Could the Dark Matter be the QCD Axion?

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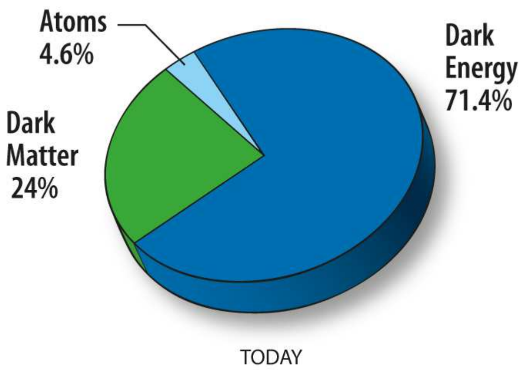
- Dark matter: how are we sure it's there?
- $\mathbf{T}$ symmetry in QCD: a mystery.
- The Axion: a possible explanation
- Cosmology of the axion: a dark matter candidate
- Relating the Dark Matter abundance to the Axion particle's mass
- The search for the axion: how knowing the mass helps

Dark Matter: a Cosmic Mystery



Atoms: Standard Model.

Dark Energy: Cosmological Constant.

Strange value, but possible

Dark Matter: MYSTERY! NOT SM!

We only know 3 things about dark matter:

- It's **Matter**: gravitationally clumps.
- It's **Dark**: negligible electric charge, interactions too feeble to be detected except by gravity
- It's **Cold**: negligible pressure by redshift $z = 3000$

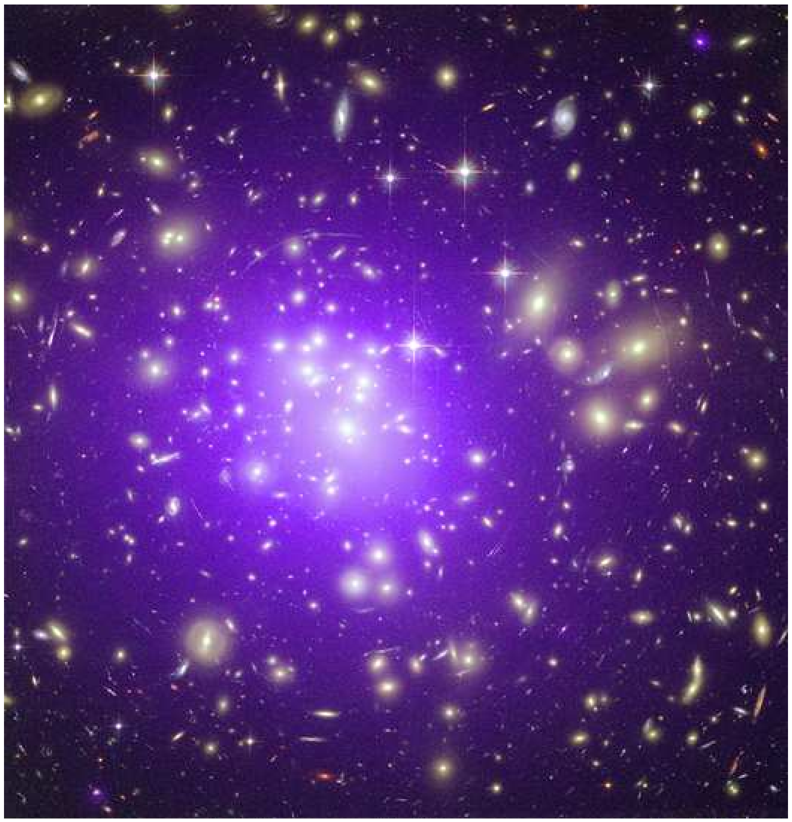
Dark matter: are you sure?

Yes we are sure! Many independent lines of evidence!

- Microwave sky: pattern of hot/cold spots requires DM to explain (models without DM fail at the $> 100\,\sigma$ level)
- X-ray appearance of rich galaxy clusters: $5\times$ more total mass than "ordinary" (baryonic) mass
- large scale structure in distribution of galaxies
- Many other lines of evidence ...

I will explain the galaxy cluster evidence as an example

Rich galaxy clusters



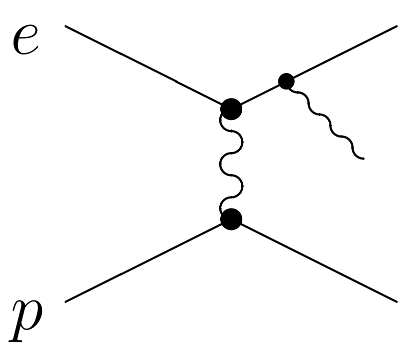
100's to 1000's of galaxies
Gravitationally bound
Full of plasma
Plasma: hydrostatic equilibrium
Glow in X-rays

Consider X-ray signal more closely

X-rays from galaxy clusters

Mohr et al ApJ 517, 627; Vikhlinin et al astro-ph/0507092; Allen et al astro-ph/0405340...

X rays emitted when e^-, p in plasma scatter inelastically



2-body scattering:

Rate $\propto \rho^2$ density^2.

Spectrum set by e^- velocity

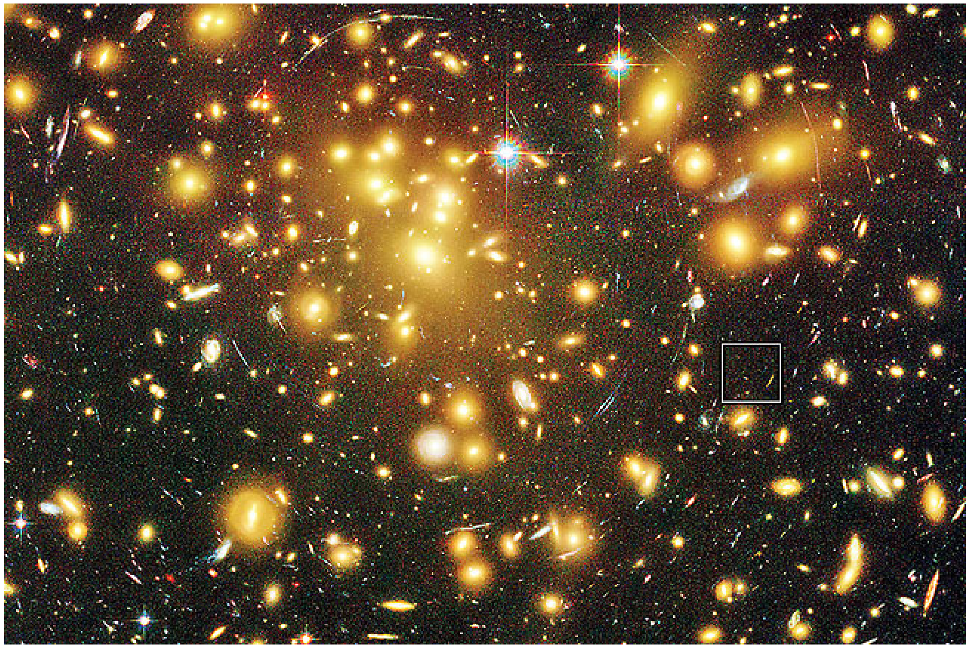
Hence by the temperature

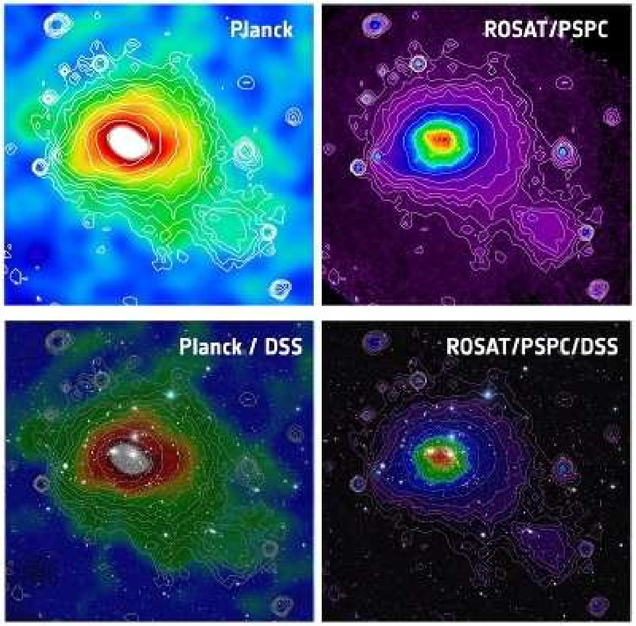
Intensity: determines $M_{\text{plasma}} \gg M_{\text{galaxies}}$.

Spectrum $+$ Hydrostatic equil.: determines $M_{\text{grav}} = M_{\text{tot}}$.

Total (grav.) mass $\simeq 5\times$ plasma mass $\gg$ mass of galaxies

Really sure? Look at cluster 2 other ways





Left: grav. lensing gives independent measure of M_{grav}

Right: Sunyaev-Zeldovich effect measures M_{plasma}

So Dark Matter really is there!

Changing the subject: $\mathbf{T}$ symmetry

Obviously wrong!

Time reversed!

“Entropic Arrow of Time”

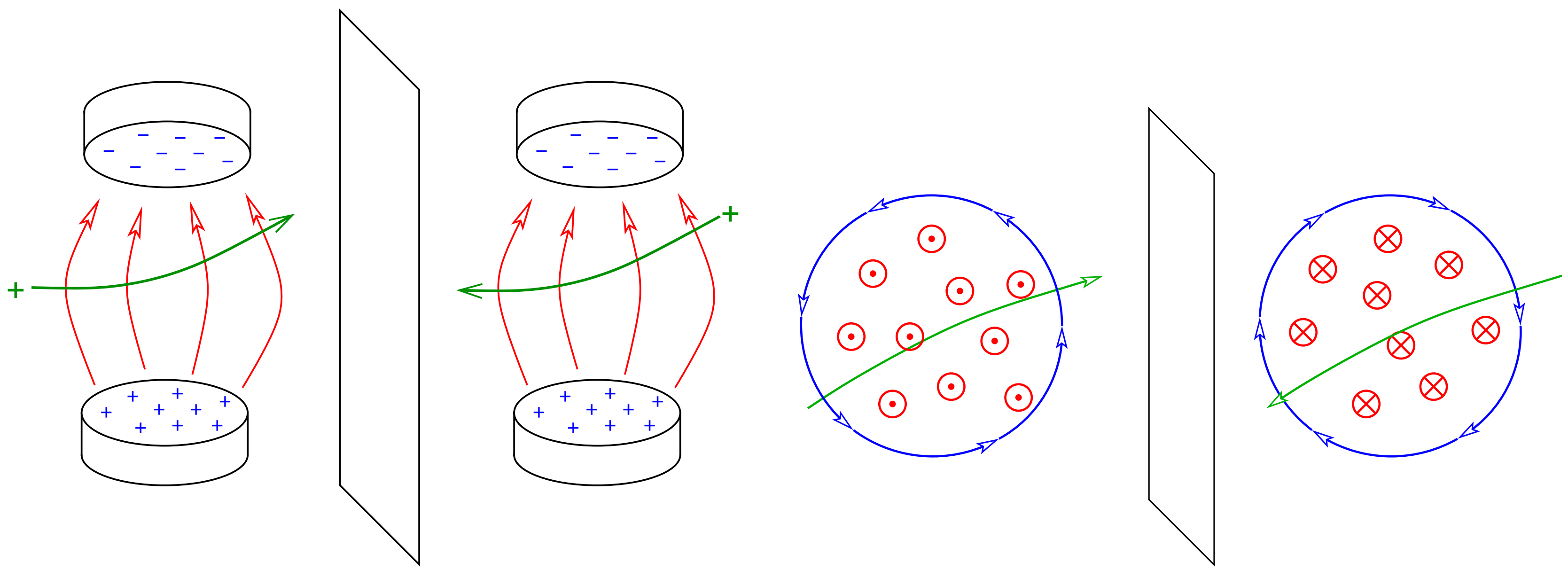
Statistical Physics

Microphysics is OK: E&M, Strong force $\mathbf{T}$ invariant

Violated by tiny weak-force effects, orders of magnitude smaller than other weak physics

$\mathbf{T}$ in E&M

How do E, B fields change when you run movie backwards?



Q's unchanged, but J's flip. E same, but B flips.

$\mathbb{T}$ in E&M or QCD

How a particle physicist thinks about E&M / QCD:

- What are the *Degrees of Freedom*?
  Electrons/quarks $\psi_e/\psi_{f,a}$, photons/gluons A^μ_A
- What are the *Symmetries*?
  Lorentz invariance, gauge invariance
- What is the *Action*?

$$S = \int d^4x \, \mathcal{L}[A^\mu_A, \psi_e, \psi_{f,a}]$$

- What is the most general $\mathcal{L}$, given fields+symmetries?

$\mathbf{T}$ in E&M, QCD Continued

Lorentz $+$ gauge symmetry: $\mathcal{L}$ is gauge-invariant scalar.

Without prejudice, write the most general answer you can!

$$\mathcal{L} = \frac{1}{4e^2\ 4g^2}\left(\vec{E}_a^2 - \vec{B}_a^2\right) + \frac{\Theta}{8\pi^2}\left(\vec{E}_a \cdot \vec{B}_a\right) + \mathcal{L}_\psi\ .$$

Here $\vec{E}_i^a = -\partial_t A_i^a - \partial_i \Phi^a + g f_{abc} A_i^b \Phi^c$ and $\vec{B}$ is ...

Both respect gauge, Lorentz invariance $\rightarrow$ should be OK!

First term: $\mathbf{T}$ respecting. Second term: $\mathbf{T}$ violating.

Time in E&M and QCD

In E&M, $\vec{E} \cdot \vec{B}$ turns out not to affect equations of motion – in fact, has no effect at all!

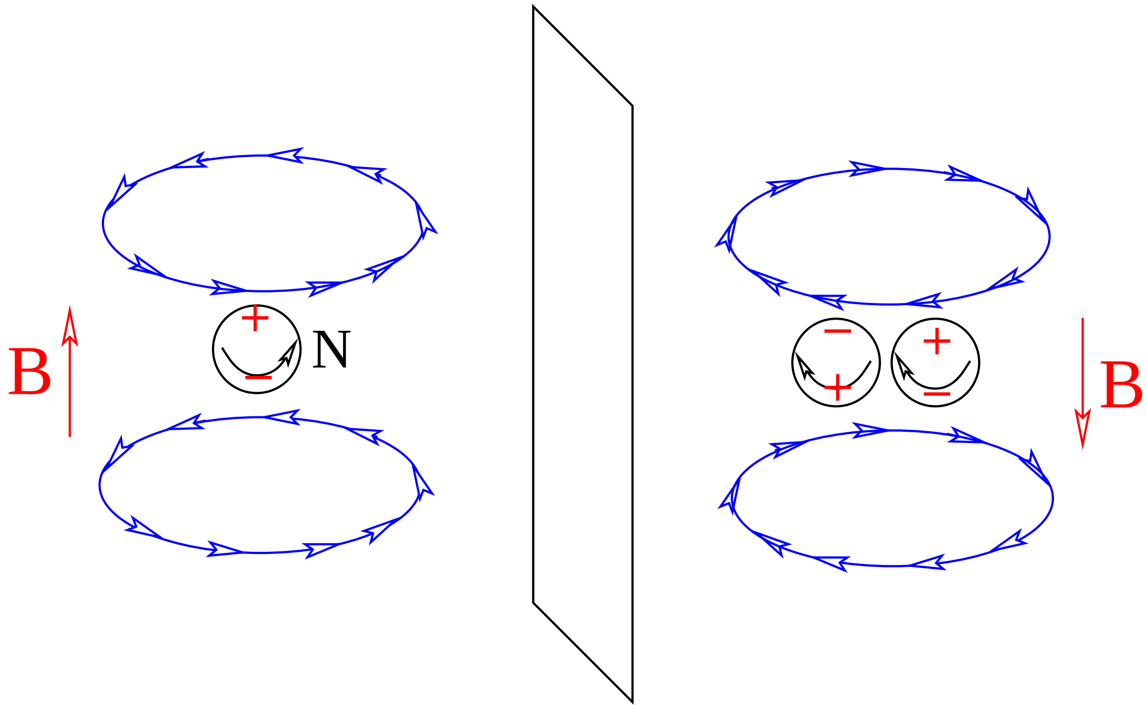
But QCD has these nonlinearities $E_i^a = ... + g f_{abc} A_i^b \Phi^c$, same argument does not apply.

QCD with $\Theta \neq 0$ is *not* equivalent to $\Theta = 0$
They have different *vacuum energy*
The $\Theta \neq 0$ version is $\mathbf{T}$ violating!

So, what do we know *experimentally*?

Neutron Electric Dipole Moment

A Test of whether **QCD** obeys **T** symmetry:



Put neutron in $\vec{B}$ field – spin lines up with $\vec{B}$.

Does it have an electric dipole moment aligned with spin?

If so: physics when you run movie backwards is different! **T** violating!

Neutron Electric Dipole Moment

Theory: Neutron electric dipole moment should exist,

$$d_n \simeq -2 \times 10^{-16}\, e\,\mathrm{cm} \times \Theta$$

so long as Θ is not zero! See arXiv:1904.00323, assumes Θ, modulo 2π, is small

Experiment: Consistent with zero! Baker *et al* (Grenoble), arXiv:hep-ex/0602020

$$|d_n| < 2.9 \times 10^{-26}\, e\,\mathrm{cm}$$

Either $|\Theta| < 10^{-10}$ by (coincidence? accident?) or there is something deep going on here.

Axion: an explanation for $\Theta = 0$

Hypothesize an extra *complex scalar* field $\varphi = \phi e^{i\theta_A}$:

- Field: takes a value at each point in space Think $\vec{E}$
- Scalar: value is a number, without direction but with units ...
- Complex scalar: value is a real and imaginary part

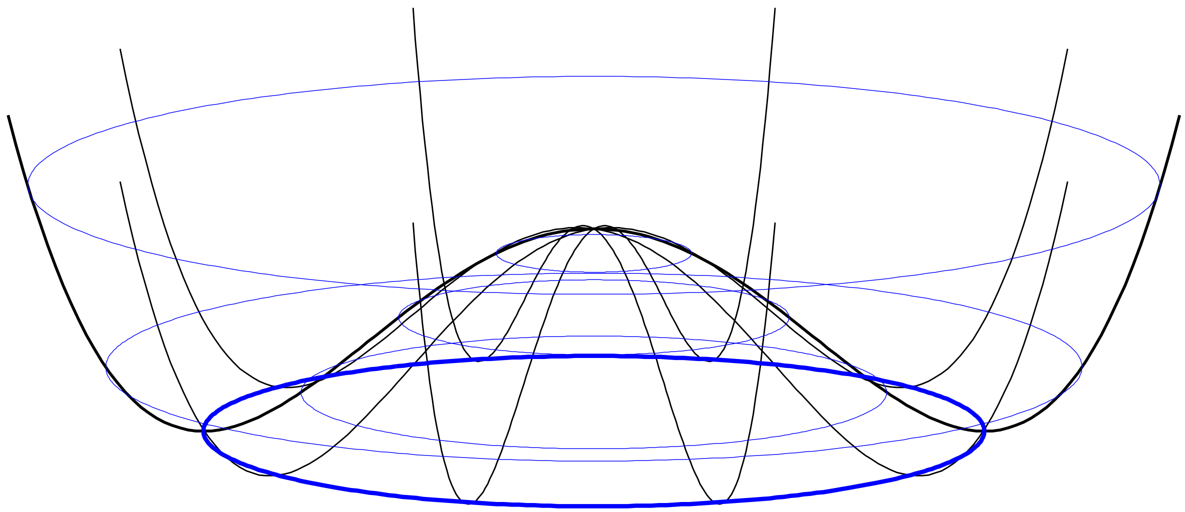
Assume a *symmetry*: $\varphi \to e^{i\theta}\varphi$

Energy should be

$$\text{Energy} = \int d^4x \; |\dot{\varphi}|^2 + |\vec{\nabla}\varphi|^2 + V(|\varphi|)$$

Axion: spontaneous symmetry breaking

Potential function $V(|\varphi|)$ can look like this:



Min of V at $\varphi \neq 0$! Call min. magnitude f_a.

Lowest energy state has $\varphi = f_a \neq 0$.

Minimum not unique; all values of θ_A equally good.

Or are they??

Indirect Interactions with QCD

Peccei, Quinn; Kim, Shifman Vainshtein Zakharov Carefully chosen high-scale physics couples the phase of this field to $\vec{E}_a \cdot \vec{B}_a$:

$$\mathcal{L} = \frac{1}{4g^2}\left(\vec{E}_a^2 - \vec{B}_a^2\right) + \frac{\Theta}{8\pi^2}\left(\vec{E}_a \cdot \vec{B}_a\right) + \frac{\theta_A}{8\pi^2}\left(\vec{E}_a \cdot \vec{B}_a\right)$$

$\mathbf{T}$ violation determined by $\Theta_{\rm eff} = \Theta + \theta_A \pmod{2\pi}$

Now φ can change values, changing $\Theta_{\rm eff}$.

Vacuum energy lower where $\Theta_{\rm eff} = 0$

Dynamics choose $\mathbf{T}$ respecting state!

Cosmological evolution of φ

Suppose φ starts
with some value
cosmologically:

Sinks to $\mathbf{T}$ respecting minimum, but
(Hubble damped) oscillations around that minimum.
Oscillations act like matter – dark matter!

Different values at different space points?

What if axion field starts at a (random) different value at different points in space? actually likely

- Energy cost to vary through space: $\int d^3x |\nabla\varphi|^2/2$
- Locally aligns to point in one direction
- Causality: cannot align globally
- *Phase-ordering dynamics*

Predicting the axion mass

Suppose I could solve these *phase-ordering dynamics*

Count axions at the end: *predict* Dark Matter density?

No, depends on φ vacuum-value f_a:

$$\rho_{\rm DM} \propto f_a$$

However the unknown axion mass also depends on f_a:

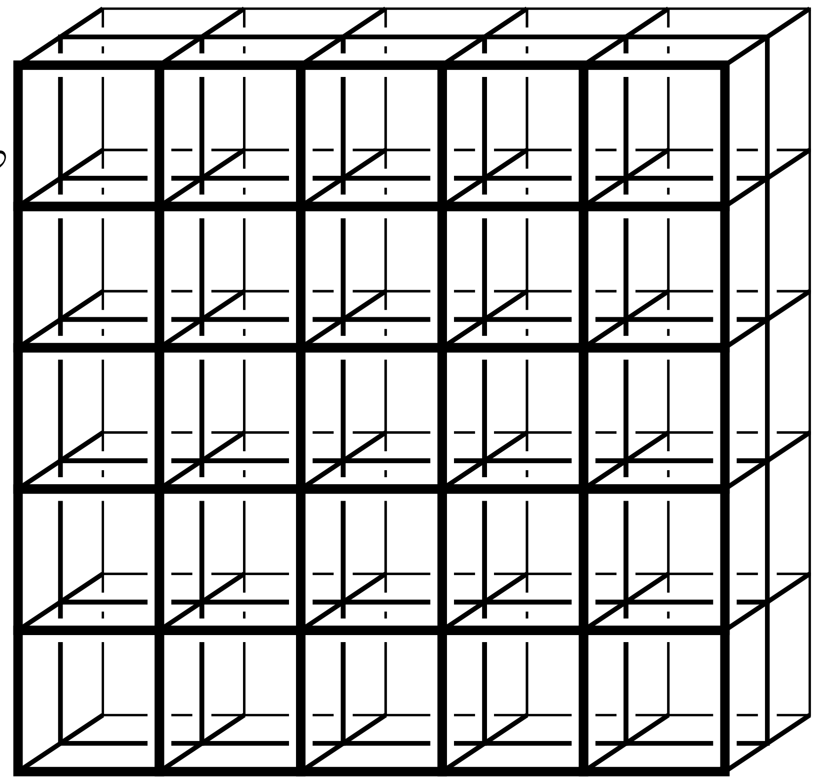
$$m_a^2 = \frac{\chi_{\rm QCD}}{f_a^2}\,, \qquad \chi_{\rm QCD} \simeq (76 \pm 1\,{\rm MeV})^4$$

If the DM is axions, computing axion production efficiency predicts the axion mass.

Needs a numerical solution

Put the Lagrangian

$$\mathcal{L} = \partial_\mu \varphi^* \partial^\mu \varphi + \frac{\lambda}{8}(2 - \varphi^* \varphi)^2 - \chi(t) \mathrm{Re}\, \varphi$$



as classical field thy.

on **real-time lattice**,

$\varphi(t = 0)$ random,

Hubble drag,

Count axions at end.

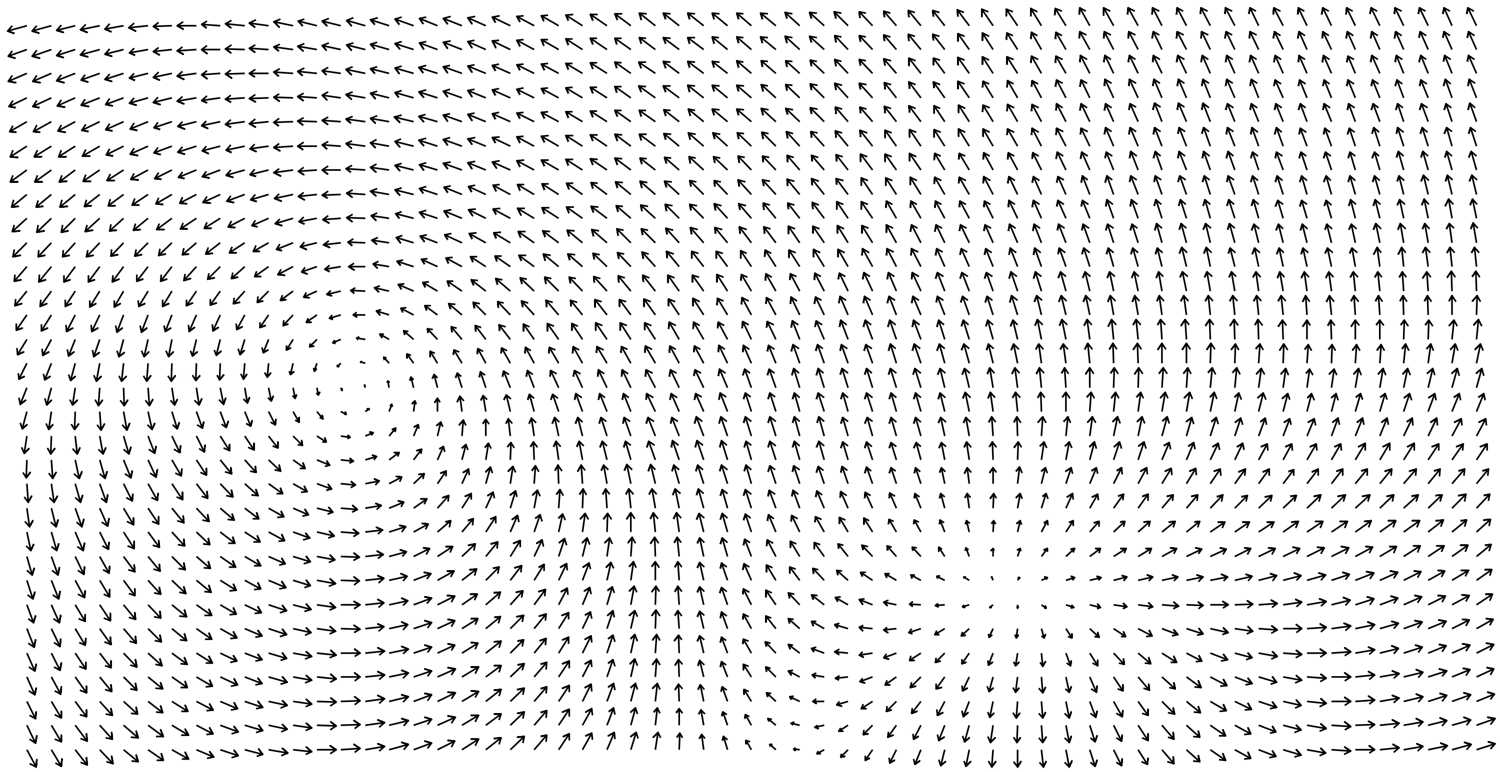
Nonperturbative approach.

$\chi(T)$ from Borsanyi *et al* 1606.07494

Axions and Topology

φ is a complex number – plot as a 2D arrow.

Axion field: a field of arrows. 2D slice for instance:



Field generically has vortices Davis, PLB180 225 (1986)

Domain walls

2D slice of evolution, When the potential tilts:

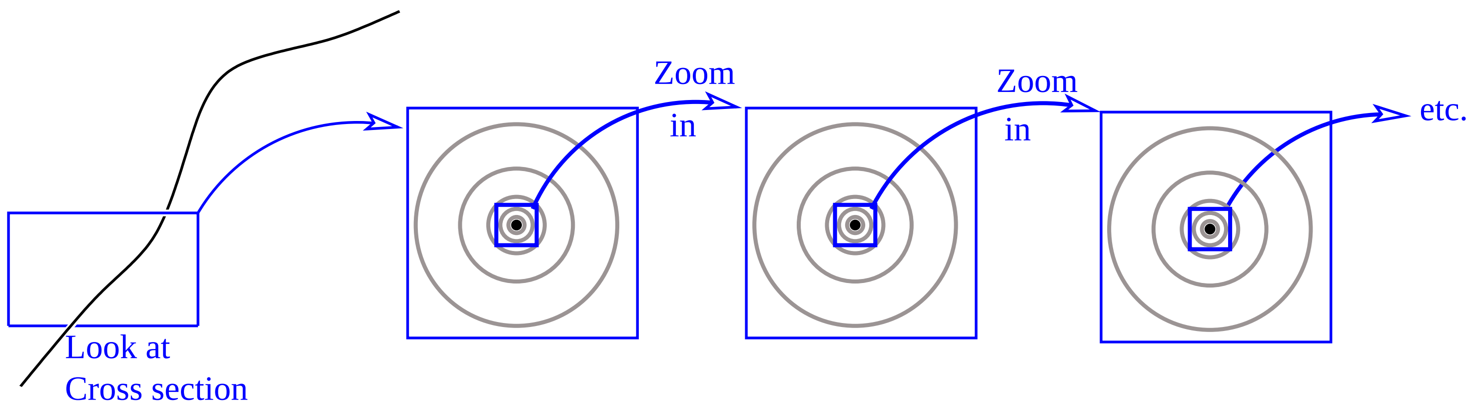
Network evolution

When potential stays untilted:

When potential tilts:

Layers of String Energy

$$E_{\rm str} = \int dz \int d\phi \int r\,dr \left(\nabla\phi^*\nabla\phi \simeq f_a^2/2r^2\right) \simeq \pi\ell f_a^2 \int_{\sim f_a^{-1}}^{\sim H^{-1}} \frac{r\;dr}{r^2}$$



Series of "sheaths" around string:
equal energy in each $\times 2$ scale, 10^{30} scale range! $\ln(10^{30}) \simeq 70$.

Log-large string tension $T_{\rm str} = \pi f_a^2 \ln(10^{30}) \equiv \pi f_a^2 \kappa$

Not reproduced by numerics (separation/core $\sim$ 400)

Another model: Abelian Higgs model

Complex scalar *plus copy of electromagnetism*

$$\mathcal{L}(\varphi, A_\mu) = \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + (D_\mu \varphi)^*(D^\mu \varphi) + \frac{\lambda}{8}\left(2\varphi^*\varphi - f_a^2\right)^2$$

with $D_\mu = \partial_\mu - ieA_\mu$ covariant derivative

Relativistic version of $\vec{B}$ fields in type-II superconductor

Tension-only (magnetic-flux) strings

Only massive fields outside of the string cores

Hybrid: global strings, local cores

Theory with one A_μ and two scalars

$$
\begin{aligned}
\mathcal{L}(\varphi_1, \varphi_2, A_\mu) \;=\; & \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 \\
& + \frac{\lambda}{8}\left[(2\varphi_1^*\varphi_1 - f^2)^2 + (2\varphi_2^*\varphi_2 - f^2)^2\right] \\
& + |(\partial_\mu - iq_1 e A_\mu)\varphi_1|^2 + |(\partial_\mu - iq_2 e A_\mu)\varphi_2|^2
\end{aligned}
$$

Pick $q_1 \neq q_2$, say, $q_1 = 4$, $q_2 = 3$.

One *global*[axions], one *local* rotation symmetry

"*glues*" Abelian-Higgs string onto axion string core.

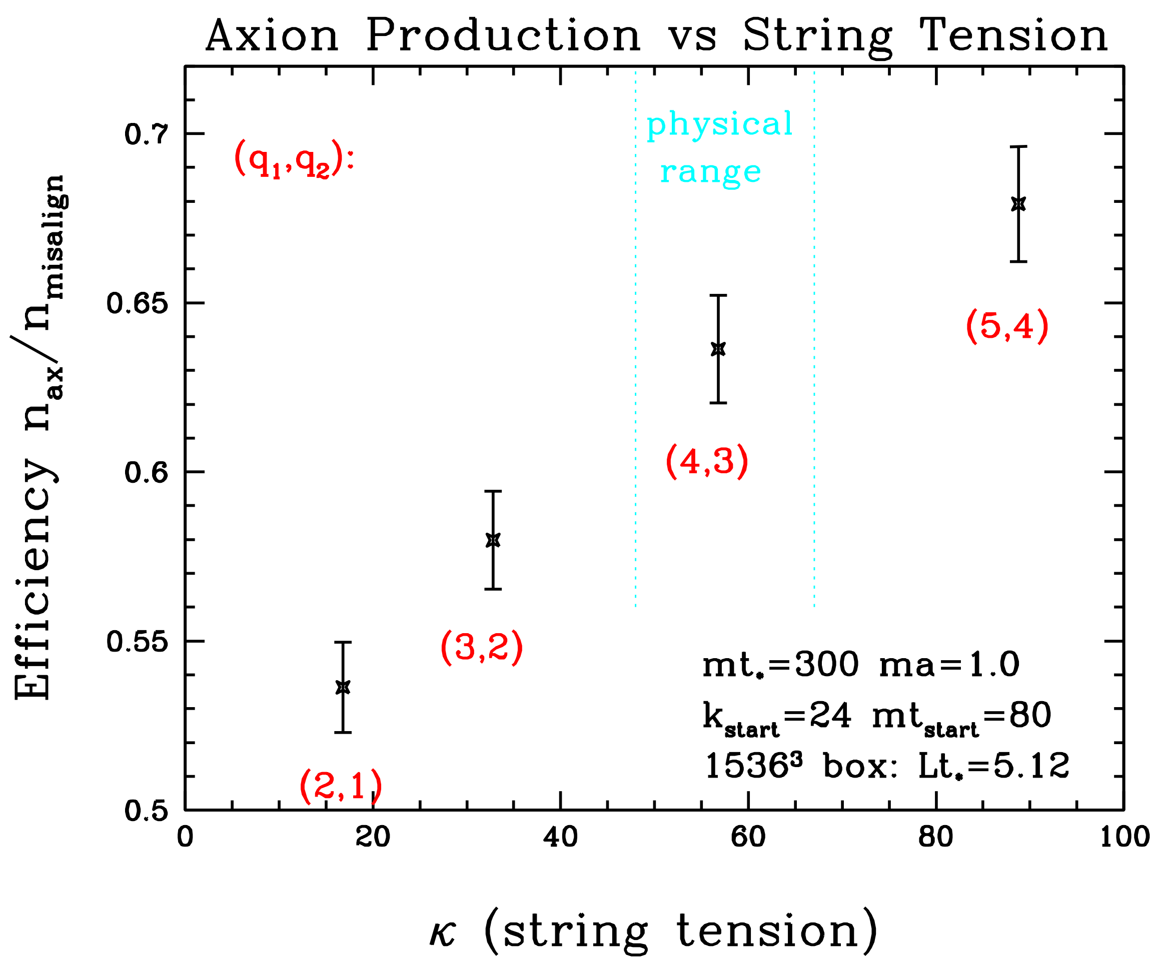
Extra tension represents effects of short-distance scales which give extra tension

Increases string tension by a completely tuneable amount!

Simulations with 4 different added tensions

Higher tension = higher initial density, longer lasting, hardier loops

Results



Axions produced vary mildly with increasing string tension

Put it all together

Axion production: $n_{\rm ax}(T=T_*) = (13 \pm 2) H(T_*) f_a^2$

Hubble law: $H^2 = \dfrac{8\pi\varepsilon}{3m_{\rm pl}^2}\,,$

Equation of state: $\varepsilon = \dfrac{\pi^2 T^4 g_*}{30}\,, \quad s = \dfrac{4\varepsilon}{3T}\,, \quad g_*(1{\rm GeV}) \simeq 73$

Susceptibility: $\chi(T) \simeq \left(\dfrac{1\,{\rm GeV}}{T}\right)^{7.6} (1.02(35)\times 10^{-11}{\rm GeV}^4)$

Dark matter: $\dfrac{\rho}{s} = 0.39\,{\rm eV}$

One finds $T_* = 1.54\,{\rm GeV}$ and $m_a = 26.2 \pm 3.4\,\mu{\rm eV}$

Summary so far

- If the QCD axion exists (solving Θ **T** problem)
- If the axion is the Dark Matter
- If it starts with random values at different space-points

Then it has $m_A = 26.2 \pm 3.4\,\mu\mathrm{eV}$ ($\simeq 6\,\mathrm{Ghz}$)

So, what are constraints and detection prospects?

Looking for DM axions today

Axion field still fluctuating today. Energy density is:

$$\begin{aligned} \varepsilon_{\rm DM} &= \frac{\chi}{2}\theta_{\max}^2 \quad \Rightarrow \quad \frac{(76\,\mathrm{MeV})^4}{2}\theta_{\max}^2 = \frac{0.3\,\mathrm{GeV}}{\mathrm{cm}^3} \\ \theta_{\max} &= 4\times 10^{-19} \end{aligned}$$

Expected $\theta_A G_{\mu\nu}\tilde{G}^{\mu\nu}$ effect: neutron EDM oscillates by $10^{-34} e\mathrm{cm}$ at $6\,\mathrm{Ghz}$

Good luck finding that! Need to consider other couplings

Axions and Electromagnetism

Axions generically also couple to E&M:

$$\mathcal{L} = \ldots + \frac{\theta_A}{8\pi^2}\left(\vec{E}_a \cdot \vec{B}_a\right) + \frac{C\alpha_{\rm EM}\theta_A}{2\pi}\left(\vec{E}_{\rm EM} \cdot \vec{B}_{\rm EM}\right)$$

with C model dependent (but never 0!).

Changes EM behavior: Ampere's law becomes

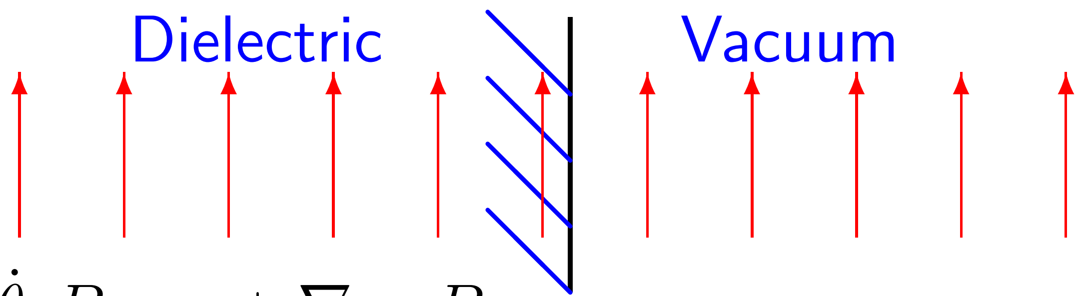
$$\vec{\nabla} \times \vec{B} = \vec{J} + \kappa\partial_0\vec{E} - \frac{C\alpha_{\rm EM}}{2\pi}(\partial_0\theta_A)\vec{B}$$

Turns $\vec{B}$ field into effective oscillating current!

Nice review article: Redondo et al arXiv:1801.08127

How to turn axions into microwaves

Consider dielectric-vacuum interface, B tangent to surface



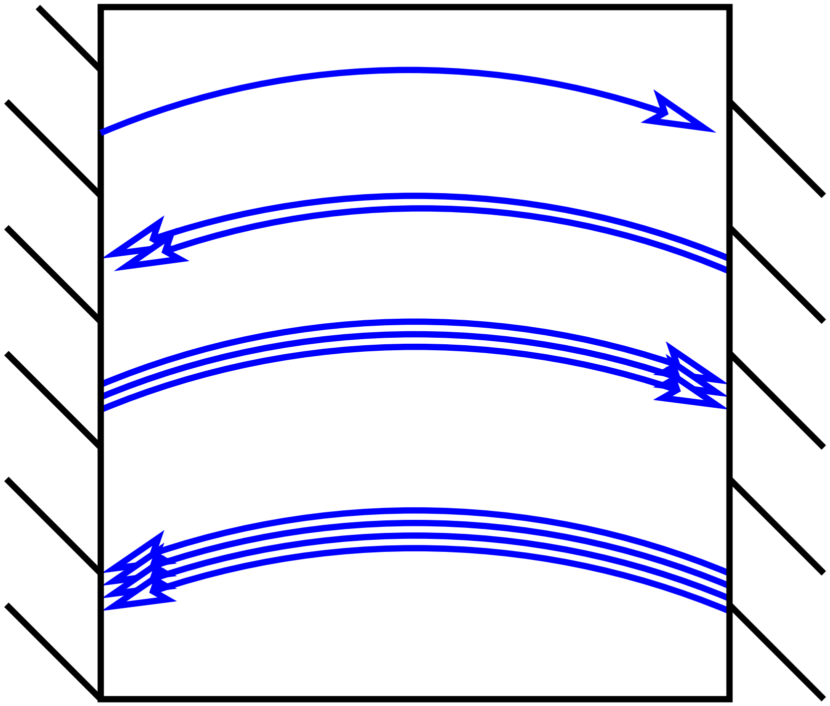
$$\dot{E}_{\|} = \frac{-g_{a\gamma}\dot{\theta}_A B_{\text{static}} + \nabla \times B}{\kappa} \qquad \dot{E}_{\|} = -\gamma_{a\gamma}\dot{\theta}_A B_{\text{static}} + \nabla \times B$$

As usual, $E_{\|}$ must match at boundary

Impossible without $\nabla \times B$ traveling wave component!

Interface, bathed in B-field, emits microwaves with $\omega = m_a$

Resonant cavity detection



Conductor: $\kappa \simeq i\infty$
Separate by $\lambda/2$:
resonant growth of
wave

Power produced enhanced by Q. Noise reduced $1/Q$
Bandwidth $\propto 1/Q$: need Q “experiments”
Ability to search enhanced by one factor of Q

Alternative: MADMAX

Parallel dielectric plates
B field: microwaves..
Emission from each plate
adds coherently if optical
dist. between plates $= \lambda$

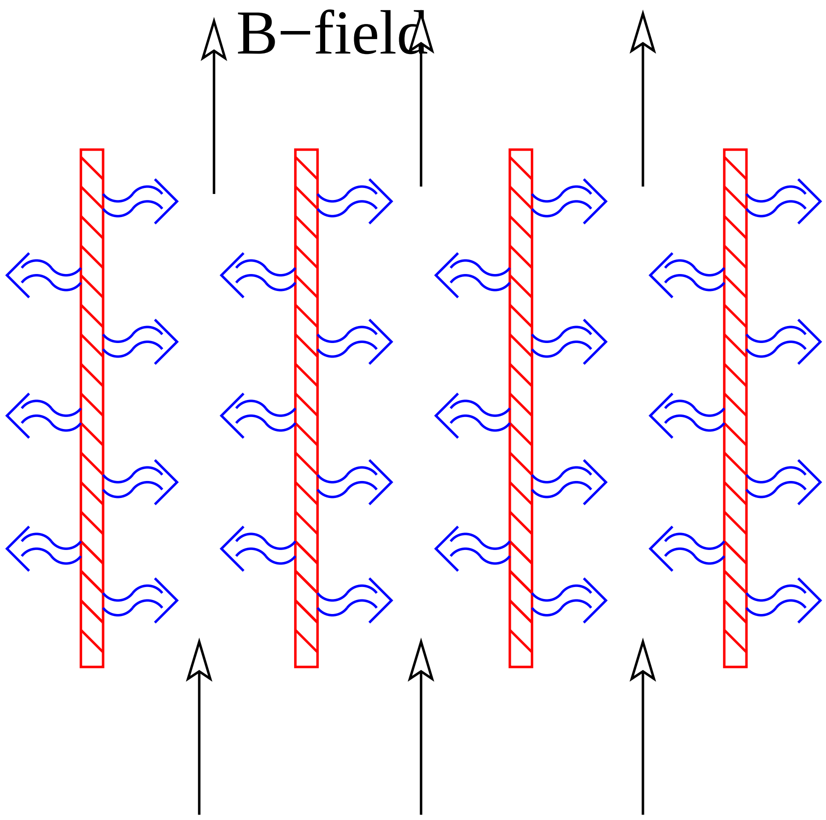


Plate separations can be actuated to tune sensitive λ
Loses Q but gains volume $\sim \mathrm{m}^3$ arXiv:1901.07401

Conclusions

- Two mysteries: Dark Matter and $\mathbf{T}$ symmetry
- Axion could explain $\mathbf{T}$ invariance of **QCD**
- Axion could also be dark matter
- Nontrivial, but now solvable, early-Universe dynamics
- *Prediction:* $m_a = 26 \pm 3\,\mu\mathrm{eV}$
- The search is on. Mass window helps design experiments and narrow the search.

What about Anthropic Principle?

Trendy Explanation for “coincidences” or “tunings”

Why is Cosmological Constant so small?
If it were 100 times bigger, matter would fly apart or collapse before life could evolve. Nature plays dice, universes with all values occur, but only universes with life get observed.

Why does **QCD** respect $\mathbf{T}$ symmetry?
If **QCD** *violated* $\mathbf{T}$*, something would go wrong with nuclear physics, which would make life impossible. Nature plays dice, only universes where life is possible get observed.*
Except that life is fine in a world where $\Theta = 10^{-2}$!

Why should the axion-EM coupling be nonzero?

Dynamics of QCD at the QCD scale are quite nontrivial.
The π^0 also has a coupling

$$\mathcal{L} = \ldots + \frac{\alpha_{\mathrm{EM}} N_c}{3\pi F_\pi} \pi^0 \vec{E} \cdot \vec{B}$$

The axion then *mixes* with π^0 in a way dependent on the m_u/m_d mass ratio. The above induces an axion-photon coupling

$$\frac{C\alpha_{\mathrm{EM}}}{2\pi} \theta_A \vec{E} \cdot \vec{B} \qquad \text{with} \qquad C = 1.92$$

For the axion-photon coupling to vanish, the "fundamental" contribution would have to be exactly -1.92.

Why random initial conditions for θ_A?

If Universe ever reached $T > f_a \sim 10^{11}\,\mathrm{GeV}$:
axion field got "symmetry restored," then randomly broke in different directions in different places.

If not: starting θ_A value "came out of inflation"

- Inflation stretches quantum fluctuations to classical ones: $\Delta\varphi \sim H_{\mathrm{infl.}}$.
- If $N_{\mathrm{efolds}} H^2 > f_a^2$, scambles field.
- If not: need $H < 10^{-5} f_a$ to avoid excess "isocurvature" fluctuations in axion field