Causal Sets, Discrete Gravity

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The Party Bag:

- Causal set theory is an approach to quantum gravity that proposes a fundamental **atomicity** of **spacetime**.
- The microscopic degrees of freedom are **discrete causal relations.**
- Causal sets were used to make a successful **prediction** (and what a prediction!) and there may be more to come.

What is Quantum Gravity?

- The phrase is a shorthand that names the biggest obstacle we currently face in our search for a unified framework for physics: General Relativity, our best theory of spacetime, does not do justice to the quantum nature of matter
- Scarcity of observational/experimental data has led to a wide range of different approaches to the problem of quantum gravity.
- Most workers think that we need to create a new framework with novel concepts perhaps radically different from the physics we know now.
- That's all very well but how to get there from here?

Quantum Gravity: what to keep? what to ditch? what's new?

To get to quantum gravity we must make a leap, but one that is grounded in our current best knowledge.

The causal set approach claims that certain aspects of General Relativity and quantum theory will have direct counterparts in quantum gravity:

- the **path integral** (sum-over-histories) from quantum theory.
- the **spacetime causal order** from General Relativity

It makes one main new hypothesis about the nature of the physical world:

- **fundamental discreteness of spacetime**.

I will argue: we already know a result in quantum gravity that points to **atomicity** and **order**

The Laws of Black Hole Mechanics

Bardeen, Carter, Hawking 1973

κ is the **surface gravity** of the horizon of a stationary black hole, A is the surface area of the horizon

	Thermo	Black Hole
Zeroth Law	T is constant throughout system in **equilibrium**	κ is constant on horizon of **stationary** black hole
1st Law	$dE = TdS - \text{ work}$	$dM = \frac{1}{8\pi G}\kappa\, dA - \text{ work}$
2nd Law	$dS \geq 0$	$dA \geq 0$ Hawking's Area Theorem
3rd Law	T cannot be reduced to zero in a finite process	κ cannot be reduced to zero in a finite process

A remarkable "analogy" ?

The Laws of Black Hole **Thermodynamics**

Bekenstein, Bardeen, Carter, Hawking, Gibbons, Perry

- A black hole has a temperature and an entropy

$$T_{BH} = \hbar \frac{\kappa}{2\pi}, \quad S_{BH} = \frac{1}{4} \frac{A}{l_p^2}$$

  where κ is the surface gravity and $l_p^2 = G\hbar$

- The Zeroth Law: T_H is constant over the horizon of a stationary black hole

- The First Law: $dM = TdS - work$

- The Generalised Second Law: $dS_{BH} + dS_{ext} \geq 0$ **Bekenstein**

Black holes are the epitome of General Relativity. It is striking that GR "knows" about quantum mechanics and thermodynamics:

The broadest unity yet achieved in physics

Seeking the Statistical Mechanics of BH Thermodynamics

- The BH entropy has a quantum gravitational value. Not only it is **finite** but its value sets a scale and suggests that spacetime itself is discrete

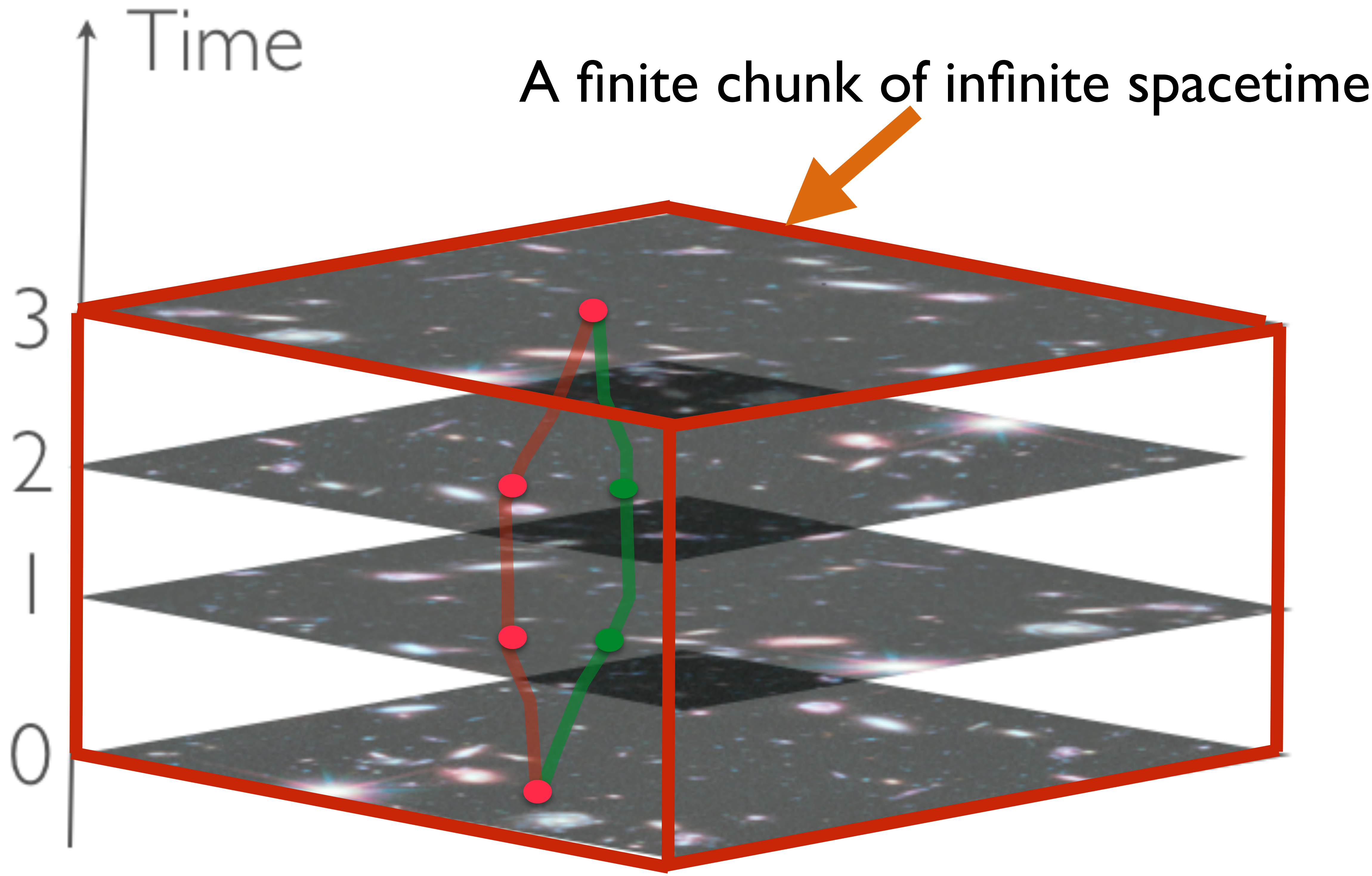
Entropy of BH is the number of Planck sized plaquettes tiling the horizon (up to factor of order one)

Entropy of a box of gas is equal to the number of molecules (up to factor of order one)

- There's nothing locally special about BH horizons. If horizons are discrete, so must **spacetime** itself be.

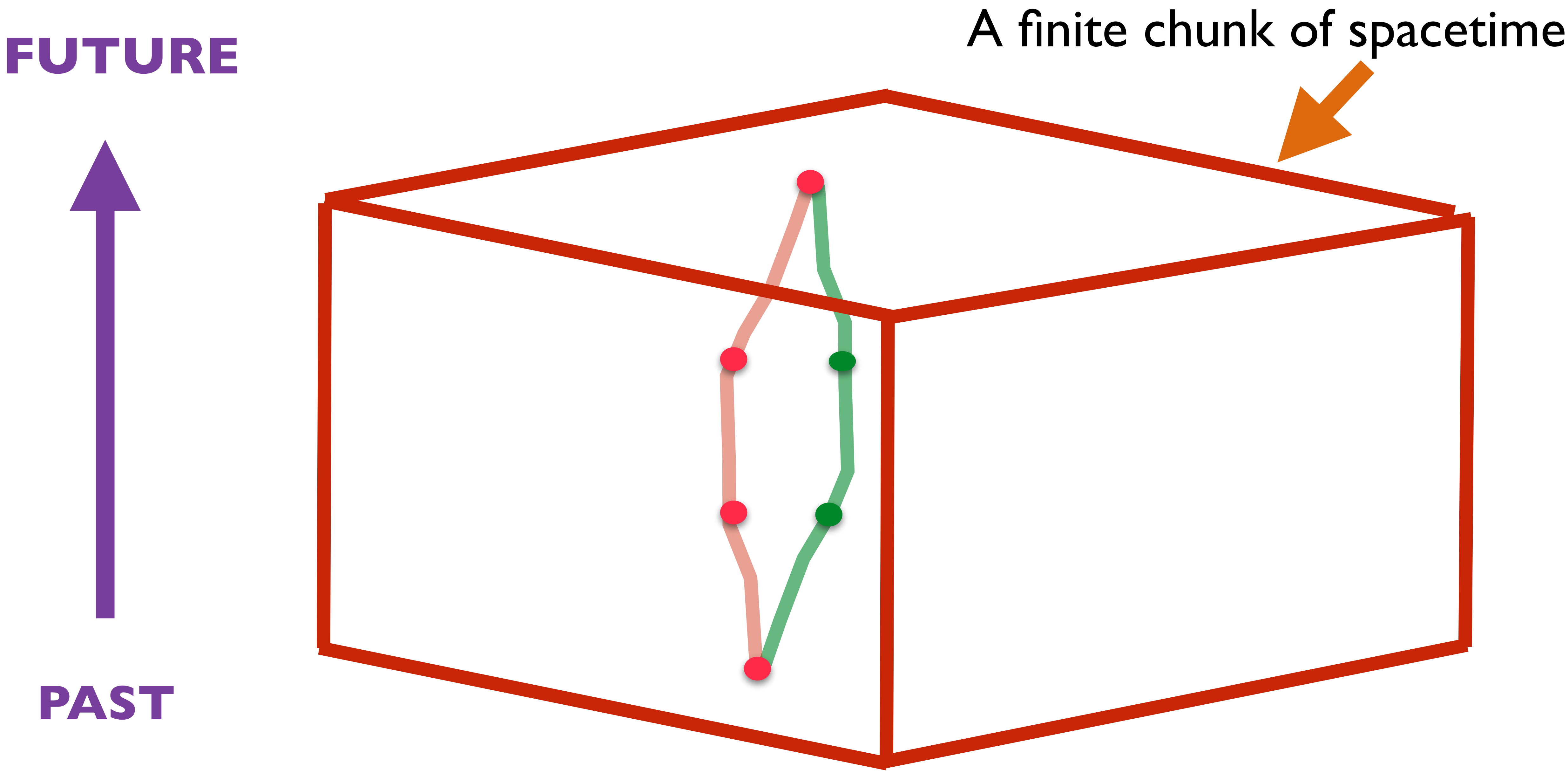
Spacetime, not space

Newtonian Spacetime: made of space



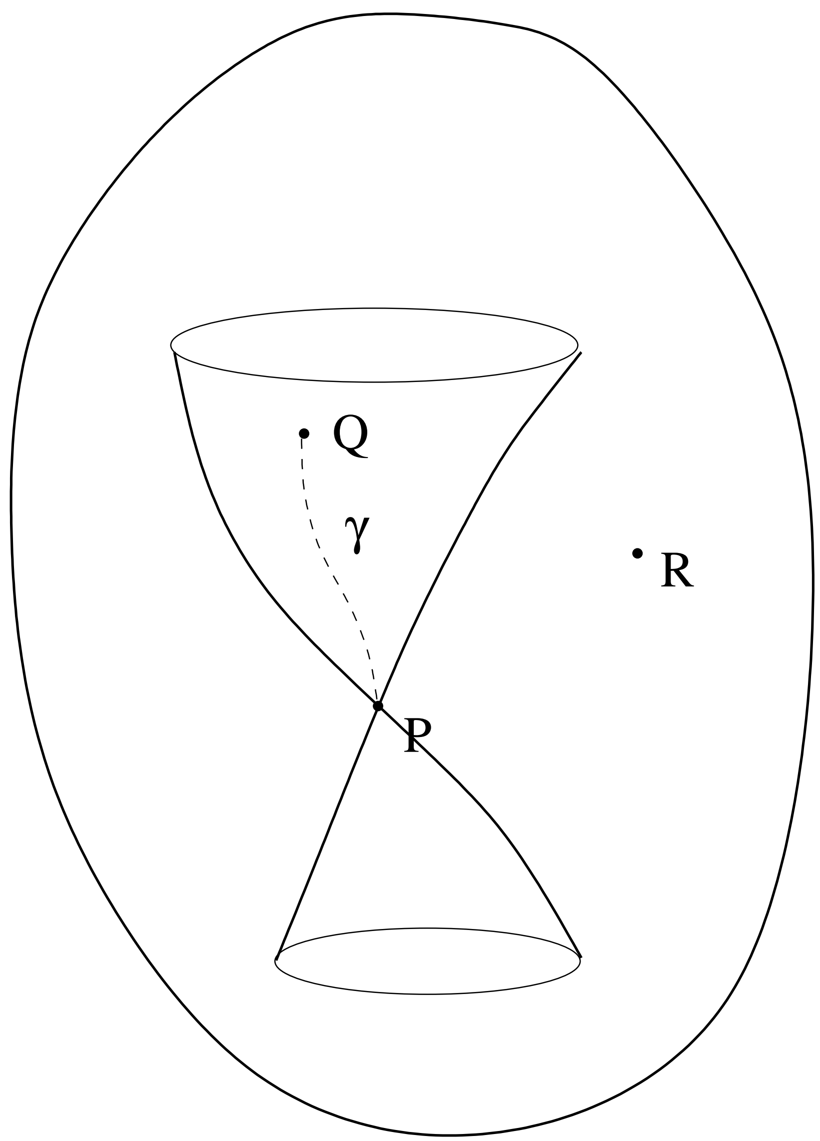
In Newtonian physics, time is “spatialised” as a “dimension”

Einsteinian Spacetime: made of events



Time passes along worldlines. There is **no space**, only spacetime.

Causal Order is central to GR and Black Hole Thermo

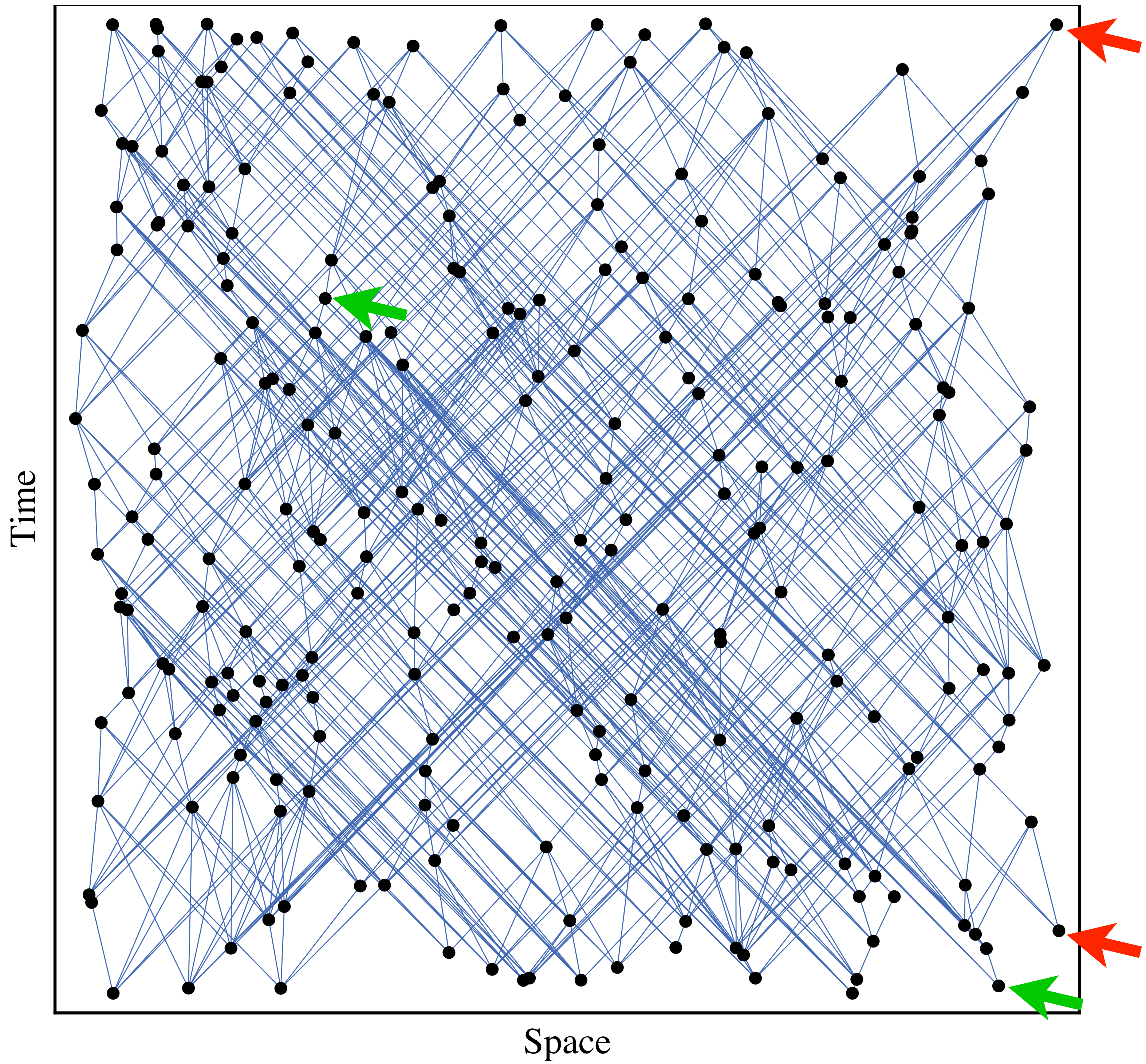


Theorem: The causal order — which events are to the causal past and future of which other events — and the volume of any region of spacetime, tells you the full geometry
(Penrose, Hawking, Malament)

A black hole horizon is the boundary of the causal past of any "eternal" worldline. Black hole thermodynamics is tied to the causal nature of the horizon

So, we postulate that **causal order** survives in quantum gravity.
Now to marry that causal order with **spacetime atomicity**……..

A discrete order (causal set) for 2-d Minkowski space



- Lorentz invariant
- **Nonlocal**
- Order => metric
- Number~Volume **statistically** (faithful embedding)
- 10^{240}

- Discrete transitive directed acyclic graph = discrete order = causal set
- There are causal sets corresponding to **each** continuum spacetime

The Causal Set Quantum Gravity Programme **(Rafael Sorkin)**

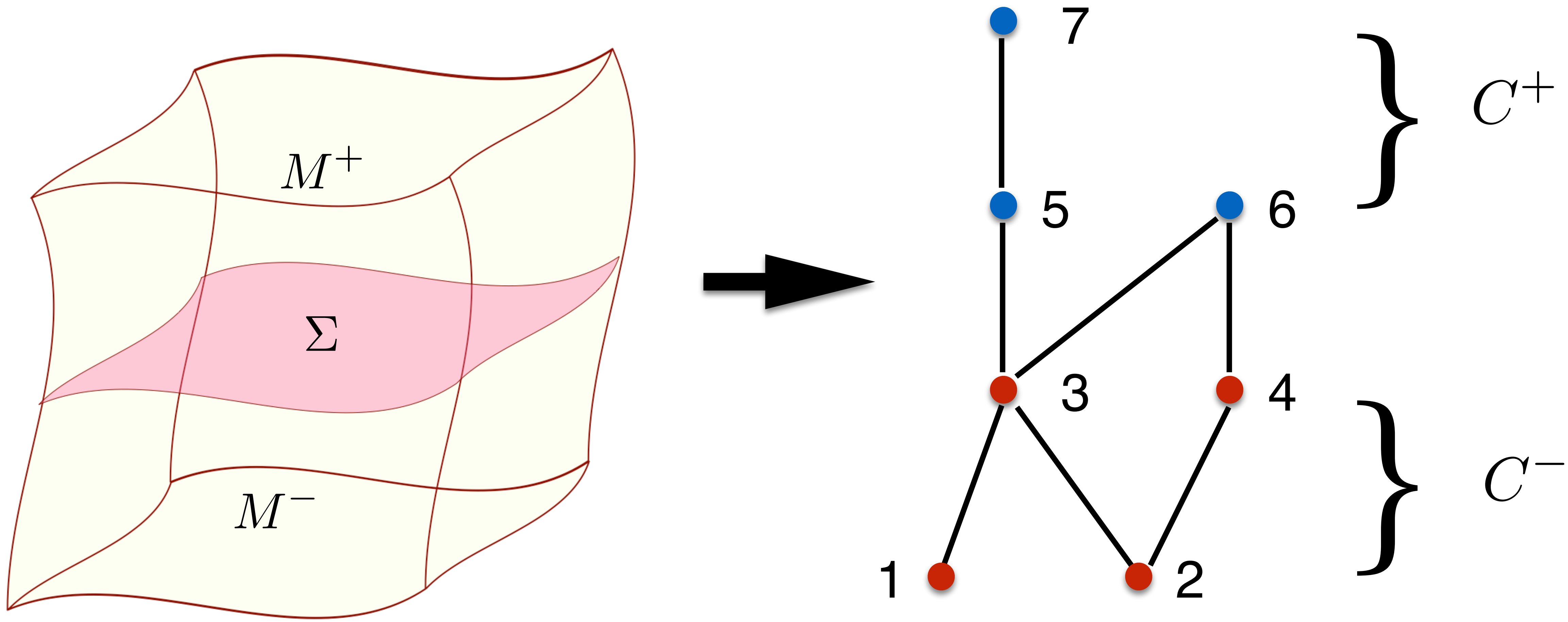
- **Kinematics:** studying the structure of the substance
- **Dynamics: on** causal sets, and **of** causal sets.

The quantum theory **of** causal sets will be based on the path integral or sum-over-histories:

$$Z = \sum_{\text{spacetime geometries g}} e^{iS(g)} \quad \longrightarrow \quad Z = \sum_{\text{causets C}} e^{iS(C)}$$

- **Phenomenology**: how could the discrete substance reveal itself to us?

Kinematics: Gibbons-Hawking boundary term



$$BT := (1/l_p^{d-2}) \int_\Sigma d\sigma K$$
$$= (1/l_p^{d-2}) \frac{\partial}{\partial n} \int_\Sigma d\sigma$$

$$CBT := \; (l/l_p)^{d-2} \; c_d \left[N^0_{max}(C^-) - N^0_{min}(C^+) \right]$$

$$c_d = \frac{\pi d(d+1)}{2(d+2)\Gamma(\frac{2}{d})\Gamma(\frac{d}{2}+1)^{\frac{2}{d}} d^{\frac{2}{d}}}$$

Michel Buck, FD, Ian Jubb, Sumati Surya

Quantum dynamics of causal sets: must be the path integral

$$\text{“} \quad Z = \sum_{\text{causets C}} e^{iS(C)} \quad \text{”}$$

- Ready made "quick and dirty" interpretation: classical behaviour results if non-classical causal sets destructively interfere
- Many questions in current causal set research revolve around trying to make technical and conceptual sense of this.
- What is the measure? What is the action?
- What is the interpretation of a quantum theory based on the path integral? (I won't open that can of worms here)

Phenomenology: Causal sets used to predict Lambda

- In the late 1980's/early 1990's Rafael Sorkin predicted the value of the Cosmological "Constant" Lambda today using a heuristic argument based on expectations of quantum causal set theory.
- Since Number ~ Volume, it is natural in a path integral for causal set quantum gravity to fix N for the causal sets summed over (Note: a justification for "unimodular" gravity c.f. Weinberg)
- Because the Number/Volume relationship is statistical, fixing N means there are fluctuations in V of order

$$\Delta V \sim \sqrt{N} \sim \sqrt{V}$$

- V and Lambda are canonically conjugate (c.f. Time and Energy) so

$$\Delta V \Delta\Lambda \sim 1$$

$$\Delta\Lambda \sim \frac{1}{\Delta V} \sim H^2 \sim 10^{-120}$$

If Lambda fluctuates about zero, then what we see is only the fluctuation

Everpresent Lambda

(Ahmed, Dodelson, Greene & Sorkin; Ahmed & Sorkin; Zwane, Afshordi, Sorkin)

There are two homogeneous cosmological models realising Sorkin's idea — stochastic models, **not GR.** In each the value of Lambda fluctuates, on a Hubble timescale, between positive and negative values that are of the same order of magnitude as the ambient matter density (Lambda is "everpresent").

Can such a cosmology fit the data?

Confrontation with observations **(Zwane, Afshordi, Sorkin))**

- Run the stochastic model to get a history of Omega_Lambda(z) and a(z)
- Calculate perturbations using CAMB: model spatial fluctuations in rho_Lambda as a quintessence field (different potential depending on history a(z), Omega_Lambda(z)),
- Input into CosmoMC, Monte Carlo Markov Chain parameter estimation
- Looked at CMB, BAO, H_0, Nucleosynthesis (Li-7 problem), Ultramassive black holes at high z.

Two models of Everpresent Λ

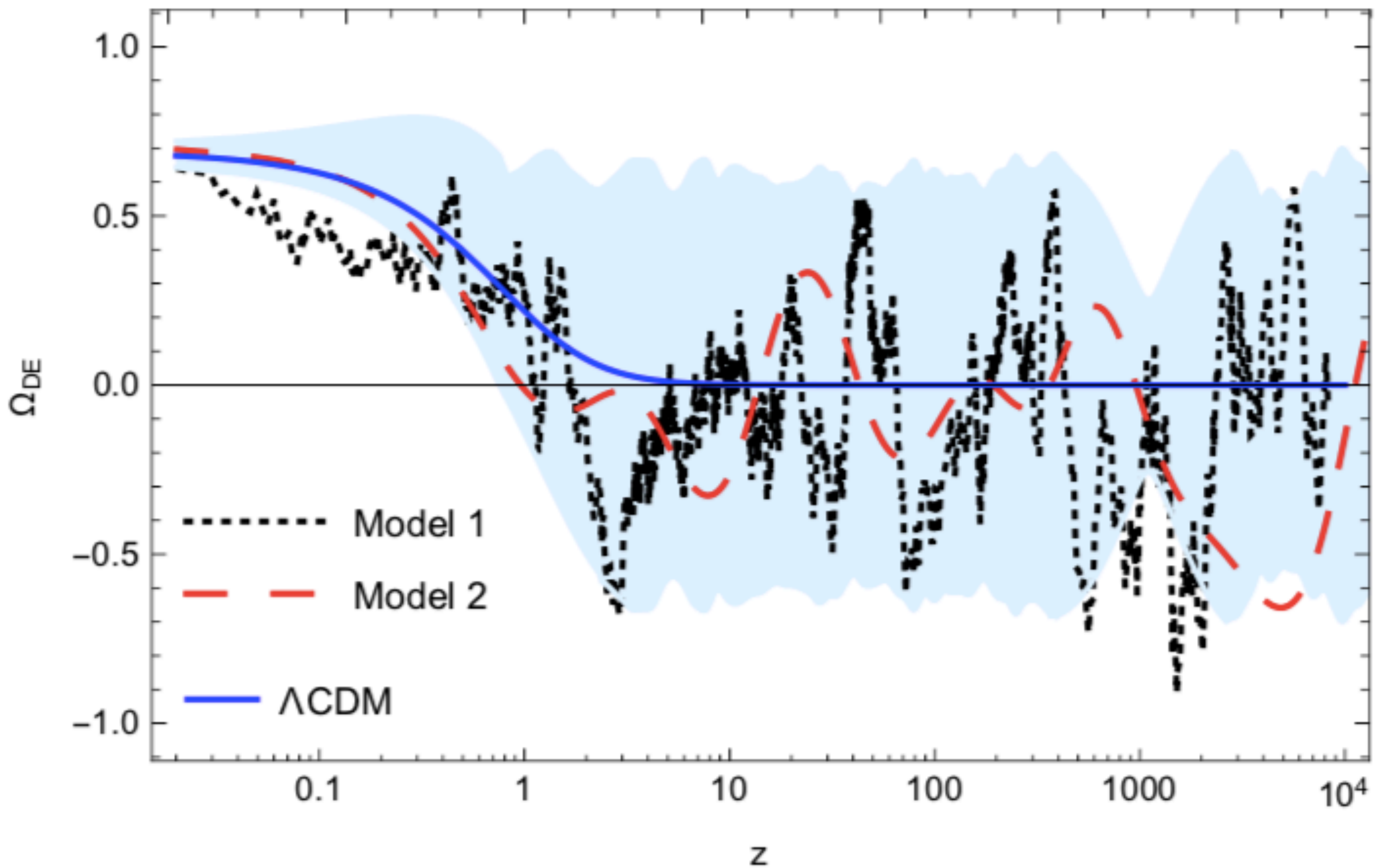


Figure : Ω_{DE} for different Cosmological Models.

Everpresent Λ and the CMB

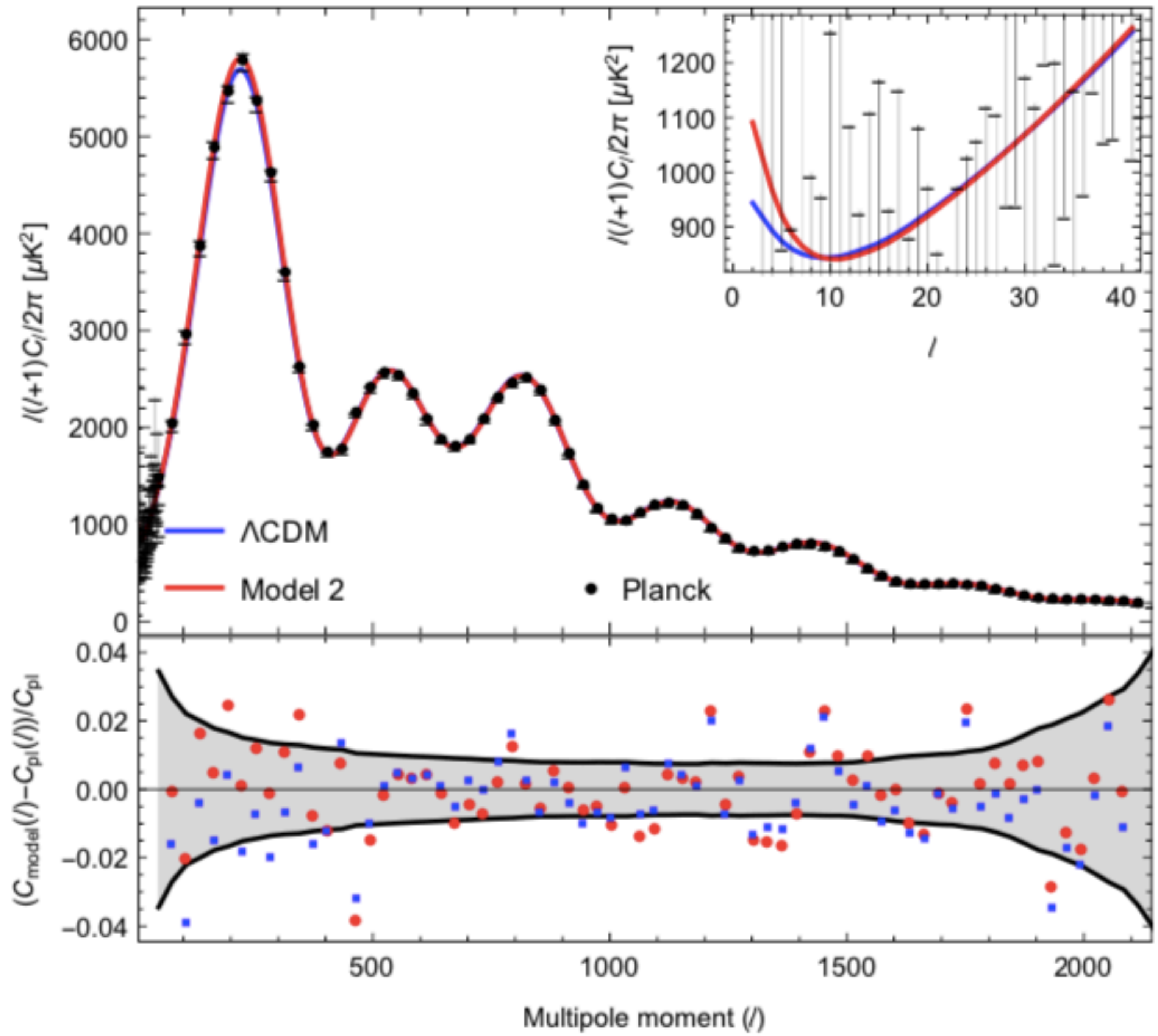


Figure : Temperature fluctuations for ΛCDM model ($\chi^2 = 11334$), everpresent Λ model 2 ($\chi^2 = 11335$) and Planck 2015 data.

How likely is best fit Everpresent-Lambda history?

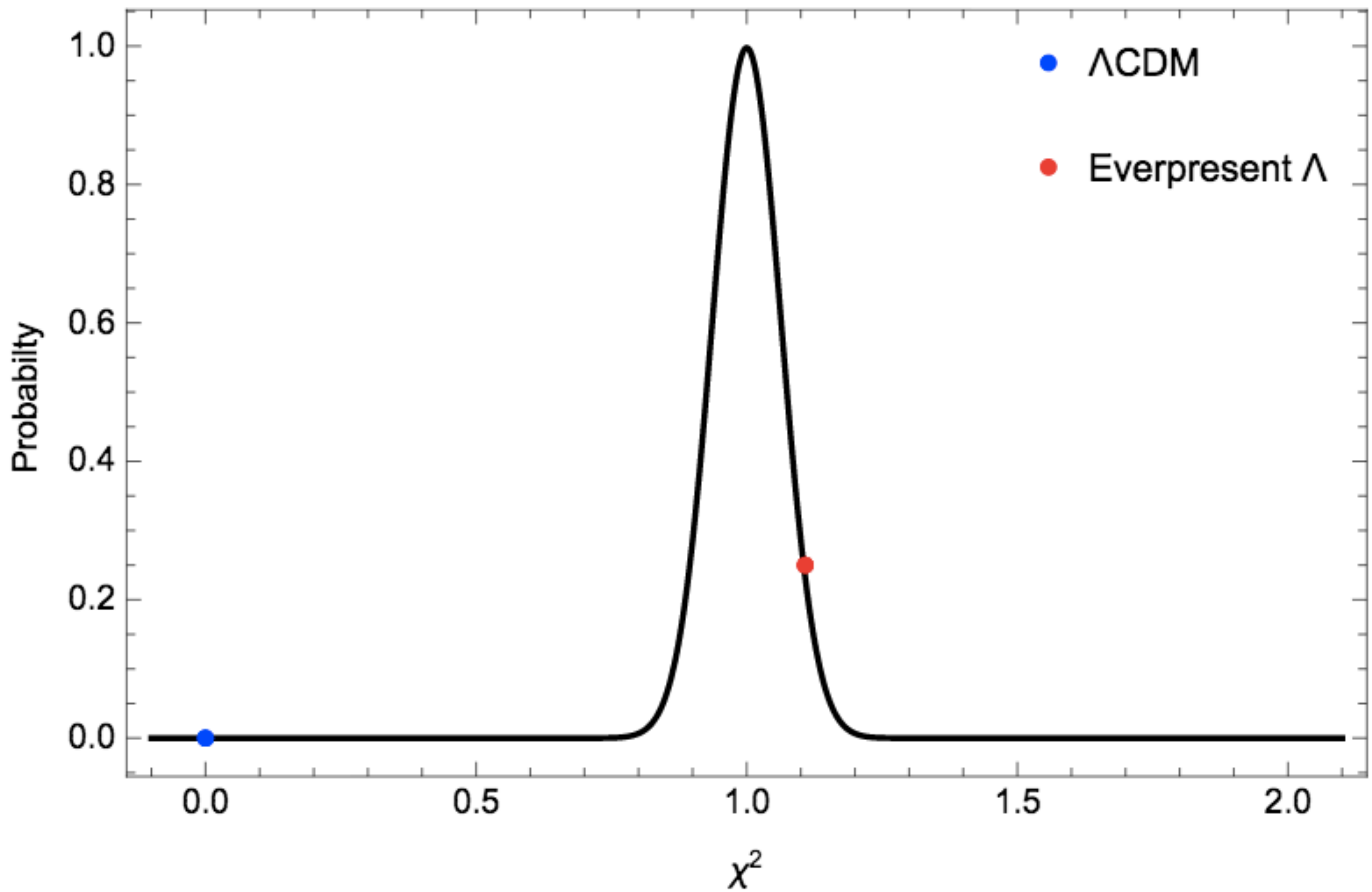


Figure 3: χ^2_{red} for stochastic dark energy with 1000 ω's sampled. This shows that the best-fit history is a typical model-2 realization, whereas ΛCDM would be a very atypical outcome of model-2.

Things to do include

- Do more of this including with new data as it comes in
- Repeat this with Model I which is closer to the underlying causal set physics
- Improve the models
- Stochastic Differential Friedmann Equation?
- Construct a quantum model: quantum Lambda and "semiclassical" a(z) or quantum Lambda and quantum a(z)….
- Make a more physical/quantum model of spatial inhomogeneities in Lambda (Stochastic PDEs?) If Lambda fluctuates independently when causal pasts have no overlap then there's a risk **(Ahmed et al, Barrow, Zuntz)** the fluctuations in space will be too large. But if quantum, then…….

Summary

- Seeking the Stat Mech of Black Hole Thermodynamics leads to the postulate of atomic causal order: Causal Sets as the basis for quantum gravity.
- Causal sets are fundamentally **discrete**: only combinatorial information.
- Lorentz invariant discreteness possible. At cost of radical nonlocality
- Kinematics: reading out geometry from the order e.g. GHY Boundary term
- Dynamics of causal sets: path integral (not canonical)
- Phenomenology: causal sets predicted (in advance of the measurement) a nonzero value for Lambda today of the right magnitude. Current homogeneous models of "Everpresent Lambda" are consistent with some cosmological data: watch out for more on this!

Thank you for listening