

# Präzisionsexperimente zur Untersuchung der Gravitation

Ulrich Schmidt

## Messung der Gravitationskonstanten

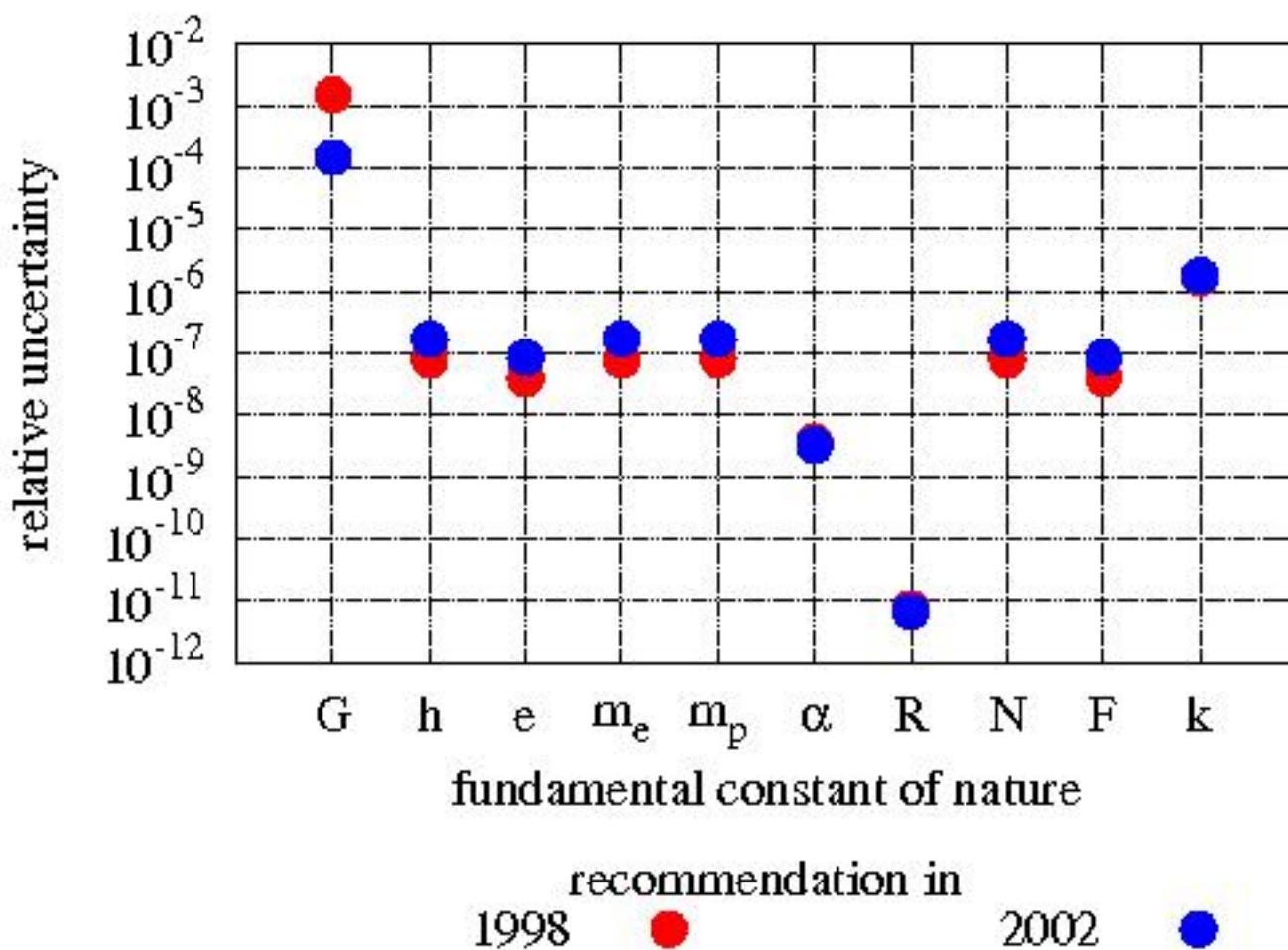
$$F_G = \gamma \frac{m_1 m_2}{r^2}$$

Newton's Gravitationsgesetz

$$m_{\text{schwere}} = m_{\text{träge}} \quad \iff \quad \begin{matrix} \text{Universalität} \\ \text{des freien Falls} \end{matrix}$$

Äquivalenzprinzip (Einstein)

# Messung der Gravitationskonstanten



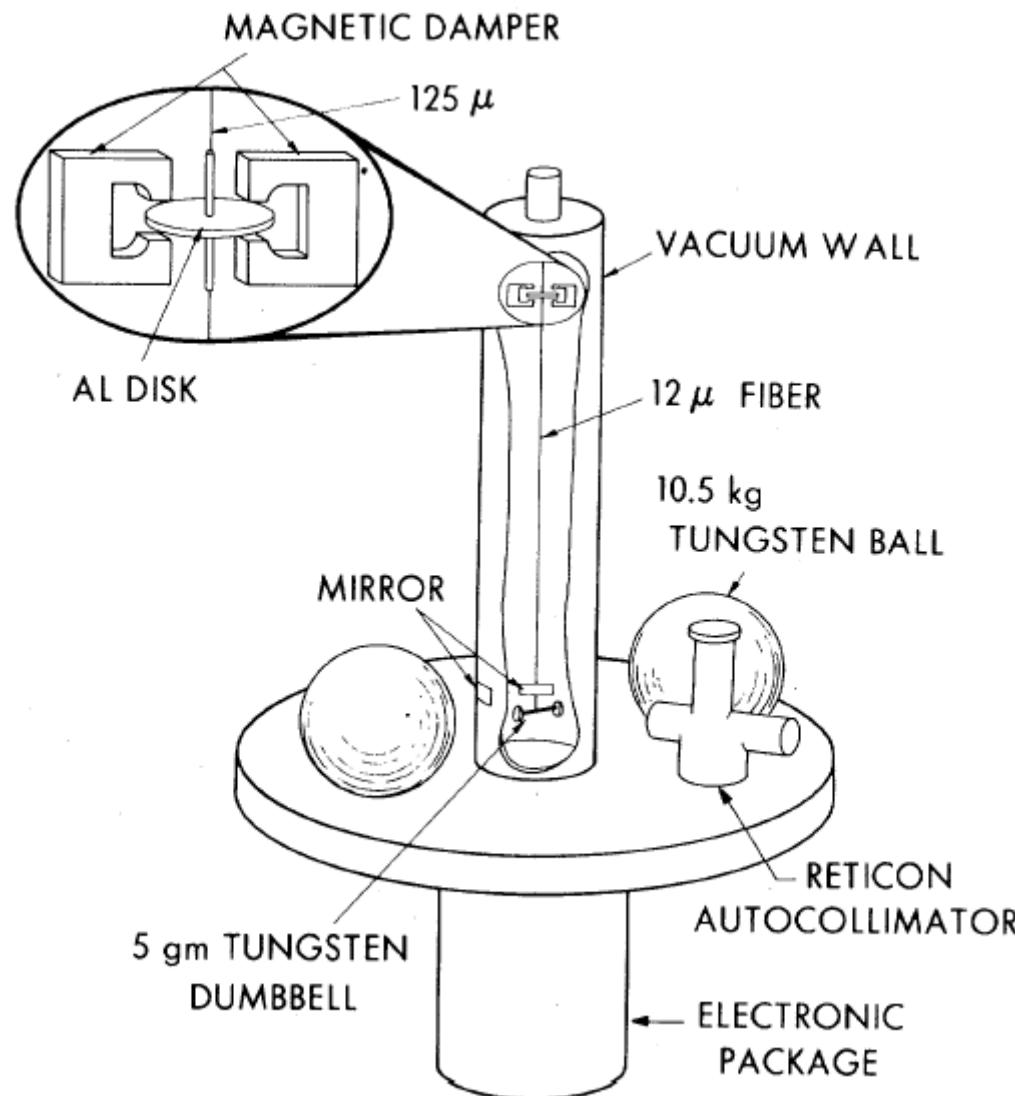
$$F_G = \gamma \frac{m_1 m_2}{r^2}$$

$$F_C = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

Für zwei Protonen

$$\left| \frac{F_G}{F_C} \right| \approx 10^{-38}$$





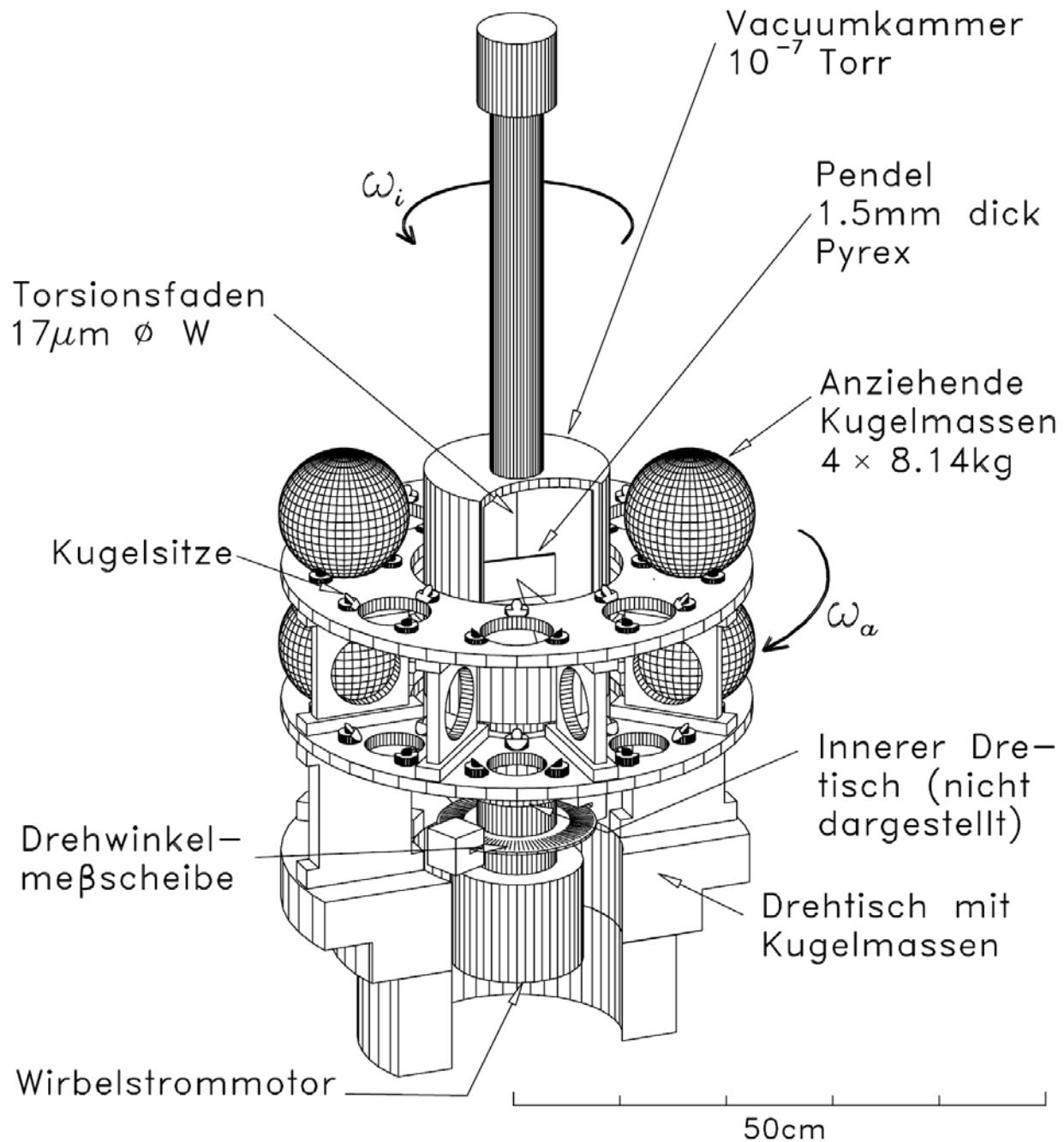
Experiment von G. G. Luther und W. R. Towler

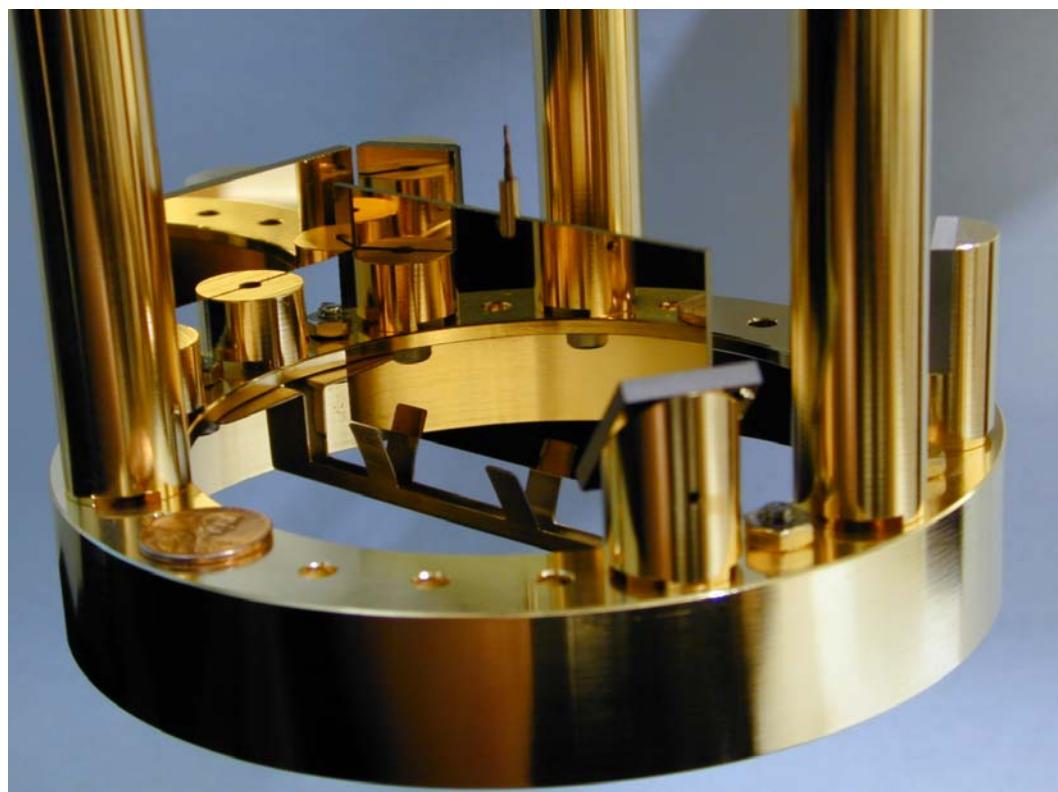
Quellmassen (Attraktor) rotieren gleichförmig um Pendel => periodisches Signal, Fourieranalyse erlaubt extreme Rauschunterdrückung

Limitierende Faktoren:

- Massenverteilung des Pendels (Dumbbell) nicht genau genug bekannt

- Nichtlinearität der Rückstellkraft (Kuroda-Effekt)





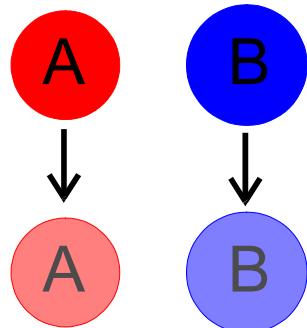
$$\gamma = (6.674215 \pm 0.000092) \cdot 10^{11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$$

Äquivalenzprinzip:  $m_{träge} = m_{schwere}$

Äquivalenzprinzip  $\Leftrightarrow$  Universalität des freien Falls

Schwaches Äquivalenzprinzip

Laborexperiment: freier Fall, punktförmige Masse



$$t = t_0$$

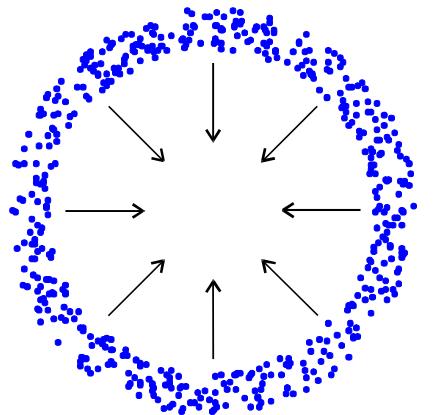
$$t = t_1$$

$$E = \frac{1}{2} m_t v^2 + m_s V$$

$$a = -\frac{m_s}{m_t} \nabla V$$

Eötvös faktor:  $\eta = \frac{a_A - a_B}{(a_A + a_B)/2} = \frac{\Delta a}{\bar{a}}$

Schwaches Äquivalenzprinzip:  $m_t = m_s \Leftrightarrow \eta_{SchwachesÄP} = 0$



Gravitationsenergie  
wird frei



Selbstenergie  $\triangleq$  Massendefekt

Selbstenergie für homogene Kugel:

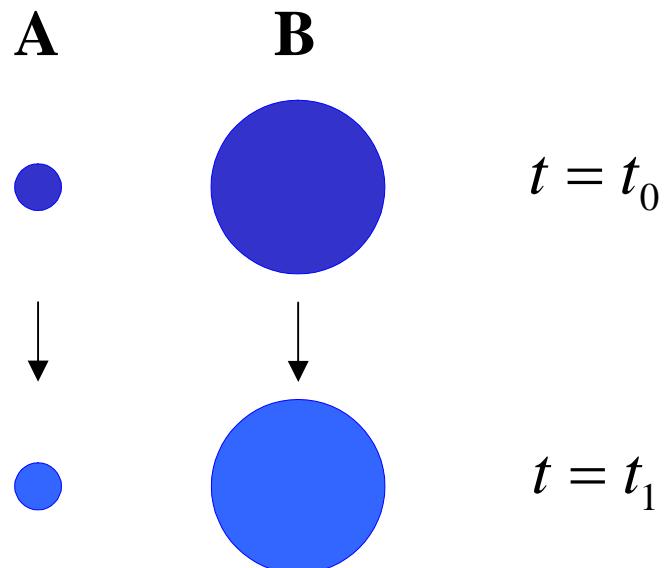
$$E_G = -\frac{3Gm^2}{5R} = -\frac{4\pi G \rho}{5} R^2 m$$

$$E_{GErde} = -4.6 \times 10^{-10} m_{Erde} c^2$$

$$E_{GMond} = -0.2 \times 10^{-10} m_{Mond} c^2$$

**Starkes Äquivalenzprinzip:**

Äquivalenzprinzip gilt auch für  
gravitative Bindungsenergie



Verletzung des Schwachen Äquivalenzprinzips  $\Rightarrow$  Fünfte Kraft

Beispiel: Quintessenz; Kosmonfeld koppelt unterschiedlich an Neutron und Proton, Vorhersage:  $\eta \approx 10^{-14}$

Reichweite der fünften Kraft endlich falls Austauschteilchen nicht masselos  
 $\Rightarrow$  Abstand zur Gravitationsquelle kann eine Rolle spielen.

Verletzung des Starken Äquivalenzprinzips  
 $\Leftrightarrow$  Allgemeine Relativitätstheorie nicht gültig

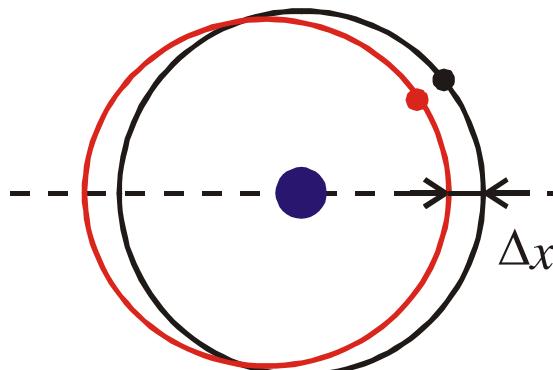
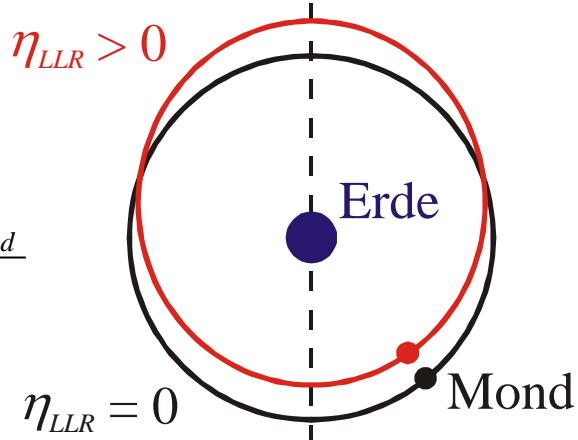
(Vermutung; C.M.Will: *The Confrontation between General Relativity and Experiment* )

# Test des Starken Äquivalenzprinzips:

## Freier Fall von Erde und Mond zur Sonne

Nordtvedteffekt

$$\eta_{LLR} = \frac{\Delta a_{LLR}}{a_s} = \frac{a_{Erde} - a_{Mond}}{a_s}$$



$$\eta_{LLR} = \frac{\Delta a_{LLR}}{a_s} = \frac{a_{Erde} - a_{Mond}}{a_s} = \frac{\Delta a_G - \Delta a_{Material}}{a_s}$$

$$\eta_{LLR_{\max}} = \frac{-E_{GEerde}}{m_{Erde}c^2} = -4.6 \cdot 10^{-10}$$

$$\Rightarrow \Delta x_{\max.} = 12 \text{ m}$$

Williams *et.al.* (2000) :

$$\eta_{LLR} = (1 \pm 2) \cdot 10^{-13}$$

Müller *et.al.* (1997) :

$$\eta_{LLR} = (3.6 \pm 4.0) \cdot 10^{-13}$$

$$\Rightarrow \Delta x \leq 8 \text{ mm}$$

## Schwaches Äquivalenzprinzip

Galileo Galilei (Pisa 1589) Fallgesetze (Genauigkeit ?)

Newton (Philosophiä naturalis Prinzipiä 1687)

$$\eta \leq 1 \cdot 10^{-3}$$

Schwebung zwischen zwei Pendeln gleicher Länge, aber unterschiedlicher Zusammensetzung

$$m_t l \ddot{\varphi} + m_s g \sin \varphi = 0 \quad \Rightarrow \quad T = 2\pi \sqrt{\frac{l}{g}} \sqrt{\frac{m_t}{m_s}}$$

Bessel (ca.1820 ?)  $\eta \leq 1.7 \cdot 10^{-5}$

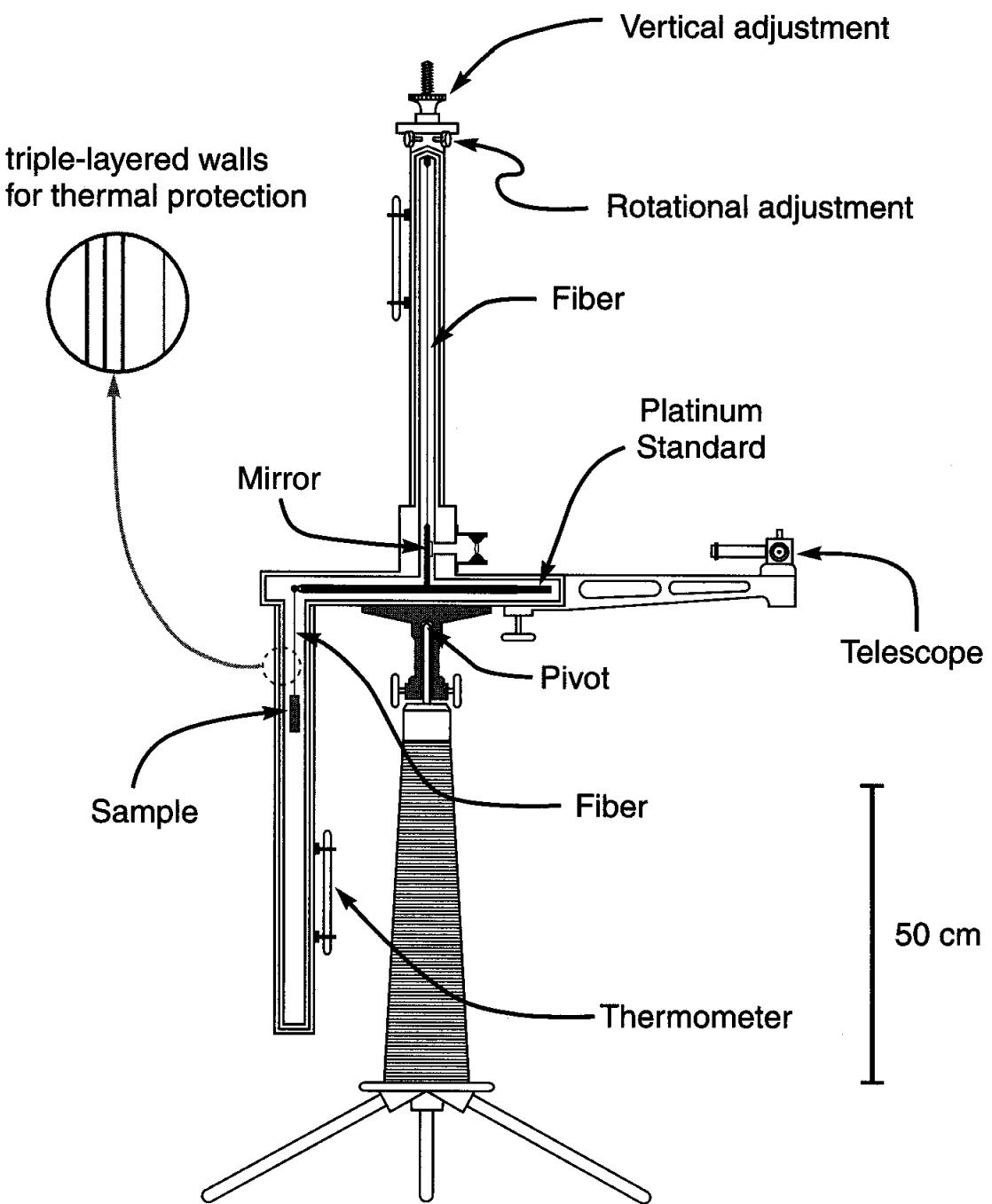


Figure 4.6: Single-arm torsion balance used by Eötvös, Pekár, and Fekete.

Eötvösh

verbesserte Drehwaage

$$(1889) \eta \leq 5 \cdot 10^{-8}$$

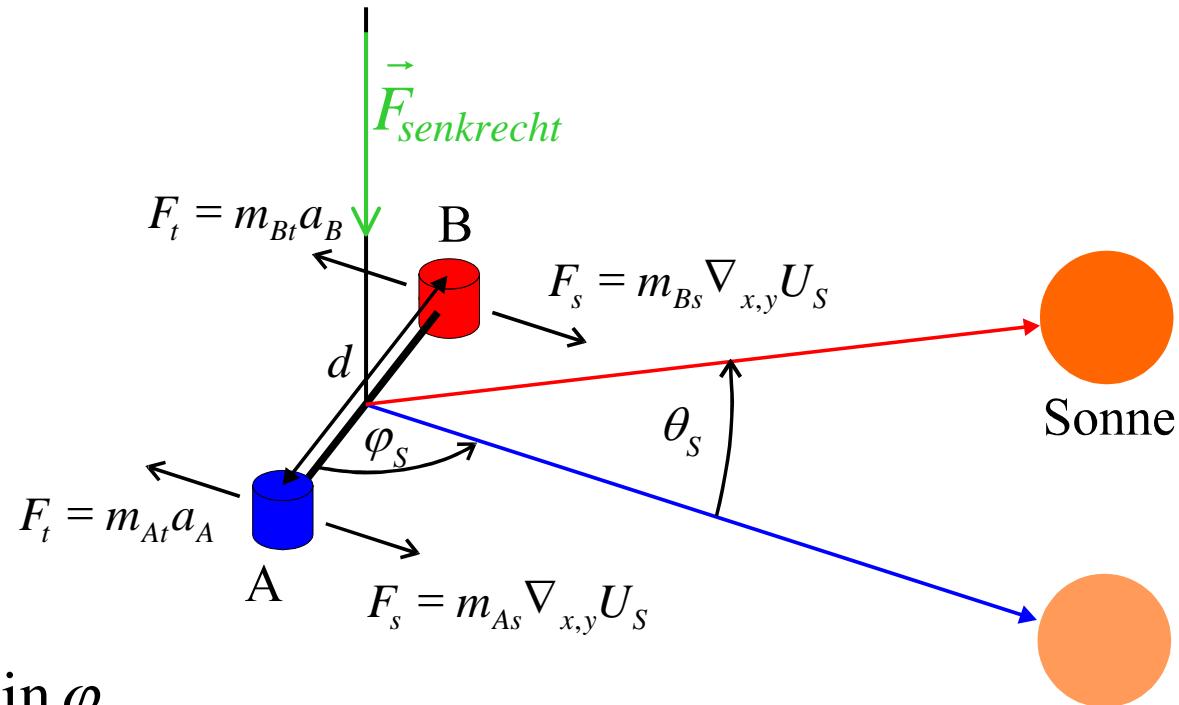
Material:

Platin, Kupfer, Wismut,  
Messing, Glas...

1922; Eötvös, Pekár  
und Fekete : um mehr  
als eine Größeordnung  
verbesserte Ergebnisse

Sonne als  
Gravitationsquelle

$$(a_S = 6.04 \cdot 10^{-4} g)$$



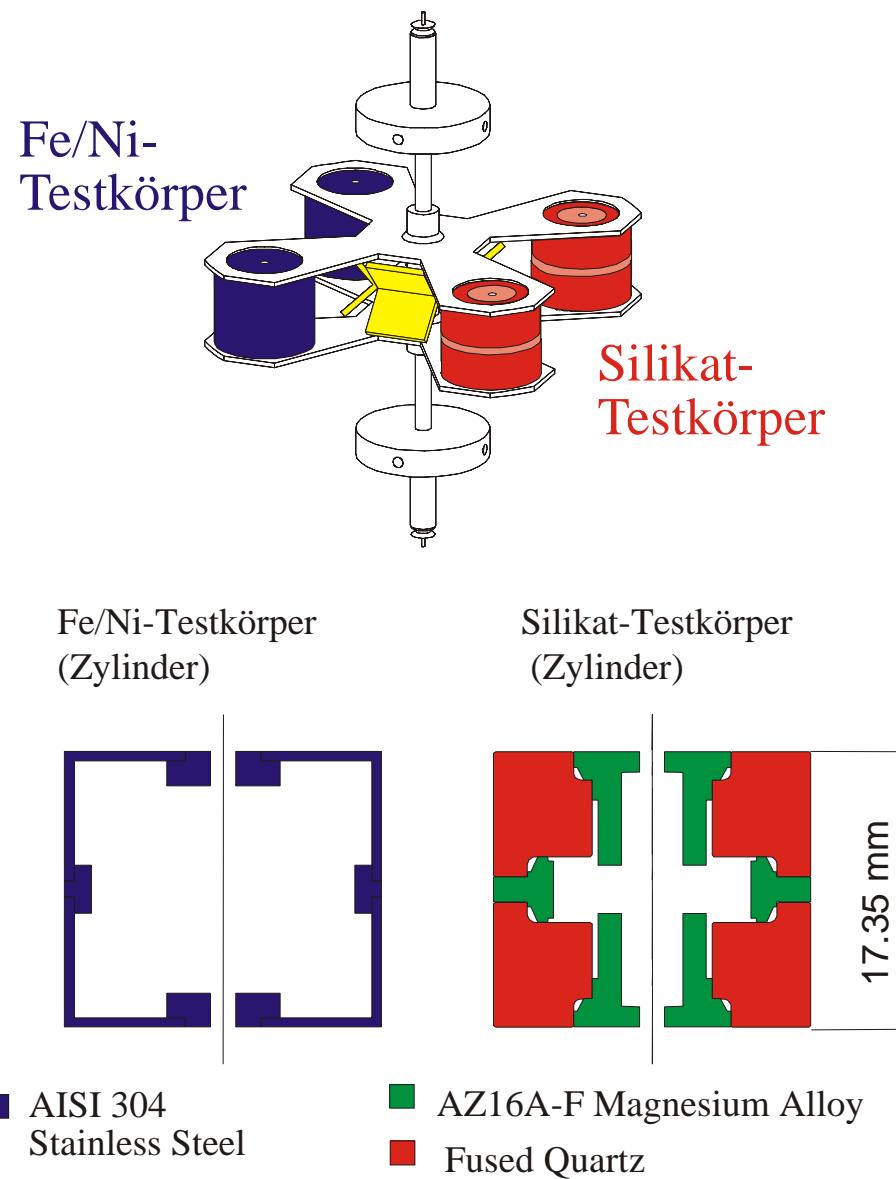
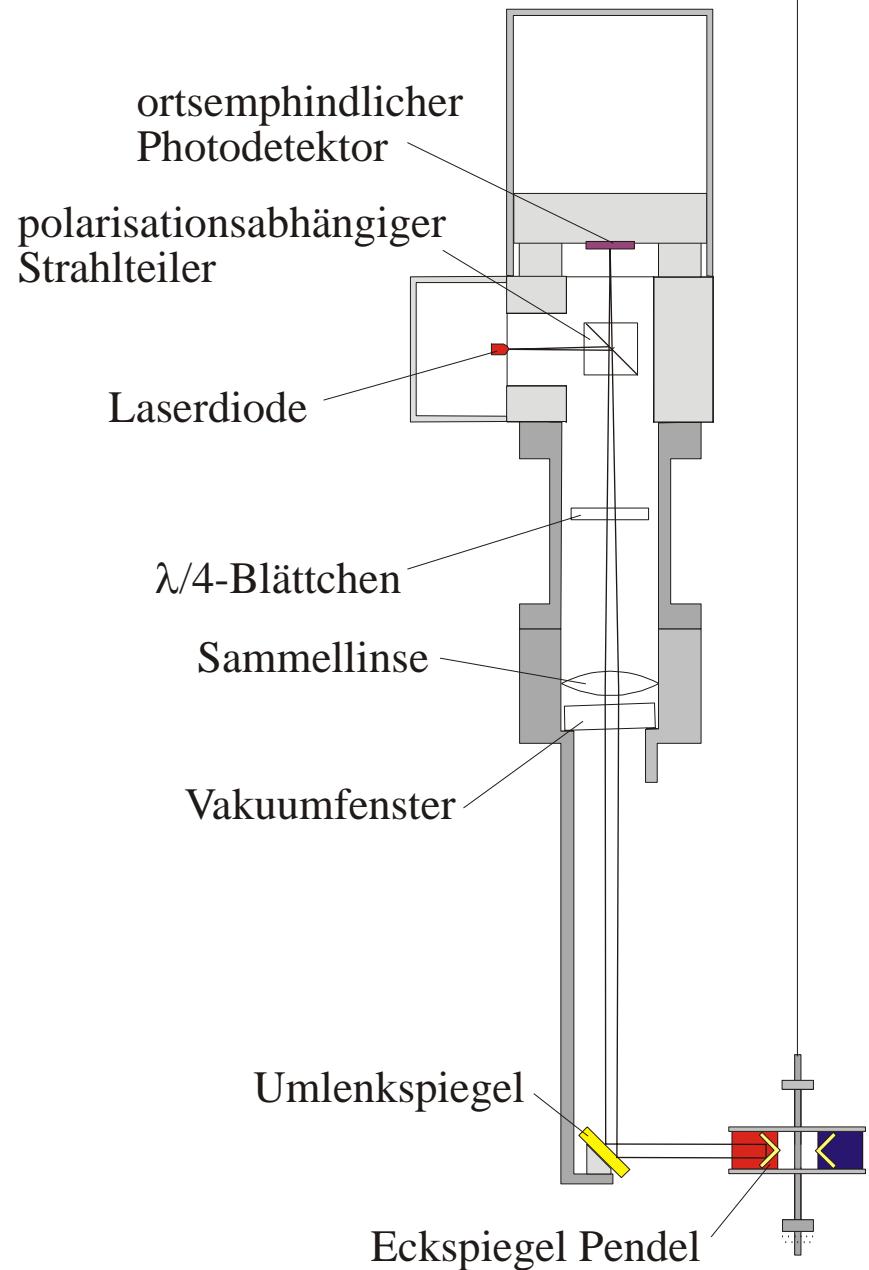
$$D = (F_A d_A - F_B d_B) \sin \varphi_S$$

$$\text{Schwerpunktsbedingung} \Rightarrow D = (a_A - a_B) \frac{m_A m_B}{m_A + m_B} d \sin \varphi_S \cos \theta_S$$

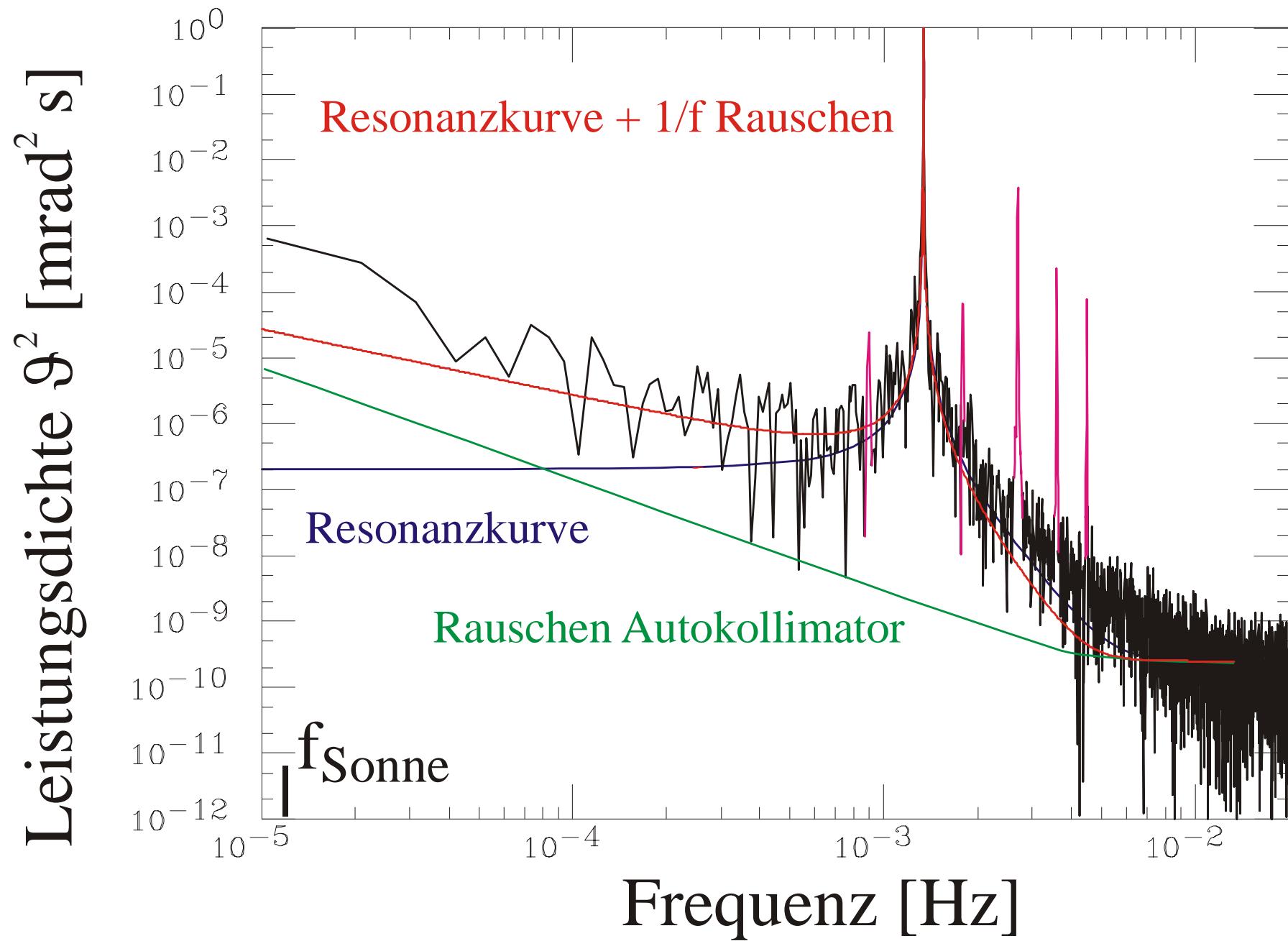
Roll, Krotov und Dicke (1964; Al-Au):  $\eta \leq (1.3 \pm 1.0) \cdot 10^{-11}$

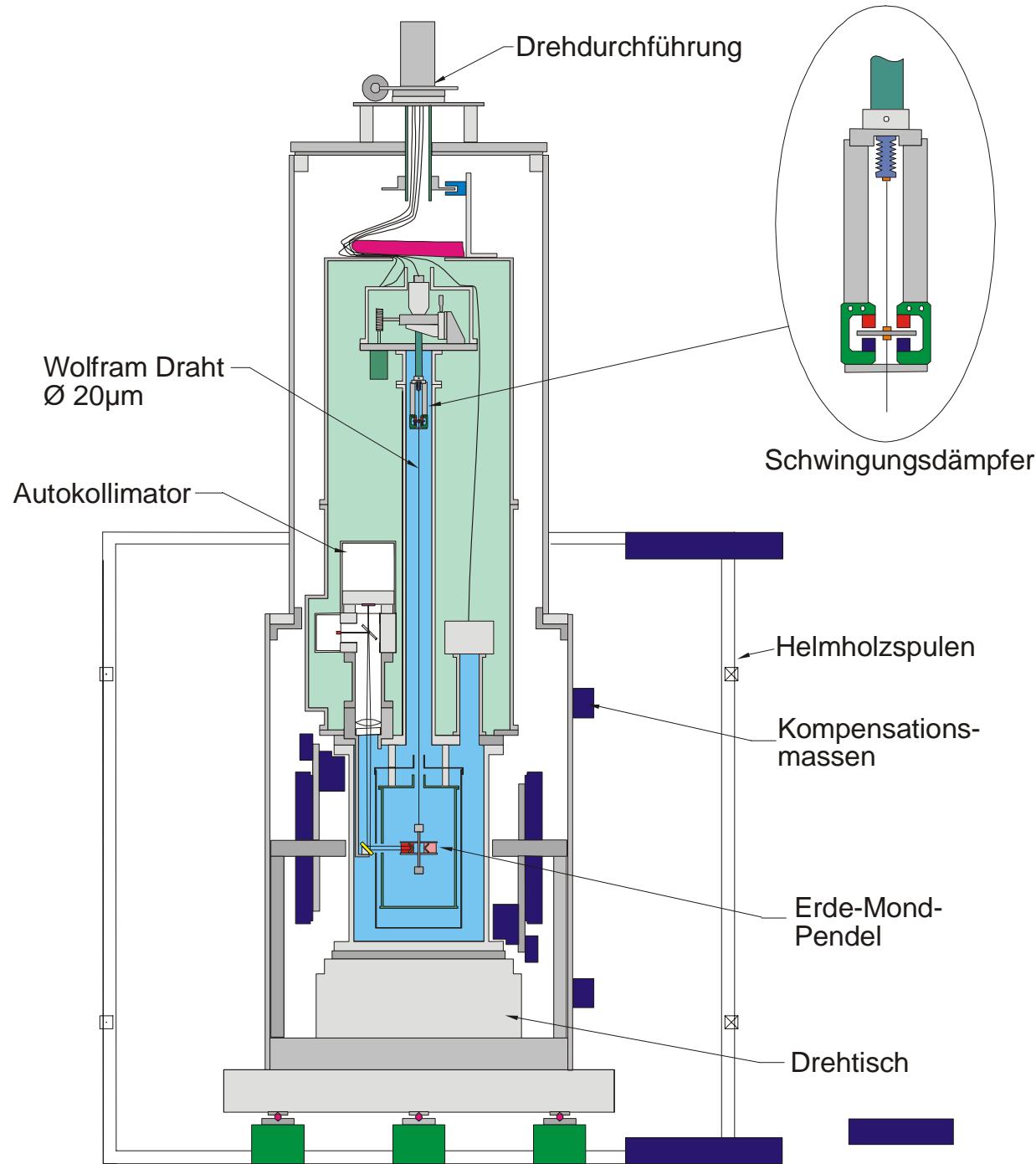
Braginsky und Panov (1971; Al-Pt):  $\eta \leq (-0.3 \pm 0.9) \cdot 10^{-12}$

Ziel Eöt-Wash-Gruppe:  $\eta_{\text{Erde-Mond}} \leq 5 \cdot 10^{-13}$



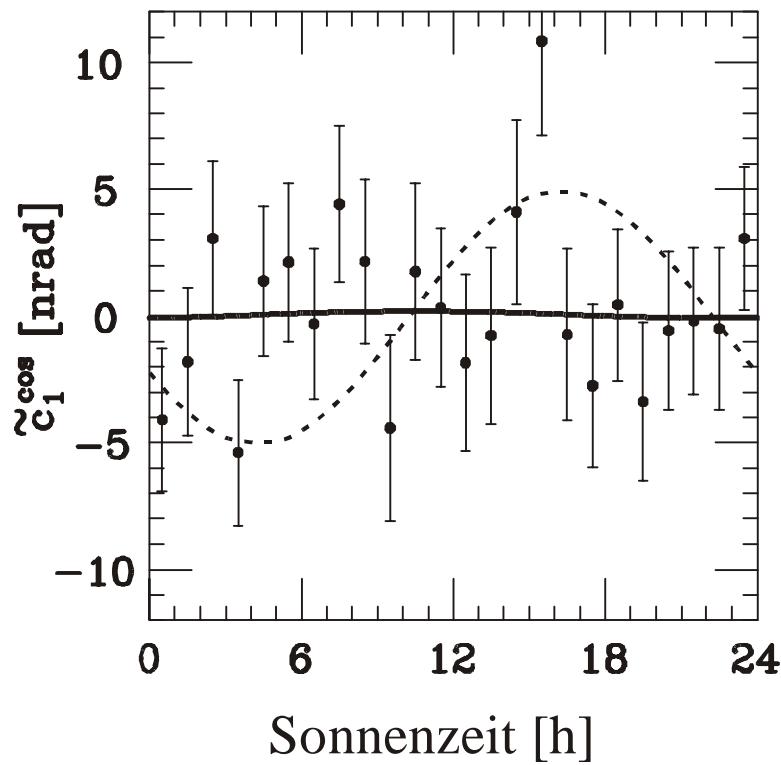
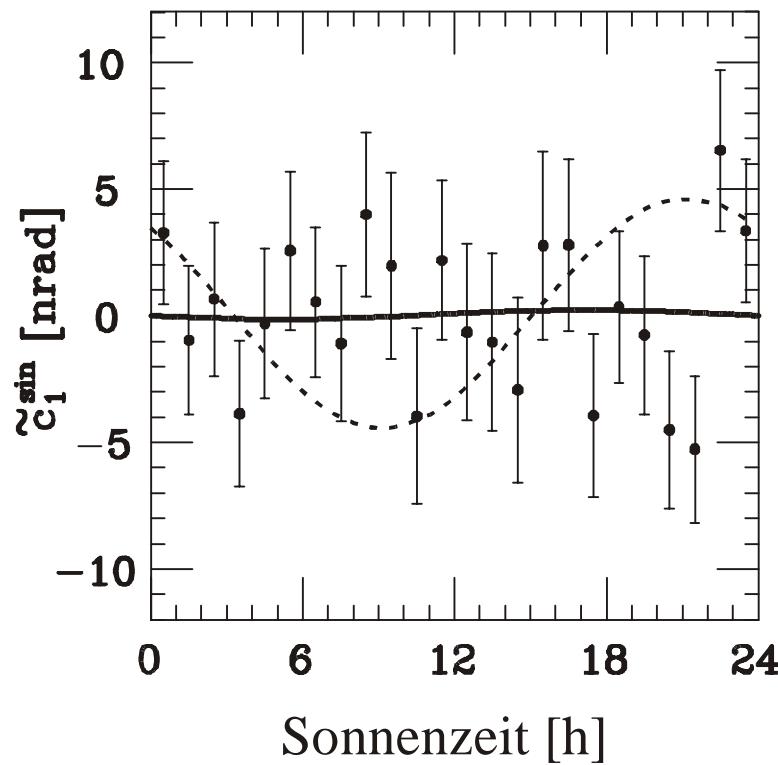
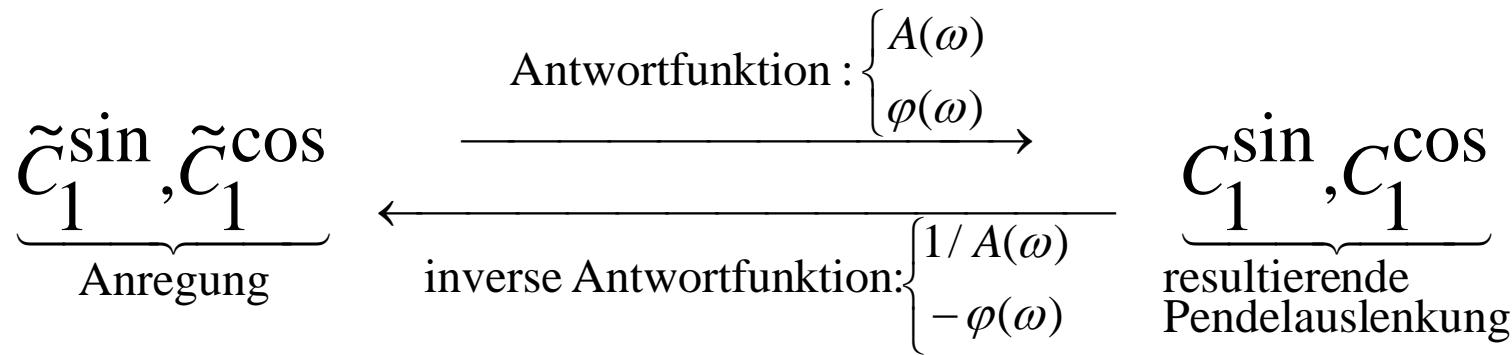
Masse eines Testkörpers: 10.04 g





## Anforderungen:

- Magnetfeld an der Pendelposition  $< 10\text{nT}$
- Temperaturänderung im inneren Teil  $< 100 \mu\text{K}$
- tägliche Komponente (sonnenstandskorreliert) der Temperaturänderung  $< 5\mu\text{K}$
- Gleichlaufschwankungen des Drehtisches  $\Delta\omega/\varpi < 10^{-6}$
- .....



gestrichelte Linie entspricht  $\eta_{EM}$  von  $20 \cdot 10^{-13}$

$$\eta_{EM} = (1.0 \pm 1.4 \pm 0.2) \cdot 10^{-13}$$

Williams *et.al.* (2000) :

Lunar Laser Ranging

$$\eta_{LLR} = (1 \pm 2) \cdot 10^{-13}$$

Müller *et.al.* (1997) :

$$\eta_{LLR} = (3.6 \pm 4.0) \cdot 10^{-13}$$

$$\Rightarrow \eta_{St\ddot{A}P} = (-2.0 \pm 2.4) \cdot 10^{-13}$$

$$\eta_{Gravitation} = \frac{|\eta_{S\ddot{A}P}|}{4.4 \cdot 10^{-10}} \leq 1.0 \cdot 10^{-3}$$

$$\frac{\Delta a_{Testkörper}}{a_{Milchstraße}} = (1.7 \pm 1.5 \pm 0.3) \cdot 10^{-5}$$

„Fünfte Kraft“ der dunklen Materie in unserer  
Milchstraße  $< 10^{-4} \times$  Gravitationskraft

# Newton's Gravitationsgesetz

Gravitationsfeld einer Punktmasse:

$$g(r) = G \frac{m}{r^2}$$

$$g(r) \propto \frac{1}{r^2}$$

folgt aus Gauß'schem Satz in 3 Raumdimensionen und Quellfreiheit ( $\operatorname{div} g = 0$ ) des Raumes

Anzahl der Raumdimensionen:  $n$

$$g(r) \propto \frac{1}{r^{n-1}}$$

Newton's gravity potential:

$$V(r) = -G \frac{m}{r}$$

Deviation from the Newtonian gravity potential:

=> Additional Yukawa potential

$$V(r) = -G \frac{m}{r} (1 + \alpha e^{-r/\lambda})$$

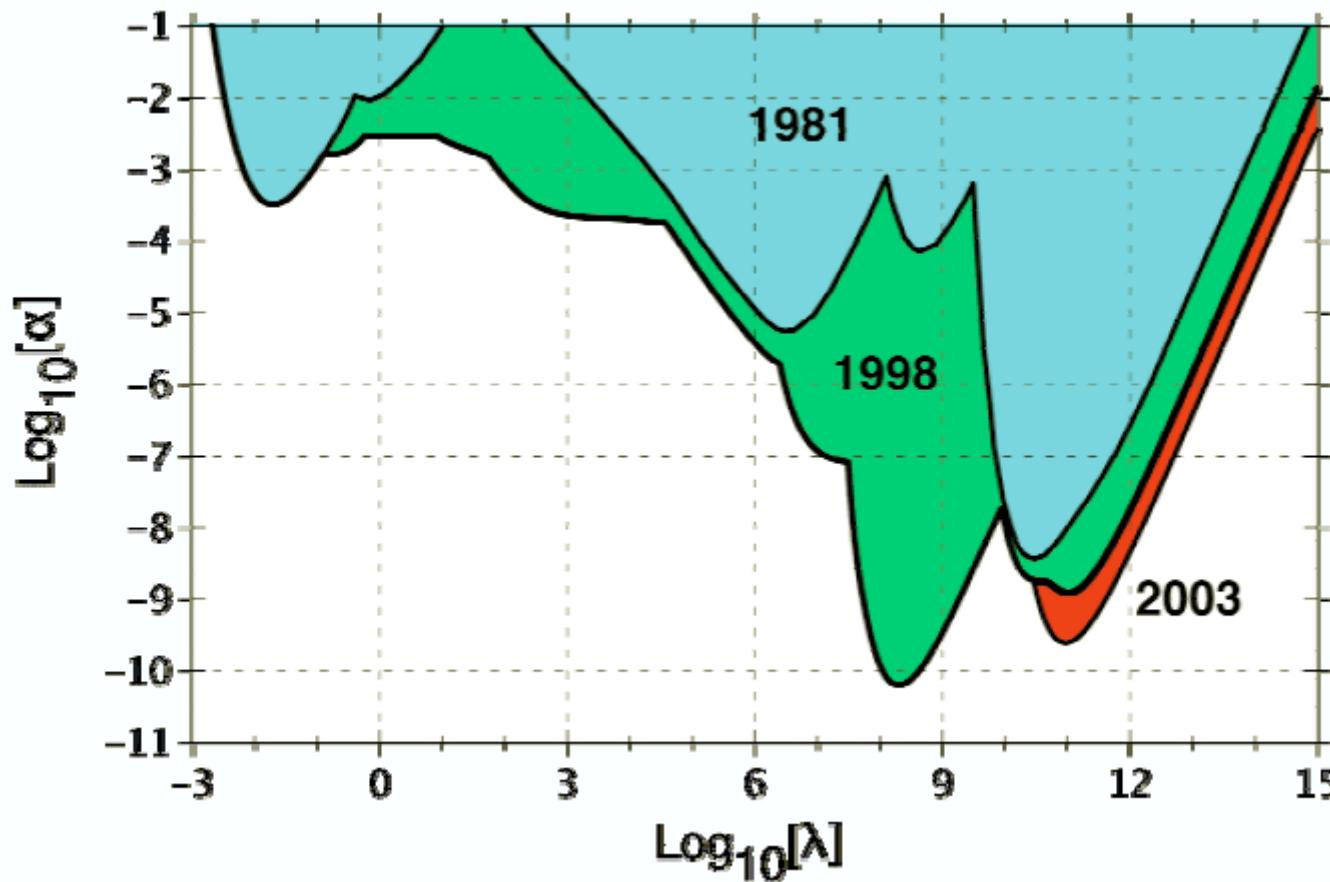
$a$  : Coupling constant (relative to  $G$ )

$l$  : Range

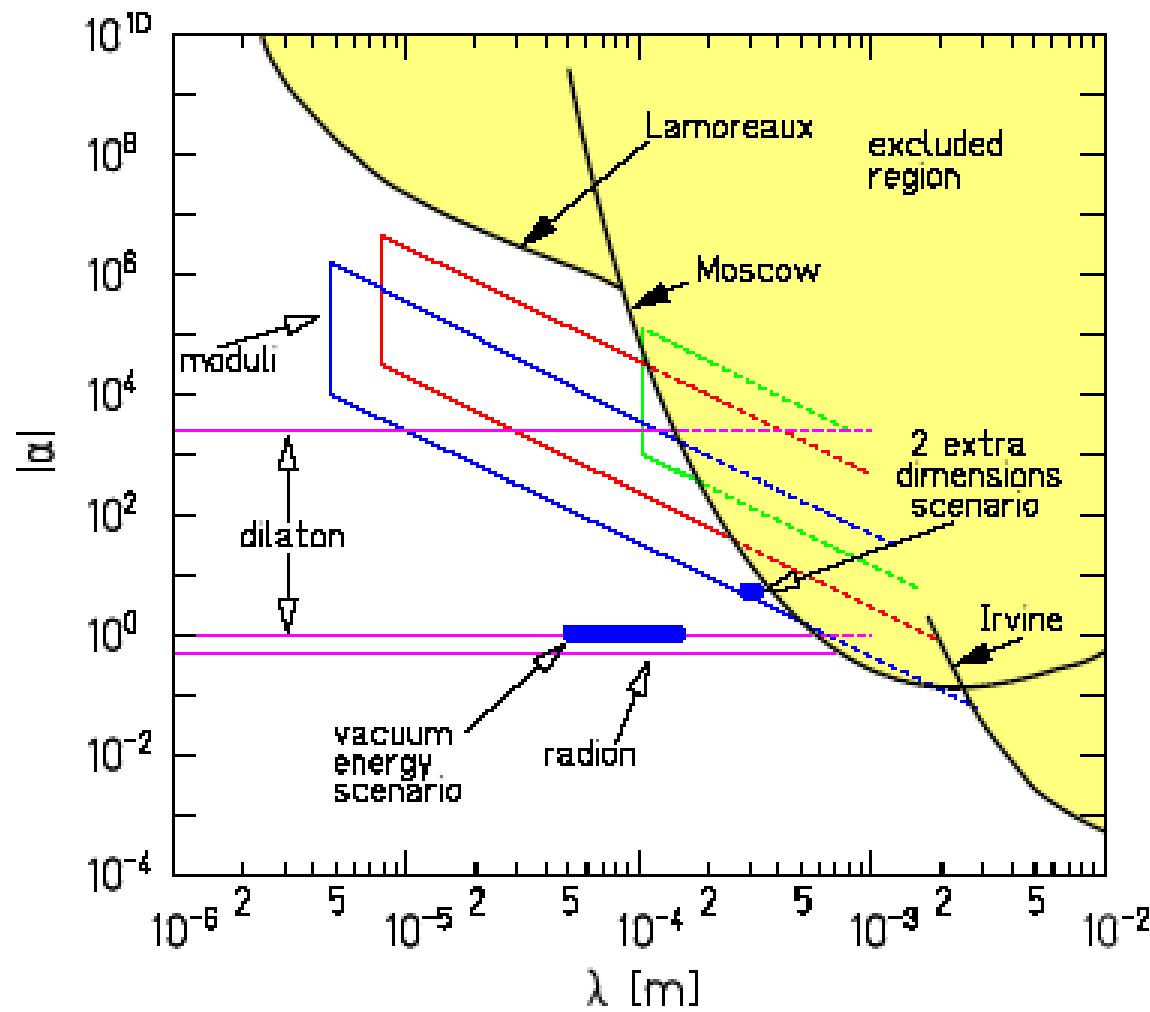
find no deviation from Newtonian physics :

=> Exclude region of the  $a-l$ -plain

# Limits on Extra Dimensions and New Forces: Long-Range Limits



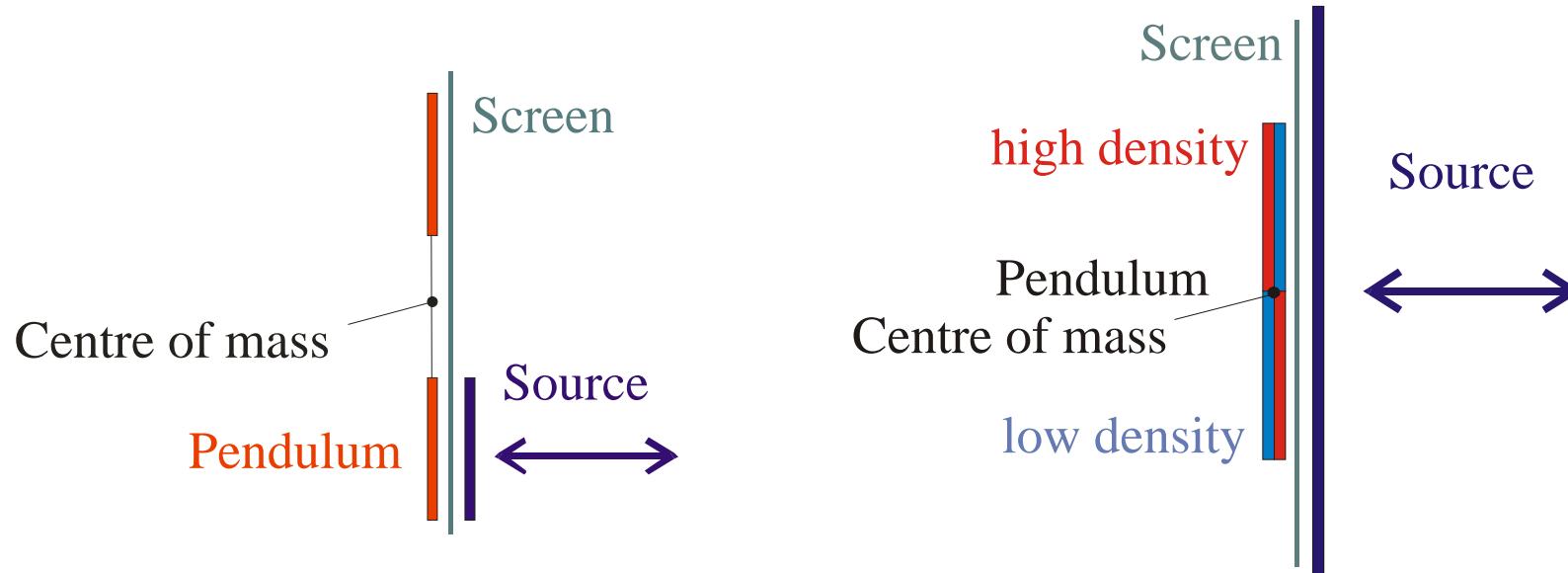
Reference: Coy, Fischbach, Hellings, Standish & Talmadge (2003)



95% confidence upper limits on a short-range Yukawa violation of the gravitational ISL as of 1999

## Measurement of a short range gravitational force

- sufficient mass of pendulum and source has to be close to each other  
=> flat plate
- patch effect: electrical conductors have a charge pattern on their surfaces  
=> screen

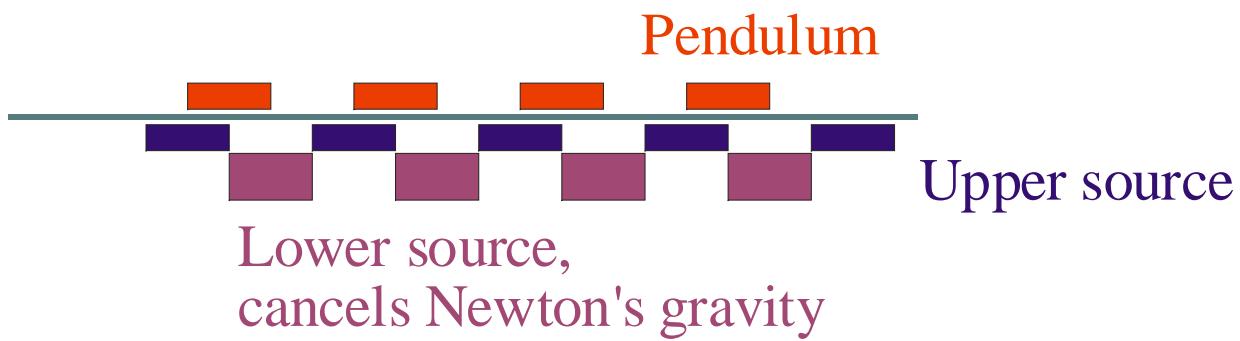
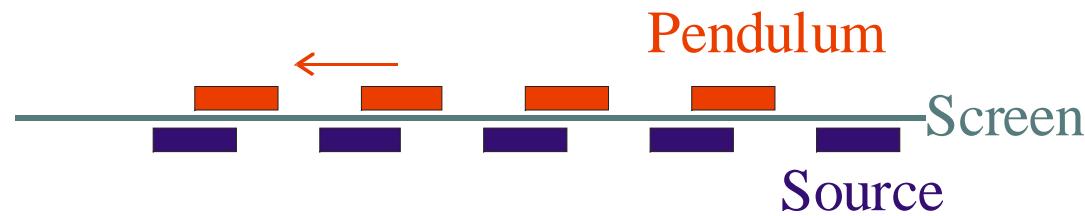
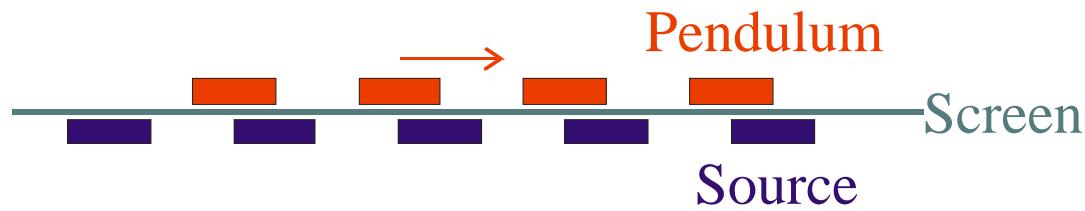


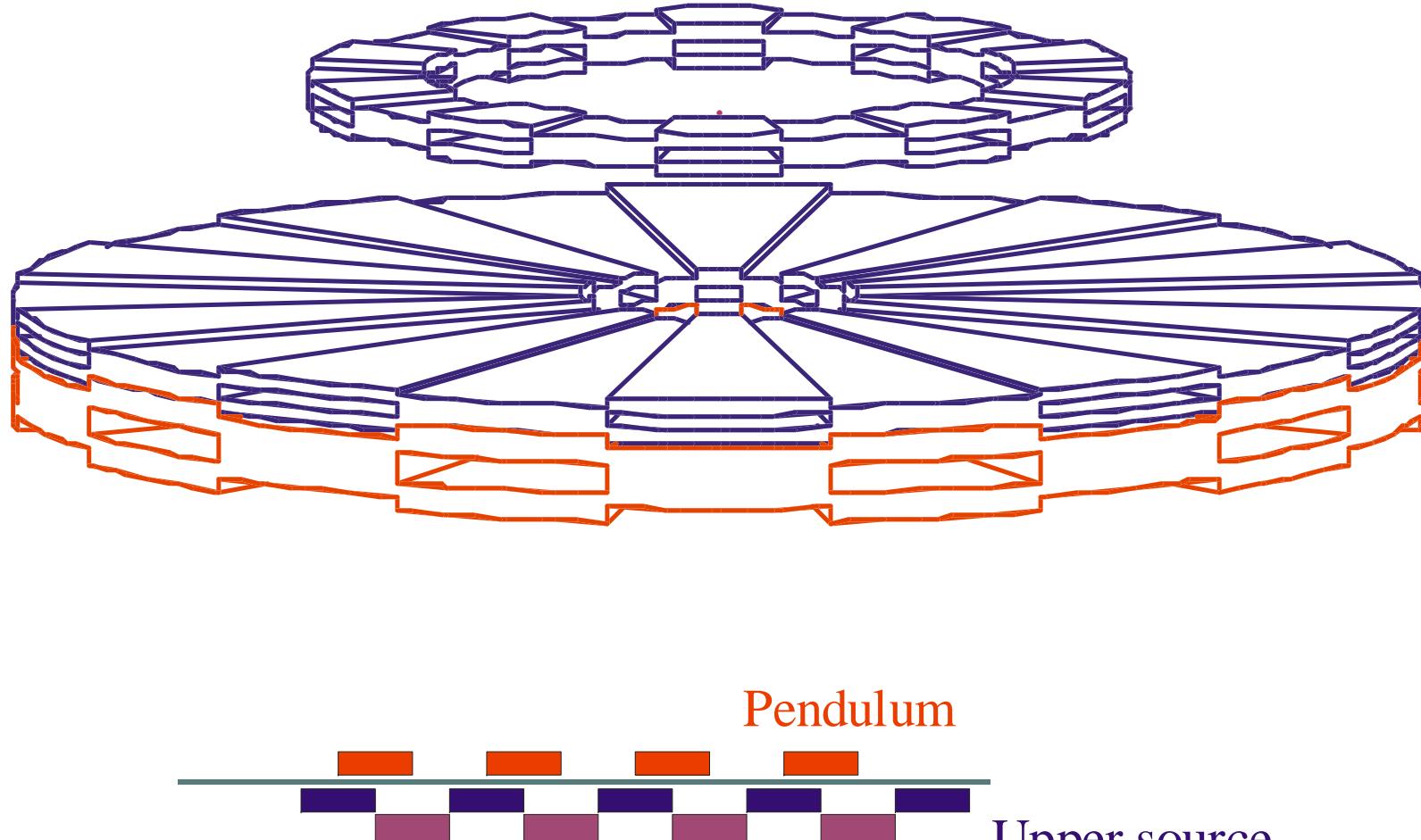
=> short range signal small compared to Newton's gravity

Infinite plate => force from Newton's gravity independent from distance

Problem: Frequency of signal = frequency of Source

=> Coupling due to vibrations, finite stiffness of apparatus etc.

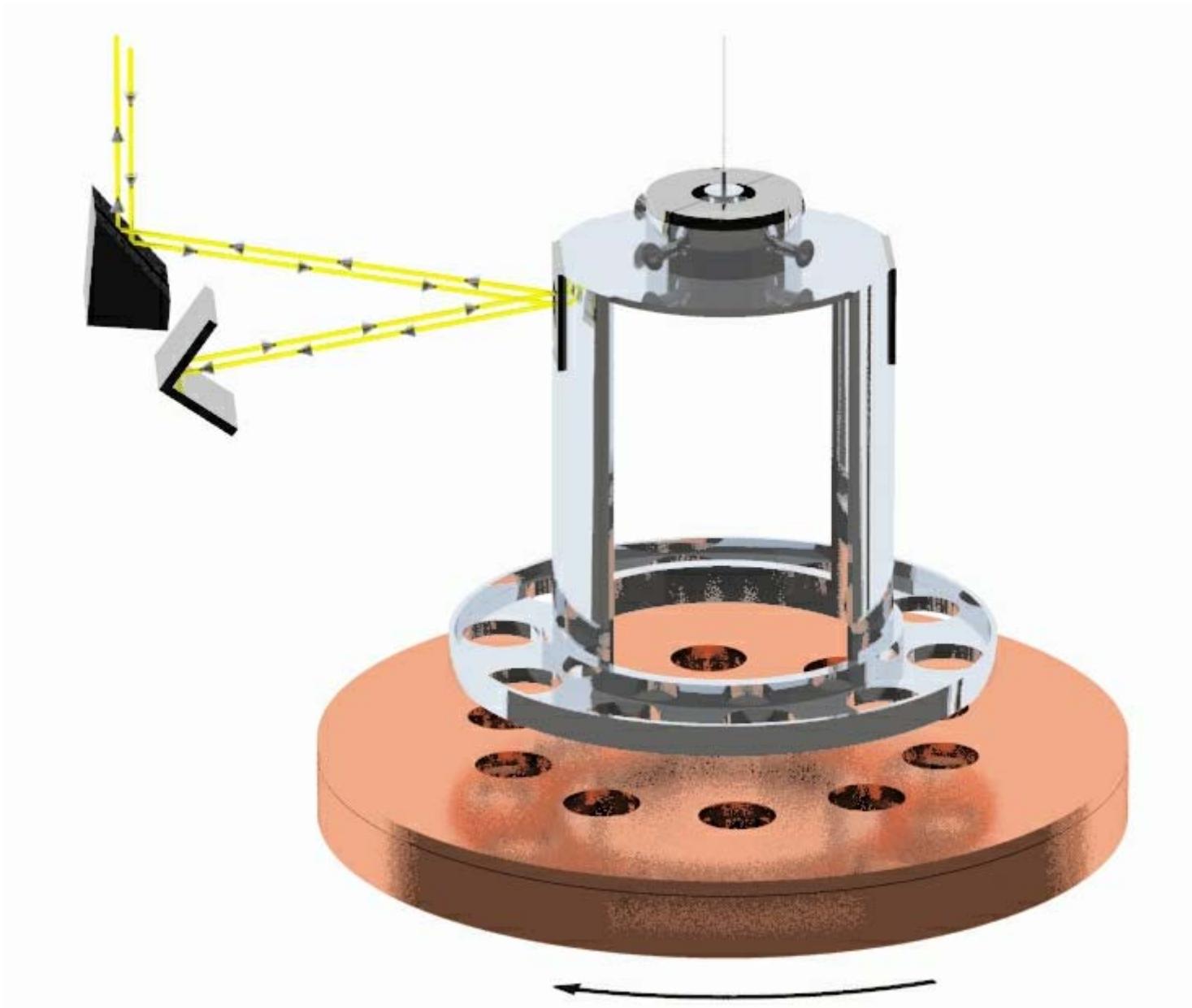


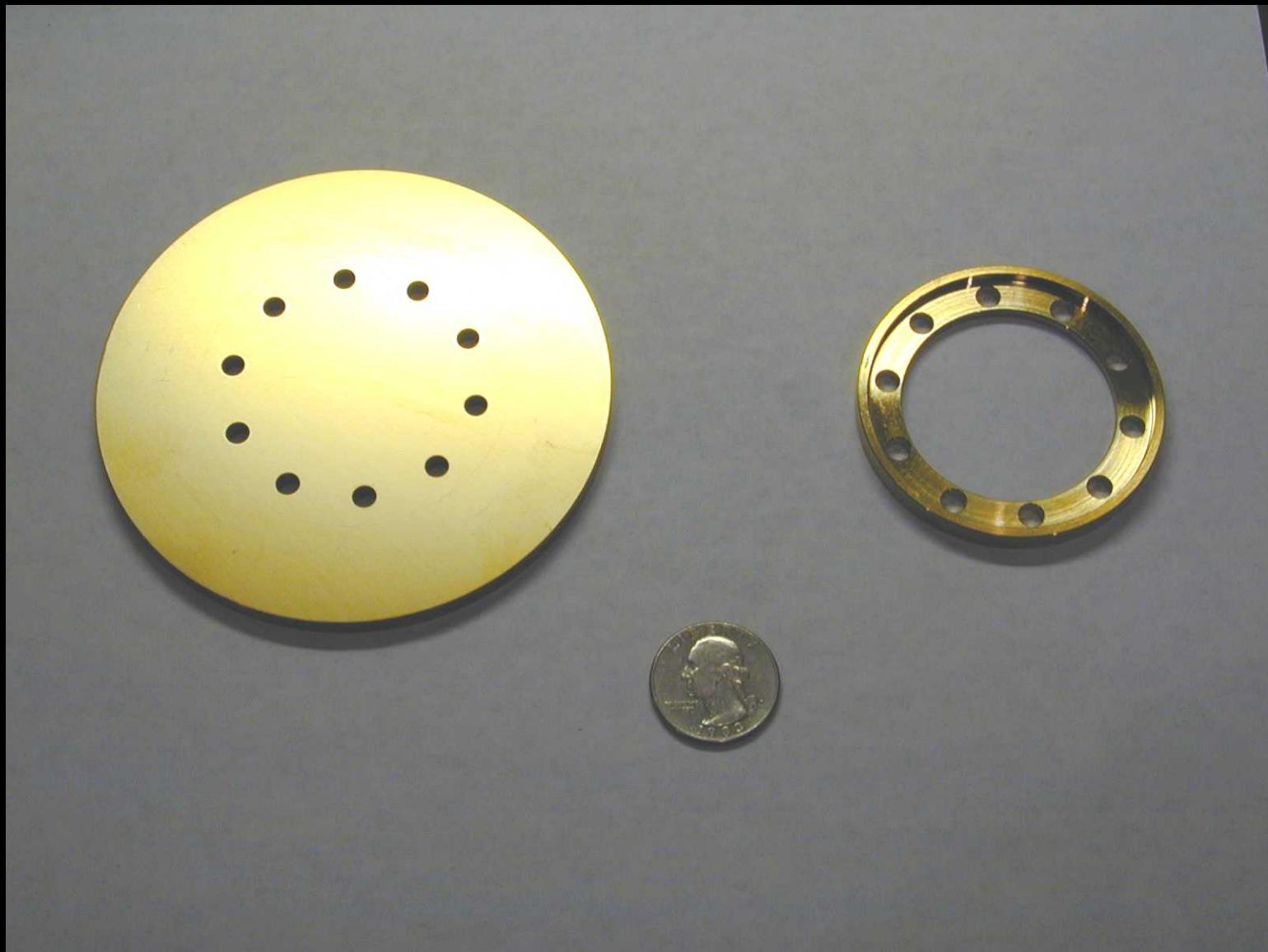


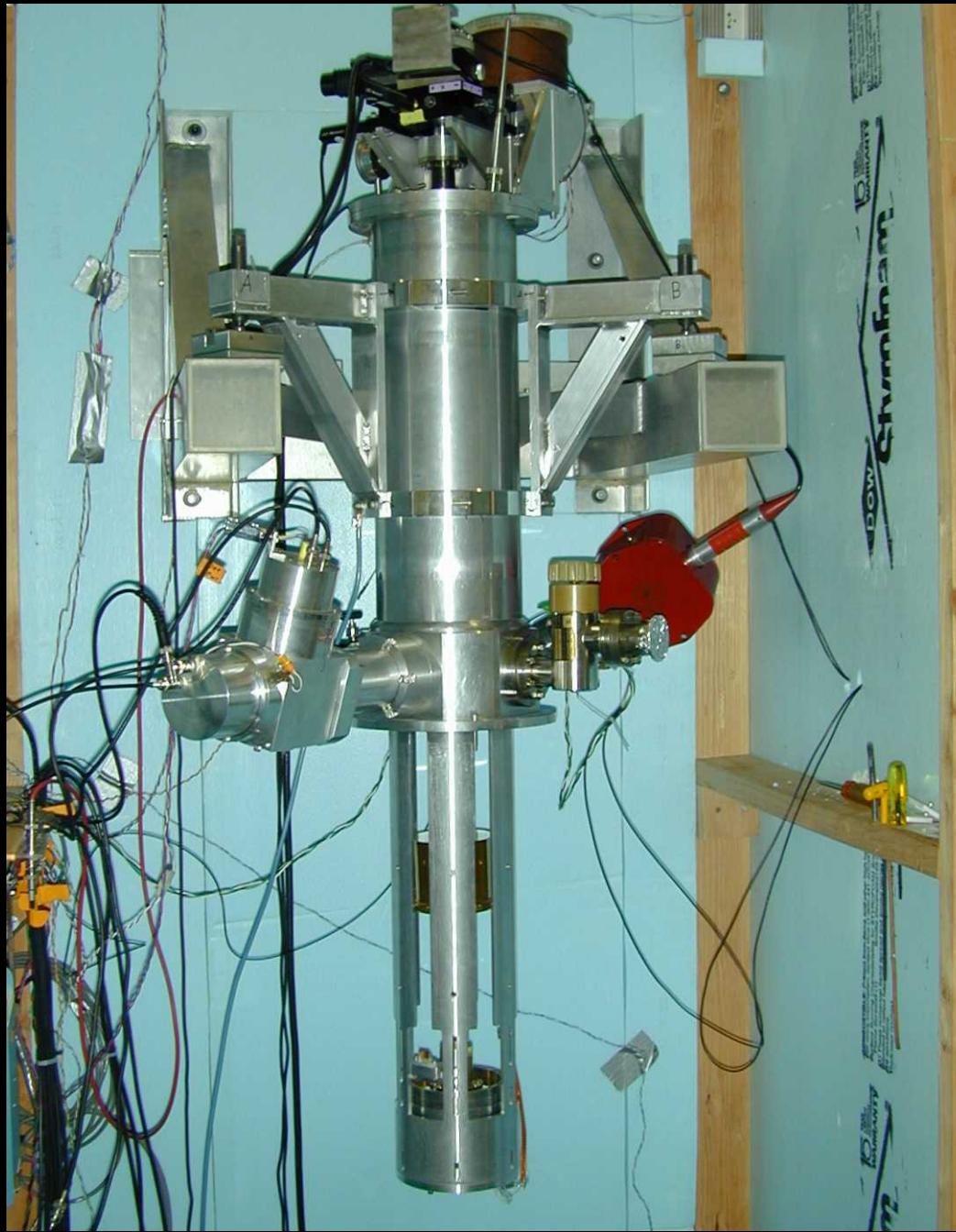
Pendulum

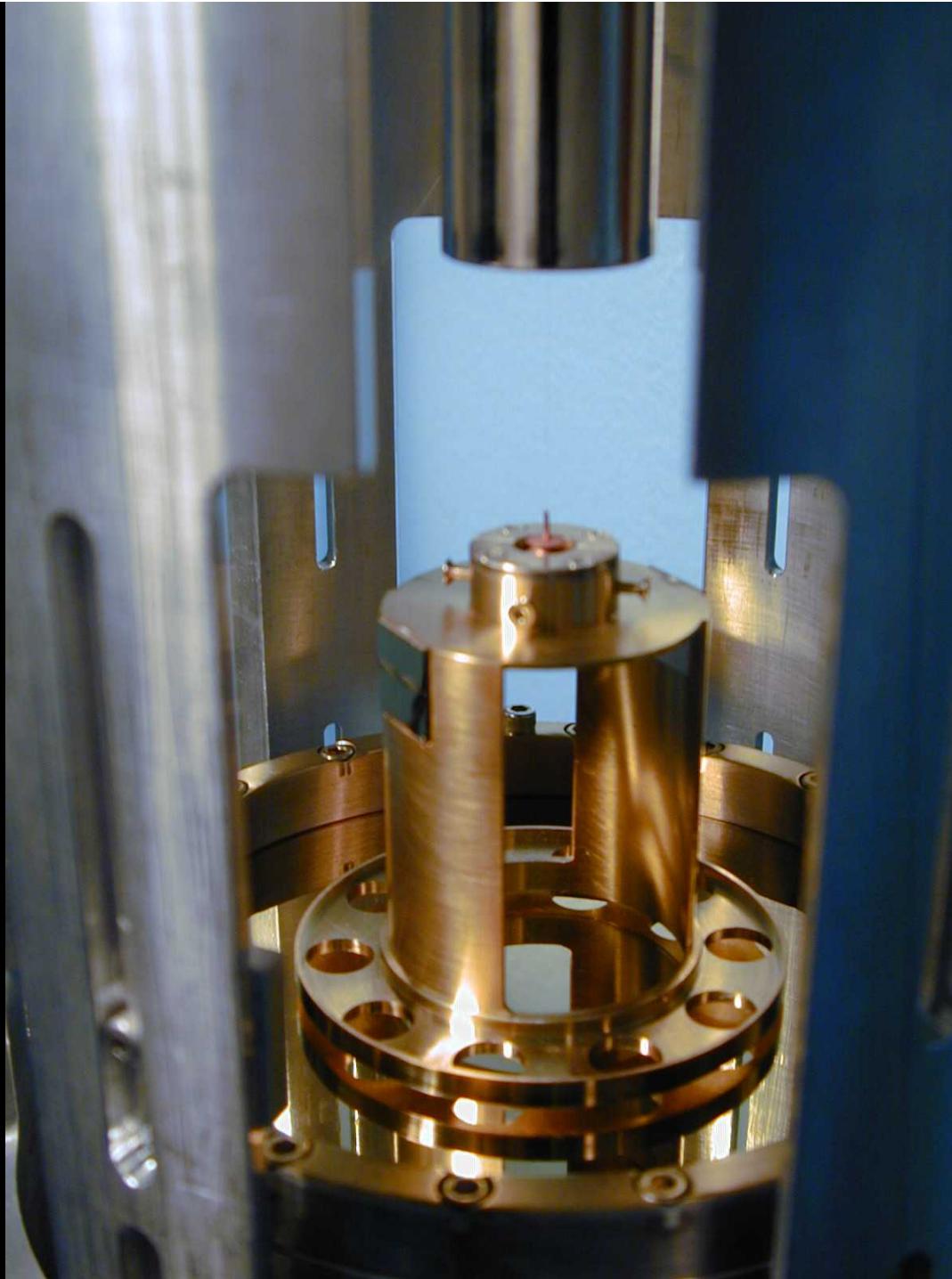
Upper source

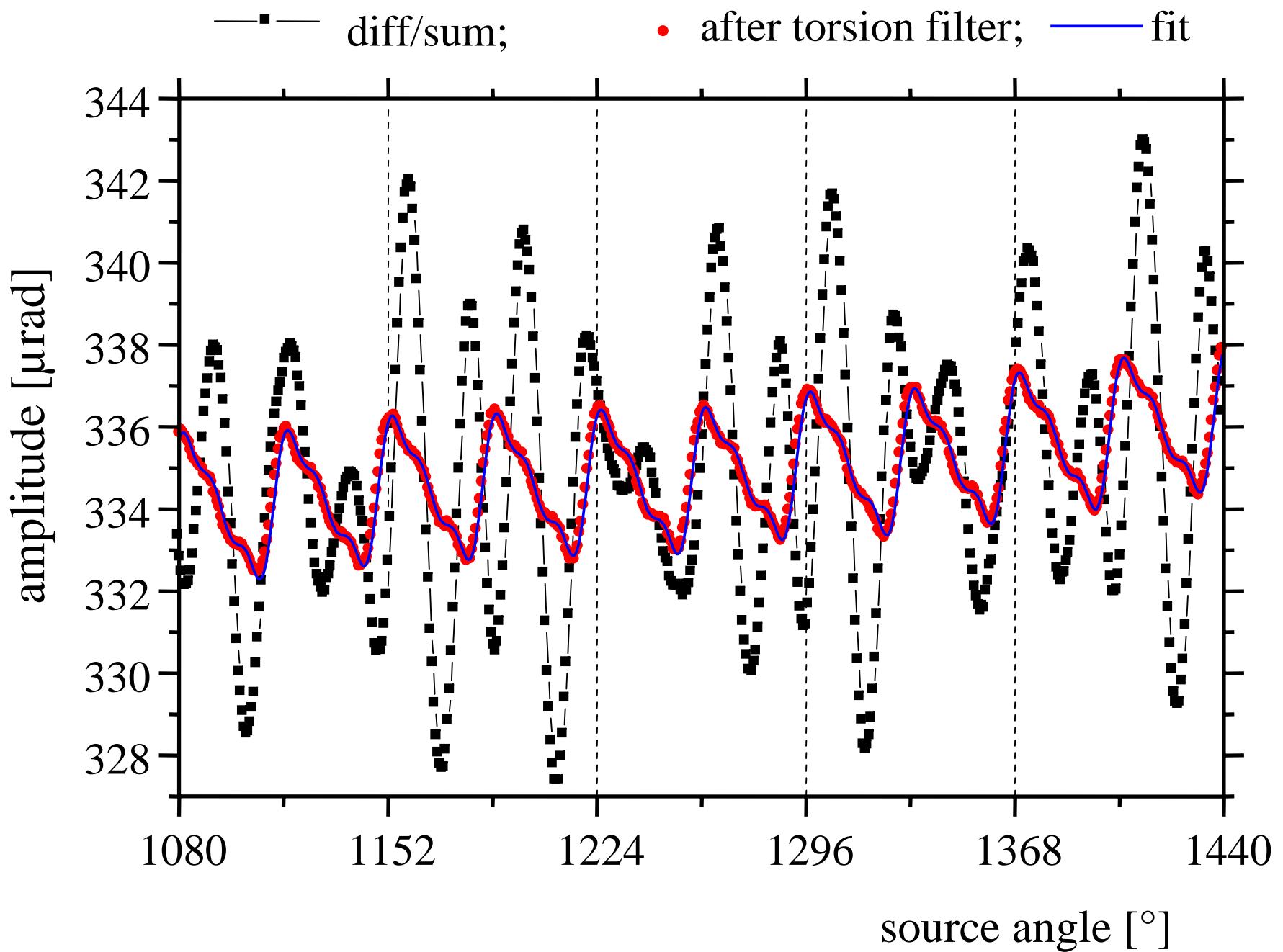
Lower source,  
cancels Newton's gravity

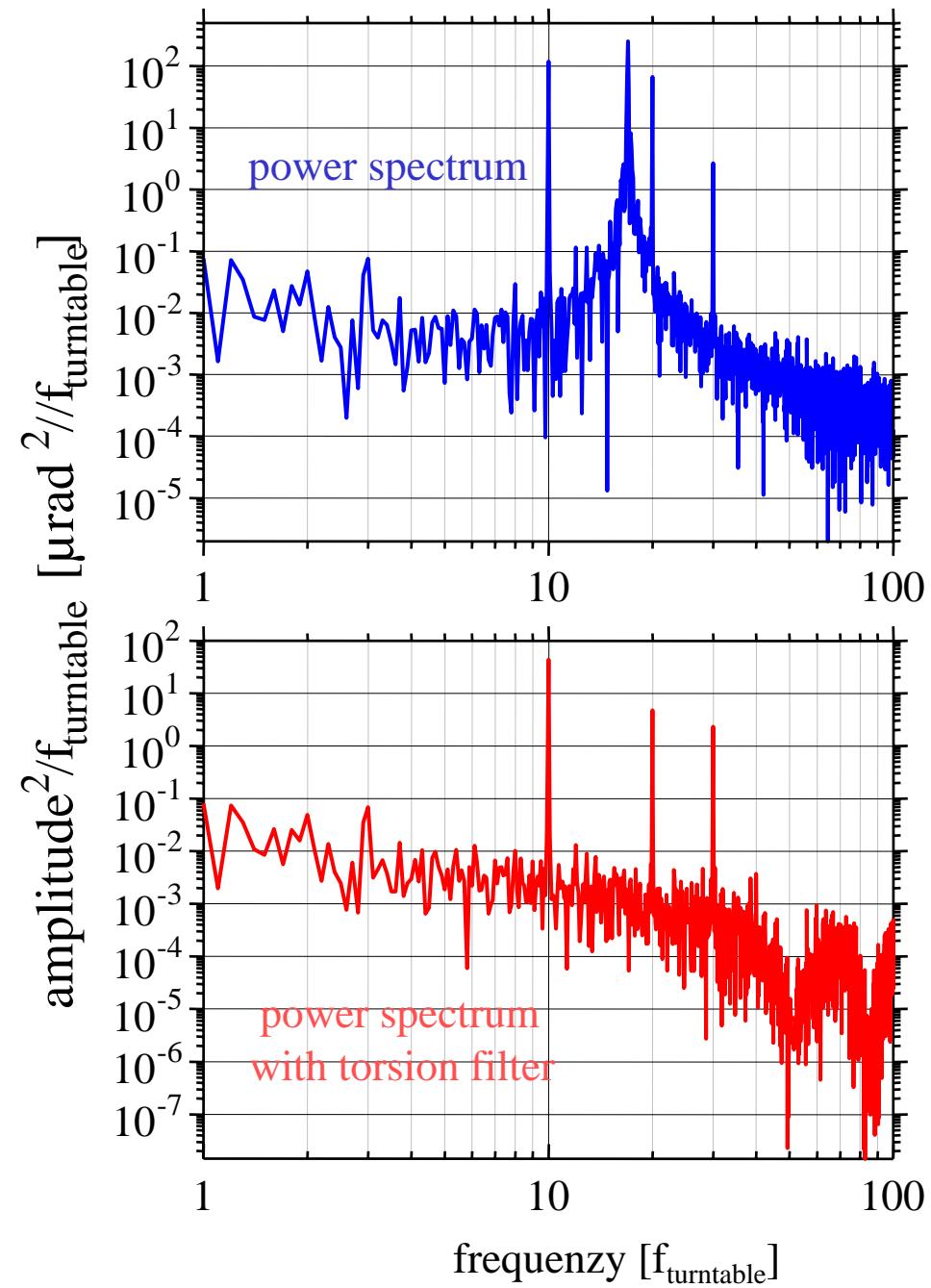


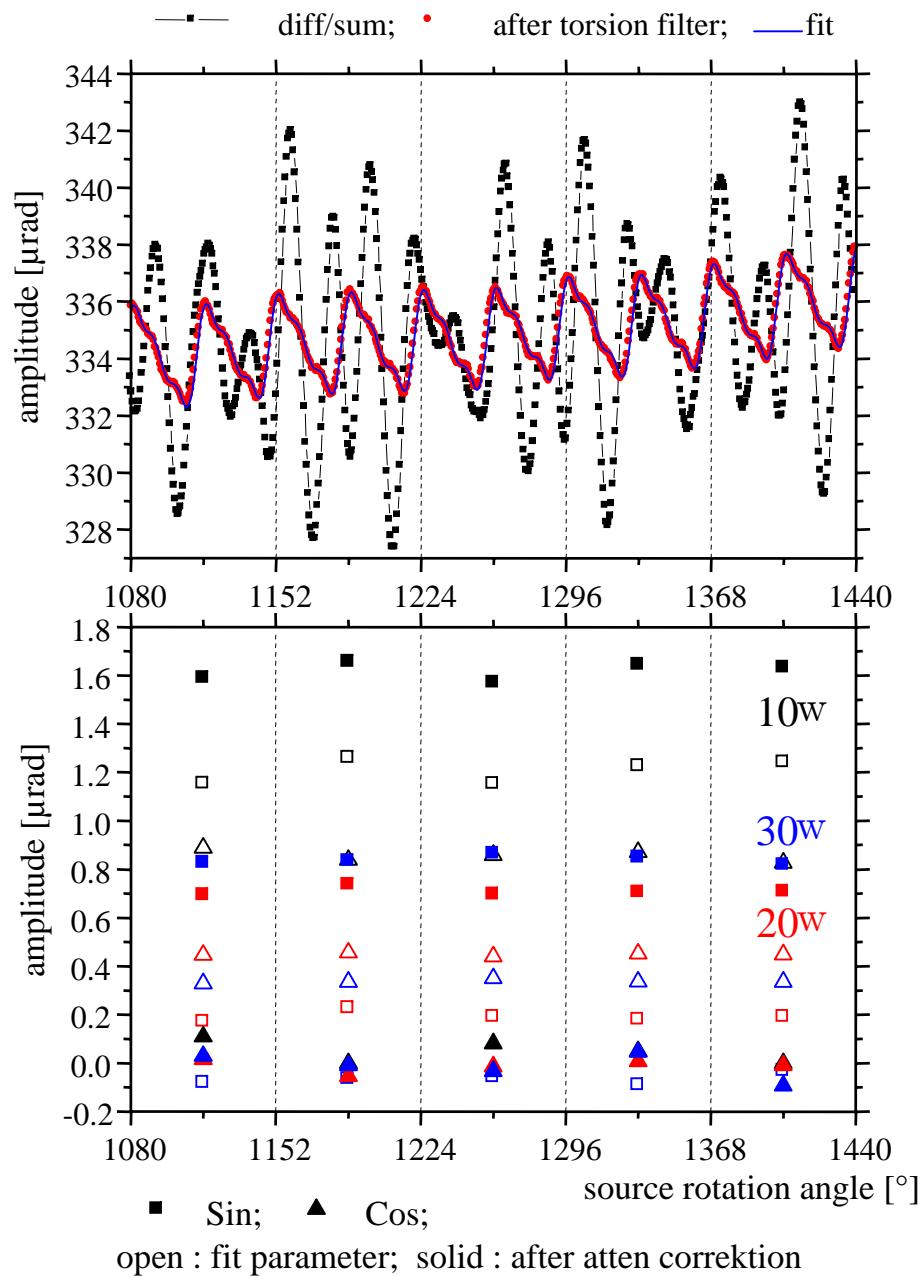


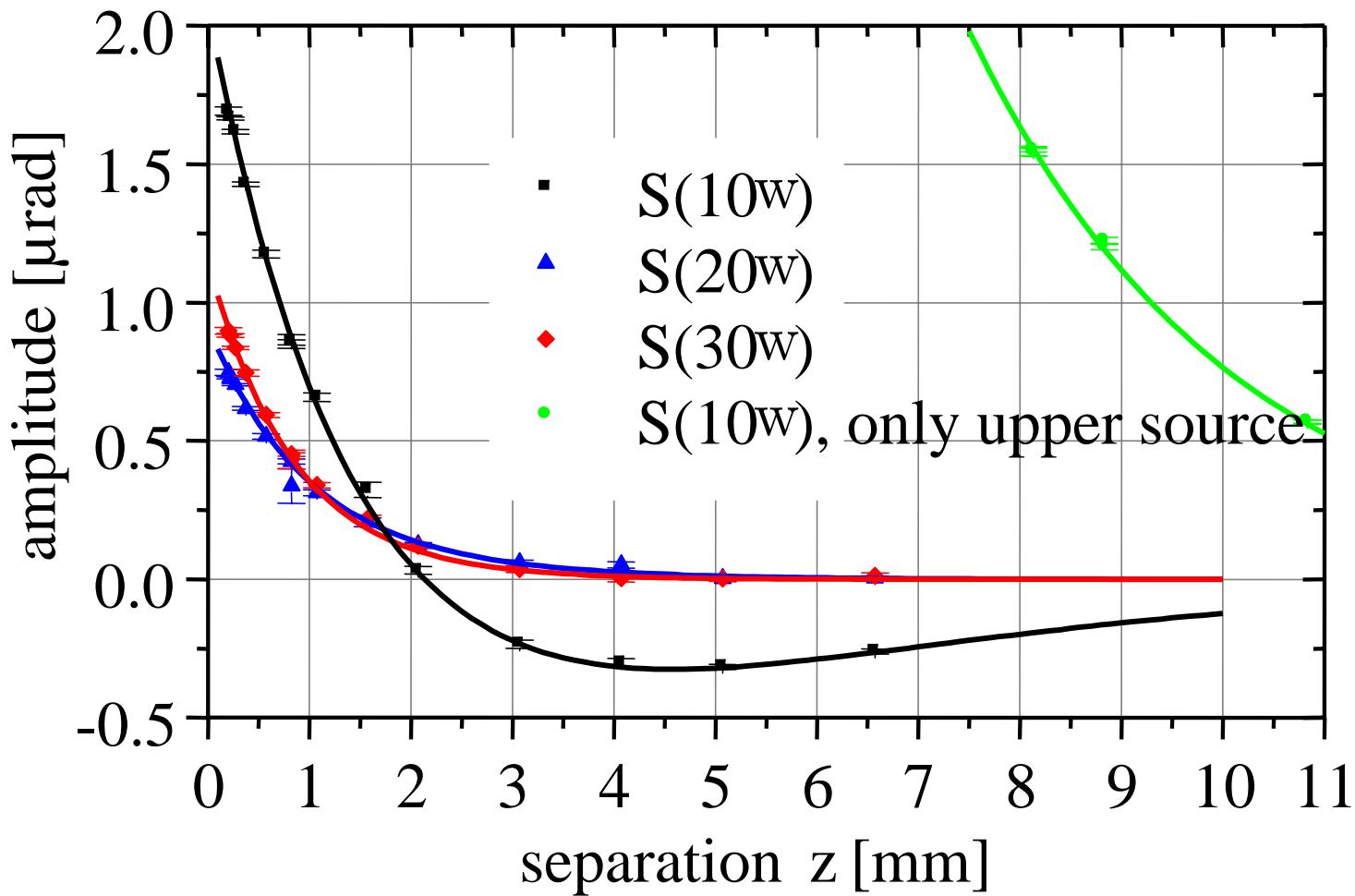


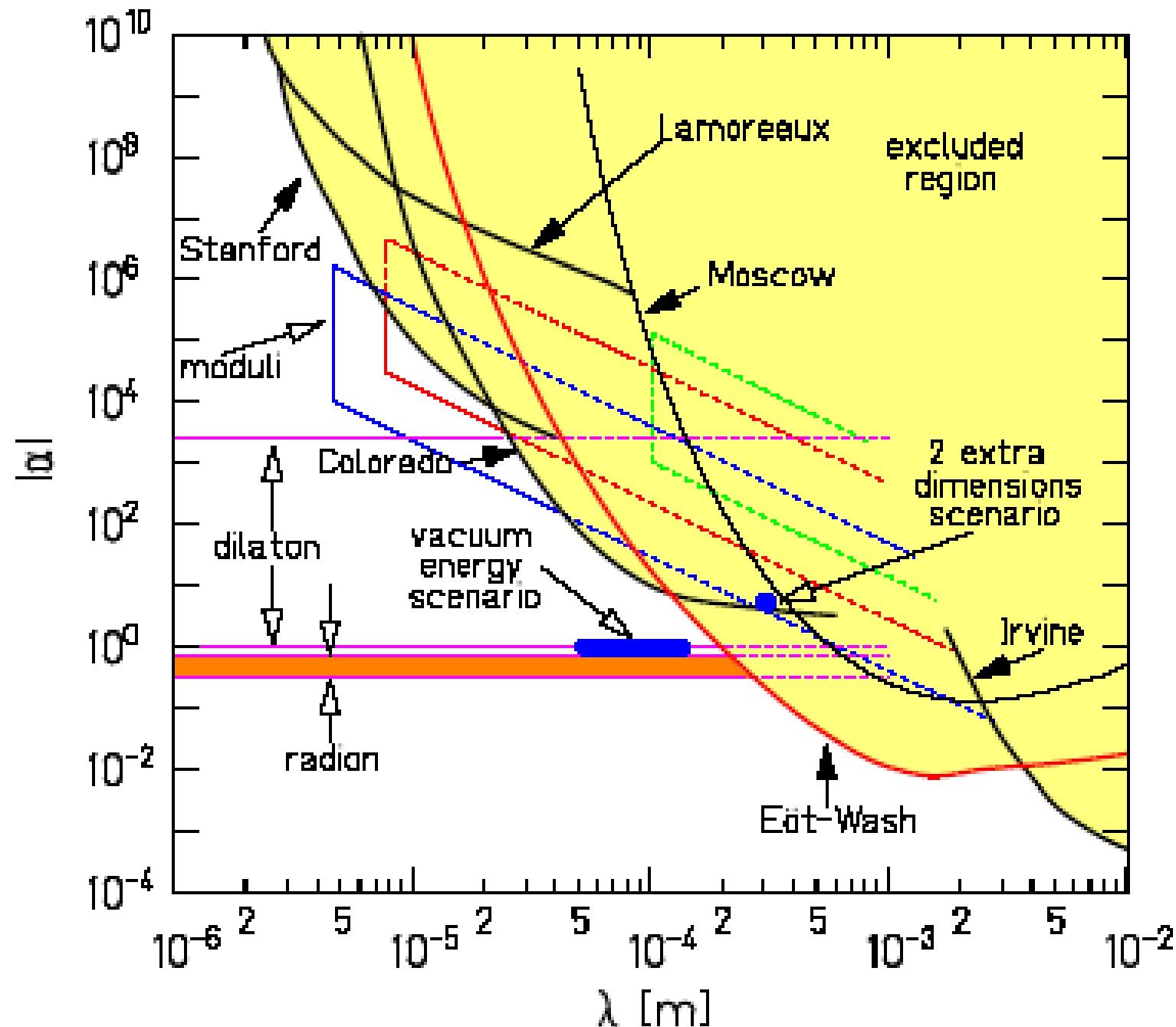


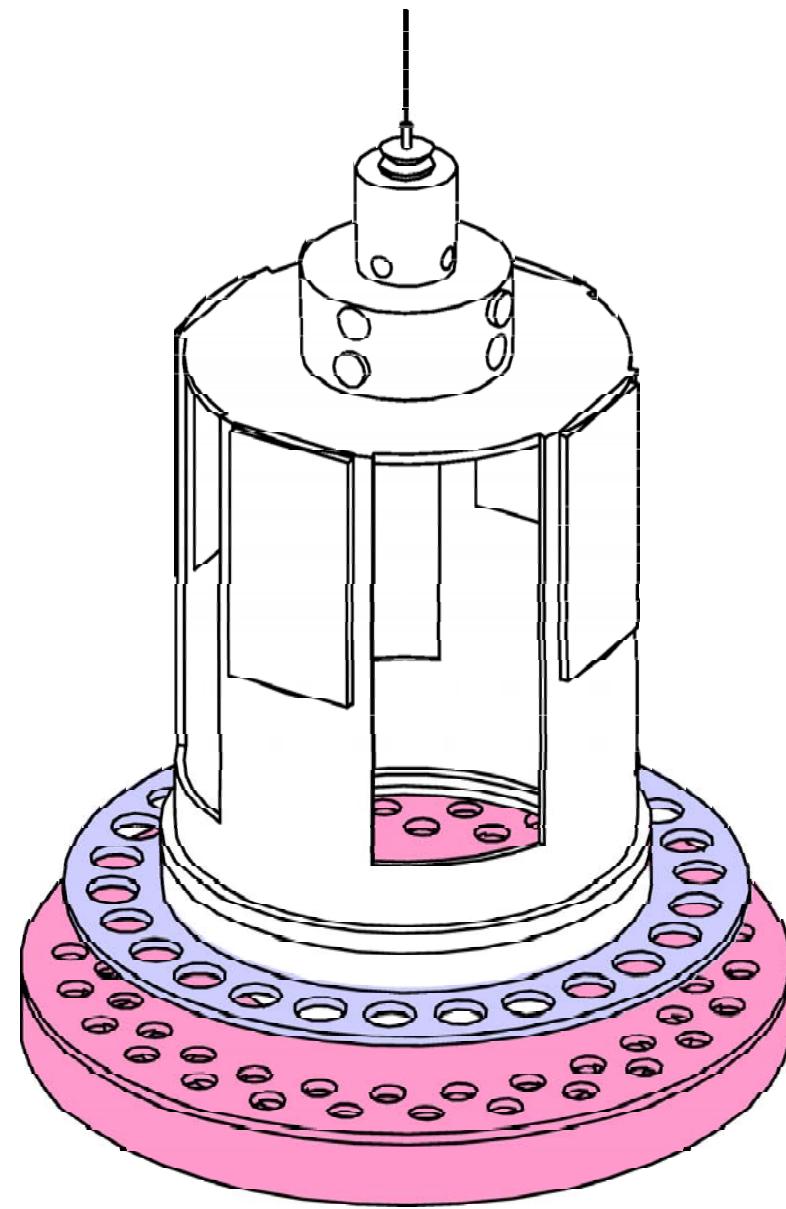


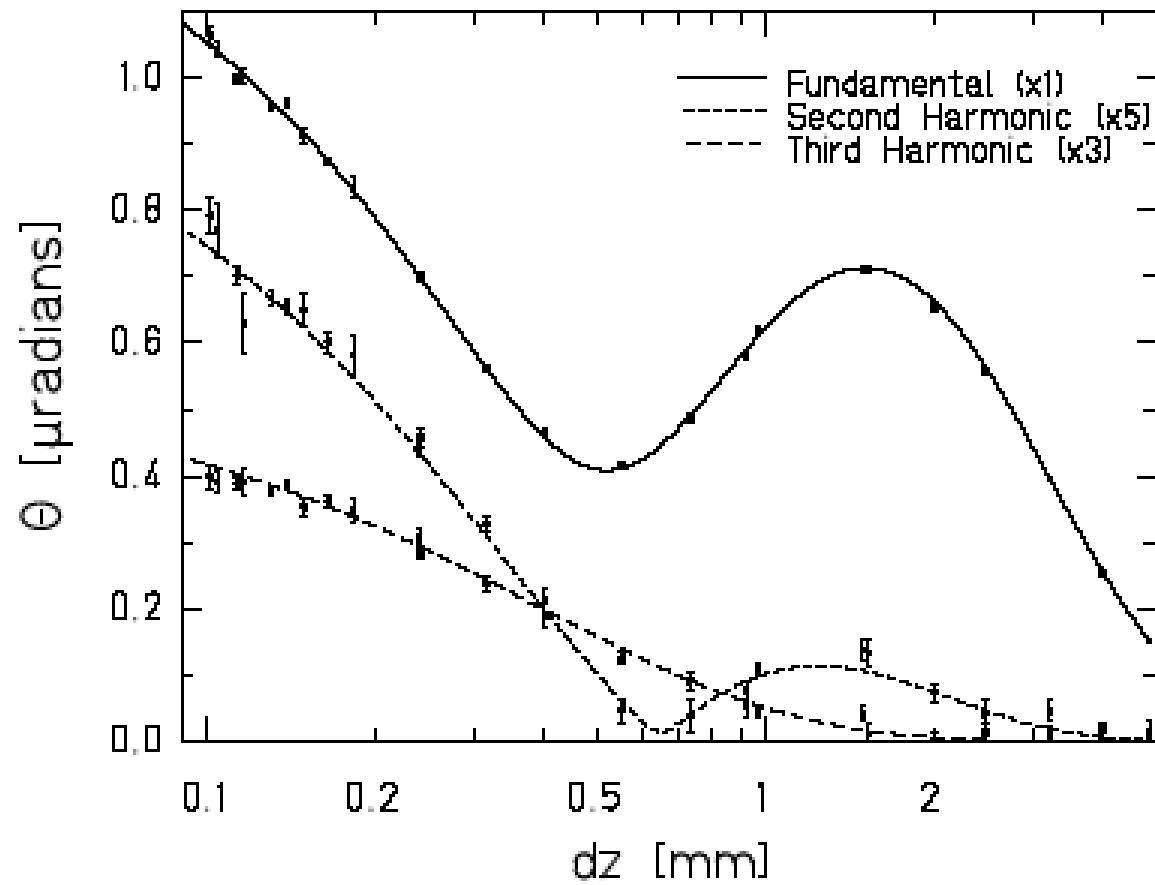
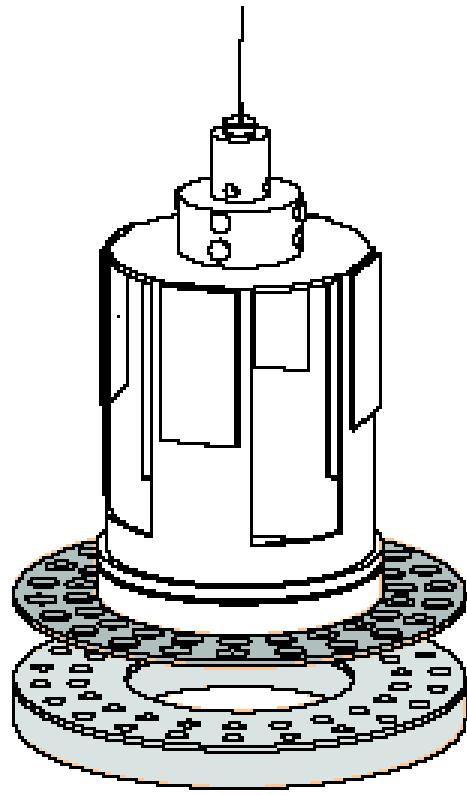


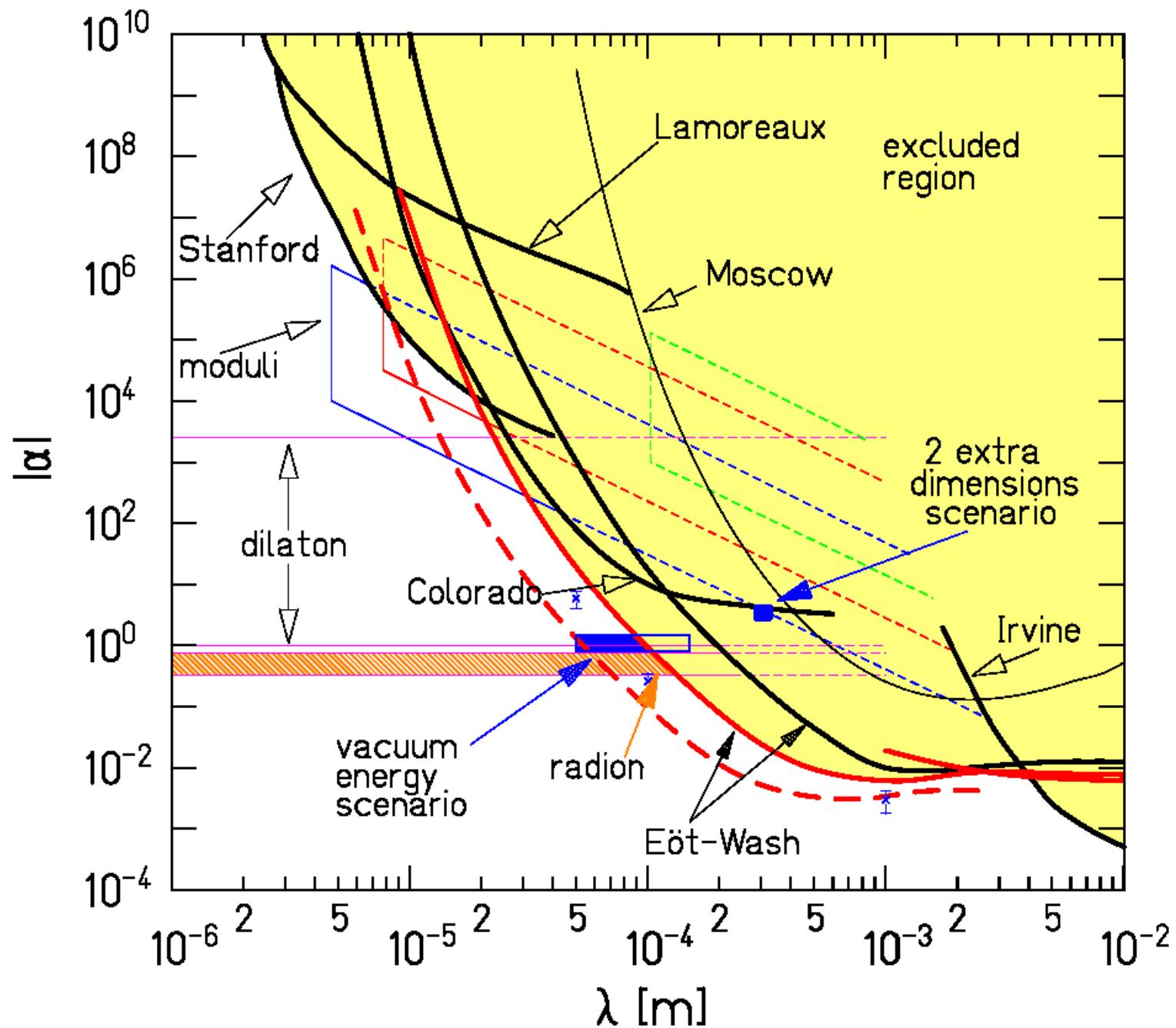


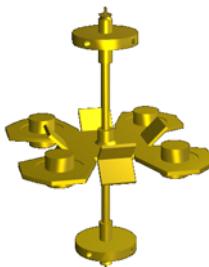












## Eöt-Wash Group

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