PT – symmetric quantum mechanics

Prof. Carl M. Bender

Physics Department, Washington University

The average quantum physicist on the street believes that a quantum-mechanical Hamiltonian must be Dirac Hermitian (invariant under combined matrix transposition and complex conjugation) in order to guarantee that the energy eigenvalues are real and that time evolution is unitary. However, the Hamiltonian $H=p^2+ix^3$, which is obviously not Dirac Hermitian, has a real positive discrete spectrum and generates unitary time evolution, and thus it defines a fully consistent and physical quantum theory!

Evidently, the axiom of Dirac Hermiticity is too restrictive. While $H=p^2+ix^3$ is not Dirac Hermitian, it is PT symmetric; that is, it is invariant under combined space reflection P and time reversal T. The quantum mechanics defined by a PT-symmetric Hamiltonian is a complex generalization of ordinary quantum mechanics. When quantum mechanics is extended into the complex domain, new kinds of theories having strange and remarkable properties emerge. In the past two years, some of these properties have been verified in laboratory experiments.

A particularly interesting PT-symmetric Hamiltonian is $H=p^2-x^4$, which contains an upside-down potential. We will discuss this potential in detail, and explain in intuitive as well as in rigorous terms why the energy levels of this potential are real, positive, and discrete.