Fractal ground state of Wigner chains in periodic potentials

The ground state of a Wigner chain in a periodic potential is a paradigmatic example of geometric frustration with long-range interactions. In an ion trap the dynamics can be modeled by a long-range elastic crystal in the presence of a periodic substrate, emulating the Frenkel-Kontorova model with Coulomb interactions. In the continuum limit, dislocations are solitons with power-law tails and their action is mapped into a massive, long-range (1+1) Thirring model. This mapping establishes a novel link between trapped ions and quantum field theoretical models, where the solitons are fermionic excitations over an effective Dirac sea and interact via the Coulomb interactions. By means of an appropriate thermodynamic limit we identify the corresponding mean field model and argue that the Coulomb interactions tend to destabilize structures commensurate with the periodic substrate, turning the commensurate-incommensurate transition into a crossover. Our predictions can be probed in state-of-the-art trapped ion experiments and show the versatility of these systems as platforms for probing quantum field theoretical models.