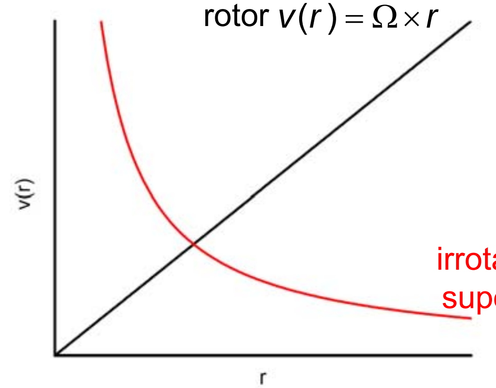
$$\Psi(r) = \sqrt{n(r)}\; e^{is\varphi} \text{ with coordinates } r, \varphi \text{ and } z \;\; (s \in \mathbb{Z})$$

is a eigenfunction of the GPE with the angular momentum $l_z = s\hbar$. Hence one Vortex carries the angular momentum: $L_z = N s \hbar$.

velocity distribution: $\vec{v_s}(\vec{r}) = \frac{\hbar}{m} \nabla (s\varphi)$

$\rightarrow$ tangential velocity: $v_s^{\,t} = \frac{\hbar}{m}\frac{s}{r}$

$\rightarrow \oint \vec{v_s}(\vec{r}) d\vec{r} = 2\pi s \frac{\hbar}{m}$ quantized in units of $\frac{\hbar}{m}$.

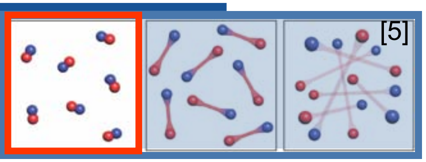


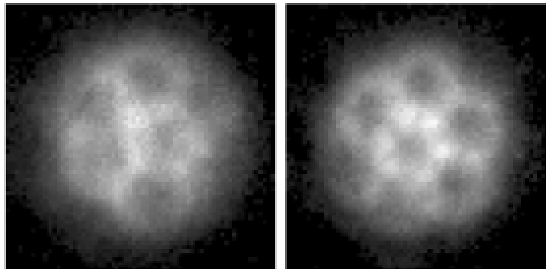
creation of vortices: In a system rotating with Ω the Hamiltonian is given by:

$$H = H_0 - \Omega L_z$$

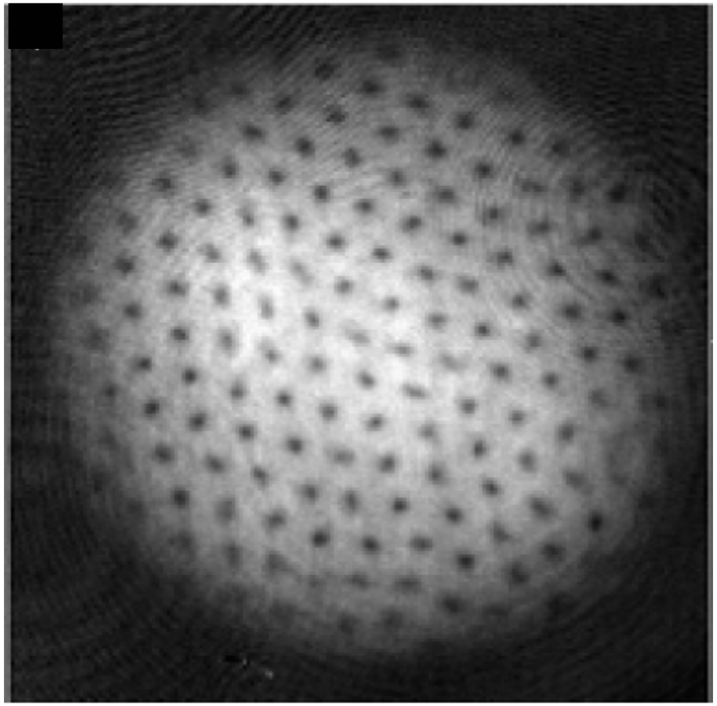
so the creation of a vortex minimizes the energy if $\Omega > \Omega_c = E_{vortex}/L_z$.
First observation in a BEC in 2000 at MIT and ENS.

Vortices



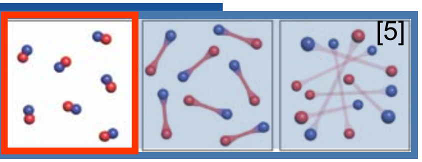


vortex lattice in ^{87}Rb BEC in magnetic trap observed in Paris (2001) [6]

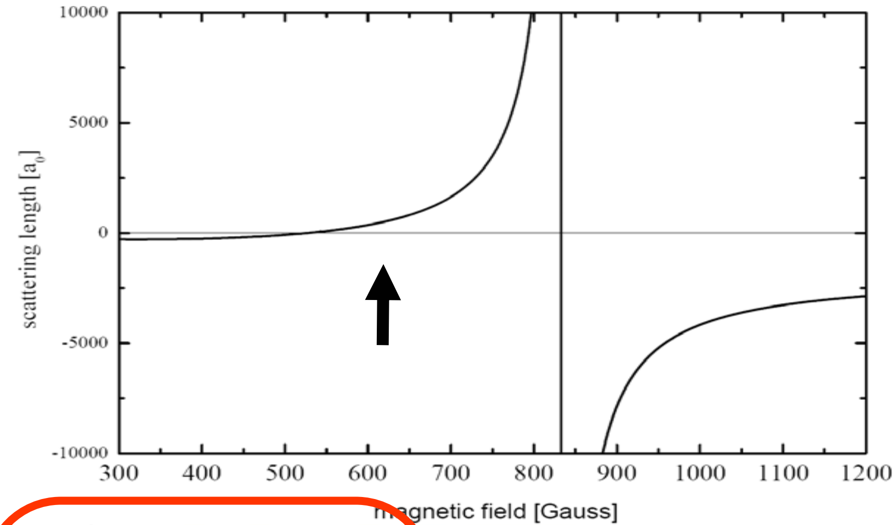


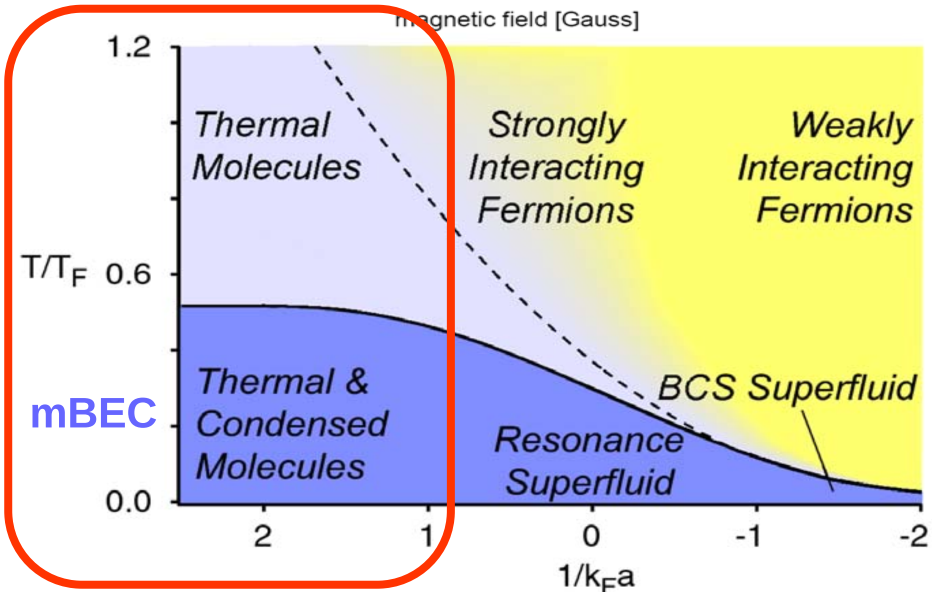
vortex lattice in Na BEC observed at MIT (2001) [7]

Phase Diagram



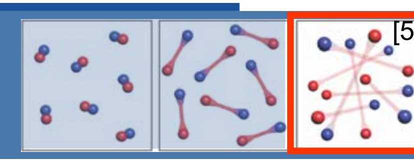
- Weak repulsive interactions
- Bosonic dimers
- Superfluidity as a two-body effect
- description via Gross-Pitaevskii Eq. (mean field description)

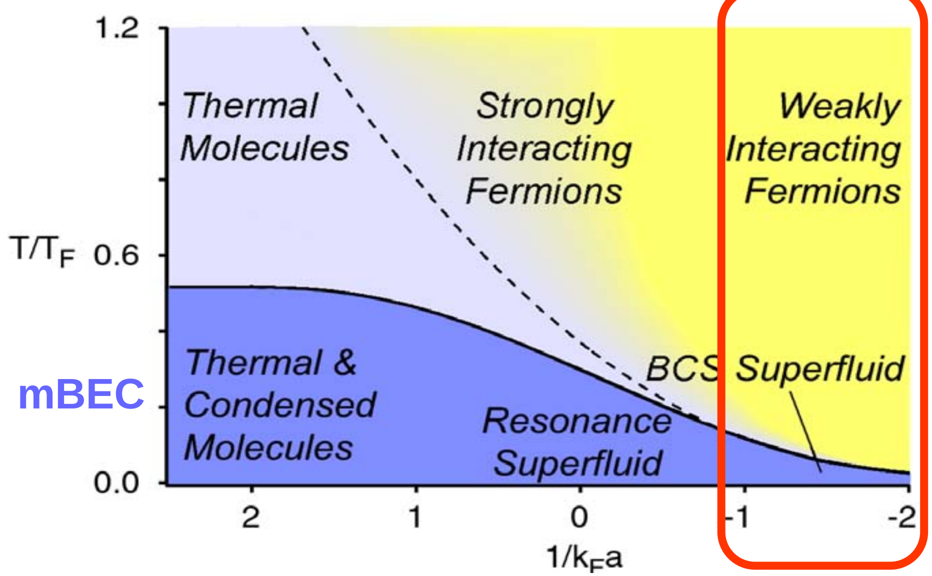




[1]

BCS Theory



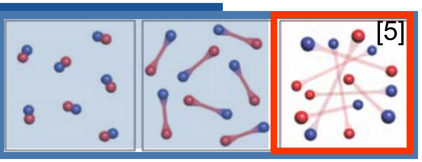


[1]

1957 **B**ardeen, **C**ooper, **S**chrieffer:
First microscopic theory of superconductivity
(superconductivity = superfluidity of electrons in a metal)

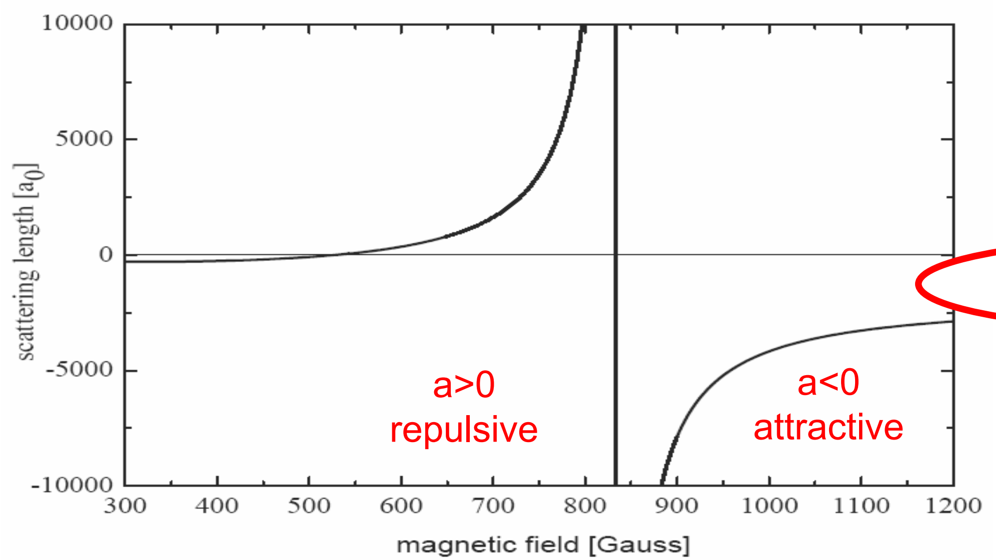
1. Weak attractive interaction
2. Presents of a filled Fermi sea
3. Coherent BCS state

BCS Theory


[5]

1. Weak attractive interaction

- In **superconducting** metals due to **electron-phonon interaction**
- In **ultracold fermionic gases** possible accessible with **Feshbach tuning**

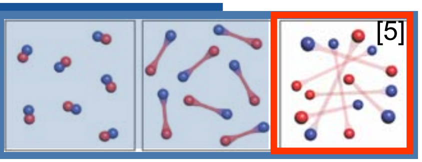


BCS theory applies

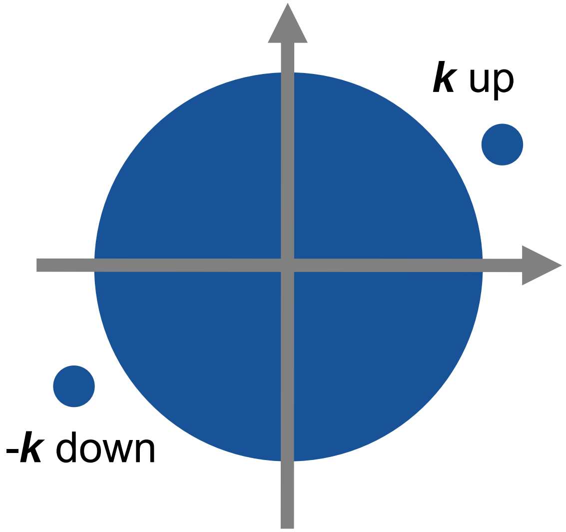
2. Presents of a filled Fermi sea

3. Coherent BCS state

BCS Theory



1. Weak attractive interaction
2. Presents of a filled Fermi sea

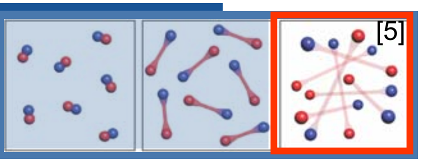


Two fermions outside a fully occupied Fermi sea form a **many-body induced bound state**!

"**Cooper problem**": A **filled Fermi sea** is **unstable** to even weak attractive interactions between the particles

3. Coherent BCS state

BCS Theory


[5]

1. Weak attractive interaction
2. Presents of a filled Fermi sea
3. Coherent BCS state

<u>Schrieffer's solution</u>:

- In the full (BCS) state **every fermion** at the Fermi surface is **part of a pair**:

  $$\hat{P}^{\dagger}_{\boldsymbol{k}} = \hat{\psi}^{\dagger}_{\boldsymbol{k}\uparrow}\hat{\psi}^{\dagger}_{-\boldsymbol{k}\downarrow}$$ (pair creation operator) $$\hat{P}^{\dagger}(\boldsymbol{R}) = \int \mathrm{d}^3 r\, \varphi(\boldsymbol{r})\hat{\psi}^{\dagger}_{\uparrow}(\boldsymbol{R}+\boldsymbol{r}/2)\hat{\psi}^{\dagger}_{\downarrow}(\boldsymbol{R}-\boldsymbol{r}/2)$$

- Cooper **pairs** are **not bosonic**!

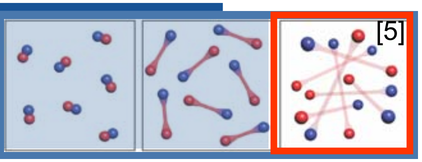
  $$\left[\hat{P}(\boldsymbol{R}), \hat{P}^{\dagger}(\boldsymbol{R}')\right] \neq \delta(\boldsymbol{R}-\boldsymbol{R}')$$ (bosonic commutation only for non-overlappping pairs)

- Full BCS state is **coherent state** of Cooper pairs:

  $$|\Phi_{\mathrm{BCS}}\rangle = \mathrm{const.}\exp\left(\sum_{\boldsymbol{k}} \alpha_{\boldsymbol{k}} \hat{P}^{\dagger}_{\boldsymbol{k}}\right)|0\rangle$$

[16]

Gap and Critical Temperature



- **Gap** in the excitation spectrum **associated with Cooper pairing**. At T=0:

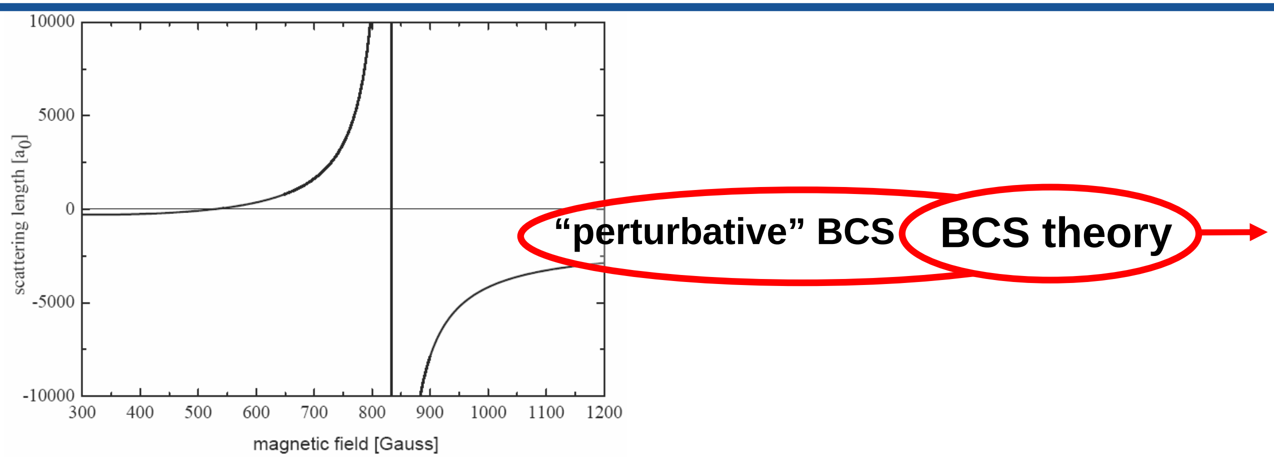
$$\Delta_0 \approx 0.49\, k_B\, T_F \, \exp\left(-\frac{\pi}{2k_F|a|}\right)$$

- **Critical temperature** T_C for a transition into the superfluid state **associated with occurence of coherence**

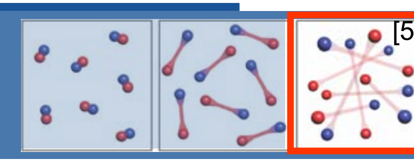
$$T_C \approx 0.28\, T_F \, \exp\left(-\frac{\pi}{2k_F|a|}\right)$$ [15]

Problem: Still today, T_C **out of reach for BCS-type dilute atomic gas** with $k_F|a|$<<1.

Solution: **Increase $k_F|a|$**. (But BCS theory no longer valid.)



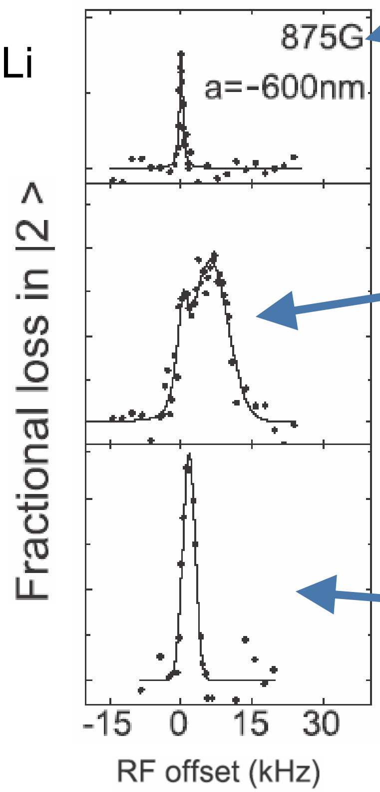
Pairing Gap



Pairing on the BCS side is a **many-body effect**

Observation of the pairing gap [11]

^{6}Li



"high" *T*

low *T*

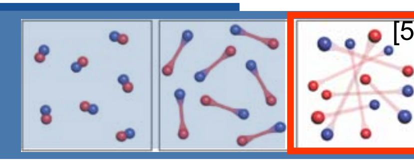
Absorption peak due to **free fermions**

Second peak shows **formation of fermionic atom pairs**

- **Only atom pairs** left.
- Drift of peak due to dependence of the gap on density and trap frequency.

|1>-|2> pairs; probe |2> → |3>

Pairing Gap & Vortices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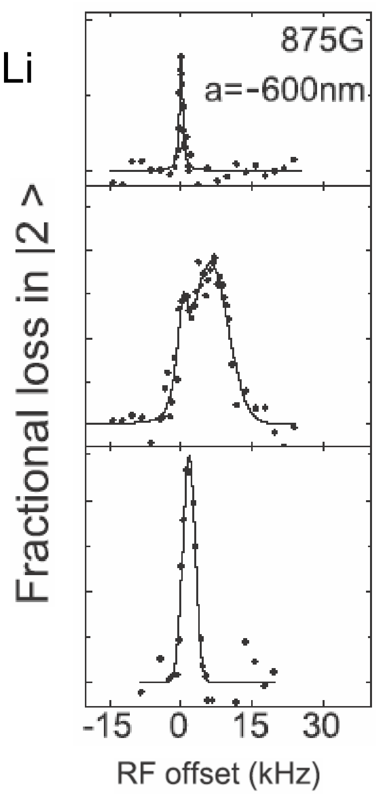


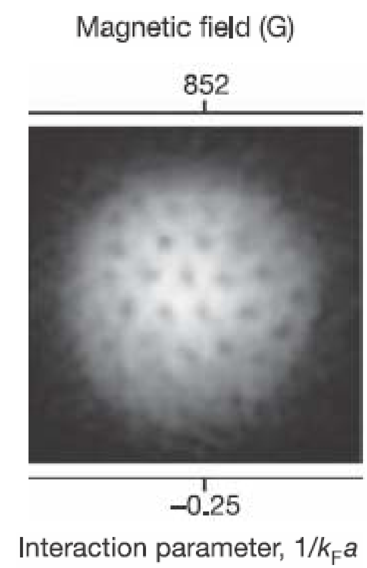
“high” *T*

low *T*

|1>-|2> pairs; probe |2> → |3>

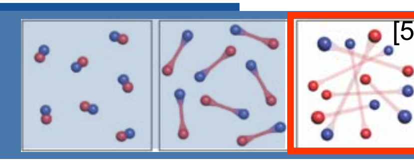
Superfluidity on the BCS side is due to the **condensation of pairs**

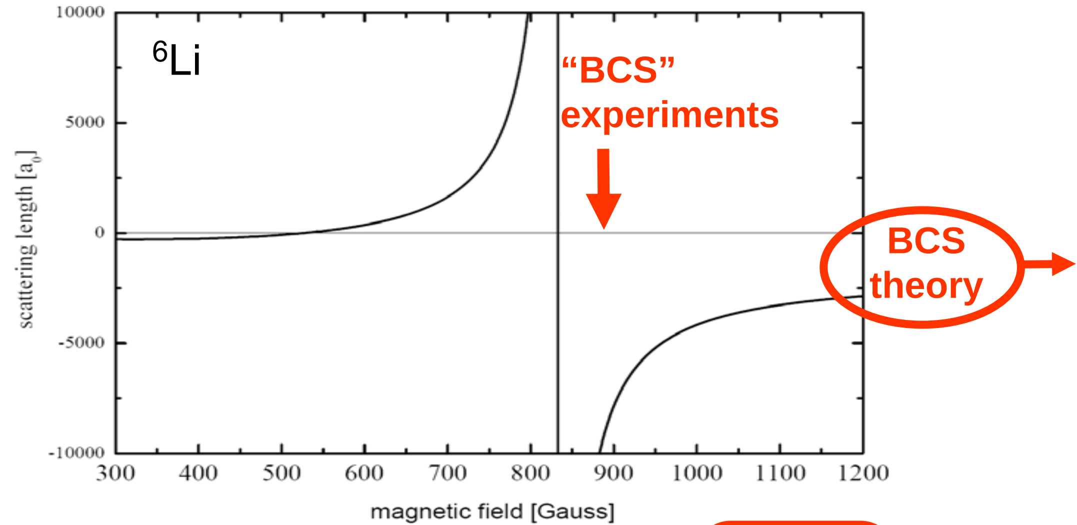
Observation of vortices [13]



^{6}Li

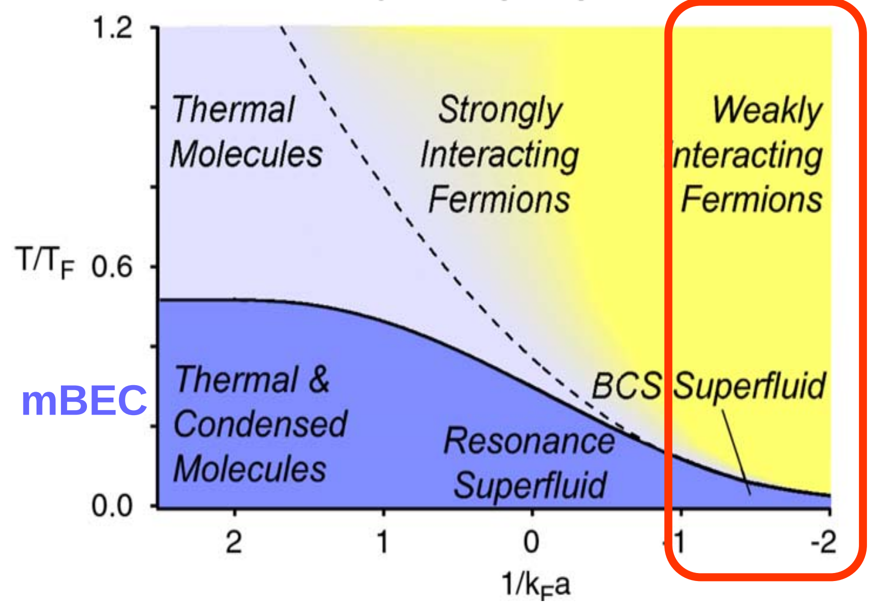
Phase Diagram





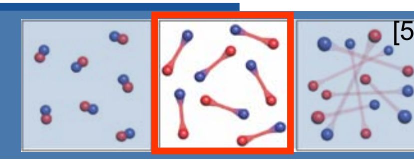
- Weak attractive interactions
- Non-bosonic cooper pairs
- Superfluidity

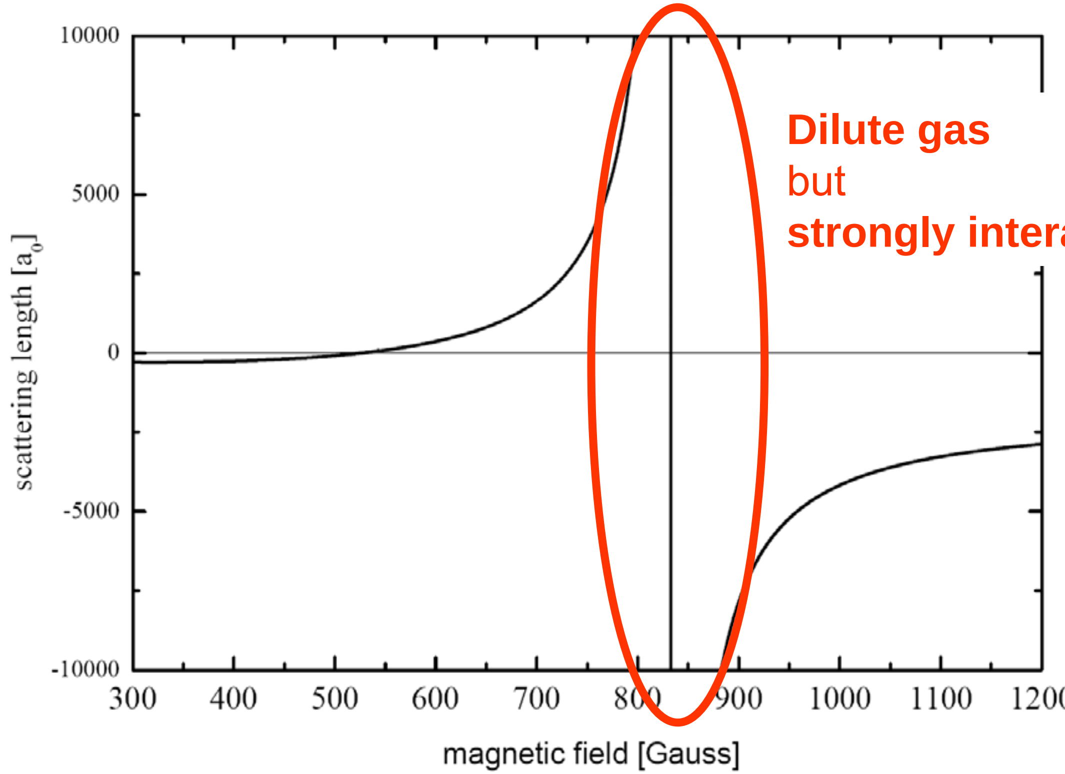
Description: („perturbative") BCS



[1]

Unitary Regime

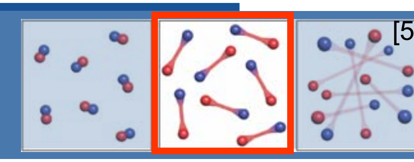




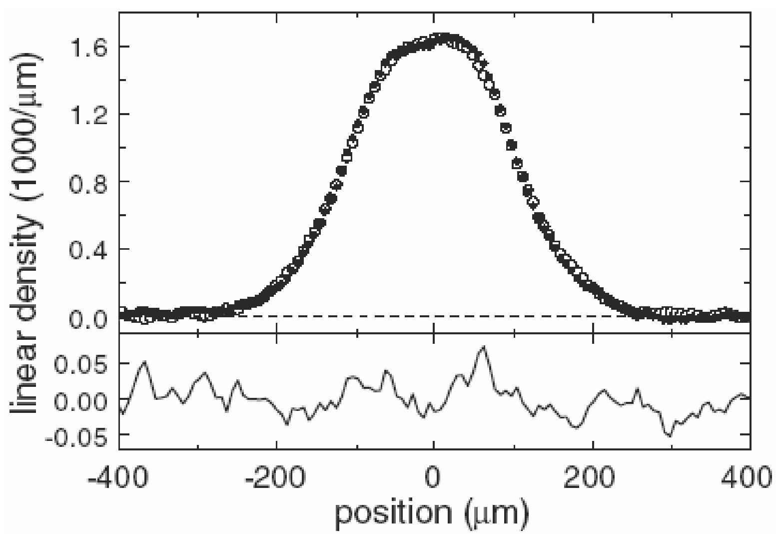
Dilute gas
but
strongly interacting.

What happens on the resonance? Stable trapping possible?
Crossover or phase transition from mBEC to BCS side?
Superfluidity present?

Crossover



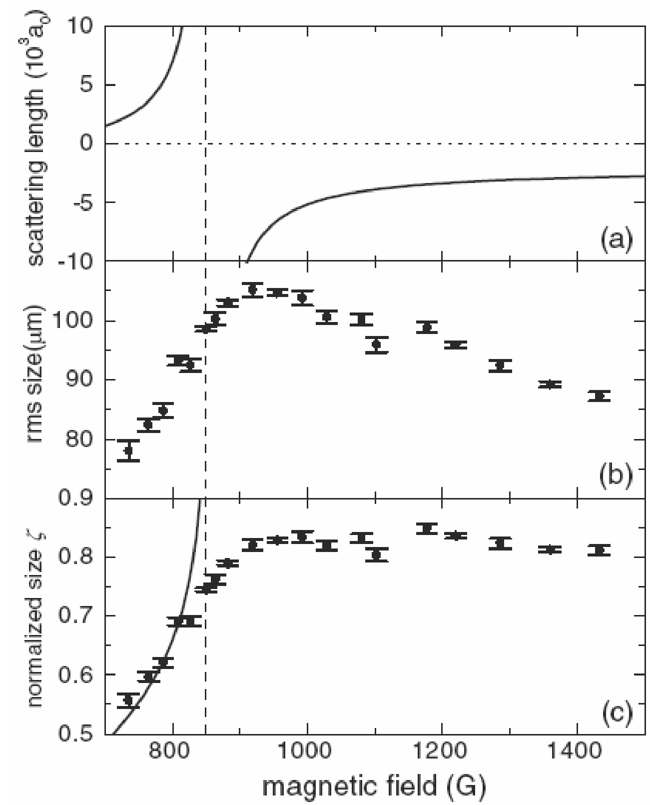
Smooth crossover from mBEC of ^{6}Li dimers to an atomic Fermi gas. [12] ^{6}Li



1. System prepared as molecular BEC.

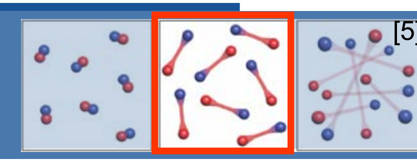
2. ● = after forth-and-back sweep over the resonance
   ○ = hold on mBEC side

3. No difference, *i.e.*, adiabatic & reversible → **isentropic**

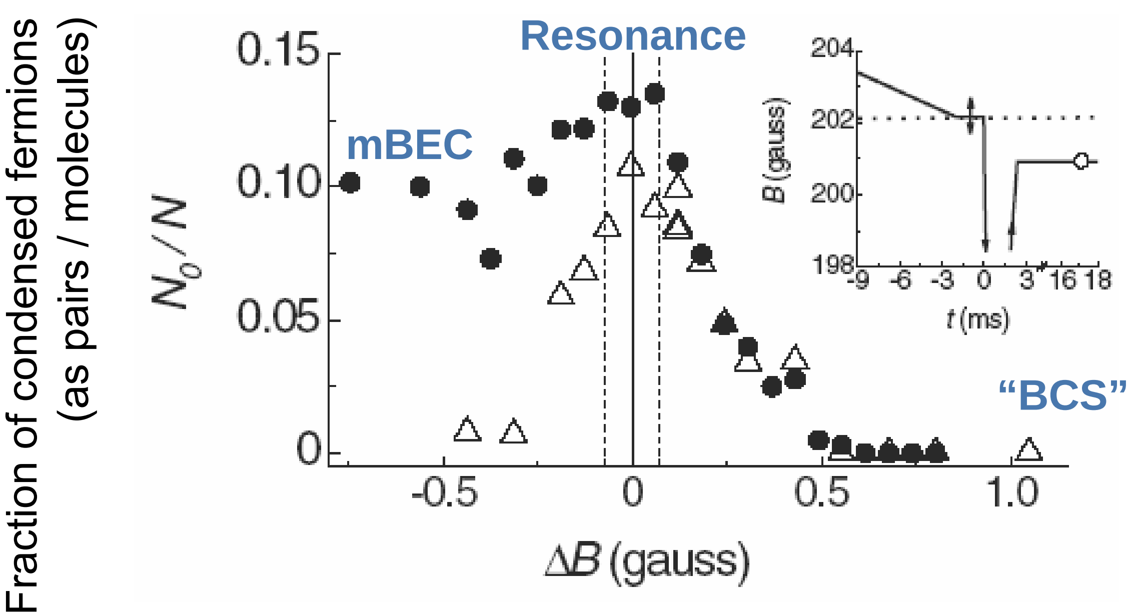


Smooth change in cloud size → **crossover**

Resonance Condensation



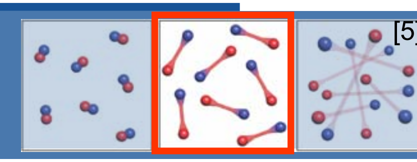
Observation of fermionic condensates in the unitary regime. [14] ^{40}K



● = short hold time (2ms)

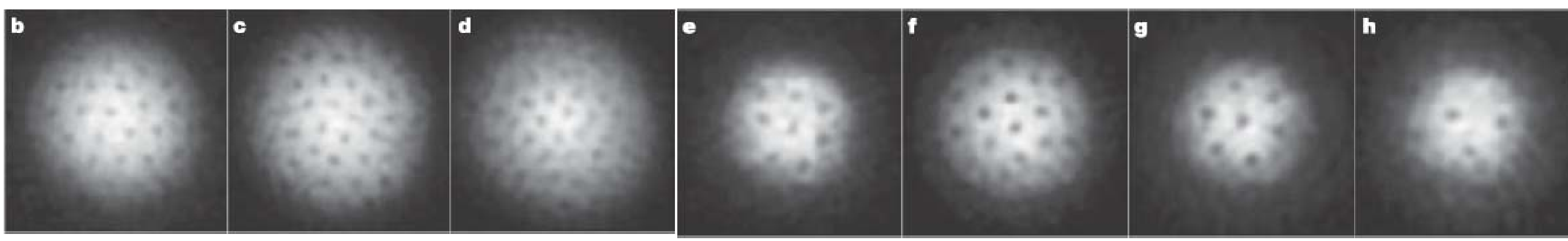
△ = long hold time (30ms)

Resonance Superfluidity



Observation of vortex lattices in a strongly interacting Fermi gas over the entire BEC-BCS crossover region.

First direct signature of superfluidity in these systems. [13] ^{6}Li



B field: 766 G | 792 G | 812 G | 833 G | 843 G | 853 G | 863 G

Molecular BEC **Resonance** **"BCS"**

Summary & Complete Phase Diagram

Summary:

In experiments with ultracold Fermi gases

- Crossover from mBEC to BCS
- Superfluidity over the entire crossover

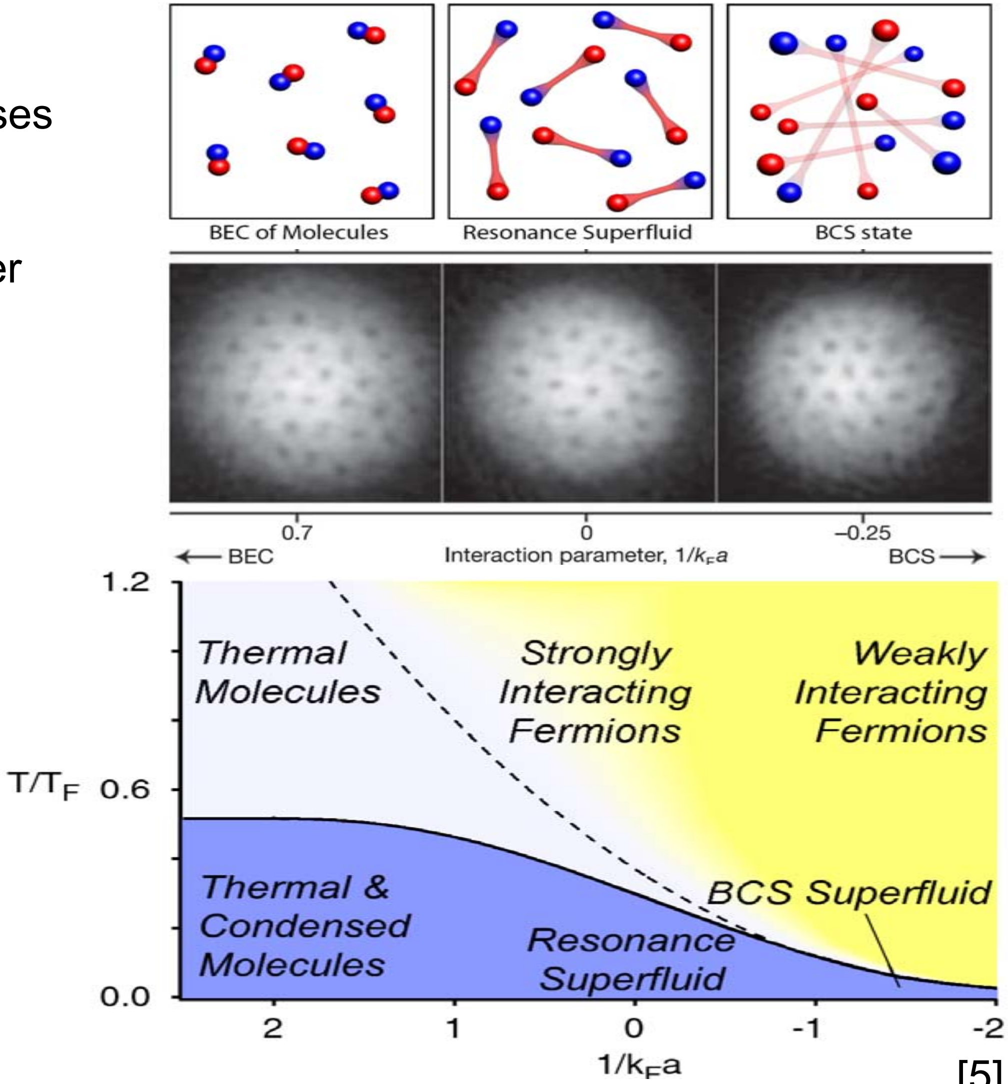
Outlook:

Ultracold Fermi gases enable to study

- Unbalanced spin-mixtures
- Collective dynamics
- ...

Interdisciplinary:

Ultracold fermions might help to better understand the Quark-Gluon Plasma.



[5]

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